

# Scalar cosmological perturbations from full quantum gravity

In collaboration with: D. Oriti, E. Wilson-Ewing, A. Pithis, A. Jercher, P. Fischer

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Radboud University

14 July 2023

Department of Mathematics and Statistics

UNB Fredericton

**Microscopic  
description**

Background independent,  
pre-geometric

**Macroscopic  
description**

**Continuum  
physics**

**Localization  
problem**

**Microscopic  
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**Continuum limit  
problem**

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**Relationality**

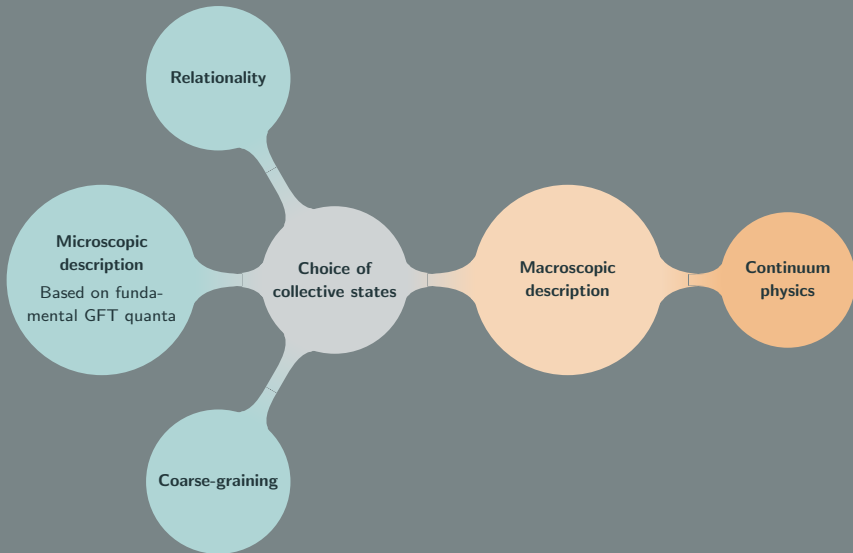
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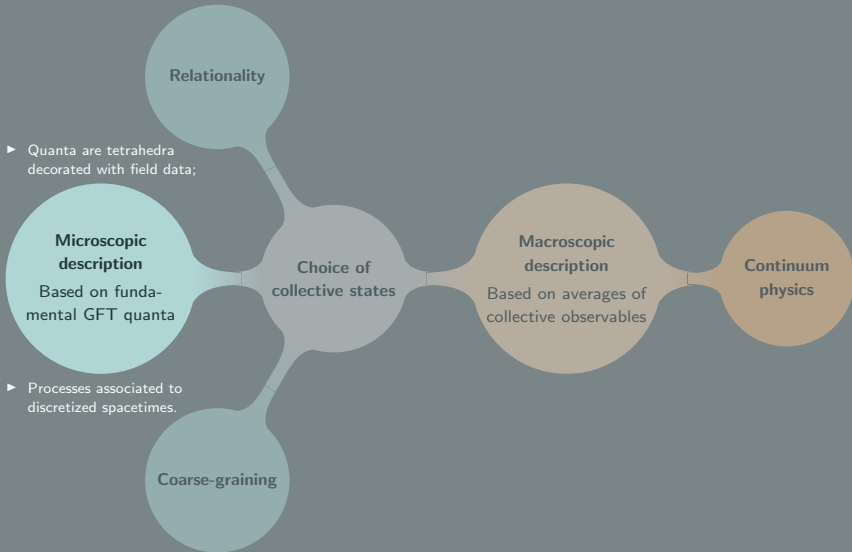
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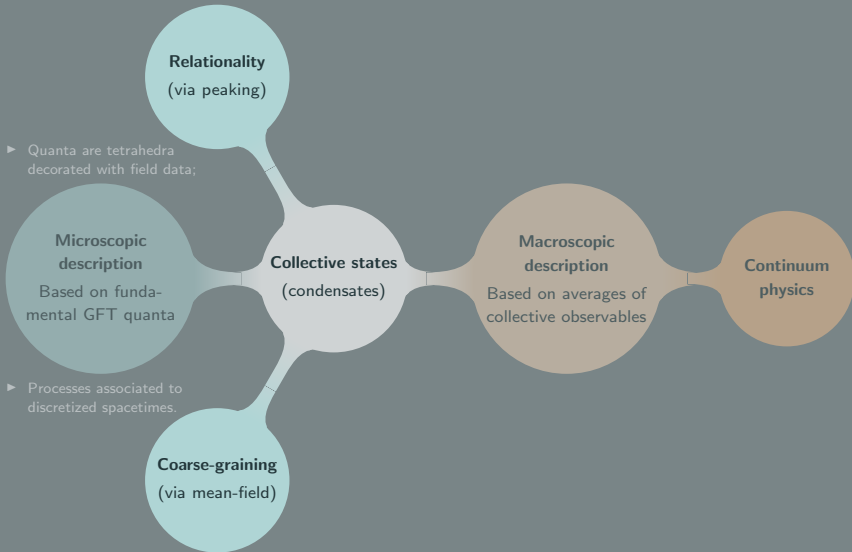
**Coarse-graining**

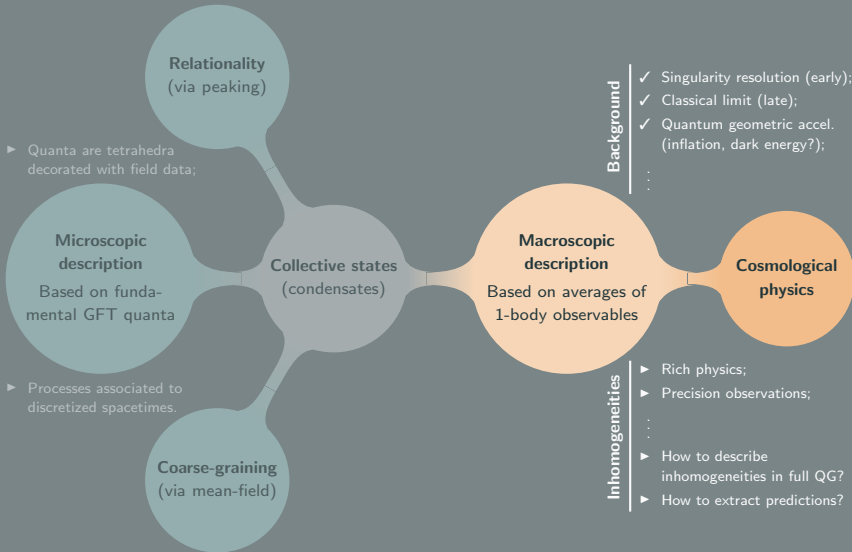
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# Inhomogeneities in GFT cosmology

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# Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

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## Classical

- ▶ 4 MCMF **reference** fields  $(\chi^0, \chi^i)$ ,
- ▶ 1 MCMF **matter** field  $\phi$  dominating the e.m. budget and **relationally inhomog.** wrt.  $\chi^i$ .

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## Quantum

- ▶  $\hat{\varphi}(\mathbf{g}_a, \chi^\mu, \phi)$  depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- ▶  $S_{\text{GFT}}$  from EPRL-like model.

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notation:  $(\cdot, \cdot) = \int d^4\chi d\phi d\mathbf{g}_a$

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### States

- ▶ Peaked  $|\sigma\rangle_x$  around  $\chi^\mu = x^\mu$ , with  $\sigma = \eta \times \tilde{\sigma}$ :
  - $\eta$ : **Isotropic** peaking on rods;
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- ▶ Small relational  $\tilde{\sigma}$ -inhomogeneities ( $\tilde{\sigma} = \rho e^{i\theta}$ ):  
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- ▶  $\langle \delta S / \delta \hat{\varphi} \rangle_{\sigma_x} = 0$  (no interactions)  $\longrightarrow$  coupled eqs. for  $(\rho, \theta)$ .
- ▶ Decoupling for a range of values of  $|\sigma\rangle_x$  and large  $N$  (**late times**).

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- ▶ **Unphysical behavior** of spatial derivative terms.

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Mat. Vol. Frame

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## Perturbing background dynamics

- ▶ Study deep super-horizon scalar pert. by perturbing background QG volume equation.

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**No matching** at early times with full effective GFT volume dynamics: **different d.o.f.!**

# Scalar perturbations from quantum correlations

## Two-body correlations

Including timelike tetrahedra: better coupling of the physical frame. **Two-sector** (+, -) GFT (BC)!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \hat{\delta}\Phi \otimes \mathbb{I}_- + \hat{\delta}\Psi + \mathbb{I}_+ \otimes \hat{\delta}\Xi) |0\rangle$$

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- ▶ 2 mean-field eqs. for 3 variables ( $\delta\Phi, \delta\Psi, \delta\Xi$ ):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ Late times and single (spacelike) rep. label.

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$$\begin{aligned} \delta V_\psi &\propto \text{Re}(\delta\Psi, \tilde{\sigma}\tilde{\tau}) + \text{Re}(\delta\Phi, \tilde{\sigma}^2) \\ \delta\phi_\psi &= \bar{\phi}_\psi (\delta V_\psi / \bar{V}_\psi) \end{aligned}$$

- ▶ Physical behavior of spatial derivative terms fixes dynamical freedom (e.g. in  $\delta\Phi$ ).

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- ▶ 2 mean-field eqs. for 3 variables ( $\delta\Phi, \delta\Psi, \delta\Xi$ ):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ Late times and single (spacelike) rep. label.

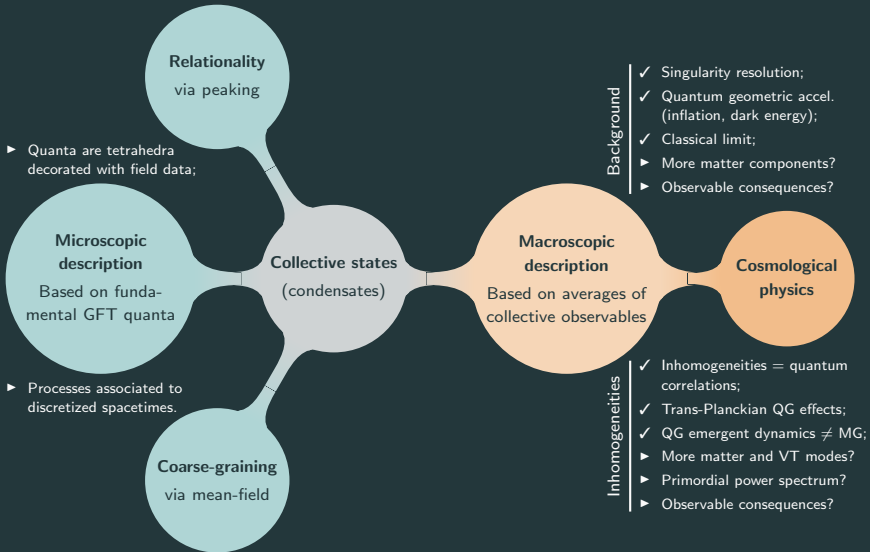
$$\delta V_\psi \propto \text{Re}(\delta\Psi, \tilde{\sigma}\tilde{\tau}) + \text{Re}(\delta\Phi, \tilde{\sigma}^2)$$

$$\delta\phi_\psi = \bar{\phi}_\psi (\delta V_\psi / \bar{V}_\psi)$$

- ▶ Physical behavior of spatial derivative terms fixes dynamical freedom (e.g. in  $\delta\Phi$ ).

### Late times scalar isotropic perturbations

- ▶ QG corrections at (relationally) trans-Planckian scales.
- ▶ GR matching at larger scales.



# Backup

---

# Group Field Theory and spinfoam models

## Definition

**Group Field Theories:** theories of a field  $\varphi : G^d \rightarrow \mathbb{C}$  defined on  $d$  copies of a group manifold  $G$ .

$d$  is the dimension of the “spacetime to be” ( $d = 4$ ) and  $G$  is the local gauge group of gravity,  $G = \text{SL}(2, \mathbb{C})$  or, in some cases,  $G = \text{SU}(2)$ .

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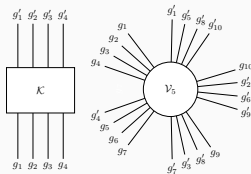
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Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] + \text{c.c.}$$

- ▶ Interaction terms are **combinatorially non-local**.
- ▶ Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph  $\gamma$ :

$$\text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}).$$





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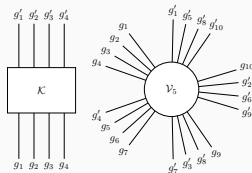
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Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma}$$

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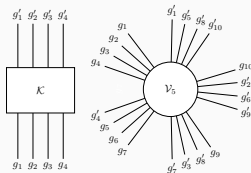
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- $\mathcal{K}$  and  $\mathcal{V}_{\gamma}$  chosen to match the desired spinfoam model.

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---

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# Group Field Theory and Loop Quantum Gravity

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### Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

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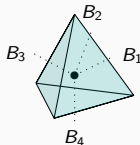
$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

#### Simplicity

$$\blacktriangleright X \cdot (B - \gamma \star B)_a = 0 \text{ (EPRL);}$$

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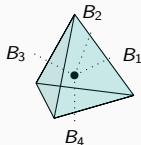
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Non-comm.

FT

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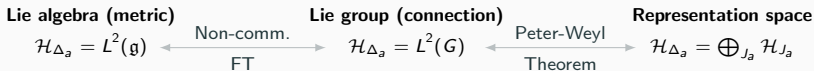
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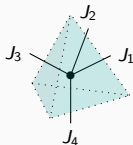
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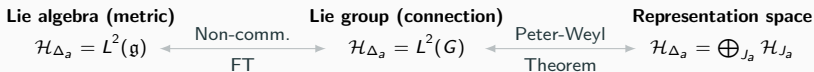
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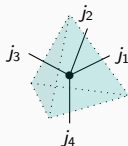
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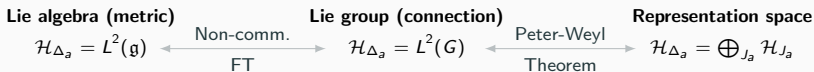
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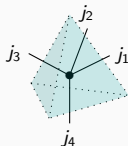
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$$\mathcal{H}_{\text{tetra}} = \bigoplus_{\vec{j}} \text{Inv} \left[ \bigotimes_{a=1}^4 \mathcal{H}_{j_a} \right]$$

= open spin-network vertex space

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**Tetrahedron wavefunction**

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- ▶  $\mathcal{F}_{\text{GFT}}$  generated by action of  $\hat{\varphi}^\dagger(g_a)$  on  $|0\rangle$ , with  $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$ .
- ▶  $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$ ,  $\mathcal{H}_\Gamma$  space of states associated to connected simplicial complexes  $\Gamma$ .
- ▶ Generic states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.
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Operators

**Volume operator**  $\hat{V} = \int dg_a^{(1)} dg_a^{(2)} V(g_a^{(1)}, g_a^{(2)}) \hat{\varphi}^\dagger(g_a^{(1)}) \hat{\varphi}(g_a^{(2)}) = \sum_{j_a, m_a, \ell} V_{j_a, \ell} \hat{\varphi}_{j_a, m_a, \ell}^\dagger \hat{\varphi}_{j_a, m_a, \ell}$

- ▶ Generic second quantization prescription to build a  $m + n$ -body operator: sandwich matrix elements between spin-network states between  $m$  powers of  $\hat{\varphi}^\dagger$  and  $n$  powers of  $\hat{\varphi}$ .

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## Relationality

► Averaged evolution wrt  $x^0$  is physical:

$$\text{Intensive} \longleftarrow \langle \hat{X}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian  $H_{\sigma_{x^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}}$ .

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Wavefunction  
 $\xrightarrow{\text{isotropy}}$

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$$\langle \hat{N} \rangle_{\sigma_{x^0}} = \sum_v |\tilde{\sigma}_v|^2(x^0)$$

►  $v = j \in \mathbb{N}/2$  (EPRL);

►  $v = \rho \in \mathbb{R}$  (ext. BC).

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## GFT condensates

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- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[ \int d^d l \chi \int d g_a \sigma(g_a, \chi^\alpha) \hat{\varphi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle .$$

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- ▶ Relational localization implemented at an **effective** level on observable **averages** on condensates.
- ▶ If  $\chi^\mu$  constitute a physical reference frame, this can be achieved by assuming

$$\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$$

# Effective relational homogeneous volume dynamics

## (Relational) Homogeneity

- ▶  $\sigma$  depends on a single clock MCFM field  $\chi^0$ .
- ▶  $\mathcal{D} = \text{minisuperspace} + \text{clock}$ .

Volume operator captures the relevant physics:

## Isotropy

- ▶  $\sigma$  depends only on a single rep. label  $v$ .
- ▶  $v \in \mathbb{N}/2$  (EPRL-like) or  $v \in \mathbb{R}$  (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \sum_v^f V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

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$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2 \sum_v V_v \rho_v \operatorname{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3 \sum_v V_v \rho_v^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2 \sum_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\sum_v V_v \rho_v^2}$$



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### Classical limit (large $N$ , late times)

- ▶ If  $\mu_v^2$  is mildly dependent on  $v$  (or one  $v$  is dominating) and equal to  $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- ▶ **Quantum fluctuations** on clock and geometric variables are **under control**.

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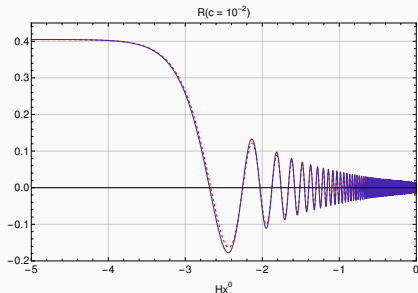
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### Bounce

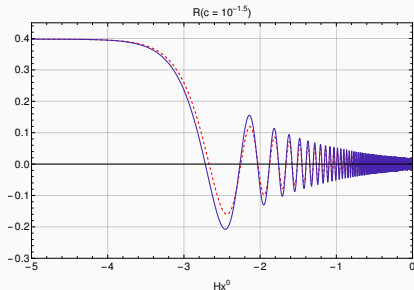
- ▶ A **non-zero volume bounce** happens for a large range of initial conditions (at least one  $Q_v \neq 0$  or one  $\mathcal{E}_v < 0$ ).
- ▶ The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

# Dynamics of a comoving curvature-like variable

$$\tilde{\mathcal{R}} \equiv -\frac{\delta V}{3\bar{V}} + \mathcal{H} \frac{\delta\phi}{\bar{\phi}'}, \quad \tilde{\mathcal{R}}'' - a^4 \nabla^2 \tilde{\mathcal{R}} = - \left[ \mathcal{H} - \frac{1}{4} (\bar{\phi}^2)' \right] \left( \frac{\delta V}{\bar{V}} \right)'.$$



- ▶  $c =$  ratio between volume and matter pert. initial conditions.



- ▶ Deep sub-horizon modes are trans-Planckian in the physical reference frame.

As  $c$  increases, QG dynamics (blue line) differs more and more from the GR one (red dotted line) at (relational) trans-Planckian scales.