Regular black holes

and the first law of BH mechanics

Sebastian Murk

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13th July, Quantum Gravity 2023
Radboud University, Nijmegen, NL

Based on arXiv:2304.05421 [gr-qc]

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Sebastian Murk[1,*] and Ioannis Soranidis[2,†]

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Hyperlink: arXiv:2304.05421 [gr-qc]

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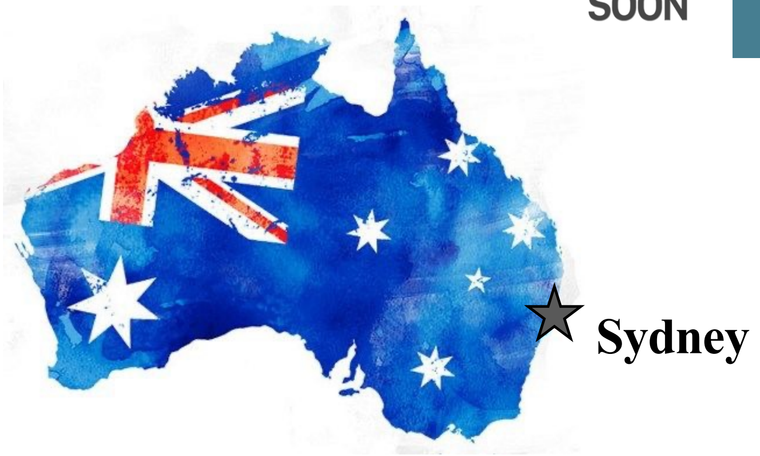
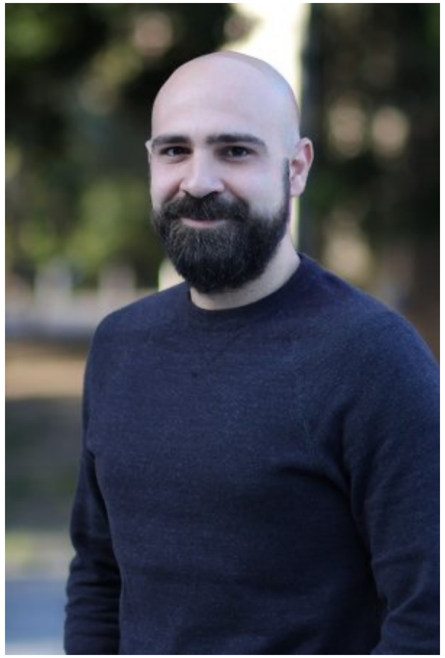


Ioannis Soranidis

MACQUARIE University
SYDNEY · AUSTRALIA



Setup: semiclassical gravity, spherical symmetry, & dynamical RBHs

Mann, SM, Terno,
Int. J. Mod. Phys. D **31**, 2230015 (2022).

$(-,+,+,+)$

$c = G = \hbar = k_B = 1$

Semiclassical gravity: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle_\psi$

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Metric functions:

$$C(v,r) = r_+(v) + \sum_{i=1}^{\infty} w_i(v) \big(r - r_+(v)\big)^i$$

$$h(v,r) = \sum_{i=1}^{\infty} \chi_i(v) \big(r - r_+(v)\big)^i$$

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Generic dynamical RBHs:

$$f(v,r) = g(v,r) \big(r - r_-(v)\big)^a \big(r - r_+(v)\big)^b$$

$$a, b \in \mathbb{N}_{>0} = \{1, 2, \ldots\}$$

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At outer horizon $r = r_+$:

$$C(v,r)\big|_{r=r_+} = r_+(v)\ , \quad h(v,r)\big|_{r=r_+} = 0$$

Surface gravity and the first law of BH mechanics

First law of BH mechanics:

(for $\delta J = \delta Q = 0$)

$$\delta M = \frac{\kappa}{8\pi}\delta A$$

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Kodama, Prog. Theor. Phys. **63**, 1217 (1980).
Abreu, Visser, PRD **82**, 044027 (2010).
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Dynamical generlizations: [1] peeling surface gravity → ill-defined for transient object!

SM, Terno, PRD **103**, 064082 (2021).
Mann, SM, Terno, PRD **105**, 124032 (2022).

[2] Kodama surface gravity:

$$\kappa_K K_\nu := \frac{1}{2}K^\mu\left(\nabla_\mu K_\nu - \nabla_\nu K_\mu\right)$$

Nielsen, Yoon, CQG **25**, 085010 (2008).
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Kodama surface gravity evaluated at outer horizon: $$\kappa_K\big|_{r=r_+} = \frac{1}{2}\partial_r f(v,r)\big|_{r=r_+}$$

$$= \lim_{r\to r_+} \frac{(r-r_+)^{-1+b} b g(v,r)(r-r_-)^a}{2}$$

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Note: Nonzero Kodama surface gravity requires that outer horizon is nondegenerate, i.e. $b = 1$.

Extended/Generalized dynamical first law

Using $f(v,r) := \partial_\mu r \partial^\mu r = 1 - \dfrac{C(v,r)}{r}$ ⇨ $\kappa_K\big|_{r=r_+} = \dfrac{1}{2}\partial_r f(v,r) = \dfrac{1}{2}\left[C(v,r) - r\partial_r C(v,r)\right]\big|_{r=r_+} = \dfrac{1-w_1}{2r_+}$

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$\left(\Rightarrow \delta A = 8\pi r_+ \delta r_+ = \frac{2}{r_+}\delta V\right)$ ⇨

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Applies generically to dynamical black holes!

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Now: Focus on dynamical RBH models $f(v,r) = g(v,r)\big(r - r_-(v)\big)^a \big(r - r_+(v)\big)^b$ with $b = 1$.

$$⇨ \quad w_1\big|_{r=r_+} = 1 - g(v,r_+)r_+(r_+ - r_-)^a$$

$$\Leftrightarrow \quad g(v,r_+)r_+\left(r_+ - r_-\right)^a = 1$$

Nondegenerate dynamical RBH models $(a = 1, b = 1)$

Popular examples: Bardeen, Dymnikova, Hayward.

Bardeen in *Proceedings of the International Conference GR5* (Tbilisi University Press, Tbilisi, 1968).

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Comparison with $f(v,r) = g(v,r)\big(r - r_-(v)\big)\big(r - r_+(v)\big)$ $\Rightarrow$ $g(v,r) = \dfrac{r - r_0}{r^3 + r_g l^2} > 0$

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Expansion of MS mass about the outher horizon $r = r_+$: $\boxed{w_1\big|_{r=r_+} = \dfrac{3l^2}{r_g^2} + \mathcal{O}(l^4) \geqslant 0}$

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$$\Rightarrow \quad f(v,r) = \frac{r - r_0}{r^3 + r_g l^2}(r - r_-)(r - r_+)$$

Comparison with $f(v,r) = g(v,r)\big(r - r_-(v)\big)\big(r - r_+(v)\big)$ $\Rightarrow$ $g(v,r) = \dfrac{r - r_0}{r^3 + r_g l^2} > 0$

Expansion of MS mass about the outher horizon $r = r_+$: $\boxed{w_1\big|_{r=r_+} = \dfrac{3l^2}{r_g^2} + \mathcal{O}(l^4) \geqslant 0}$

Analogous expressions are obtained for other nondegenerate models following the same procedure.

Degenerate dynamical RBH models $(a > 1, b = 1)$

$$f(v,r) = g(v,r)\big(r - r_-(v)\big)^a \big(r - r_+(v)\big)$$

Assume:

$$f(v,r) = \frac{\mathcal{P}_n(r)}{\tilde{\mathcal{P}}_n(r)} \quad \& \quad g(v,r) = \frac{1}{c_0 + c_1 r + \cdots + c_{a+1} r^{a+1}}$$

Frolov, PRD **94**, 104056 (2016).

Carballo-Rubio, Di Filippo, Liberati, Pacilio, Visser, JHEP **118** (2022).

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Consistency condition:

$$g\left(v, r_+\right) r_+ \left(r_+ - r_-\right)^a = 1 \quad \Leftrightarrow \quad \boxed{\left(r_+ - r_-\right)^a r_+ = \frac{1}{g\left(v, r_+\right)} = \sum_{i=0}^{a+1} c_i\left(r_-, r_+\right) r_+^i}$$

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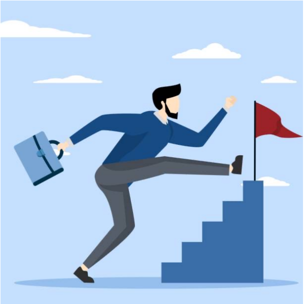
[...]



$$\Leftrightarrow \quad \sum_{i=0}^{a} d_{ij} = \binom{a}{a-j} (-1)^j$$

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Carballo-Rubio, Di Filippo, Liberati, Pacilio, Visser, [JHEP **118** (2022).]

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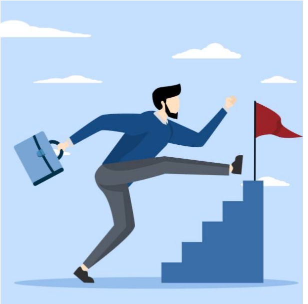

$$\boxed{\Leftrightarrow \quad \sum_{i=0}^{a} d_{ij} = \binom{a}{a-j} (-1)^j}$$

Test CDLPV model $(a = 3)$: $\Rightarrow \quad 3r_+^2 - 3r_- r_+ + r_-^2 < 0$

[[JHEP **118** (2022)]]

Discriminant: $-3r_-^2 < 0$ ➡ Two distinct complex conjugate roots.

Summary & arXiv:2304.05421 QR code

Regular black holes and the first law of black hole mechanics

Sebastian Murk[1,*] and Ioannis Soranidis[2,†]

[1] *Okinawa Institute of Science and Technology, 1919-1 Tancha, Onna-son, Okinawa 904-0495, Japan*
[2] *School of Mathematical and Physical Sciences, Macquarie University, Sydney, New South Wales 2109, Australia*

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Results:

1. First law receives corrections that can be interpreted as an additional work term of an extended first law:

$$\delta\left(\frac{r_+}{2}\right) = \frac{1-w_1}{16\pi r_+}\delta A + \boxed{\frac{w_1}{8\pi r_+^2}\delta V}$$

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2. Linear coefficient of Misner–Sharp suffices to determine the relevant thermodynamic properties.

3. Need for corrections is linked to introduction of minimal length scale (consequence of spacetime regularization).

arXiv:2304.05421

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Review article

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International Journal of Modern Physics D | Vol. 31, No. 09, 2230015 (2022) | Review Paper

Black holes and their horizons in semiclassical and modified theories of gravity

Robert B. Mann, Sebastian Murk and Daniel R. Terno

https://doi.org/10.1142/S0218271822300154

arXiv:2112.06515 [gr-qc]

Int. J. Mod. Phys. D **31**, 2230015 (2022)

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Abstract

For distant observers, black holes are trapped spacetime domains bounded by apparent horizons. We review properties of the near-horizon geometry emphasizing the consequences of two common implicit assumptions of semiclassical physics. The first is a consequence of the cosmic censorship conjecture, namely, that curvature scalars are finite at apparent horizons. The second is that horizons form in finite asymptotic time (i.e. according to distant observers), a property implicitly assumed in conventional descriptions of black hole formation and evaporation. Taking these as the only requirements within the semiclassical framework, we find that in spherical symmetry only two classes of dynamic solutions are admissible, both describing evaporating black holes and expanding white holes. We review their properties and present the implications. The null energy condition is violated in the vicinity of the outer horizon and satisfied in the vicinity of the inner apparent/anti-trapping horizon. Apparent and anti-trapping horizons are timelike surfaces of intermediately singular behavior, which manifests itself in negative energy density firewalls. These and other properties are also present in axially symmetric solutions. Different generalizations of surface gravity to dynamic spacetimes are discordant and do not match the semiclassical results. We conclude by discussing signatures of these models and implications for the identification of observed ultra-compact objects.

Keywords: Semiclassical gravity ▪ modified gravity ▪ black holes ▪ apparent horizon ▪ evaporation ▪ white holes ▪ energy conditions ▪ thin shell collapse ▪ surface gravity ▪ information loss

arXiv

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Existence of a trapped region

Hayward,
Phys. Rev. D **49**, 6467 (1994).

Geodesic congruences: $\theta_- = -\frac{2}{r}\,, \quad \theta_+ = \frac{f(v,r)}{r}$ ⇨ Existence of trapped region: $\theta_-\theta_+ \overset{?}{\lesseqgtr} 0$

Presence of trapped is signified by $\theta_-\theta_+ > 0$, which implies $f < 0$ inside, $f > 0$ outside of the trapped region.

$$f(v,r) = g(v,r)\big(r - r_-(v)\big)^a \big(r - r_+(v)\big)^b \quad ⇨ \quad g > 0 \text{ and } b \text{ odd.}$$

"Disappearance point": $\theta_-\theta_+\big|_{v=v_d} = -\frac{2}{r^2} g\left(v_d, r\right)\left(r - r_+(v_d)\right)^{a+b} \leq 0 \quad \forall\, r$

$r_-\left(v_d\right) \equiv r_+\left(v_d\right)$

⇨ Sum $a + b$ must be even.

⇨ a odd.

SM, Soranidis,
arXiv:2304.05421 [gr-qc].

Generalized dynamical charged Hayward–Frolov RBH

SM, Soranidis, arXiv:2304.05421 [gr-qc].

Generalized dynamical metric function: $f(v,r) = 1 - \frac{\left(r_g(v)r - q(v)^2\right) r^2}{r^4 + \left(r_g(v)r + q(v)^2\right) l(v)^2}$

Kodama surface gravity: $\kappa_{K_{\mathrm{HF}}} = \frac{1 - w_1(v,l)}{2r_+(v,l)}$

Horizons:

$$r_-(v,l) = r_-(v) + \beta_-(v)l^2 + \mathcal{O}\left(l^3\right)$$
$$r_+(v,l) = r_+(v) + \beta_+(v)l^2 + \mathcal{O}\left(l^4\right)$$

$$r_-(v) = m(v) - \sqrt{m(v)^2 - q(v)^2}$$
$$r_+(v) = m(v) + \sqrt{m(v)^2 + q(v)^2}$$

Consider MS expansion: $w_1(v,l) = \frac{q(v)^2}{r_+(v)^2} + \beta(v)l^2 + \mathcal{O}\left(l^4\right) \quad \Rightarrow \quad \kappa_{K_{\mathrm{HF}}} = \frac{r_+(v) - r_-(v)}{2r_+(v)^2} + \mathcal{O}\left(l^2\right)$

Differences:

1. Inner horizon $r_- \neq 0$ even if $l = 0$ due to the presence of a charged term that is independent of l.
2. Compatibility with the first law is no longer encoded by $w_1 = 0$. For $l \to 0$, the new compatibility condition can be stated as $w_1(v,0) = \frac{q(v)^2}{r_+(v)^2}$

Page evaporation law

SM, Soranidis, arXiv:2304.05421 [gr-qc].

Mass loss due to emission of Hawking radiation: $$\frac{dM}{dt} = -\sum_{j,\ell,m,p} \frac{1}{2\pi} \int_0^\infty \frac{\omega \Gamma_{j\omega\ell mp}}{e^{2\pi\omega/\kappa} - 1} d\omega$$

Simplifying assumptions: $m = \ell = 0$

$$\Gamma \simeq \omega^2 r_g^2$$

Note:
Effects of Hawking radiation are described by ingoing Vaidya metric with decreasing mass $(C'(v) < 0)$.

$$\frac{dM}{dv} \simeq -\frac{a}{M^2} \Leftrightarrow \frac{dr_+}{dv} \simeq -\frac{\alpha}{r_+^2} \quad \Rightarrow \quad t_e \sim M_0^3$$

Explicit form of the coefficients and their expansion about $w_1 = 0$:

$$\alpha = 8a = -\frac{4}{\pi} \frac{1}{e^{\frac{4\pi}{1-w_1}} - 1},$$

$$\alpha = -\frac{4}{\pi} \frac{1}{e^{4\pi} - 1} + \mathcal{O}(w_1),$$

⇨ Standard Page evaporation law is modified if $w_1 = 0$ is not satisfied.

Surface gravity in stationary spacetimes

Hawking temperature: $T_{\mathrm{H}} = \frac{\kappa}{2\pi}$ (for observer at infinity)

Several equivalent definitions, related to either

Killing vector field

with norm $\sqrt{\xi^\mu \xi_\mu} = 0$

Inaffinity of null geodesics on the horizon: $\xi^\mu{}_{;\nu}\xi^\nu := \kappa\xi^\mu$

or

$x := r - r_{\mathrm{g}}$

$r \gtrsim r_{\mathrm{g}}$

Peeling off properties of null geodesics near the horizon: $\frac{dr}{dt} = \pm 2\kappa_{\mathrm{peel}}(t)x + \mathcal{O}\left(x^2\right)$

Surface gravity in dynamical spacetimes

In general dynamical spacetimes: no asymptotically timelike Killing vector.

Kodama, Prog. Theor. Phys. **63**, 1217 (1980).
Abreu, Visser, Phys. Rev. D **82**, 044027 (2010).
Kurpicz, Pinamonti, Verch, Lett. Math. Phys. **111**, 110 (2021).

Role of Hawking temperate captured either by **peeling** or **Kodama surface gravity**.

Barceló, Liberati, Sonego, Visser,
Phys. Rev. D **83**, 041501(R) (2011).

Indistinguishable for sufficiently slowly evolving horizons with properties close to their classical counterparts.

However: the similarity fails for dynamic spherically symmetric solutions!

Mann, SM, Terno,
Phys. Rev. D **105**, 124032 (2022).

Surface gravity in dynamic spacetimes: peeling surface gravity

Nielsen, Yoon, Class. Quantum Gravity **25**, 085010 (2008).
Cropp, Liberati, Visser, Class. Quantum Gravity **30**, 125001 (2013).

For example: k=0

$$C = r_g - c_{12}\sqrt{x} + \sum_{j\geqslant 1}^{\infty} c_j x^j$$

$$h = -\frac{1}{2}\ln\frac{x}{\xi} + \sum_{j\geqslant \frac{1}{2}}^{\infty} h_j x^j$$

Consider **peeling surface gravity:** $\kappa_{\text{peel}} = \dfrac{e^{h(t,r_g)}\left(1 - C'(t,r_g)\right)}{2r_g}$

With the metric functions C and h of the k=0 and k=1 solutions: $\kappa_{\text{peel}} \to \infty$

$$\frac{dr}{dt} = \pm r_g' + a_{12}(t)\sqrt{x} + \mathcal{O}(x)$$

Cf. stationary expression: $\dfrac{dr}{dt} = \pm 2\kappa_{\text{peel}}(t)x + \mathcal{O}\left(x^2\right)$

Nielsen, Visser,
Class. Quantum Gravity **23**, 4637 (2006).

Using Painlevé–Gullstrand coordinates $(\bar{t}, r)$: $\kappa_{\text{PG}_1} = \dfrac{1}{2r_g}\left(1 - \partial_r \bar{C}\right)\Big|_{r=r_g} \longrightarrow \kappa_{\text{PG}_1} = 0$

Mann, SM, Terno,
Phys. Rev. D **105**, 124032 (2022).

$\kappa_{\text{PG}_2} = \dfrac{1}{2r_g}\left(1 - \partial_r \bar{C} + \partial_{\bar{t}} \bar{C}\right)\Big|_{r=r} \longrightarrow$ 3 possibilities (0,∞,finite) depending on behaviour of $\bar{t}$

Surface gravity in dynamic spacetimes: Kodama surface gravity

Defined via $\frac{1}{2}K^{\mu}\left(\nabla_{\mu}K_{\nu}-\nabla_{\nu}K_{\mu}\right):=\kappa_{\mathrm{K}}K_{\nu}$ evaluated at horizon.

Kodama vector field: $K^{\mu}=\left(e^{-h_{+}},0,0,0\right)$ ((v,r) coordinates)

covariantly conserved: $\nabla_{\mu}K^{\mu}=0,$

$\nabla_{\mu}J^{\mu}=0, \quad J^{\mu}:=G^{\mu\nu}K_{\nu}$

Result: $\kappa_{\mathrm{K}}=\frac{1}{2}\left(\frac{C_{+}(v,r)}{r^{2}}-\frac{\partial_{r}C_{+}(v,r)}{r}\right)\bigg|_{r=r_{+}}=\frac{(1-w_{1})}{2r_{+}}$

→ 0 at formation of black hole.

→ Approaches static value $\kappa=1/(4M)$ only if metric is close to pure Vaidya metric. → **Contradicts semiclassical results.**

Mann, SM, Terno, Phys. Rev. D **105**, 124032 (2022).