A gauge for graviton fluctuations in the WKB approach

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Based on: GM, G. Montani, S. Antonini, Phys. Rev. D 107 (2023) L061901 (arXiv 2302.10832)

Talk @ Quantum Gravity 2023 conference, Nijmegen

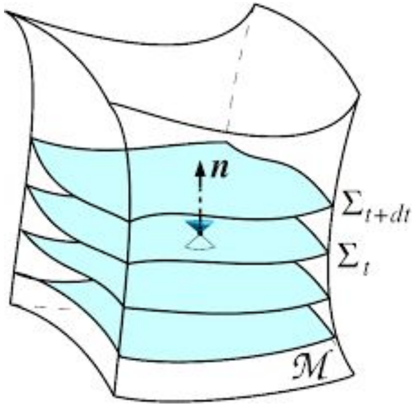
Quantum gravity and Wheeler-deWitt theory

Hamiltonian formulation of GR via ADM foliation: N, N^i, h_{ij}

Wheeler-deWitt equation (scalar constraint): $\hat{H}^g \Psi \approx 0$

("frozen formalism")

→ different time implementations

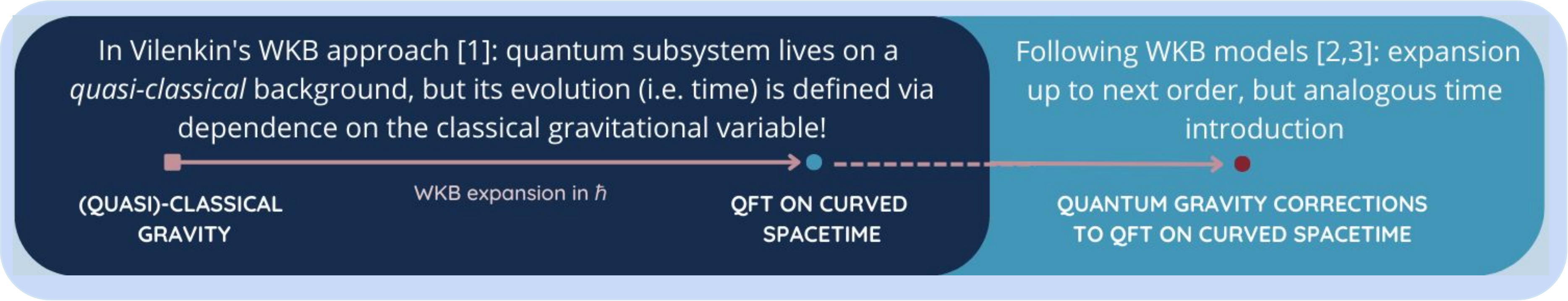


- ❖ Aim: investigate quantum gravity effects at low (< Planck) scales
- ❖ Requirement: recover Quantum Field Theory (QFT) in curved spacetime via some limit (top-down approach)

The WKB path

Wave functional of the Universe (subsystem+background) is expanded in a perturbative parameter

→ approximate equations of motion order by order from WDW



[1] A. Vilenkin, Phys. Rev. D 39 (1989)
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Proposal: gravitons take the stage

We propose a WKB implementation based on 4 points:

1) gravitational variables are categorized in classical ones and (graviton) quantum fluctuations;
2) independence of the small quantum fluctuations;
3) symmetry associated with wave functional separability (no gravitational constraints *a priori*);
4) QFT limit must be obtained using the classical gravitational variables.

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Classical gravitational background (Bianchi I, vacuum)	+	Quantum gravitational perturbations (gravitons)	+	Quantum matter scalar field (inflaton)

$$H = \frac{4}{3M} e^{-\frac{3}{2}\alpha} N \left(-p_\alpha^2 + p_+^2 + p_-^2\right) + \sum_{\mathbf{k},\lambda} \frac{1}{2}\left[-\partial^2_{v^\lambda_{\mathbf{k}}} + \omega_k^2(\eta)(v^\lambda_{\mathbf{k}})^2 + \mathcal{V}_{\lambda,\bar{\lambda}}\right] + \sum_{\mathbf{k}} \frac{1}{2}\left[-\partial^2_{\phi_{\mathbf{k}}} + \nu_k^2(\eta)\left(\phi_{\mathbf{k}}\right)^2\right]$$

$\overline{\text{B}}$-O symmetry and geometrical phase

WKB assumption on total wave functional:

$$\Psi = \psi_g(\alpha, \beta_\pm, v^\lambda_{\mathbf{k}})\, \chi_m(\phi_{\mathbf{k}}; \alpha, \beta_\pm, v^\lambda_{\mathbf{k}}) = e^{\frac{i}{\hbar} M S_0}\, e^{\frac{i}{\hbar}(S_1 + \mathcal{O}(M^{-1}))}\, e^{\frac{i}{\hbar}(Q_1 + \mathcal{O}(M^{-1}))}$$

with expansion parameter $M = \dfrac{c^2}{32\pi G} = \dfrac{c m_{Pl}^2}{4\hbar}$

Due to point 3): wave functional is invariant under rescaling $\psi \to \psi\, e^{-\frac{i}{\hbar}\theta}\,,\ \chi \to e^{\frac{i}{\hbar}\theta}\chi$

Questions:

- How to address point 4) (QFT limit) ?
  → *average* over tensor fluctuations, but then theory is transparent to dynamics of fluctuations.
- What about point 2) ?

$\bar{\text{A}}$ gauge for gravitons and QFT limit

Make use of geometric phase to select a gauge:

$$e^{\frac{i}{\hbar}S_1}\left[i\hbar\partial_T e^{-\frac{i}{\hbar}S_1^*} + \frac{1}{2}\sum_{\mathbf{k},\lambda}\left(\omega_k^2(v_{\mathbf{k}}^\lambda)^2 + \mathcal{V}_{\lambda,\bar{\lambda}} - \partial^2_{v_{\mathbf{k}}^\lambda}\right)e^{-\frac{i}{\hbar}S_1^*}\right] = 0$$

with WKB time is defined as $-i\hbar\partial_T = \frac{8}{3}e^{-\frac{1}{2}\alpha}\left(\partial_\alpha S_0\,\partial_\alpha + \partial_{\beta_+}S_0\,\partial_{\beta_+} + \partial_{\beta_-}S_0\,\partial_{\beta_-}\right)$

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Averaged matter wave functional $\widetilde{\Theta}(\phi_{\mathbf{k}};\alpha,\beta_+,\beta_-)=\int\prod_{\mathbf{k},\lambda}dv_{\mathbf{k}}^\lambda\left|e^{\frac{i}{\hbar}S_1}\right|^2 e^{\frac{i}{\hbar}Q_1}$

satisfies (up to boundary terms):

$$i\hbar\,\partial_T\widetilde{\Theta}=\frac{1}{2}\sum_{\mathbf{k}}\left[\nu_k^2(\phi_{\mathbf{k}})^2-\partial^2_{\phi_{\mathbf{k}}}\right]\widetilde{\Theta}=N\hat{H}^{(\phi)}\widetilde{\Theta}$$

QFT limit is properly recovered

Interpretation of the gravitons' gauge

To preserve B-O symmetry, we did not impose initial constraints on the gravitational wave functional.

In [1]: WDW equation for gravity only is also imposed
In [2], [3]: gravitational dynamics is related to a gauge

Meaning of the gauge?

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reconstructs at this order the (c.c. of) gravitational WDW constraint for the *gravitational fluctuations*
→ justification for the additional hypothesis in [1]

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Conclusions and outlooks

- ➢ The non-classical behaviour of the gravitational component emerges as graviton fields with their own dynamics;
- ➢ The gauge freedom typical of the B-O approach can be used instead of the additional hypothesis of gravitational WDW;
- ➢ The matter evolution is recovered only after *averaging over* the fluctuating gravitational sector, as in effective field theory, and fixing their gauge.
- Next order expansion? Different time implementations? *to be continued*

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Thank you for your attention!