The problem of time in quantum cosmology from different perspectives

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My papers

Quantum Gravity 2023

Introduction

Our model

- Flat FLRW universe with scale volume v
- Massless scalar field Φ
- Unimodular cosmological constant λ

The canonical pairs of our model are: $\{t, \lambda\} = \{\varphi, \pi_\varphi\}, = \{v, \pi_v\} = 1$

Unimodular gravity

The Hamiltonian of the system is: $\mathcal{H} = \tilde{N} \underbrace{\left[-\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda \right]}_{\mathcal{C} = 0}$

Lapse

Classically this model has a big bang and a big crunch singularity

Quantisation I: Canonical Quantisation

Classical constraint with gauge $\tilde{N} = 1$

$$-\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda = 0$$

Wheeler-DeWitt equation

$$\Downarrow$$

$$\left(\hbar^2 \frac{\partial^2}{\partial v^2} + \frac{\hbar^2}{v}\frac{\partial}{\partial v} - \frac{\hbar^2}{v^2}\frac{\partial^2}{\partial \varphi^2} - i\hbar\frac{\partial}{\partial t}\right)\Psi(v, \varphi, t) = 0$$

Suited to use t as relational clock

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Classical constraint with gauge $\tilde{N} = v^2$

$$-\pi_v^2 v^2 + \pi_\varphi^2 + \lambda v^2 = 0$$

Wheeler-DeWitt equation

$$\left(\hbar^2 v^2 \frac{\partial^2}{\partial v^2} + \hbar^2 v \frac{\partial}{\partial v} - \hbar^2 \frac{\partial^2}{\partial \varphi^2} - i\hbar v^2 \frac{\partial}{\partial t}\right)\Psi(v, \varphi, t) = 0$$

Suited to use Φ as relational clock

Dynamics of the t-clock theory

Gauge $\tilde{N} = 1$

Dynamics are governed by the Schrödinger equation $i\hbar\frac{\partial\Psi}{\partial t} = \hat{\mathcal{H}}\Psi$

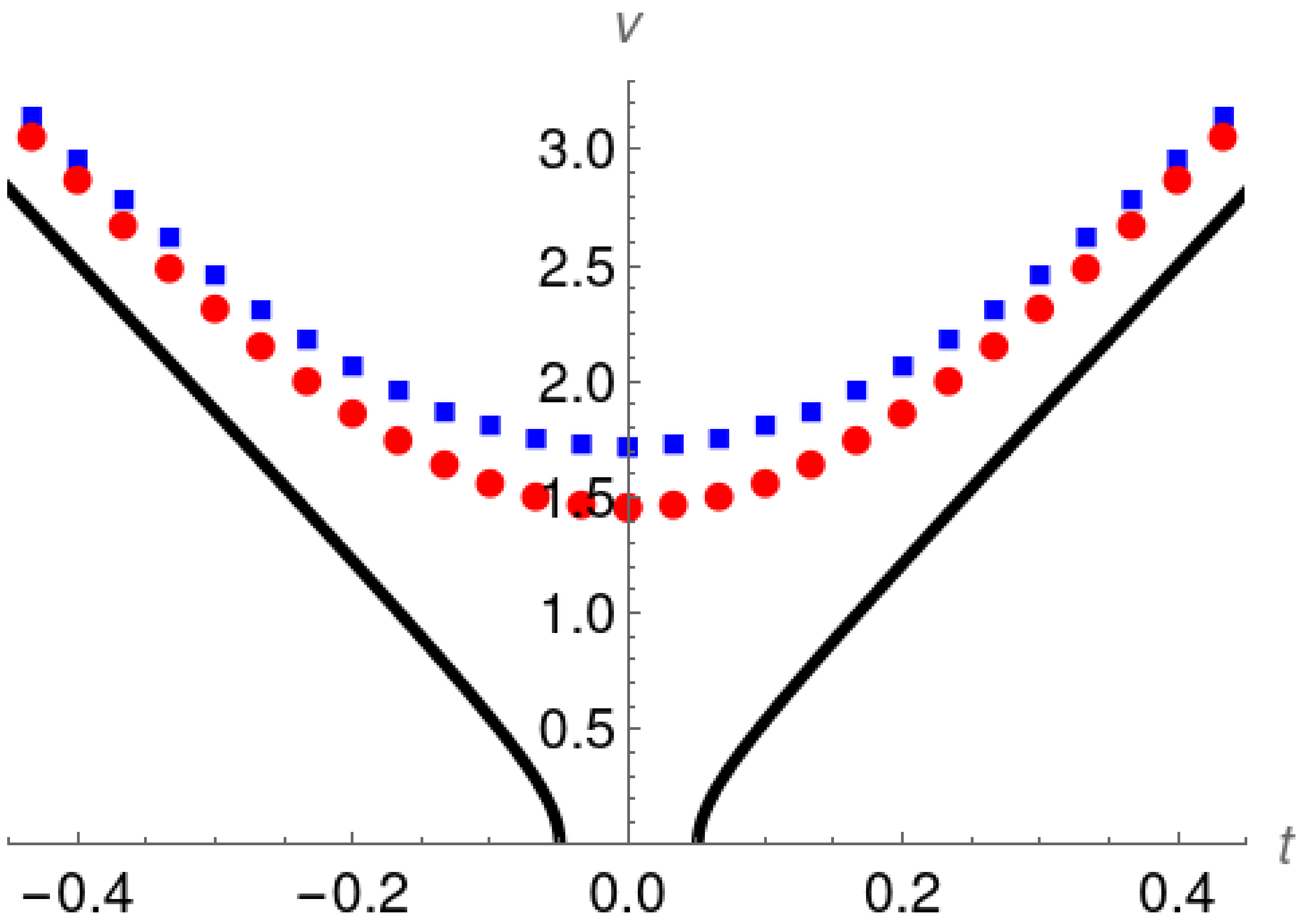
The Hilbert space is defined by $\langle\Phi|\Psi\rangle_t = \int \mathrm{d}v\mathrm{d}\varphi\; v\bar{\Phi}\Psi$

To ensure unitary dynamics, the Hamiltonian must be self-adjoint. In this case, the Hamiltonian is not self-adjoint, but admits a family of self-adjoint extensions parametrised by the solutions to the boundary condition:

$$\left[v\bar{\Phi}\partial_v\Psi - v\Psi\partial_v\bar{\Phi}\right]_{v=0} = 0 \longleftarrow \text{Reflection around v=0}$$

This boundary condition leads to singularity resolution

Dynamics of the t-clock theory



Dynamics of the Φ-clock theory

Gauge $\tilde{N} = v^2$

Dynamics are governed by the Klein-Gordon equation $\hat{\mathcal{G}} = \hbar^2 \frac{\partial^2}{\partial \varphi^2} \Psi$

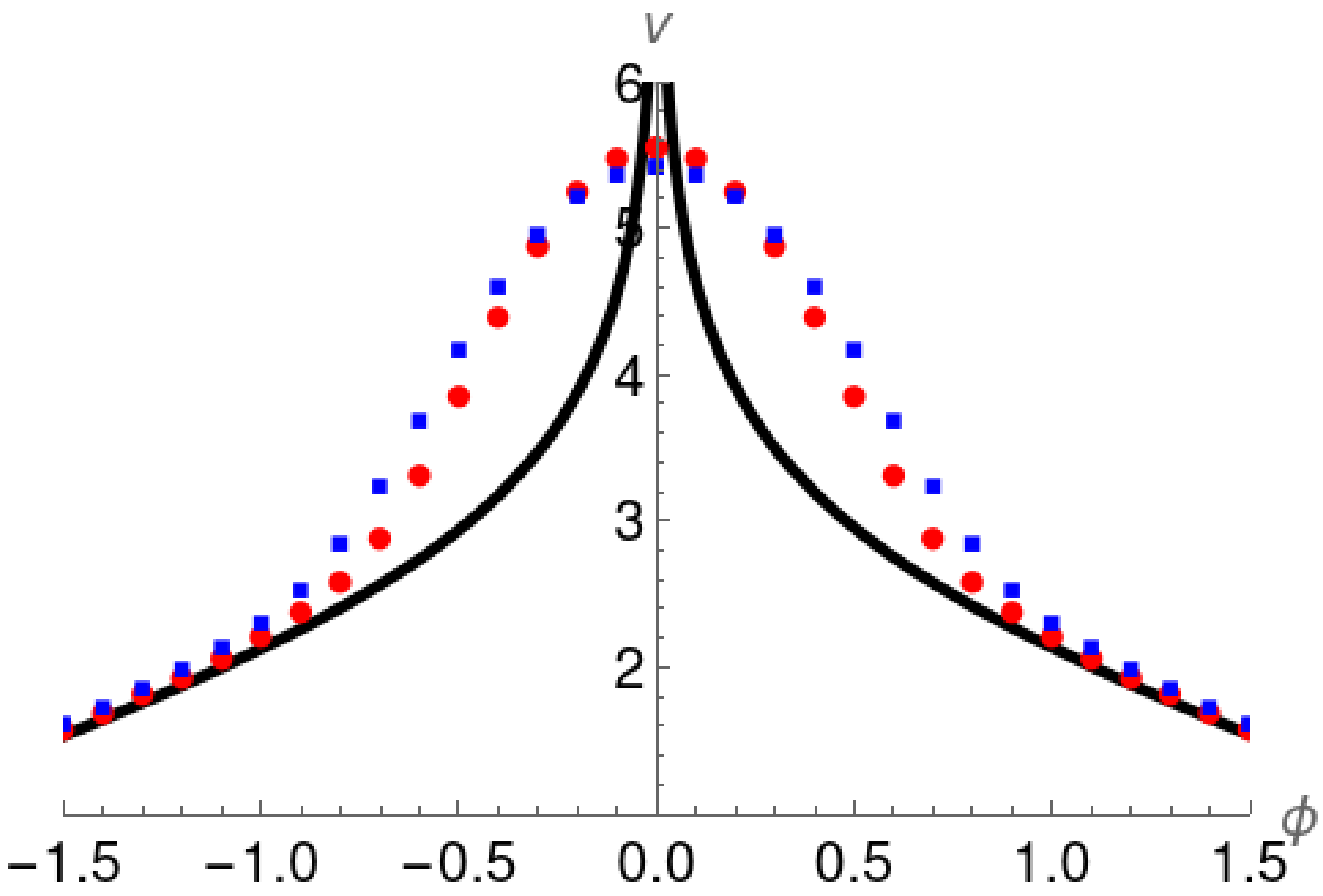
The Hilbert space is defined by $\langle \Phi | \Psi \rangle_\varphi = i \int \frac{\mathrm{d}v}{v} \mathrm{d}t \; (\bar{\Phi} \partial_\varphi \Psi - \Psi \partial_\varphi \bar{\Phi})$

To ensure unitary dynamics, the ev. operator must be self-adjoint. In this case, the ev. operator is not self-adjoint, but admits a one dimensional family of self-adjoint extensions parametrised by the solutions to the boundary condition:

$$\left[v \bar{\Phi} \partial_v \Psi - v \Psi \partial_v \bar{\Phi} \right]_{v=\infty} = 0 \longleftarrow \text{Reflection around } v=\infty$$

This boundary condition leads to spatial infinity resolution

Dynamics of the Φ-clock theory



Conclusions I

Conclusions:

- To ensure unitarity evolution, one must impose additional non trivial boundary conditions.
- Gauge (Lapse $\tilde{N}$) choices lead to different boundary conditions.
- The dynamics (in particular singularity resolution) are theory dependent

However, the path integral formalism is supposedly covariant

Questions:

- How to analyse the problem of time in the path integral formalism?
- What happens to unitarity and these boundary conditions?

Quantisation II: Path Integral quantisation

Generic form of a path integral in GR:

weight: EH action

$$\langle g_{\mu\nu,f} | g_{\mu\nu,in} \rangle = \int \mathcal{D} g_{\mu\nu} e^{iS_{EH}(g_{\mu\nu})}$$

"Probability" of going from an initial to a final configuration

functional integral

Our model

In our model this simplifies to:

$$\langle v_f, \varphi_f, t_f | v_{in}, \varphi_{in}, t_{in} \rangle = \int \mathrm{d}\tilde{N}(\tau_f - \tau_{in}) \int \mathcal{D}v\mathcal{D}\varphi\mathcal{D}t\mathcal{D}\pi_v\mathcal{D}\pi_\varphi\mathcal{D}\lambda \; e^{i\mathcal{S}_{class}}$$

Terms you need to add to preserve Gauge invariance

This action is covariant under change of Lapse

t-clock theory gauge: $$\mathcal{S}_{class} = \int_{\tau_{in}}^{\tau_f} \mathrm{d}\tau \left\{ \pi_v \dot{v} + \pi_\varphi \dot{\varphi} + \lambda \dot{t} - \tilde{N} \left[-\pi_v^2 + \frac{\pi_\varphi^2}{v^2} + \lambda \right] \right\}$$

Φ-clock theory gauge: $$\mathcal{S}_{class} = \int_{\tau_{in}}^{\tau_f} \mathrm{d}\tau \left\{ \pi_v \dot{v} + \pi_\varphi \dot{\varphi} + \lambda \dot{t} - N \left[-\pi_v^2 v^2 + \pi_\varphi^2 + \lambda v^2 \right] \right\}$$

What is the situation?

- Finding an analitical expression of the P.I. is very hard because of the v integral
- The P.I. quantisation should be equivalent to the canonical quantisation via formulas like

$$\langle v_f, \varphi_f, t_f | v_{in}, \varphi_{in}, t_{in} \rangle = \int \mathrm{d}E e^{iE(t_f - t_i)} \bar{\Psi}_E(v_f, \varphi_f, t_f) \Psi_E(v_{in}, \varphi_{in}, t_{in})$$

Energy eigenstate of the canonical theory

- But in our case, we have two different canonical theories for two different Lapse choices... What is going on?
- In simpler models with similar b.c. in the canonical theory, these b.c. can be incorporated by adding extra terms in the P.I., but these terms depend on the specific canonical theory... We lose covariance!

Conclusions II

In this talk we have seen:

- How, in a simple model different Lapse ($\tilde{N}$) choices affect the resulting canonical quantum theories, by introducing extra boundary conditions to ensure unitarity. These extra condition break covariance.
- How this loss of covariance can also be made manifest in the P.I. quantisation formalism.
- It seems that the issue of unitarity is present in both the P.I. and the canonical approaches.

Future work:

- Further study the P.I. formalism of this model

Conclusions II

Thanks!