Emergence of geometry from fluctuations

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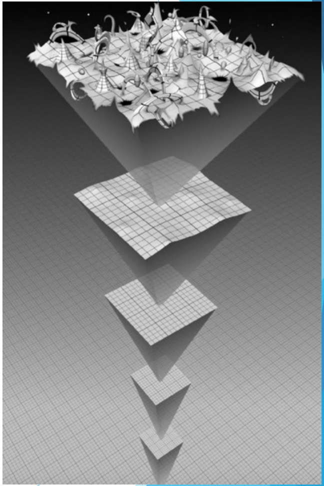
Motivation: lesson of quantum gravity - 1/1

- QG approaches agree: no manifold $\mathcal{M}$ at high energies.



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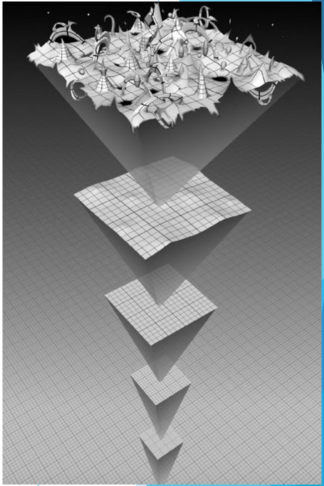
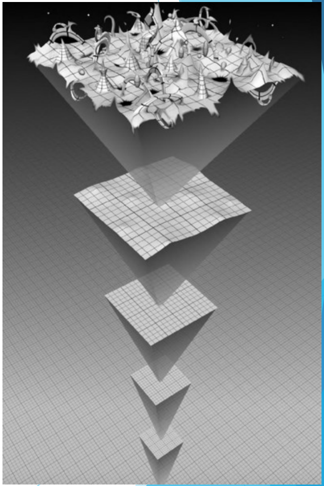


Motivation: lesson of quantum gravity - 1/1

- QG approaches agree: no manifold $\mathcal{M}$ at high energies.
- Picture of manifold with fields emerges at low energy.
- Here: we investigate a mechanism of emergence of spacetime from path-integral Z with matter/gravity action S.



Fluctuations as rods and clocks - 1/1

- Motivation to take non-geometric starting point: Hilbert space $\mathcal{H}$, bosonic ϕ and fermionic ψ matter fields and associated Laplacian Δ and Dirac D operators.

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- Motivation to take non-geometric starting point: Hilbert space $\mathcal{H}$, bosonic ϕ and fermionic ψ matter fields and associated Laplacian Δ and Dirac D operators.
- Kac's question in spectral geometry: "Can you hear the shape of a drum?"
- Is knowing Laplacian Δ through spectrum $\{\lambda_i\}$ sufficient to reconstruct $\mathcal{M}$?
- No. Yes, if interactions are present using basis where interactions are diagonal, metric $g_{\mu\nu}$ from propagator $G_F(x,y)$:

$$g_{\mu\nu}(x) = -\frac{1}{2}\left(\frac{\Gamma(d/2-1)}{4\pi^{d/2}}\right)^{\frac{2}{d-2}} \lim_{x\to y} \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\left(G_F(x,y)^{\frac{2}{2-d}}\right)$$

Kempf 2021.

An action (gravity) - 1/4

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- Makes dimension N of $\mathcal{H}$ finite with:

$$N = \frac{1}{16\pi^2} \int d^4x \sqrt{g} \left(\frac{\bar{\Lambda}^2}{2} + \frac{\bar{\Lambda}}{6} R + O(R^2) \right)$$

Gilkey 1975, Hawking 1978.

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- Gravity action $S_g = \mu N$, contains $S_{EH} + \mathcal{O}(R^2)$ when $\mu = \frac{6\pi}{\bar{\Lambda}}$.

Gilkey 1975, Hawking 1978.

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- N_b, N_f copies of $\{\phi_n^{(i)}\}$, Grassmann components $\theta_n^i, \bar{\theta}_n^i$ of ψ and eigenvalues $\{\lambda_n\}$, of $(\Delta + m^2)$ and D, all mass m.
- Note: λ_n of $(\Delta + m^2)$ is related to $\sqrt{\lambda_n}$ of D for simplicity.

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- Next, path-integral Z with action S
- Standard matter path-integral, Gravitational path-integral: sum over dimension N, integral over spectra $\{\lambda_i\}$ with UV cut-off Λ.

The path-integral - 4/4

- Combined gravity/matter path-integral:

$$Z = \sum_{N=1}^{\infty} \int_{m^2}^{\Lambda} \mathcal{D}\lambda \int \mathcal{D}\phi \int \mathcal{D}\theta \mathcal{D}\bar{\theta} e^{-\beta S} \frac{\Lambda^{N(\frac{N_f}{2}-1)}}{(N-1)!}$$

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- Choice: with or without zero-mode, here with, to investigate spectral gap.
- Note: Z is finite.

Effective dimension - 1/6

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- Note, $d_{eff} > 0$ only if $N_f > N_b - 2$.

Effective dimension - 2/6

- Scale-dependent effective dimension from varying masses:
- $$p(\lambda) \propto \prod_{k}^{N_b} \sqrt{(\lambda + m_{b_k}^2)} \prod_{l}^{N_f} (\sqrt{\lambda} + m_{f_k})$$

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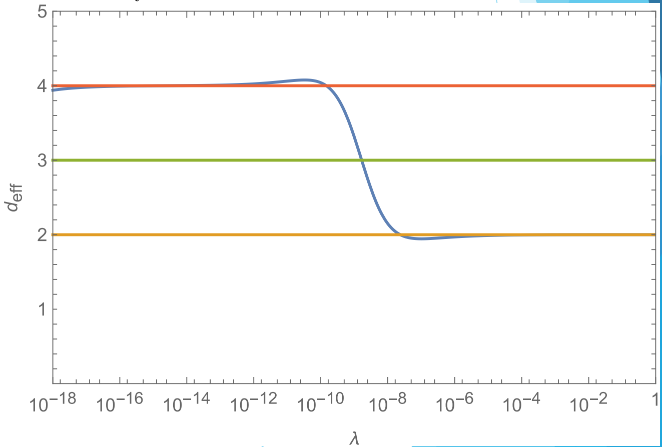
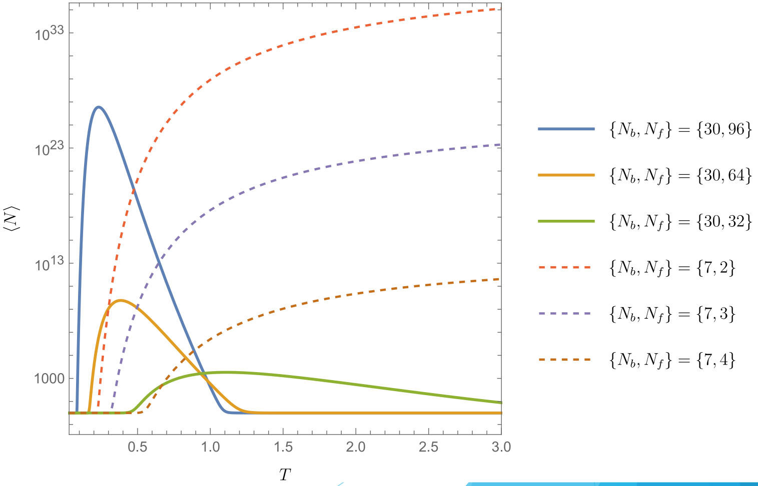
- $$p(\lambda) \propto \prod_{k}^{N_b} \sqrt{(\lambda + m_{b_k}^2)} \prod_{l}^{N_f} (\sqrt{\lambda} + m_{f_k})$$
- Typical dimensional reduction in quantum gravity.



Density of degrees of freedom - 3/6

- Calculate expectation value $\langle N \rangle$:

$$\langle N \rangle = \frac{-Z^{-1}}{\beta} \frac{\partial Z}{\partial \mu}$$



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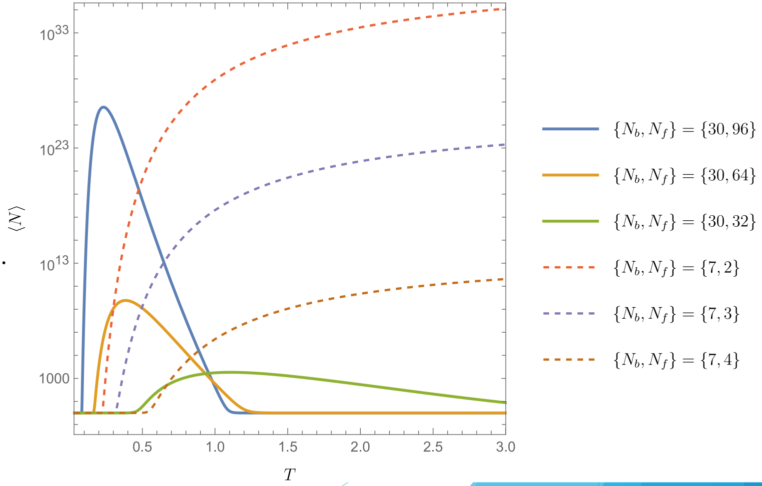
- Fermions dominant:
  $\rightarrow d_{eff} > 0$, $\langle N \rangle$ bounded.
  geometric.

  Bosons dominant:
  $\rightarrow d_{eff} < 0$, $\langle N \rangle$ unbounded.
  non-geometric.

  physical picture:
  Fermions span effective manifold, but constrain dimension of $\mathcal{H}$.



Effective volume - 4/6

- Estimate effective volume V_{eff} from expected gap $\langle\lambda_2\rangle$ from relation between diameter ℓ and λ_2 on a manifold:

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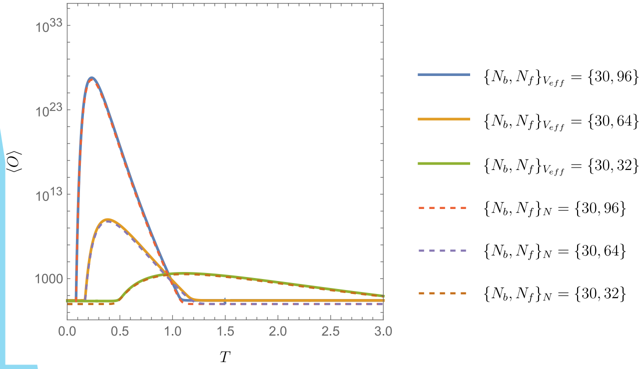
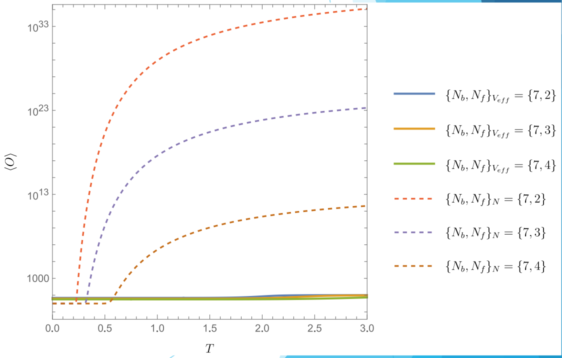
- So we define the effective volume V_{eff},

$$\langle \lambda_2 \rangle^{-d_{eff}/2} \equiv V_{eff}$$

Effective density of degrees of freedom - 5/6

- Compare $\langle O \rangle = \langle N \rangle$ and $\langle O \rangle = \langle \lambda_2 \rangle^{-d_{eff}/2} \equiv V_{eff}$.





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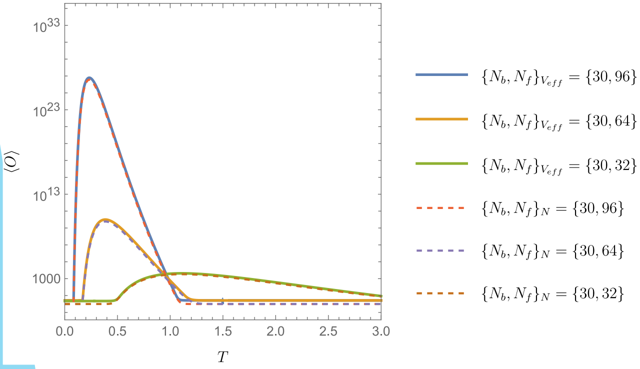
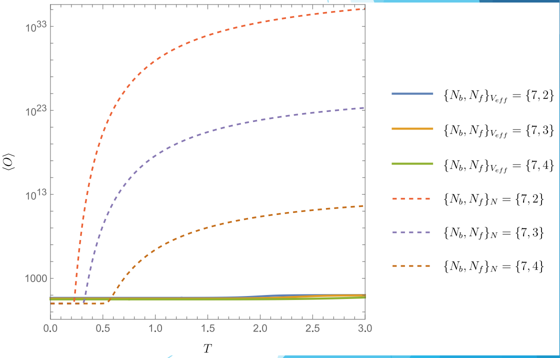
- $\langle N \rangle$ and V_{eff} match closely, ratio ~ 1.





Effective density of degrees of freedom - 5/6

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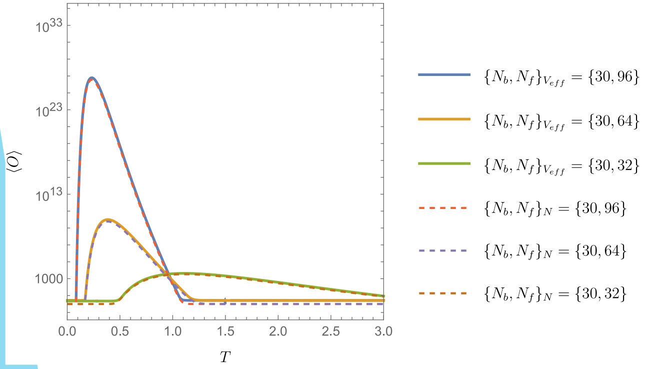
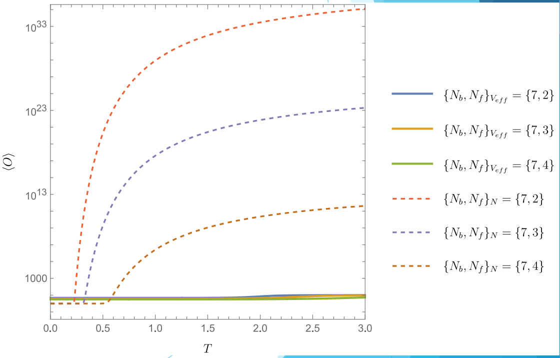
- $\langle N\rangle$ and V_{eff} match closely, ratio ~ 1.
- No agreement between $\langle N\rangle$ and V_{eff} when bosons dominate.





Effective density of degrees of freedom - 6/6

- Remember Hawking-Gilkey:

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- Remarkable consistent geometric picture when fermions dominate without initial geometric structure.

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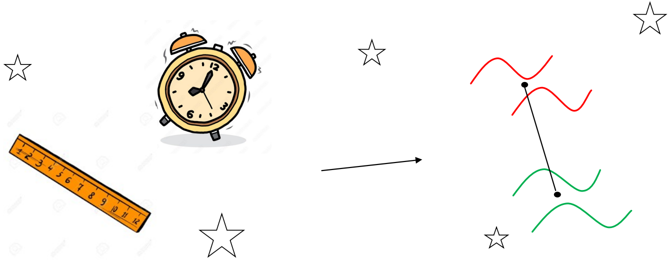
- New mechanism by which spacetime with matter emerges from a pre-geometric model, no modding out of diff. group.



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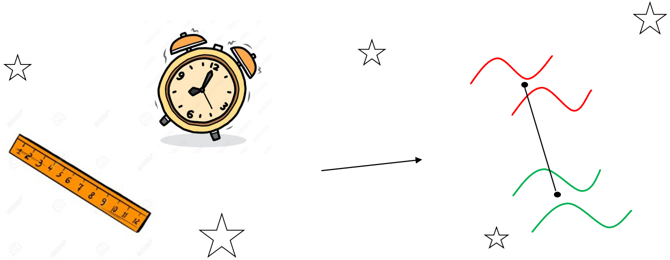

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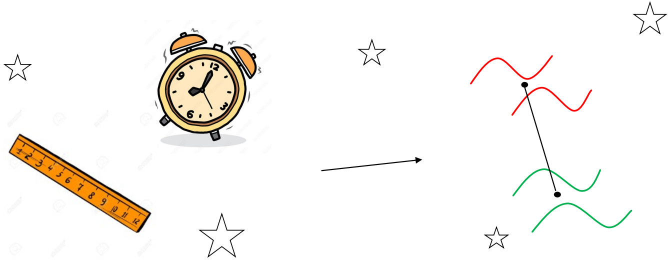

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- Future: work in Lorentzian signature, include interactions, varying masses, decouple eigenvalues Δ and D, etc.