Running relational observables

in Asymptotic Safety

Renata Ferrero

Based on:

A. Baldazzi, K. Falls and R. Ferrero, Annals Phys. ***80*** *(2022), 168822 [arXiv:2112.02118 [hep-th]]*

and work in progress in collaboration with Kevin Falls

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JG|U
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Asymptotic Safety

[Wetterich, Reuter, Saueressig, Percacci and many more]

[Höhn, Thiemann, Dittrich, Fröb, Chataignier, Rejzner, Marchetti, Gielen and many more]

Asymptotic Safety

Relational observables

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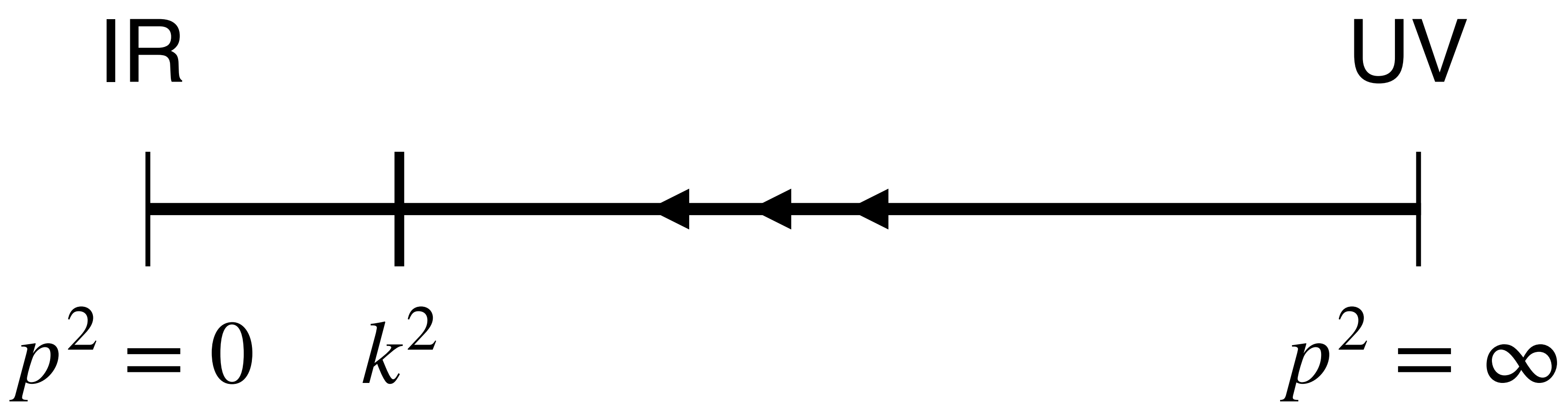
Asymptotic Safety

EAA

$$\lim_{k\to 0} \Gamma_k \to \Gamma$$



$$\lim_{k\to \infty} \Gamma_k \to S$$

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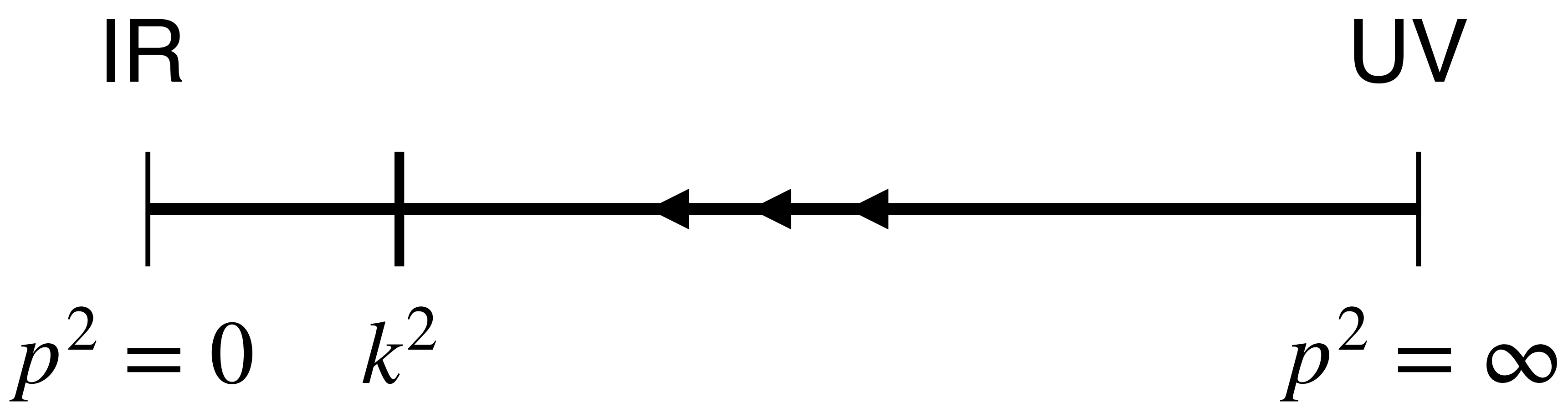

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FRGE

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

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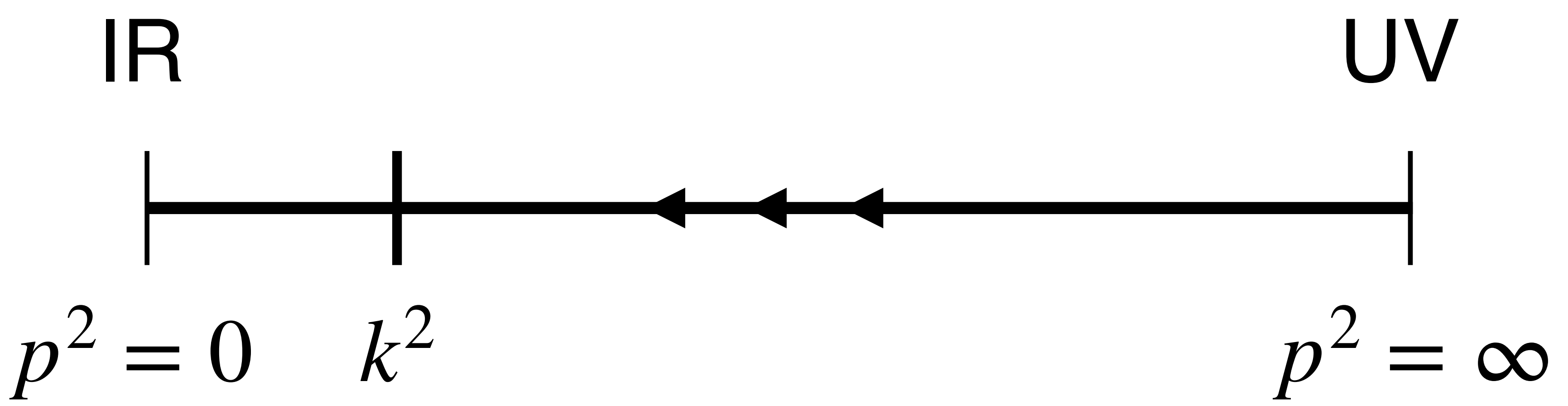

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Weinberg's conjecture

physical scattering amplitudes are finite (but non-vanishing) at energy scales exceeding the Planck scale

Asymptotic Safety

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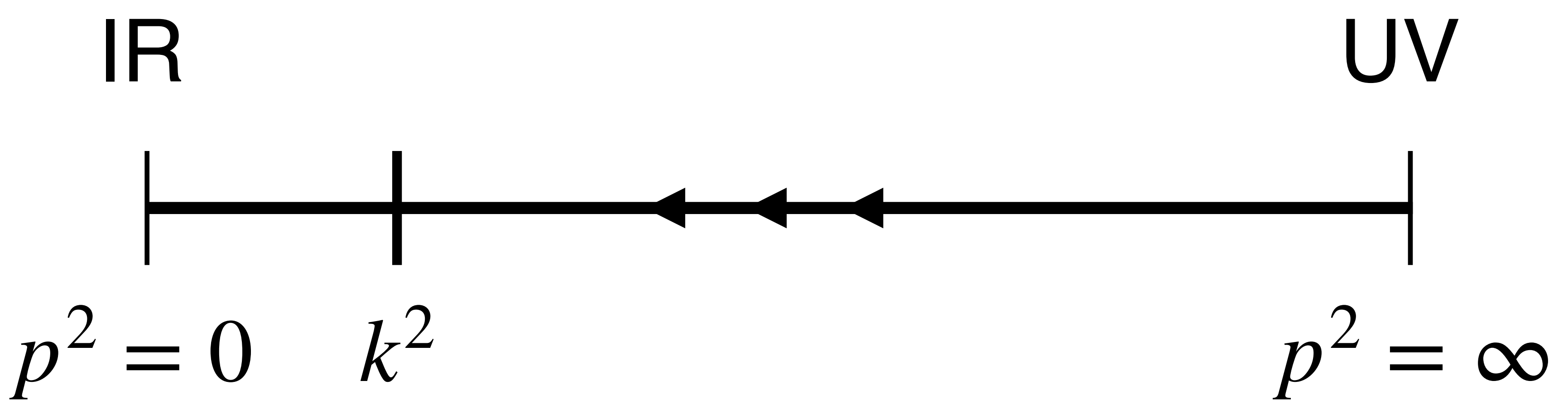

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Fixed points

$$\Gamma_k[\phi] = \sum_{i=1}^{\infty} U^i(k) P_i[\phi] \longrightarrow \beta^i(u_*) = 0$$

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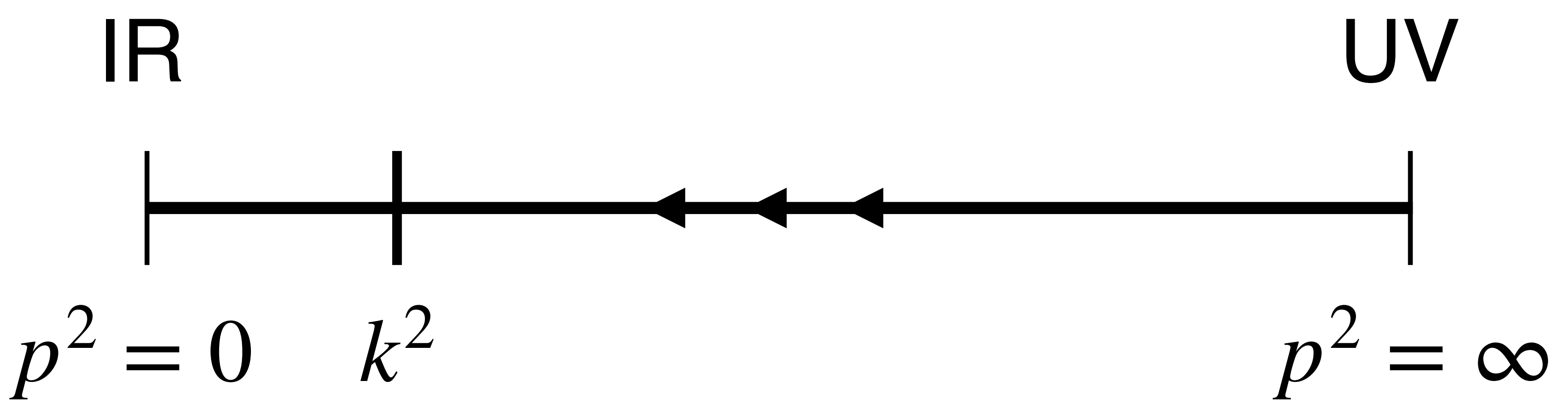

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Observables and AS

$$\mathcal{O} = \mathcal{O}(E, U_a(\mu), \mu) \longrightarrow \mathcal{O} = \mu^D \tilde{\mathcal{O}}(E/\mu, u_a(\mu))$$

$\mathcal{O} = E^D$

$\mu = E \to \infty$ well-defined limit $\Longleftrightarrow$ $\lim_{\mu\to\infty} u_a(\mu) = u_a^*$

Asymptotic Safety

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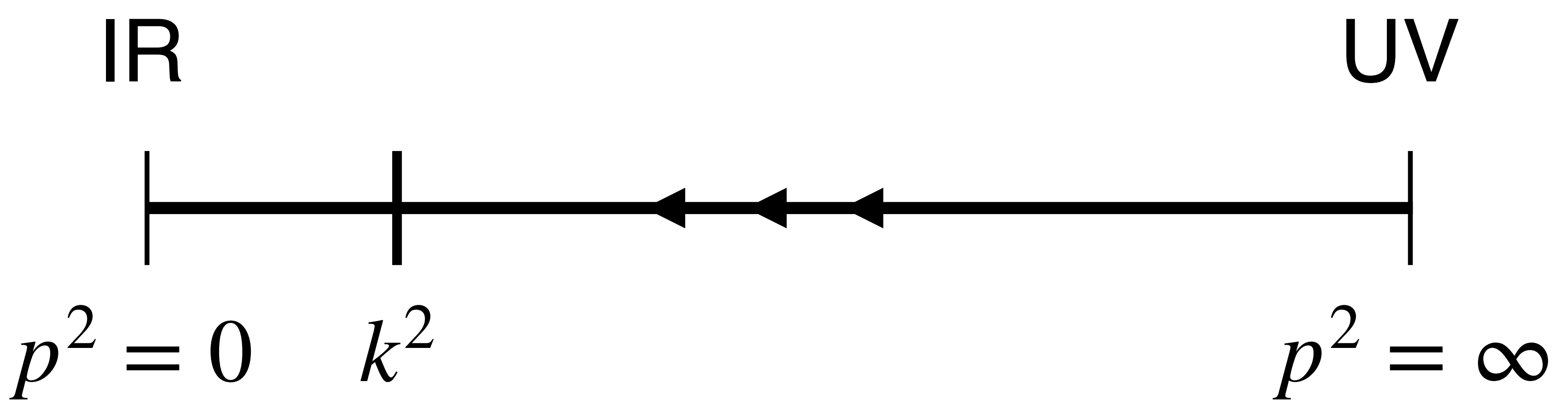

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what is the meaning of *k*?

Asymptotic Safety

Which is the behaviour of geometric or relational quantities at the quantum level?

Asymptotic Safety

Composite operators

[Pagani, Becker]

Which is the behaviour of geometric or relational quantities at the quantum level?

$$\Gamma_k[\phi, \varepsilon] = \Gamma_k[\phi] + \int \mathrm{d}^d x\, \varepsilon(x)\, \mathcal{O}_k(x) + O(\varepsilon^2)$$

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Concrete method to compute expectation values of observables.

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Critical exponents

The critical exponents do not depend on the cutoff scheme, they are **universal**.

$$u^i(k) = u^i_* + \sum_I C_I V^i_I \left(\frac{k_0}{k}\right)^{\theta_I}$$

[Höhn’s terminology:
covariant representation]

Relational observables

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In GR there are no local diffeomorphism-invariant observables.

$$R(x) \mapsto \varphi * R(x)$$

Diffeomorphism invariance

Relational observables

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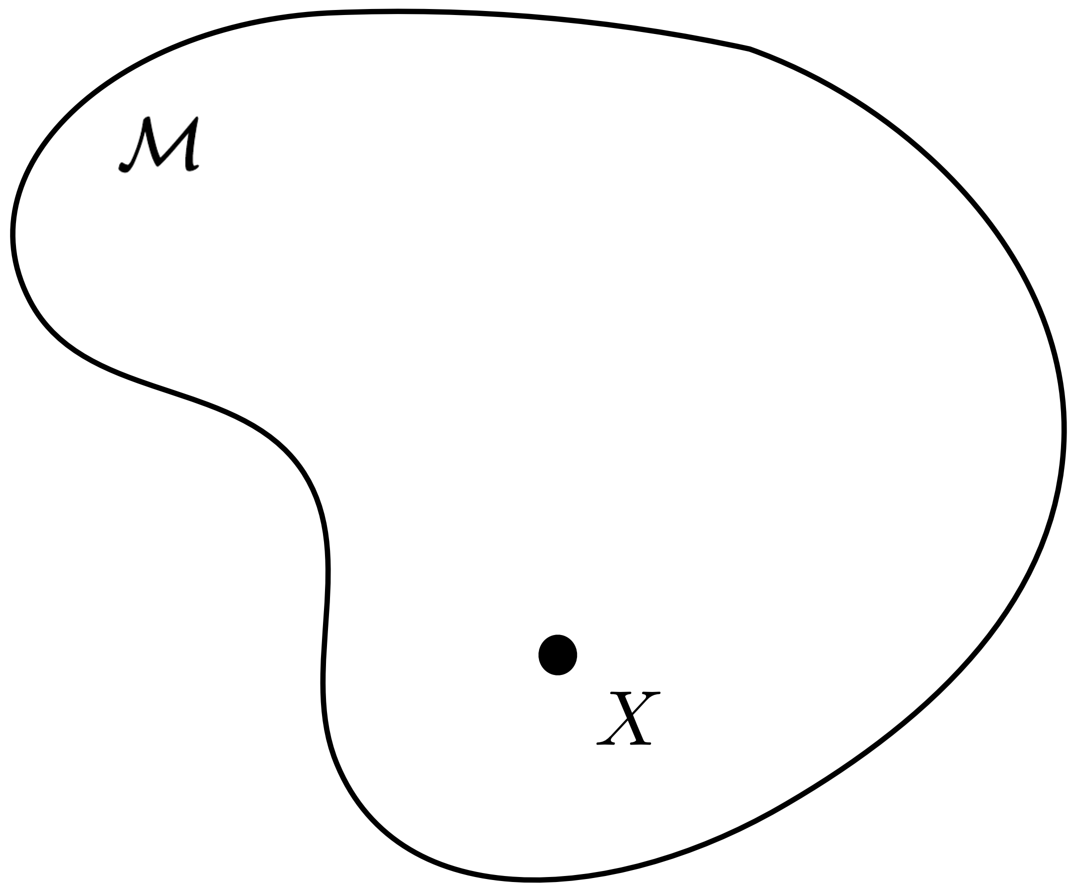
However, if X denotes the (spacetime) position of a particle, a diffeomorphism will map

$$X \mapsto \varphi^{-1}(X)$$



Thus $R(X)$ at the position of the particle, is diffeomorphism invariant, and hence observable.

$$R(X) \mapsto \varphi * R(\varphi^{-1}(X)) = R(X)$$

Diffeomorphism invariance

Relational formalism

Relational observables

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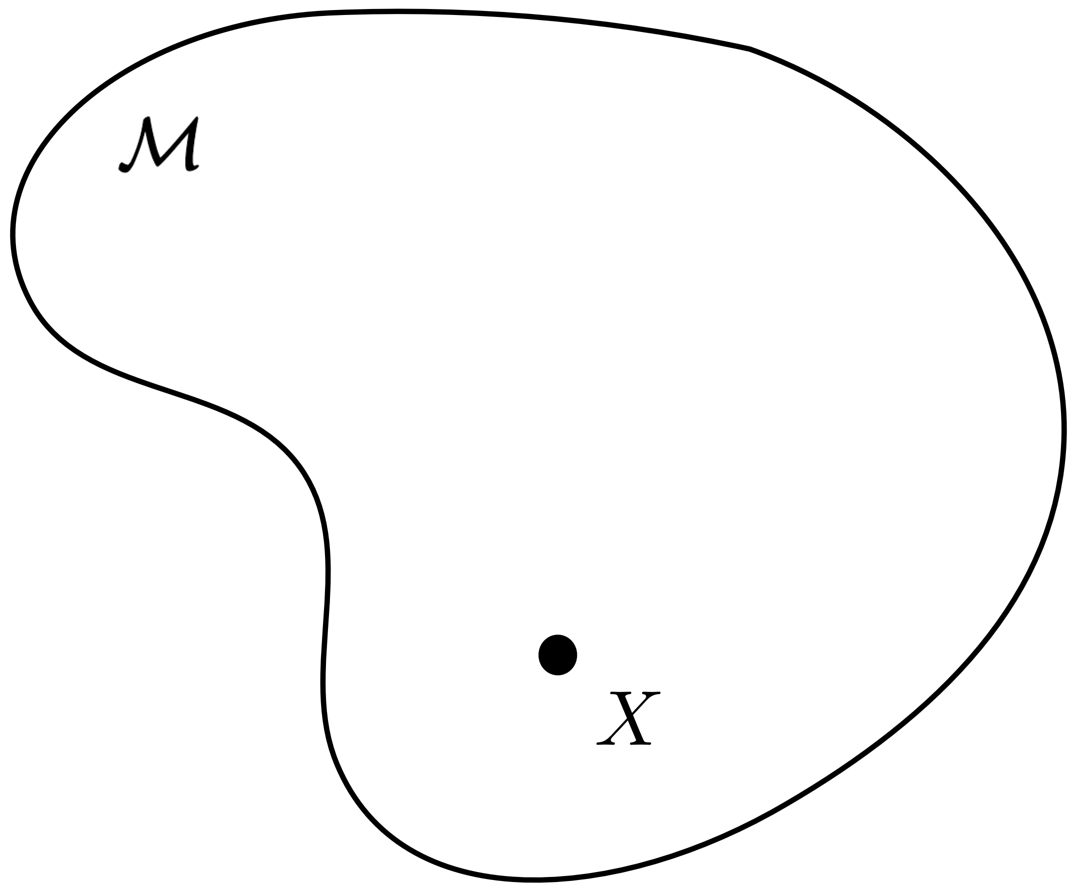

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Construct a physical coordinate frame,
s.t. composed transformation leaves the tensor invariant.

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Physical coordinate system

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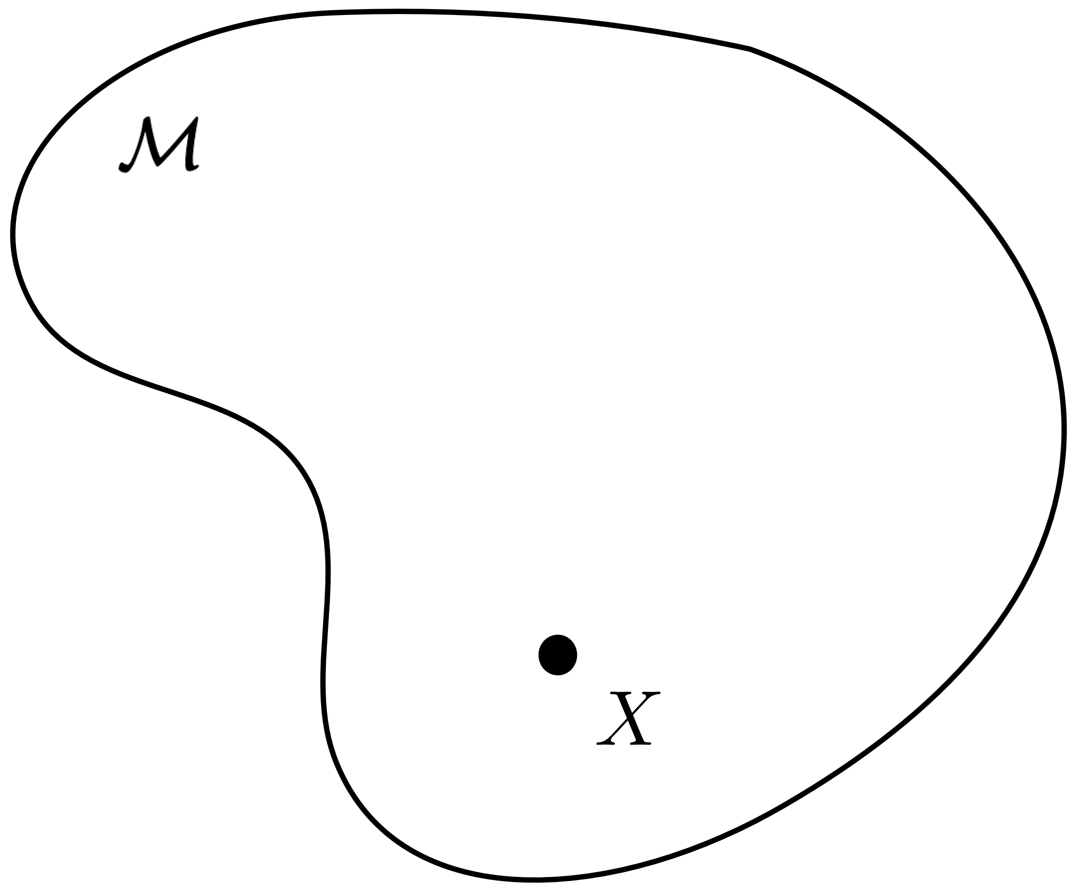

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Physical coordinate system

Relational observables

add matter fields


Asymptotic Safety
Relational observables

Asymptotic Safety

Relational EAA

$$\Gamma_k^{\text{rel.}} \equiv \int \mathrm{d}^4x \, \tilde{e}(x) \mathcal{L}_k^{\text{rel.}}(x) := \int \mathrm{d}^4x \, \tilde{e}(x) \sum_i a_{\hat{I}_i}(k) O^{\hat{I}_i}(x)$$

Relational observables

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Flow of the relational EAA

$$k\partial_k \Gamma_k^{\text{rel.}} = -\frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \left(\Gamma_k^{\text{rel.}(2)} \right) \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right]$$

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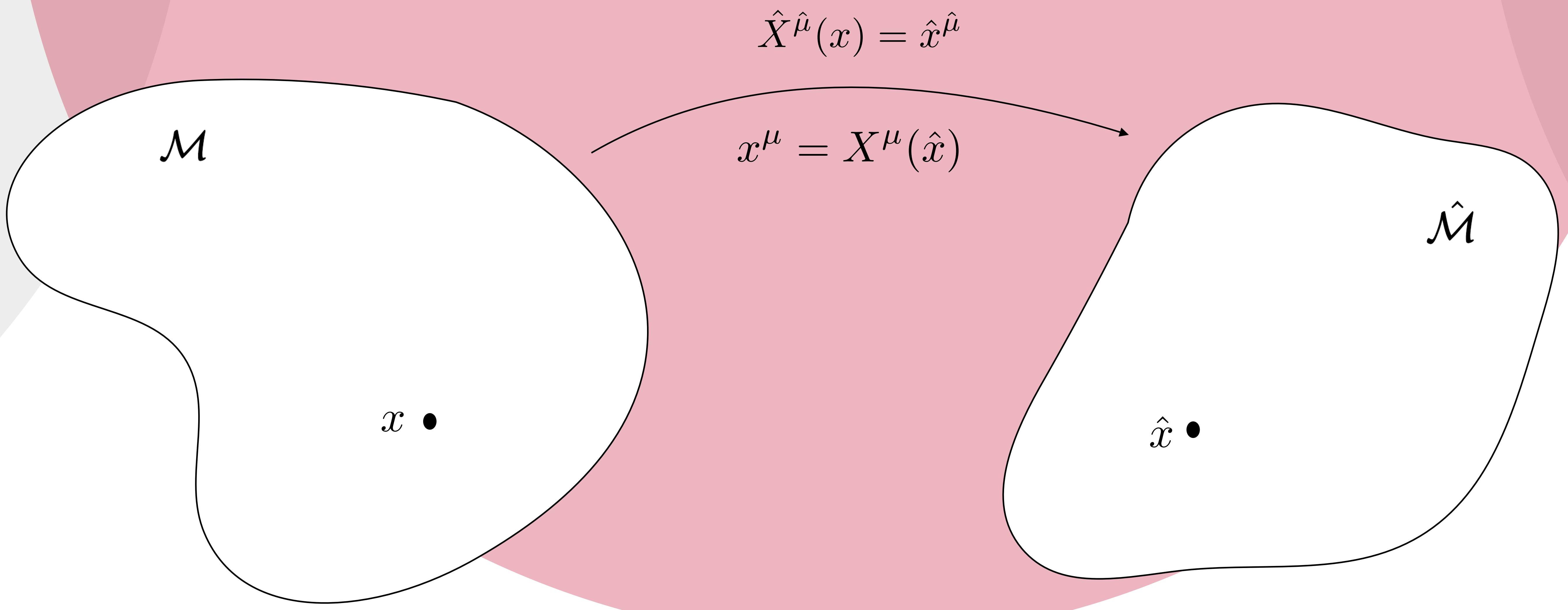
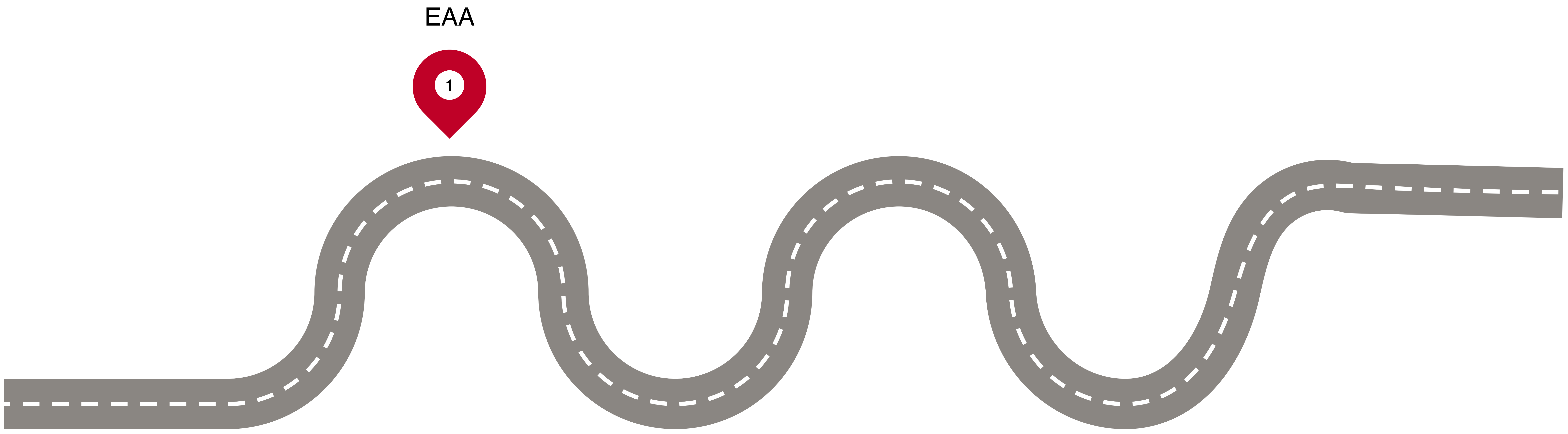


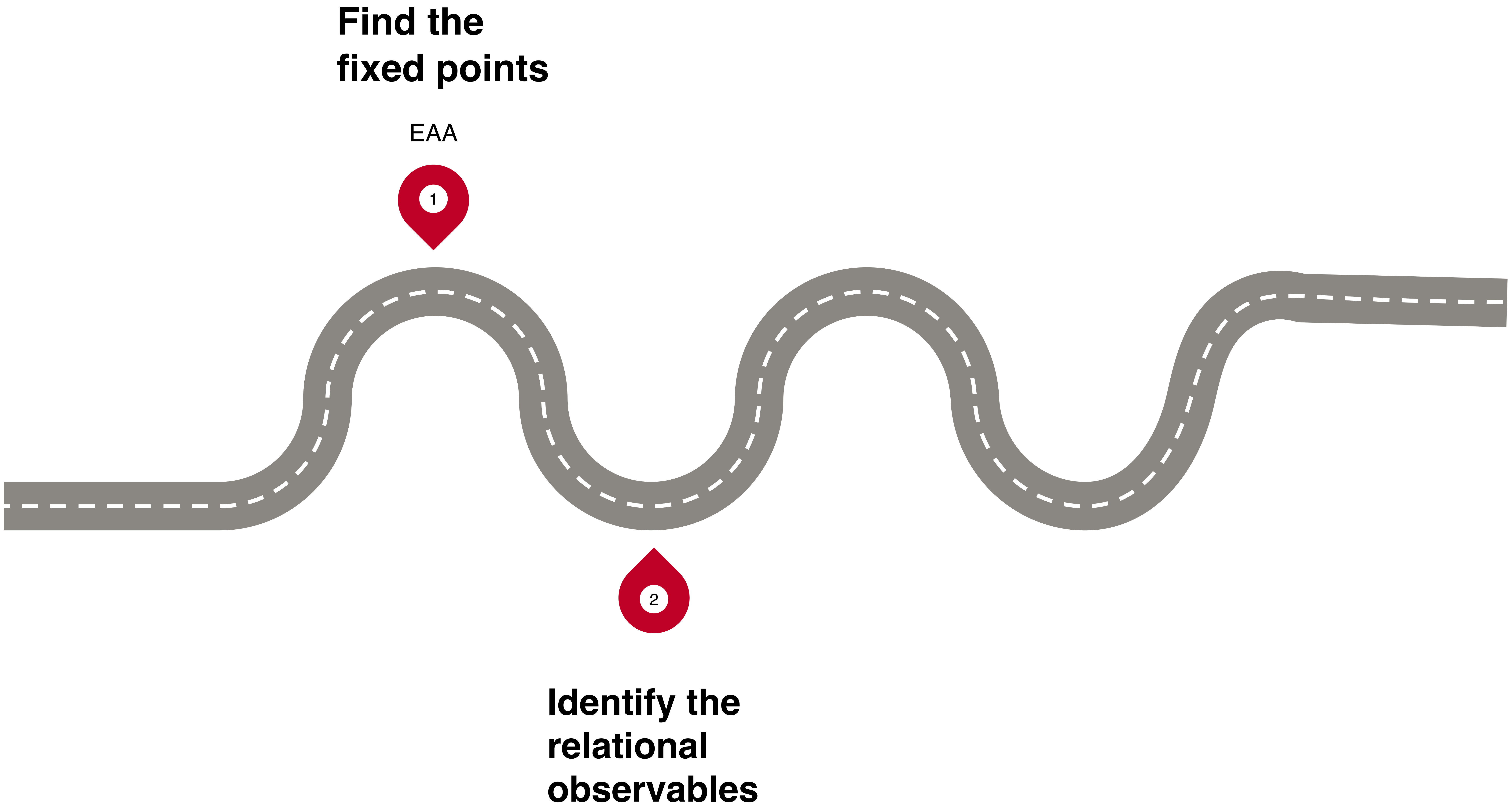
Relational observables

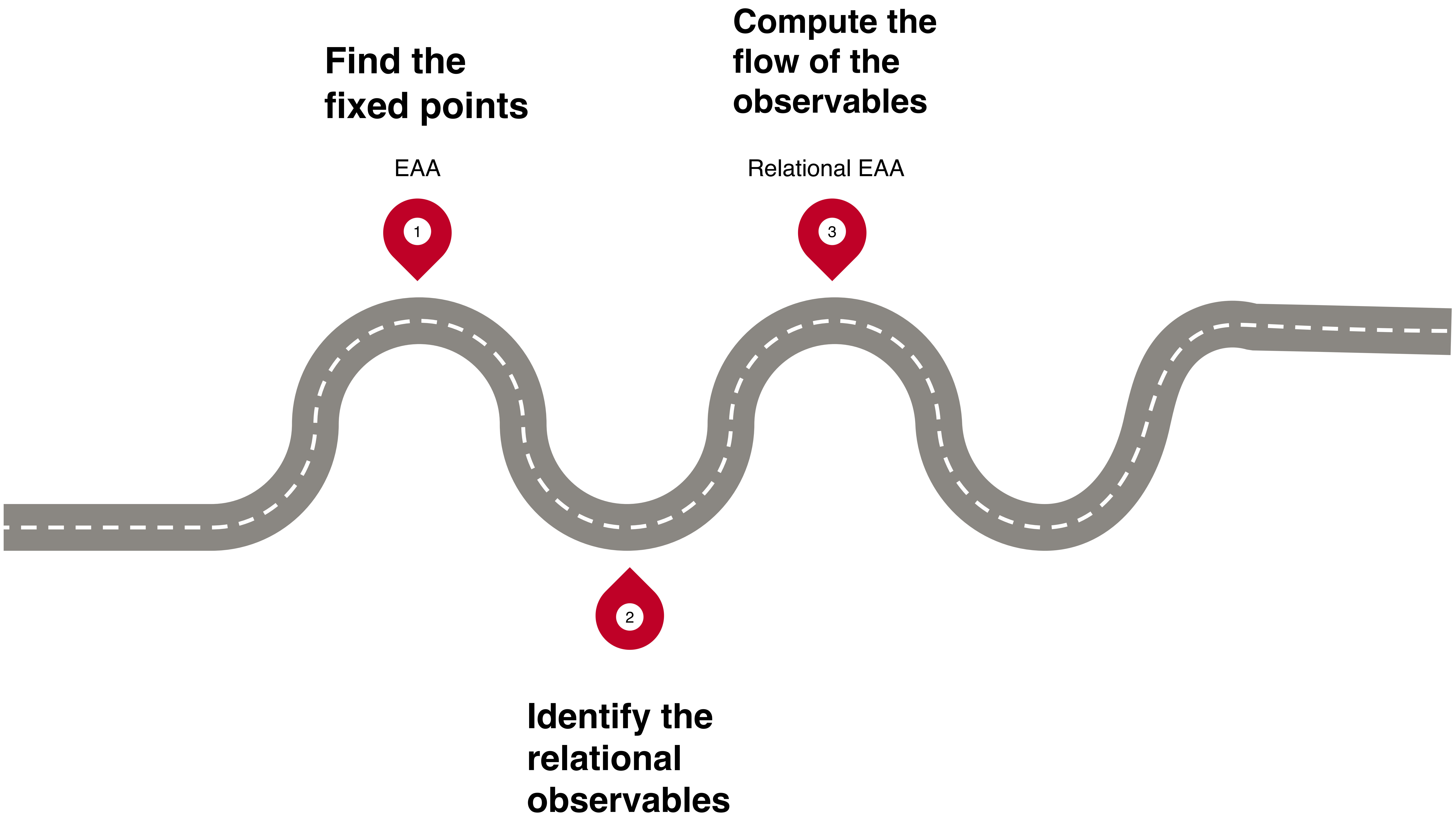
$$e_\mu^{\hat{\mu}}(x) = \partial_\mu \hat{X}^{\hat{\mu}}(x)$$

$$\tilde{e} = \det e_\mu^{\hat{\mu}}$$

Find the fixed points




Find the
fixed points
EAA
1
2
Identify the
relational
observables



Find the
fixed points
EAA
1
Compute the
flow of the
observables
Relational EAA
3
2
Identify the
relational
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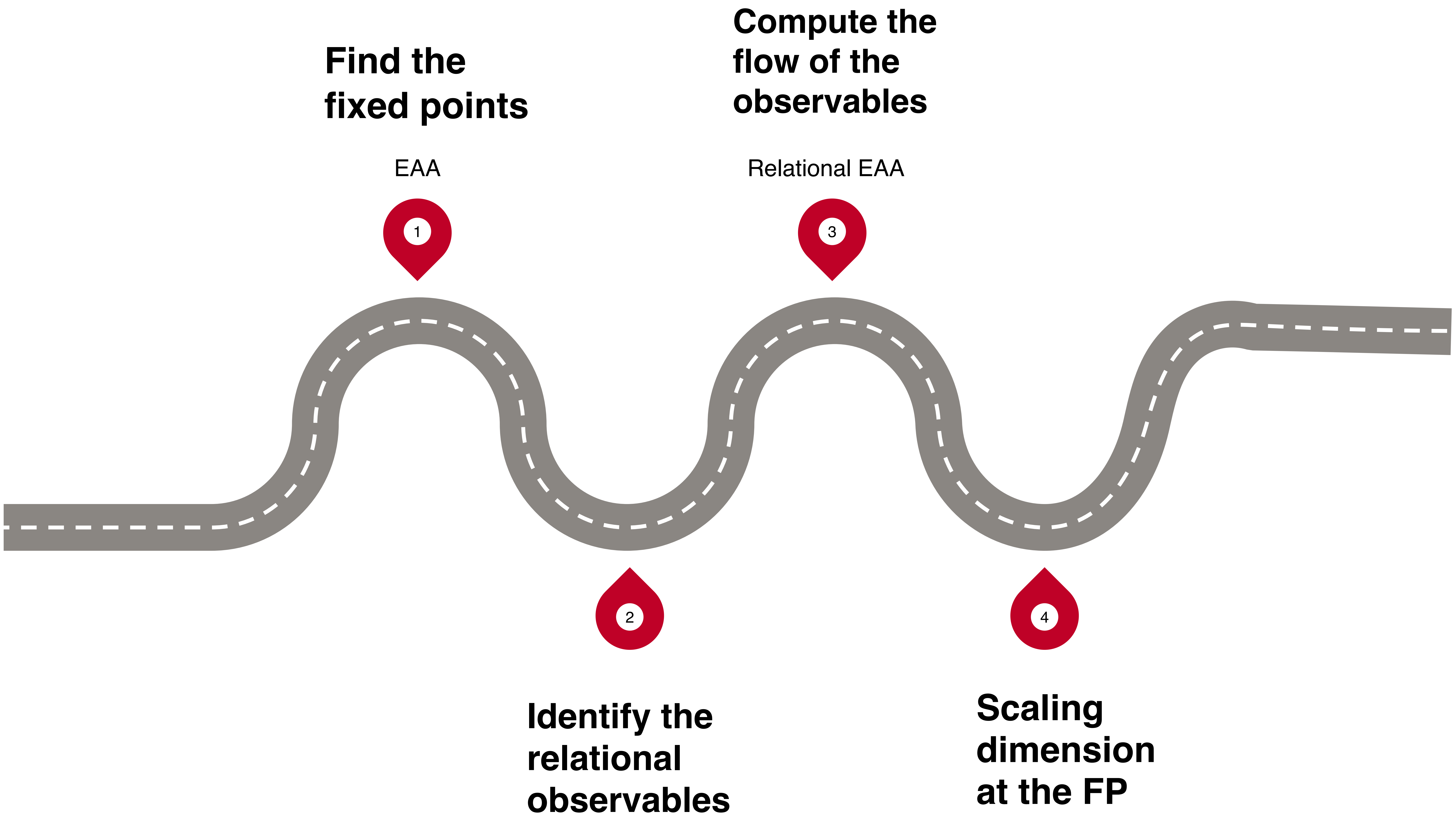
Find the fixed points
EAA
1
Identify the relational observables
2
Compute the flow of the observables
Relational EAA
3
Scaling dimension at the FP
4

Application

Find the fixed points — 1

Identify the relational observables — 2

Compute the flow of the observables — 3

Scaling dimension at the FP — 4

Application

Find the fixed points 1

$$\Gamma_k = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{16\pi G_N(k)}(R-2\Lambda(k)) + \frac{1}{2}\delta_{\mathrm{AB}}g^{\mu\nu}\partial_\mu\varphi^{\mathrm{A}}\partial_\nu\varphi^{\mathrm{B}}\right) + \dots$$

Identify the relational observables 2

Compute the flow of the observables 3

Scaling dimension at the FP 4

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Identify the relational observables 2

$$\varphi^{\hat{\mu}} = \hat{X}^{\hat{\mu}}$$

Compute the flow of the observables 3

Scaling dimension at the FP 4

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Identify the relational observables — 2

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Compute the flow of the observables — 3

$$\Gamma_k^{\mathrm{rel.}} = \int \mathrm{d}^4x\,\tilde{e}\left(\alpha_0(k) + \alpha_R(k)R + \alpha_1(k)\delta_{\hat{\mu}\hat{\nu}}g^{\mu\nu}(\partial_\mu\hat{X}^{\hat{\mu}})(\partial_\nu\hat{X}^{\hat{\nu}})\right)$$

Scaling dimension at the FP — 4

Application

Find the fixed points (1)

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Identify the relational observables (2)

$$\varphi^{\hat{\mu}} = \hat{X}^{\hat{\mu}}$$

Compute the flow of the observables (3)

$$\Gamma_k^{\text{rel.}} = \int d^4x\,\tilde{e}\left(\alpha_0(k) + \alpha_R(k)R + \alpha_1(k)\delta_{\hat{\mu}\hat{\nu}}g^{\mu\nu}(\partial_\mu\hat{X}^{\hat{\mu}})(\partial_\nu\hat{X}^{\hat{\nu}})\right)$$

Scaling dimension at the FP (4)

Matter content	θ_0	θ_R	θ_1
SM (type II)	-4	-5.97643	-7.92358
SM (type I)	-4	-5.97467	-7.8177
SM + **SF** (type II)	-4	-5.97505	-7.80603
SM + **3** ν (type II)	-4	-5.98015	-7.78084

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Small quantum corrections

Observables
as
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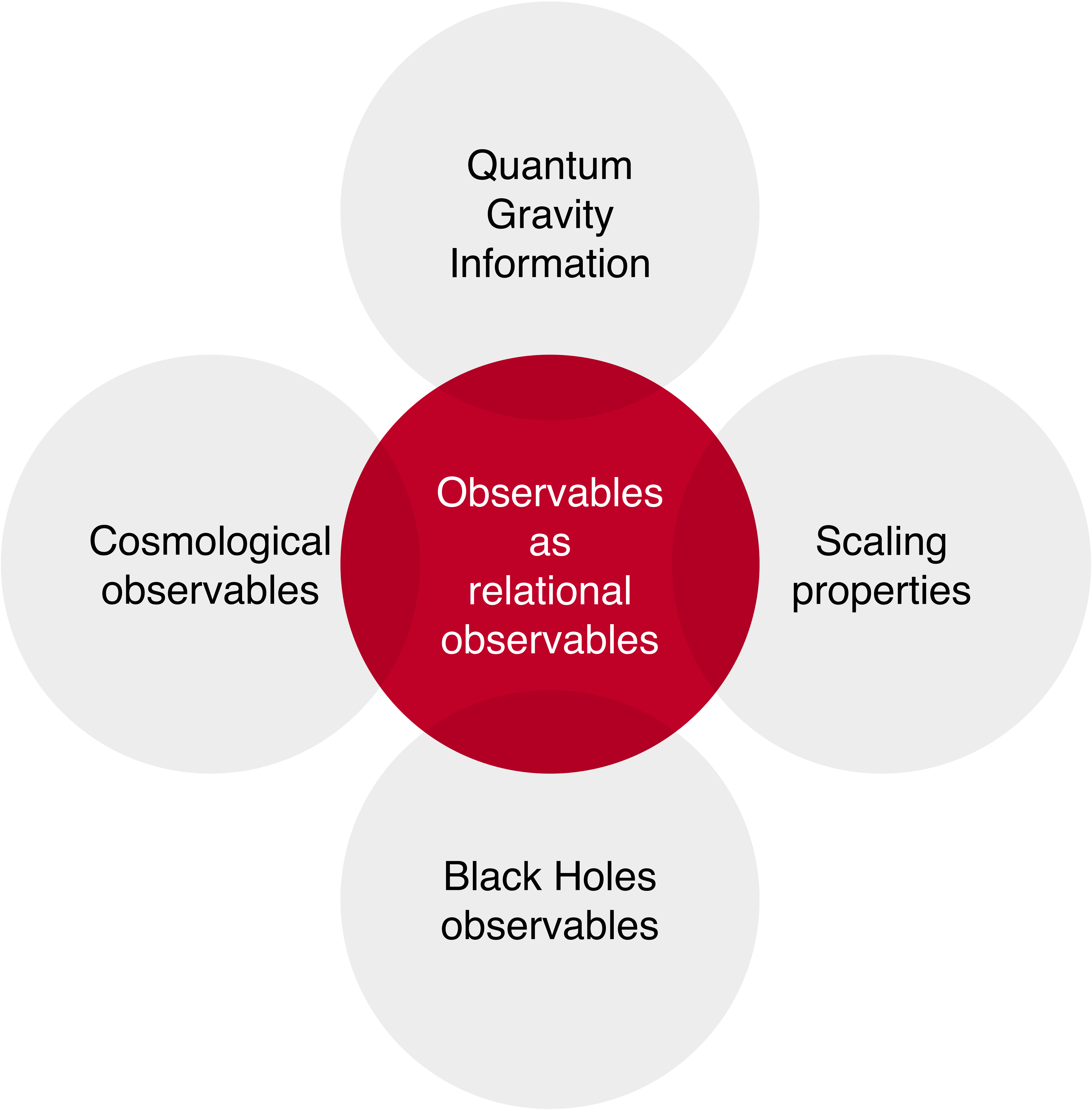
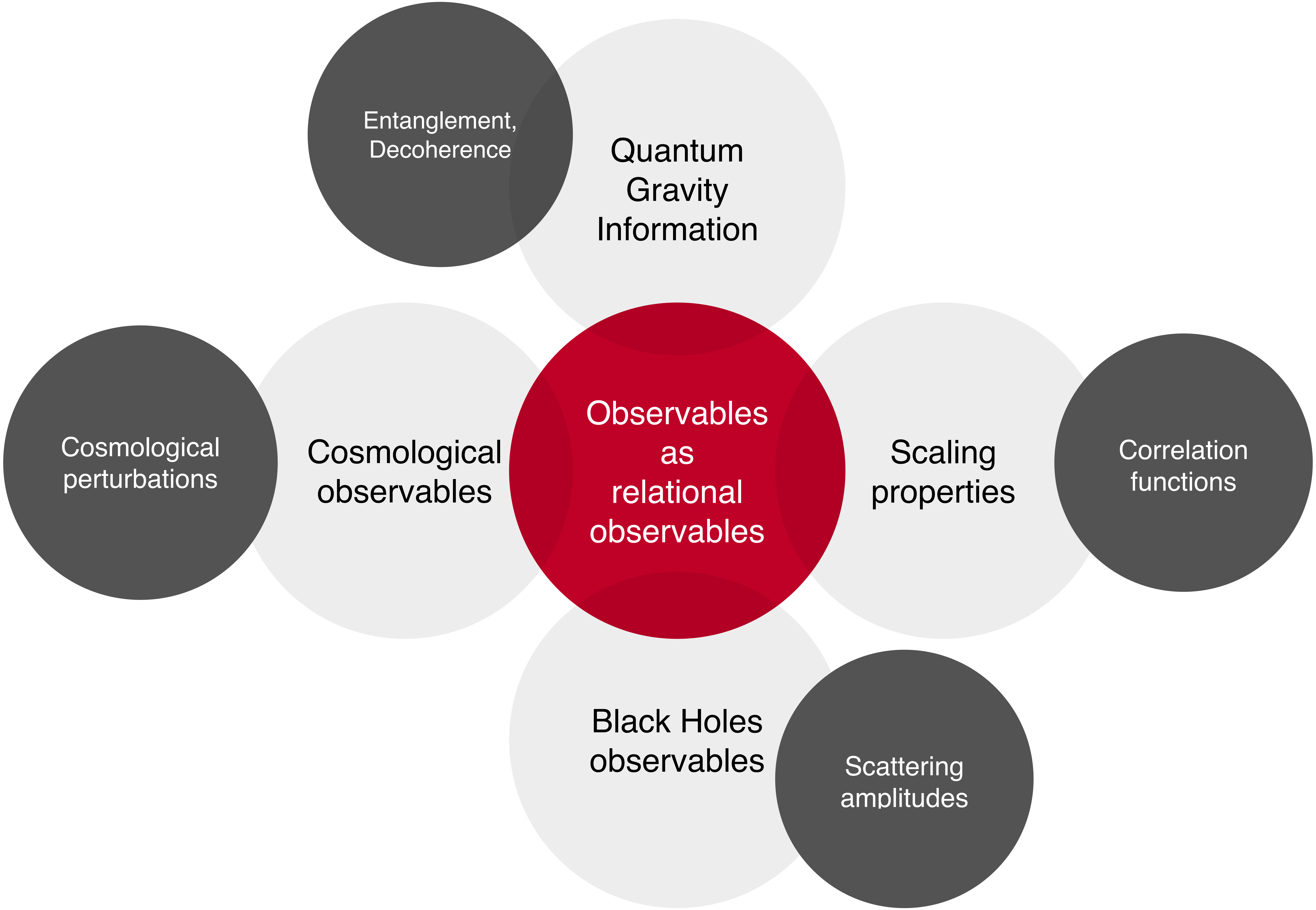

Quantum Gravity Information
Cosmological observables
Observables as relational observables
Scaling properties
Black Holes observables


Entanglement,
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Black Holes
observables
Scattering
amplitudes


Quantum Reference Systems
Canonical Formalism
Entanglement, Decoherence
Quantum Gravity Information
Lattice methods: CDT
Cosmological perturbations
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Loop Quantum Gravity
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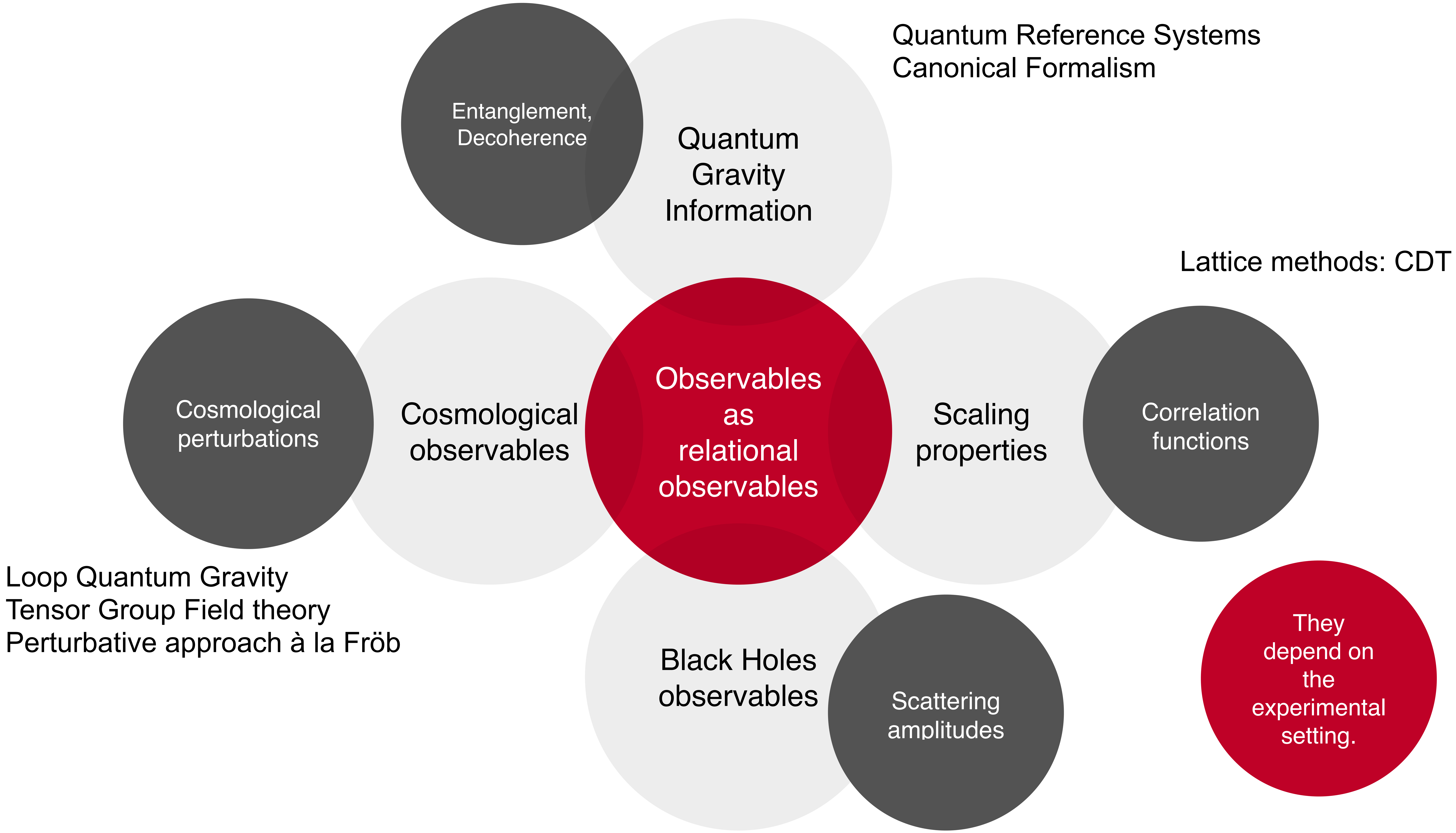
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They depend on the experimental setting.


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Running observables in AS

- Comparing in detail the consequences of the Asymptotic Safety Scenario with observations represents one of the main challenges.

- Starting from the **safe harbor of asympotically safe gravity**, we developed new methods for extracting physical contents, by constructing **relational observables.**

- Which are “good observables”? How are they constructed? Inspiration from cosmology?

- Contact with different approaches of Quantum Gravity?
  - At the level of the scaling exponents, e.g. scaling of the correlation functions
  - At the level of the RG computations

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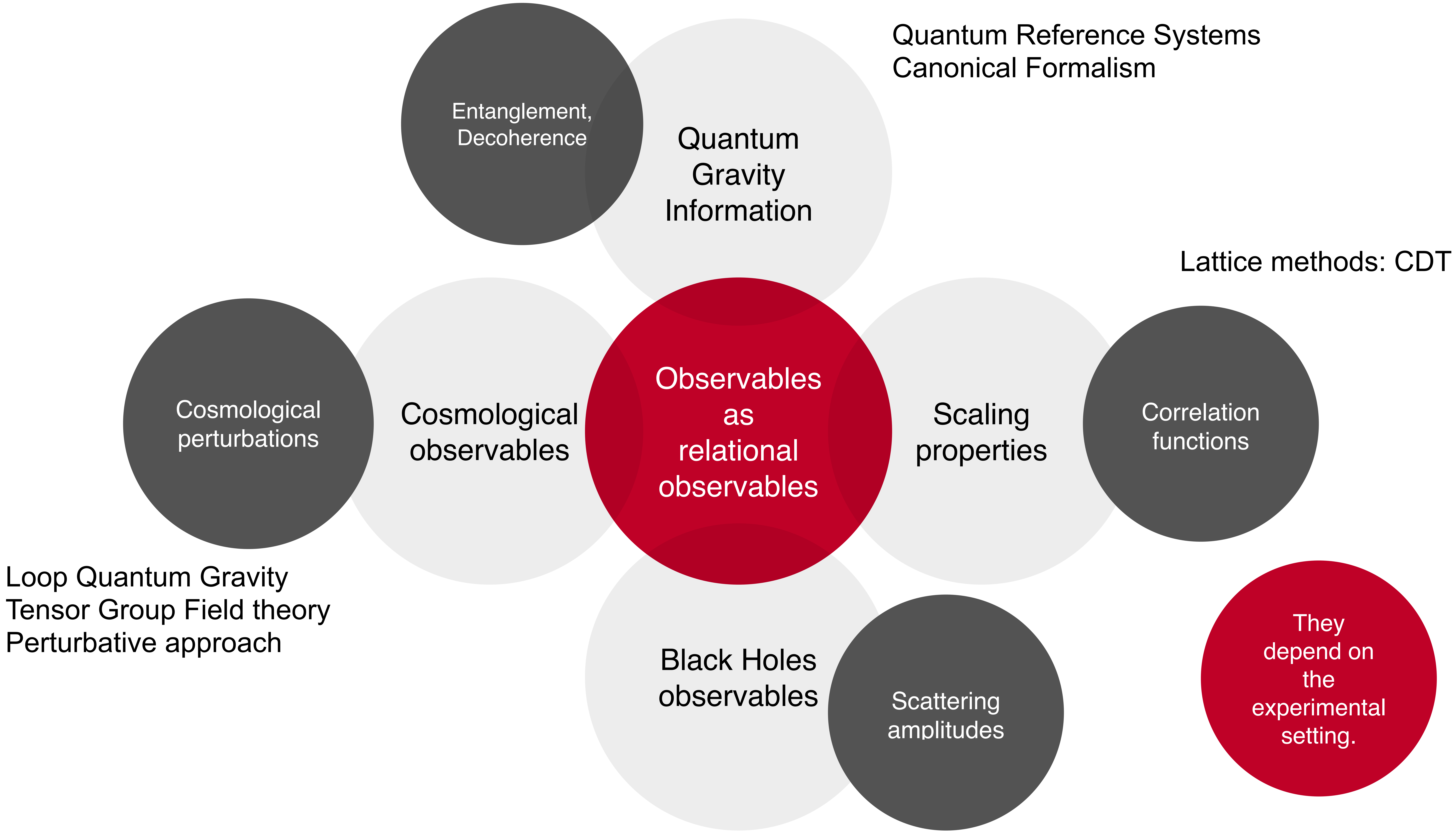
Introduce an additional scale (fixed volume) and study the scaling wrt. this scale



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