

# Vacuum energy, Casimir effect, and Newton's non-constant

B. Koch

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# Content

- What we know & what we would like to know
- Hypothesis 1 & 2
- Scale-dependent (SD) framework
- SD-Casimir
- Towards experiment?
- Discussion and Conclusion



# Vacuum energy

What we know so far?



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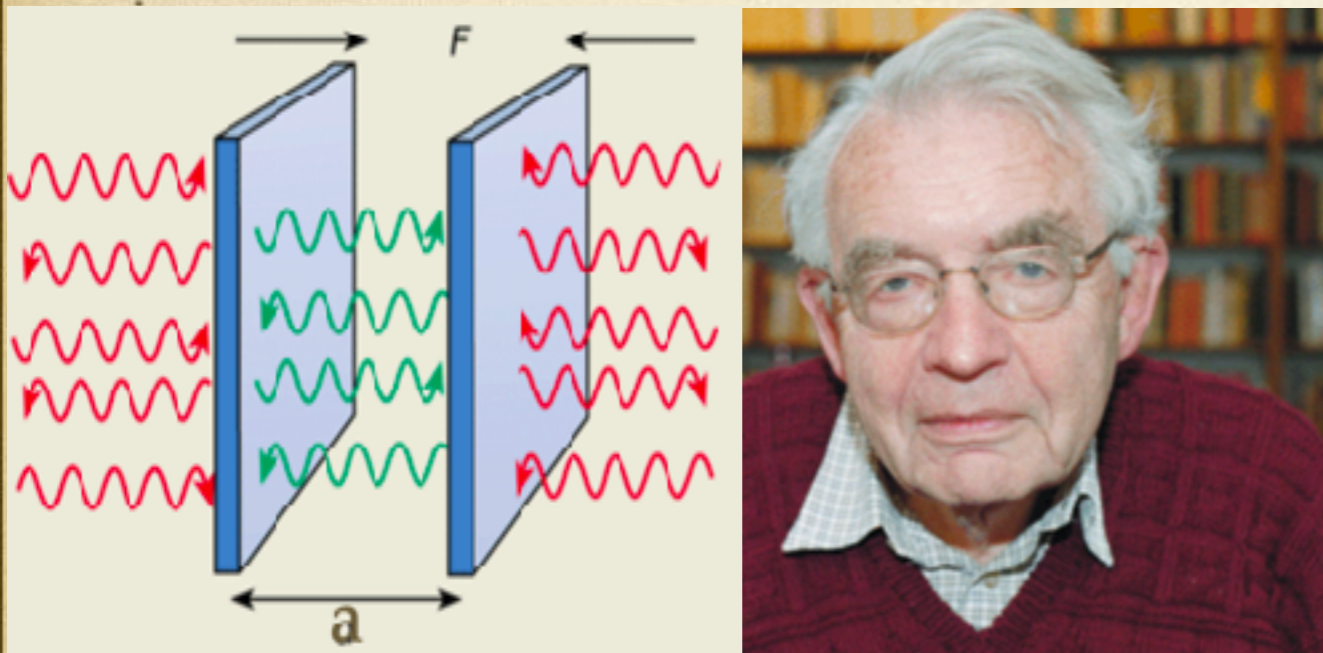
Quantum vacuum:

$$\rho_Q$$



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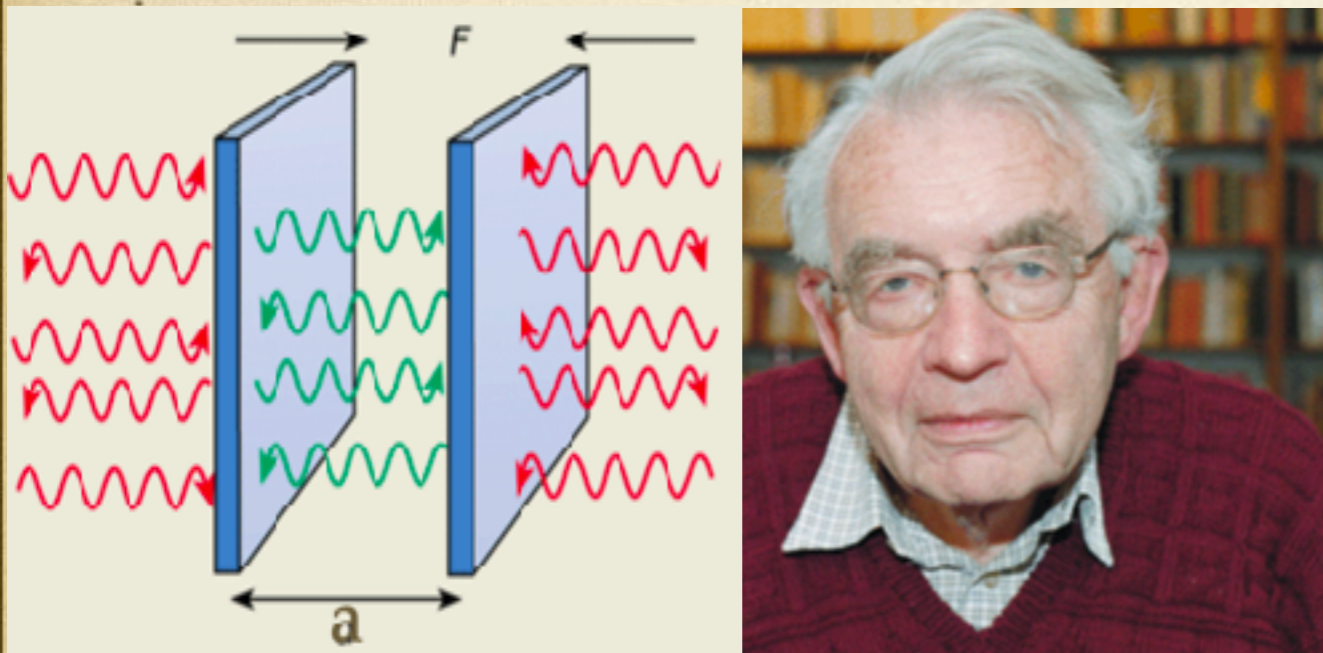
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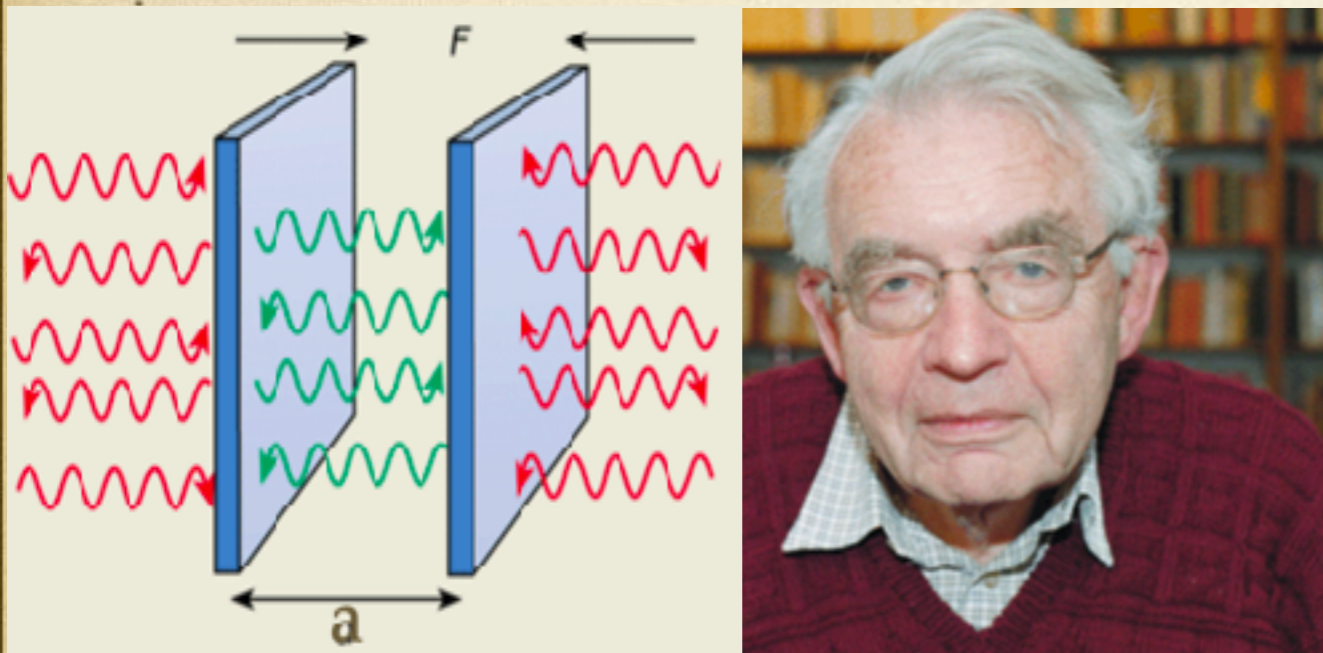
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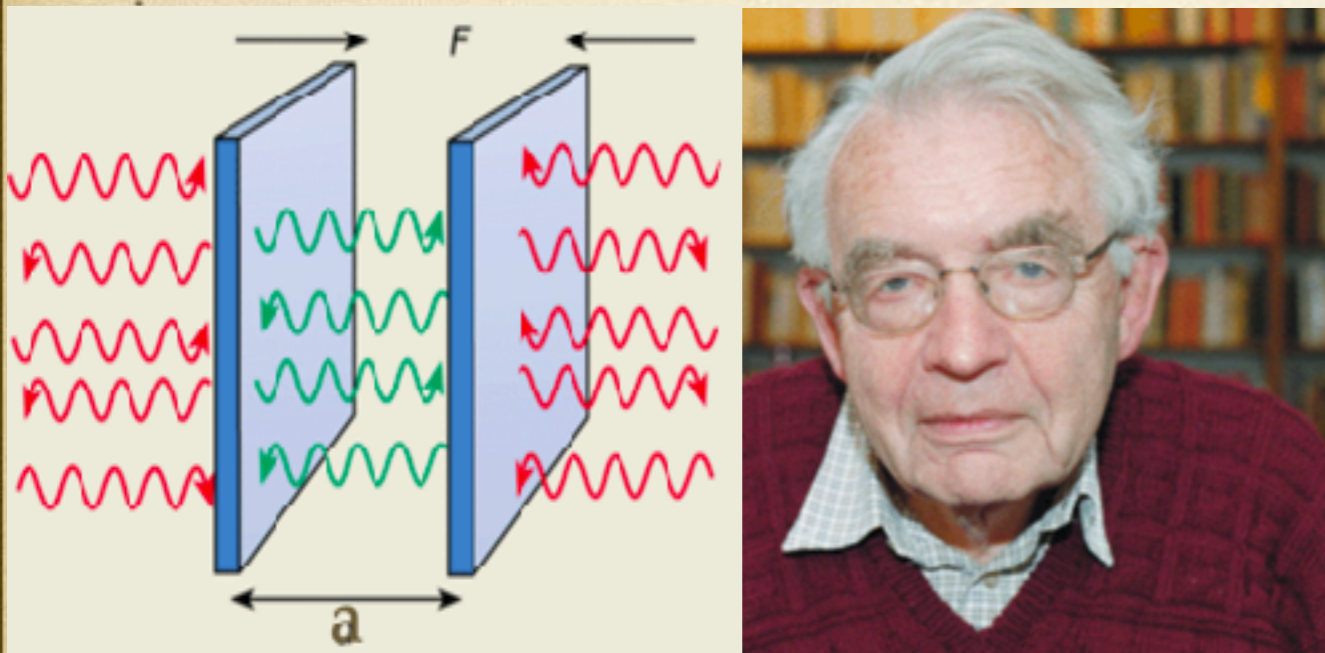
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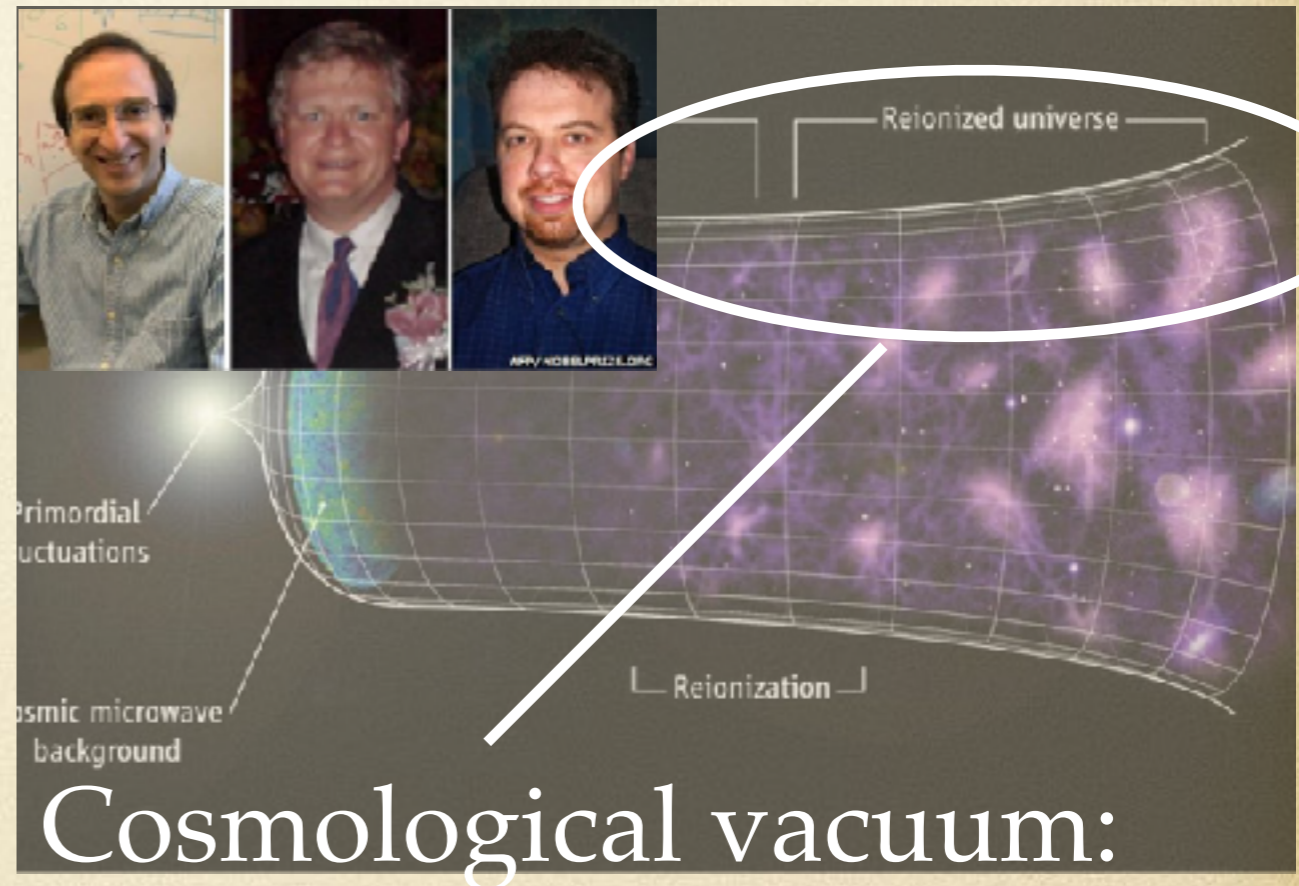


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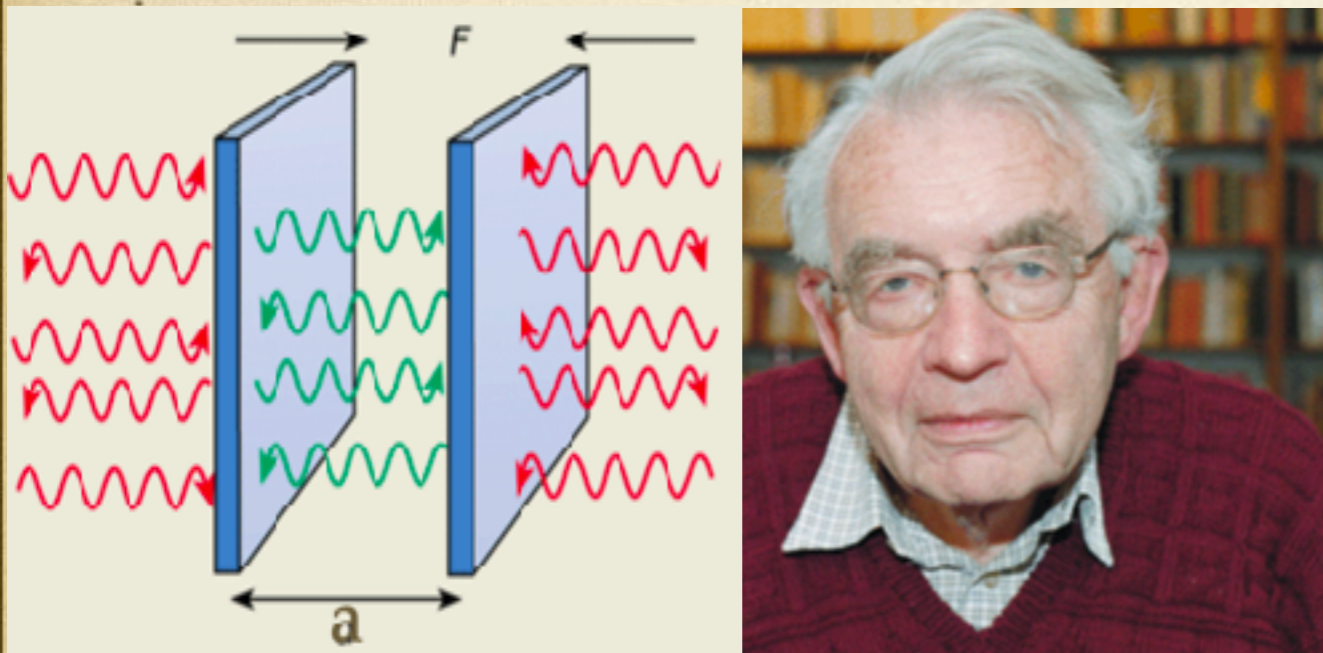
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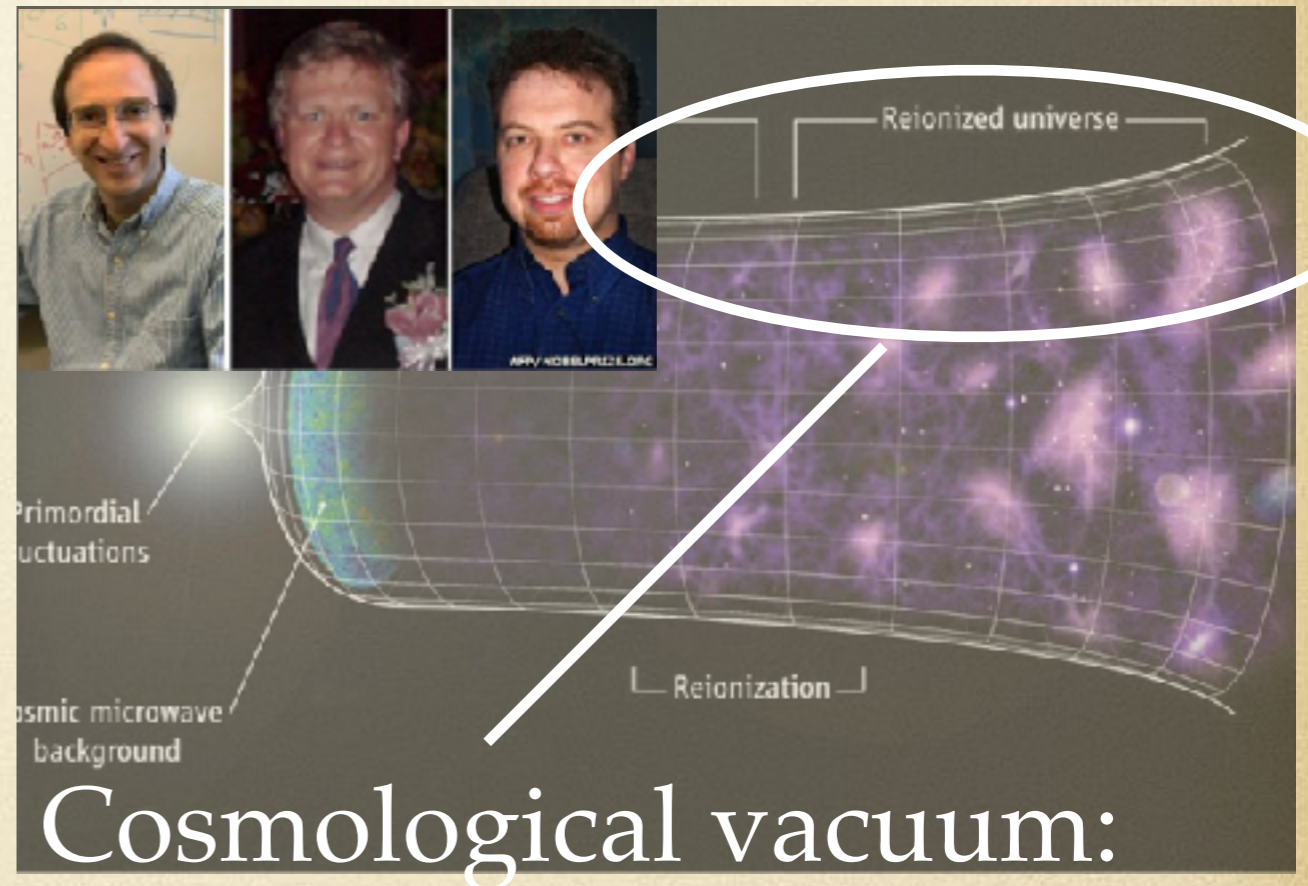


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Quantum vacuum:

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What we would like to know:

How are they related?

 $\rho_Q$  $\rho_\Lambda$



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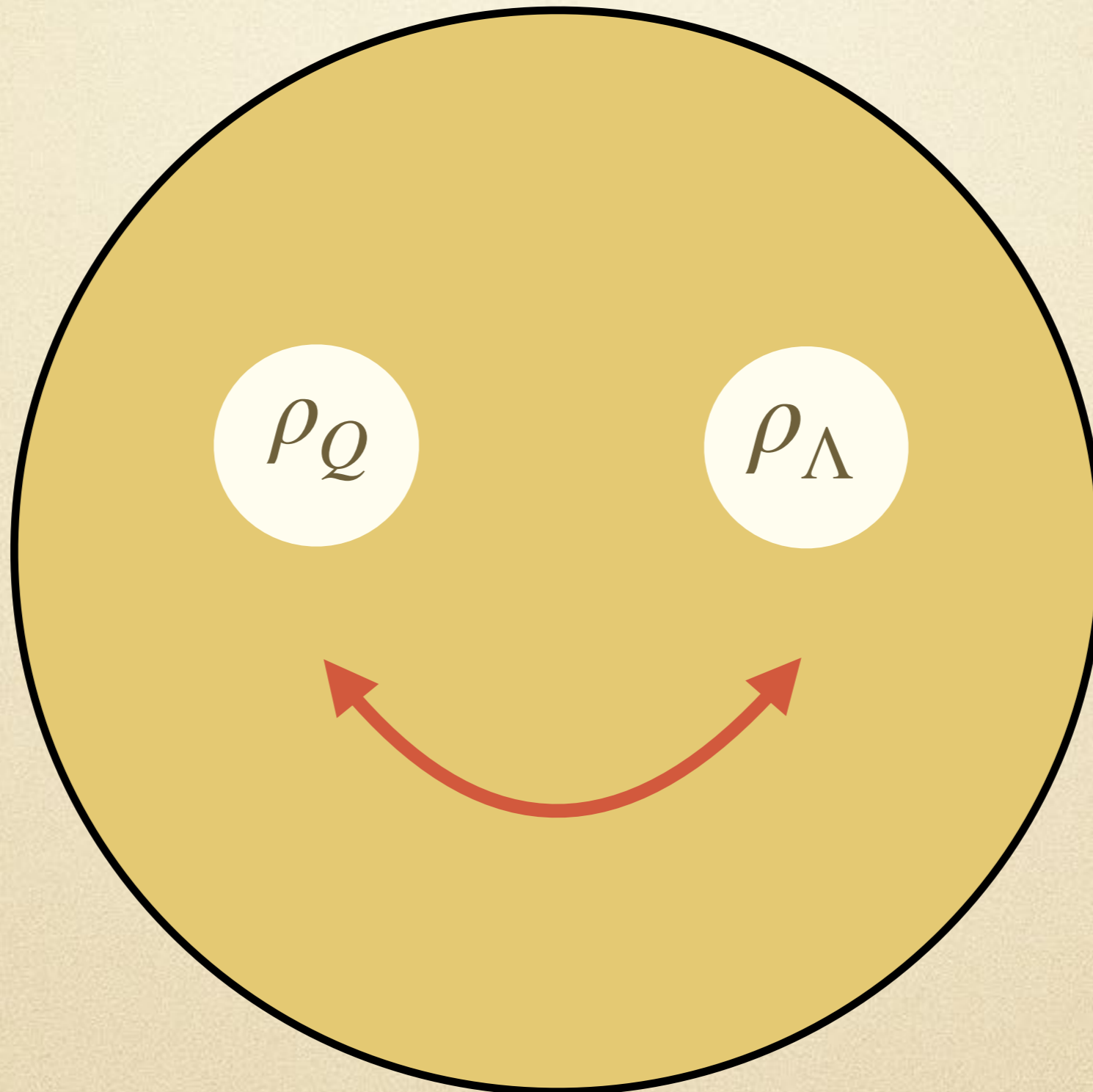
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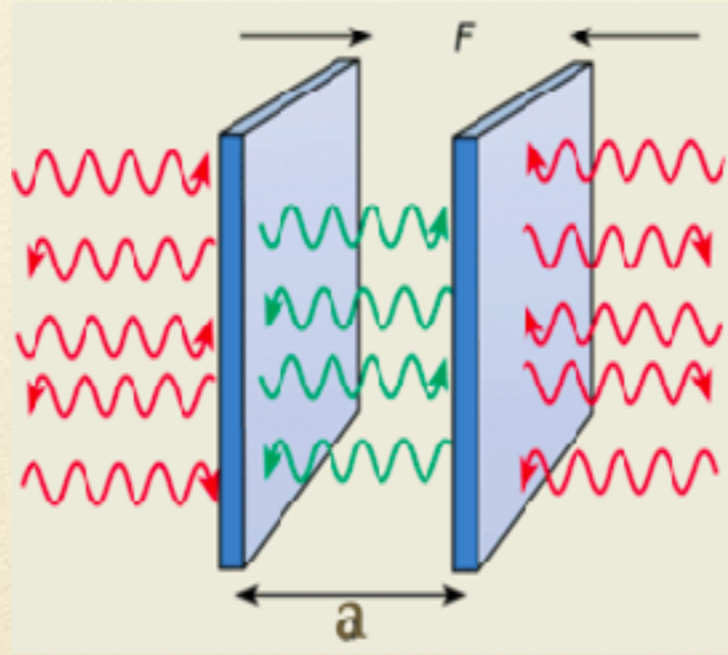




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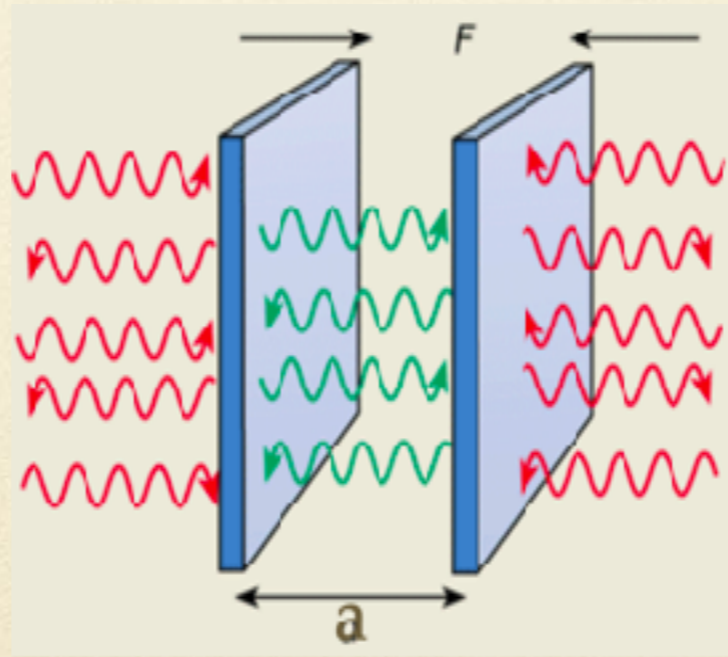
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Casimir effect

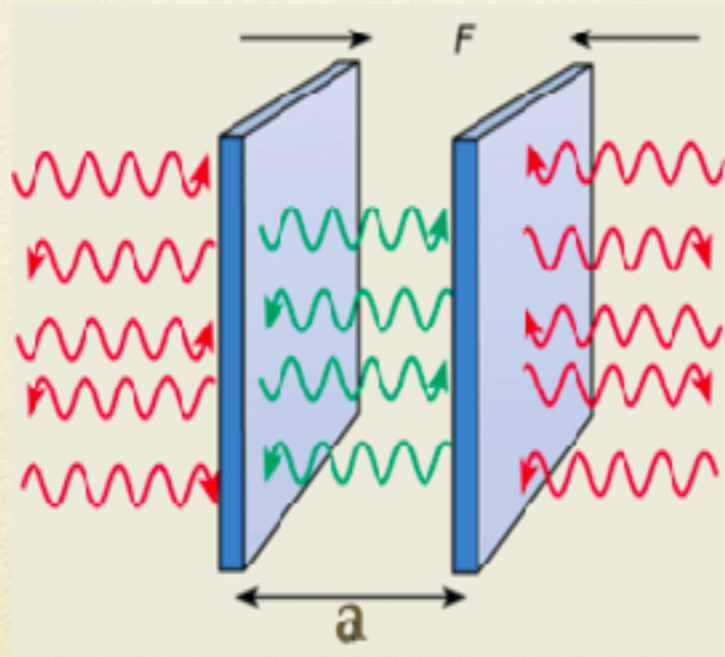




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Predicted 1948



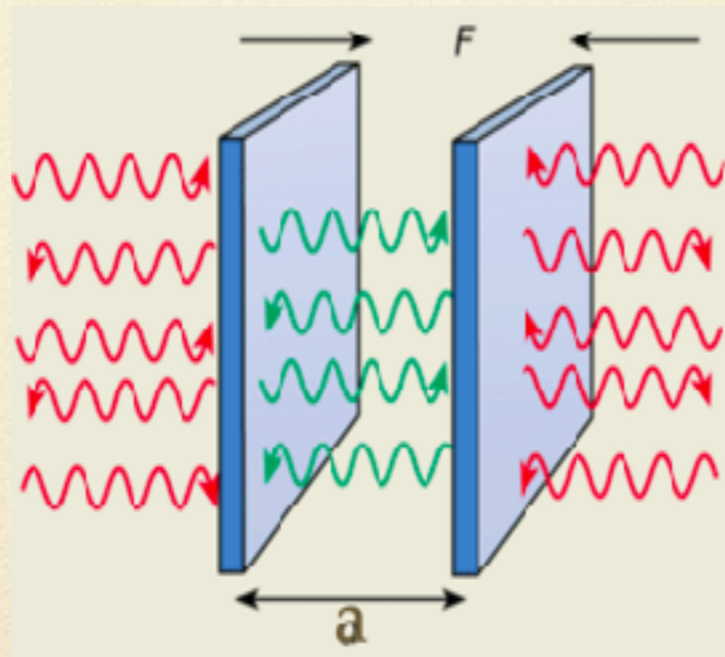


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Observed 1997<sup>ref [10]</sup>



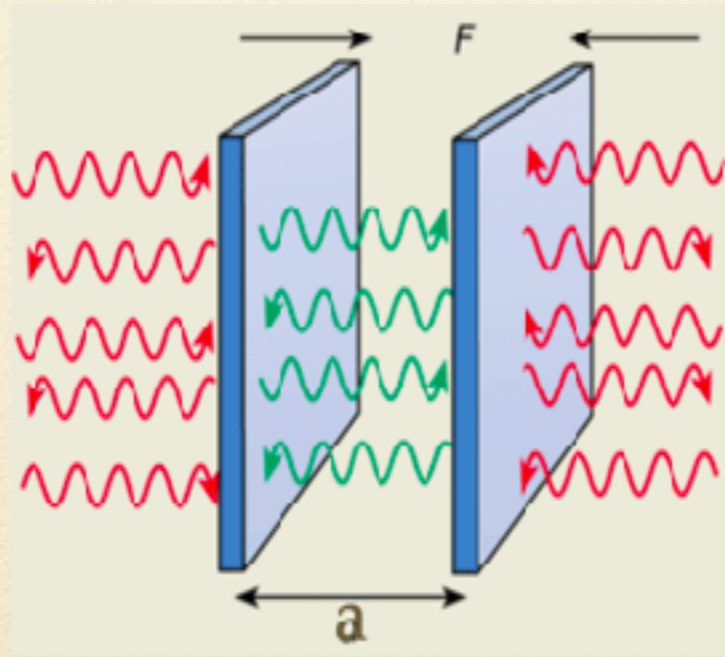


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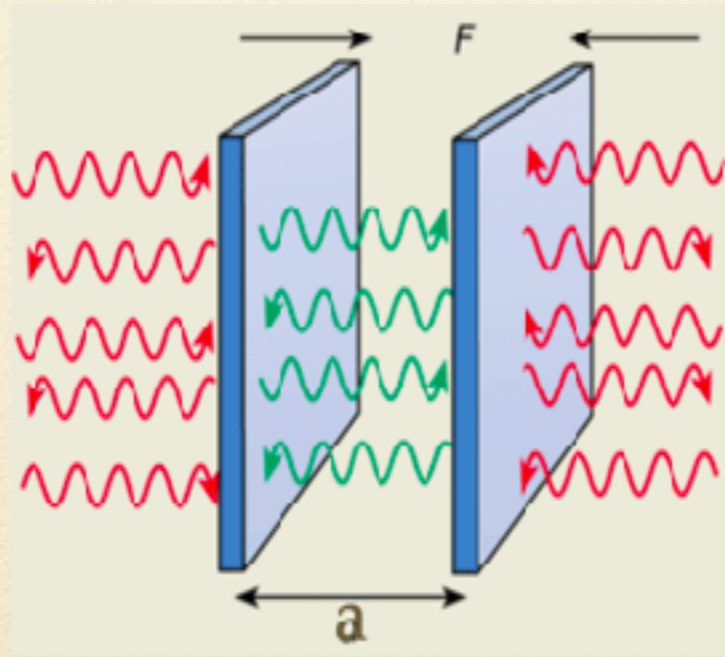


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$$\rho_C = -\frac{\hbar\pi^2}{720a^4}$$
$$\Rightarrow \frac{F_Q}{A} \approx \rho_Q \cdot a$$



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In reality, additional effects



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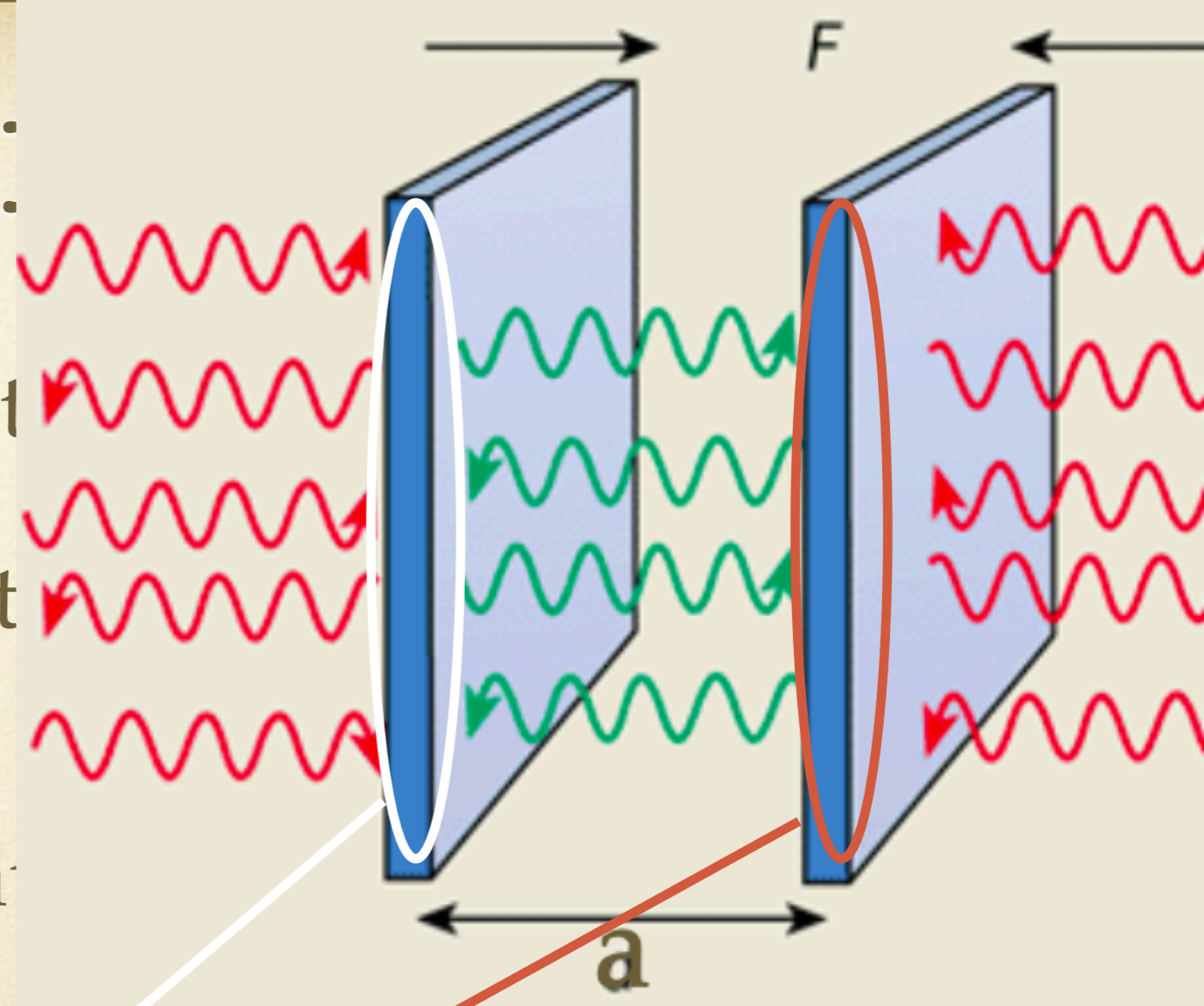
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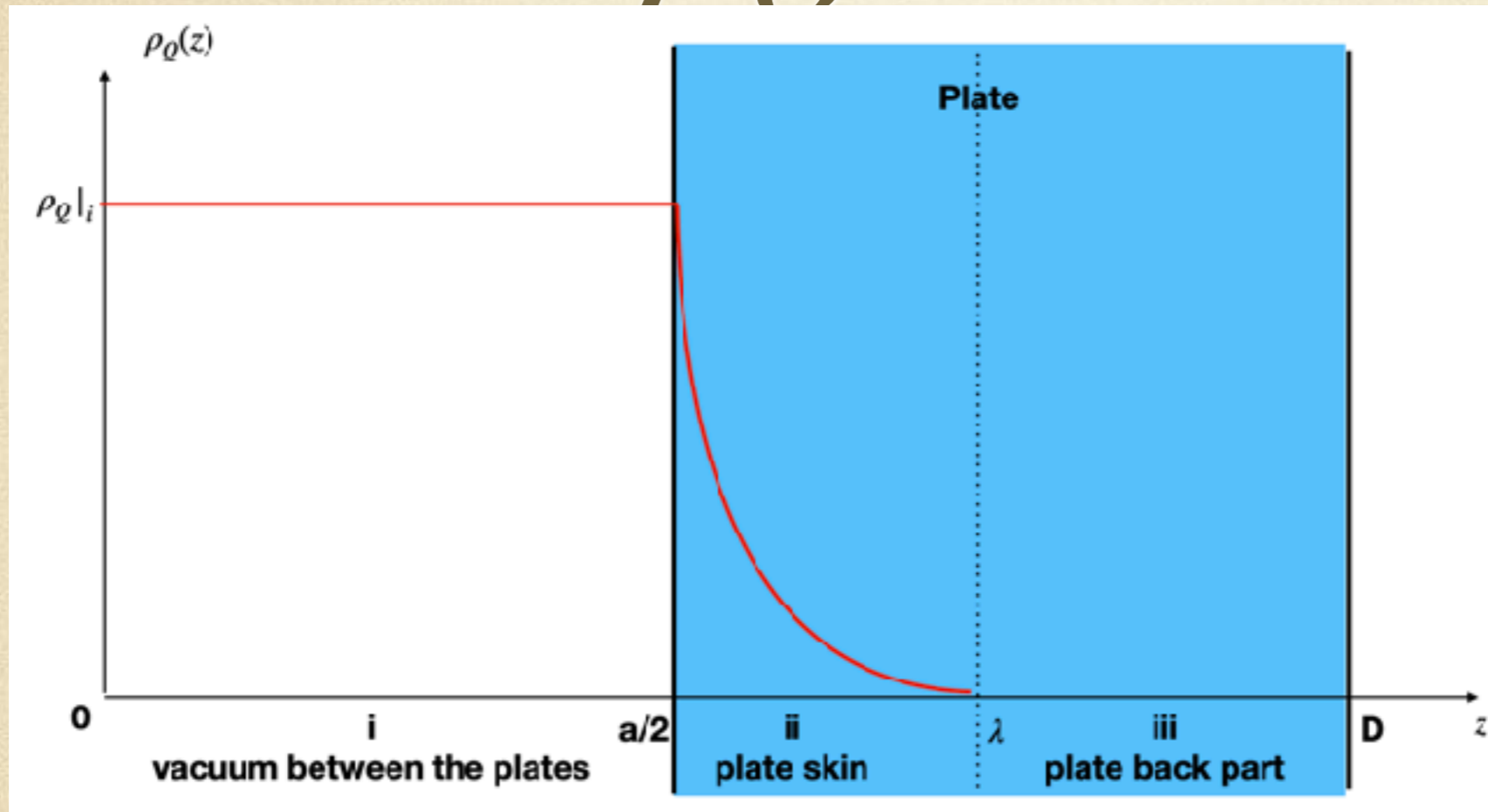
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 $\lambda \approx 10^{-8} m$ , thus  $\rho_C = \rho_C(\vec{x})$



# $\rho_0$ in lab

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veen

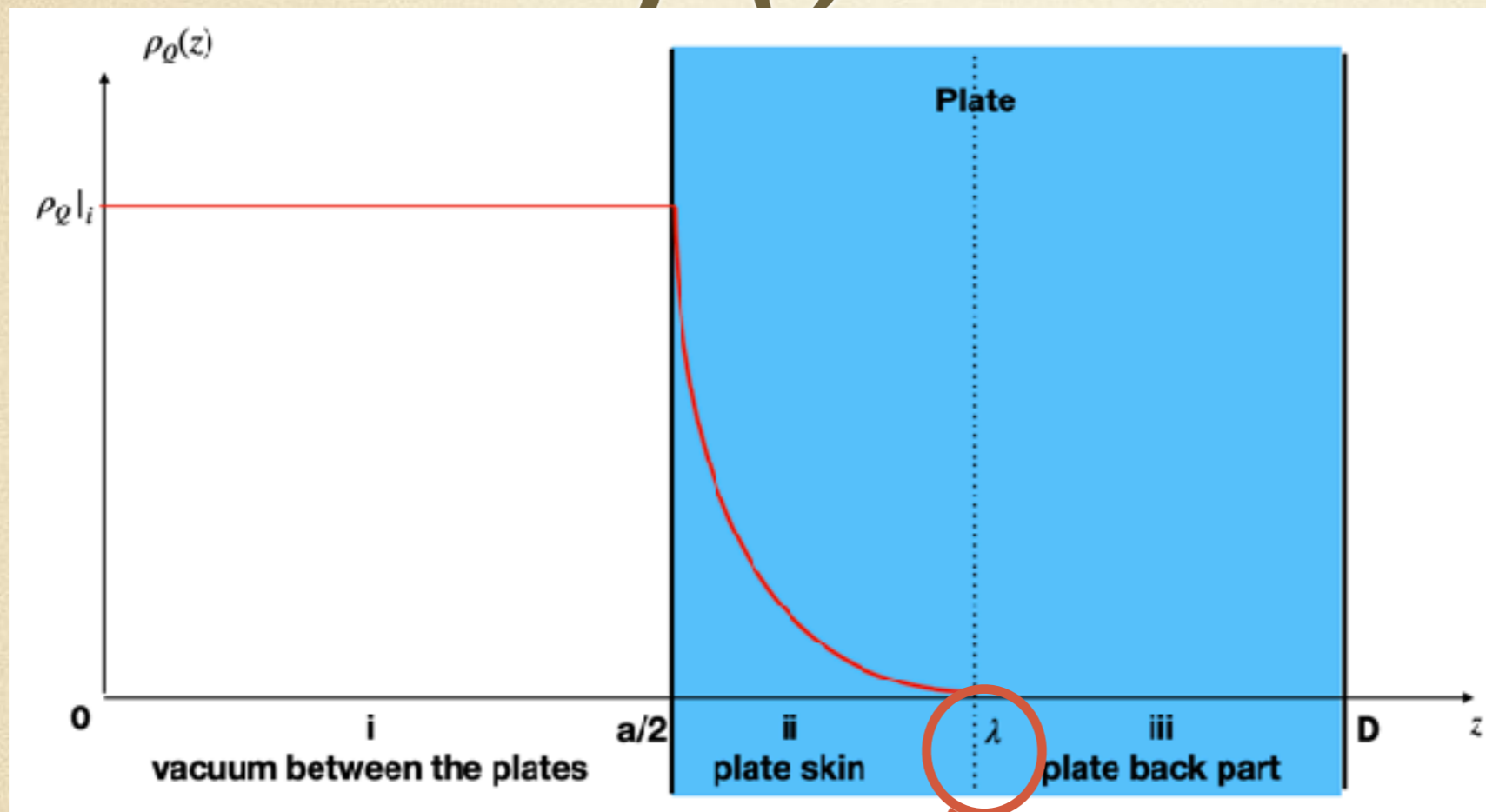
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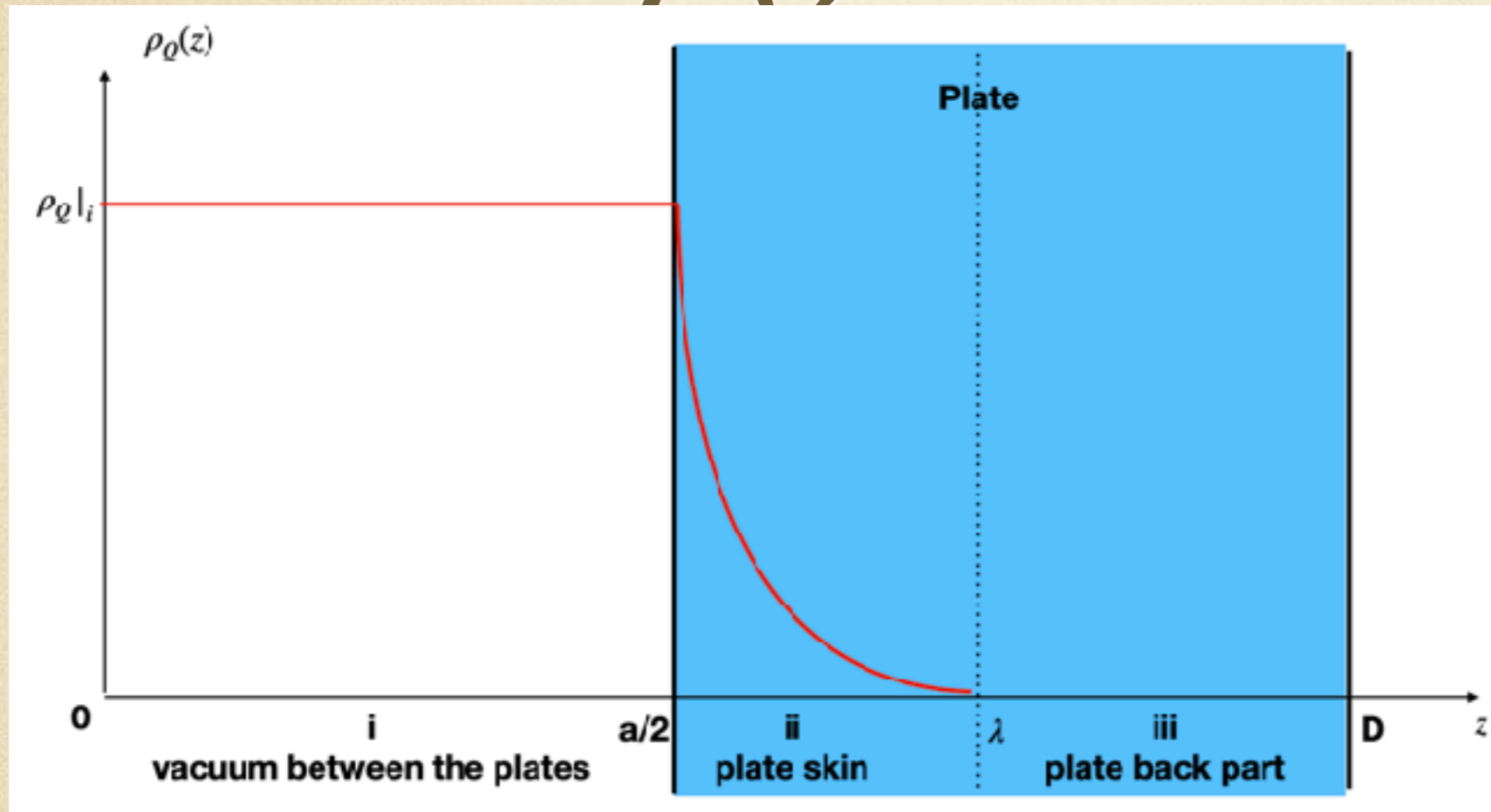
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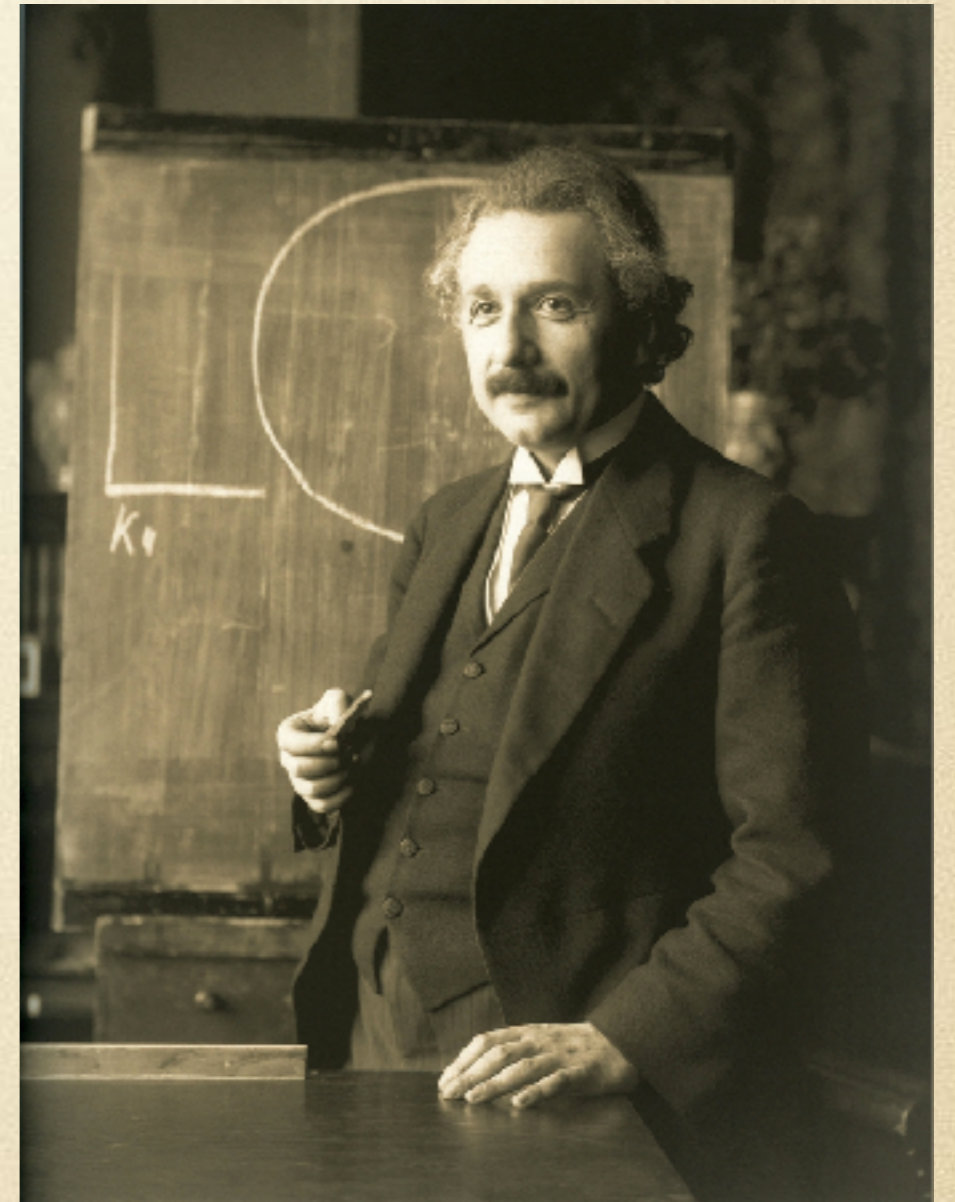


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Albert Einstein

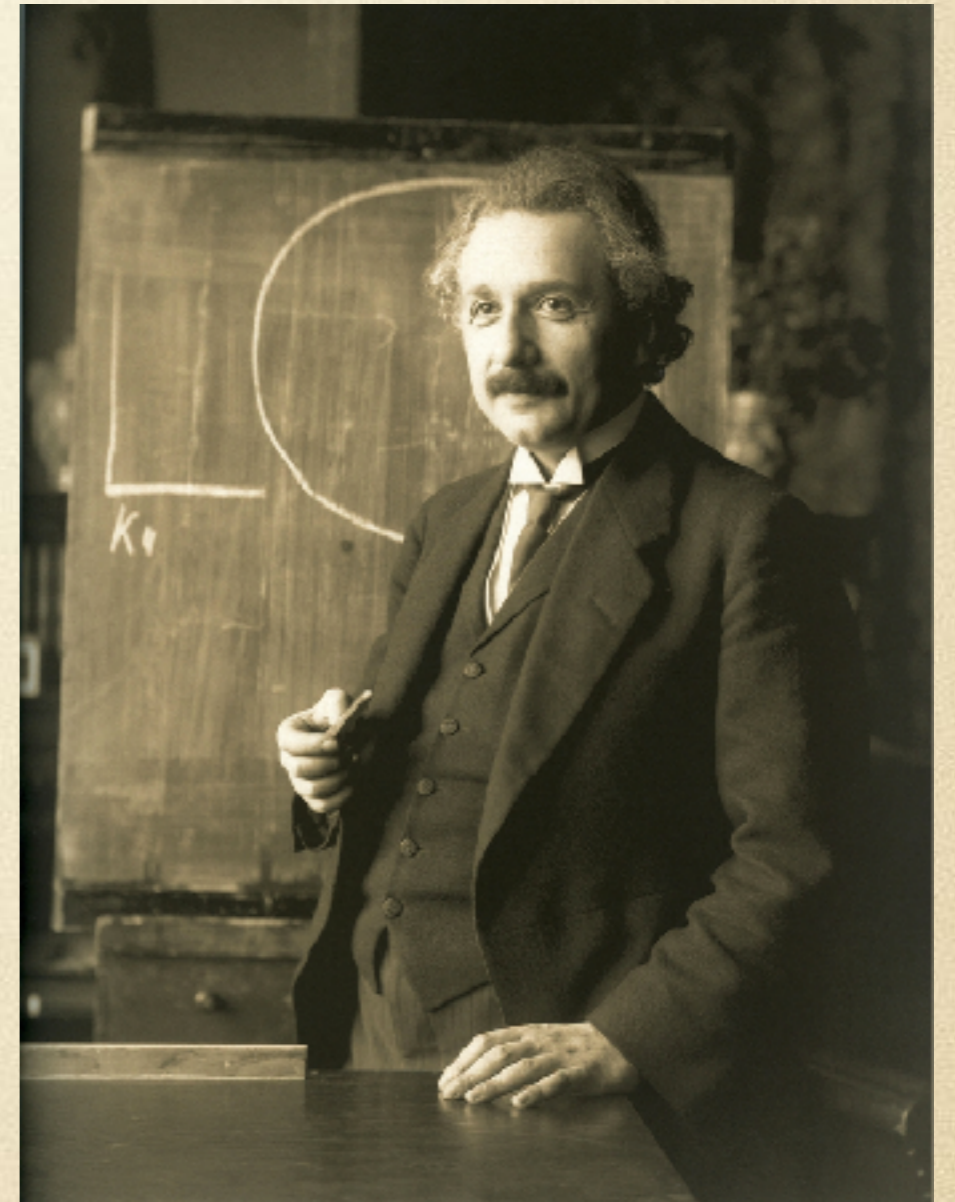




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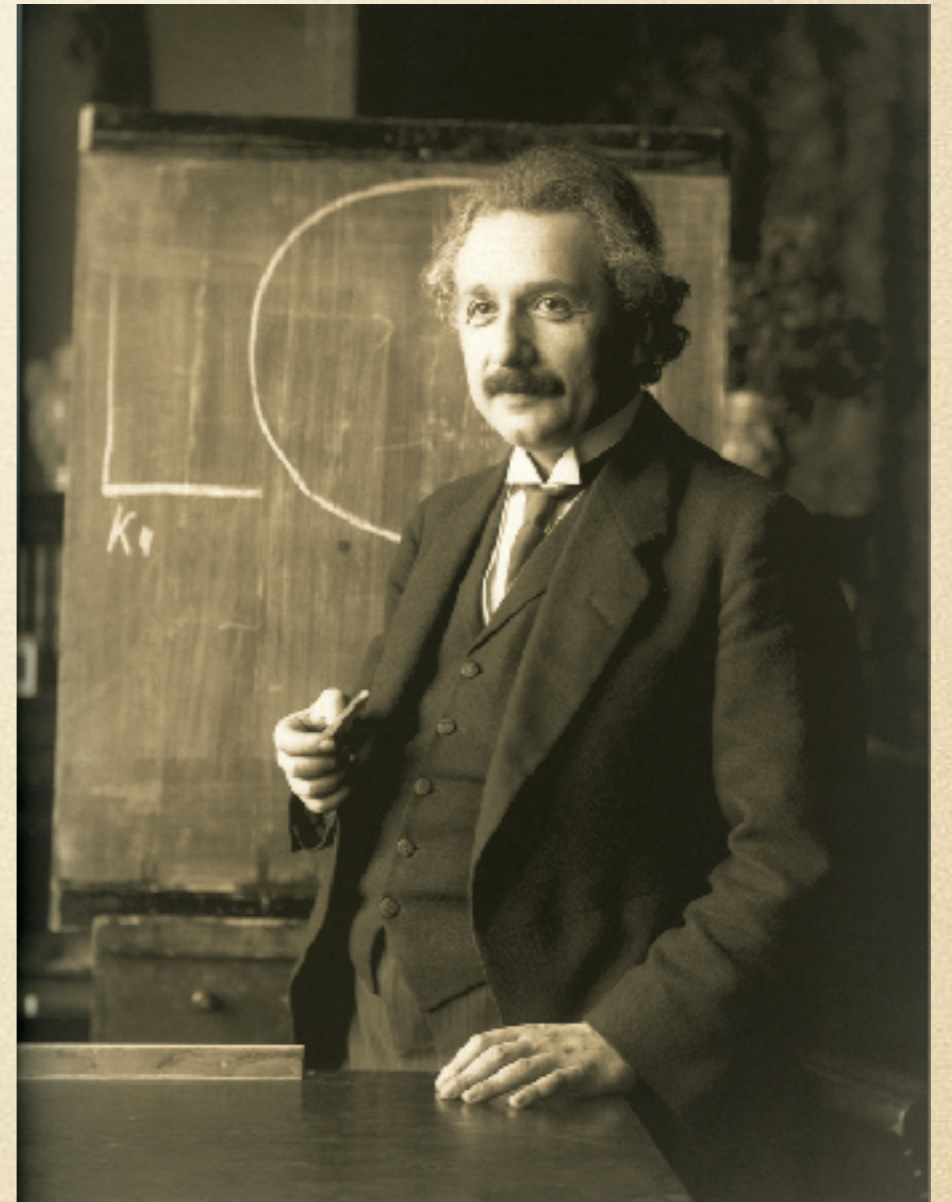
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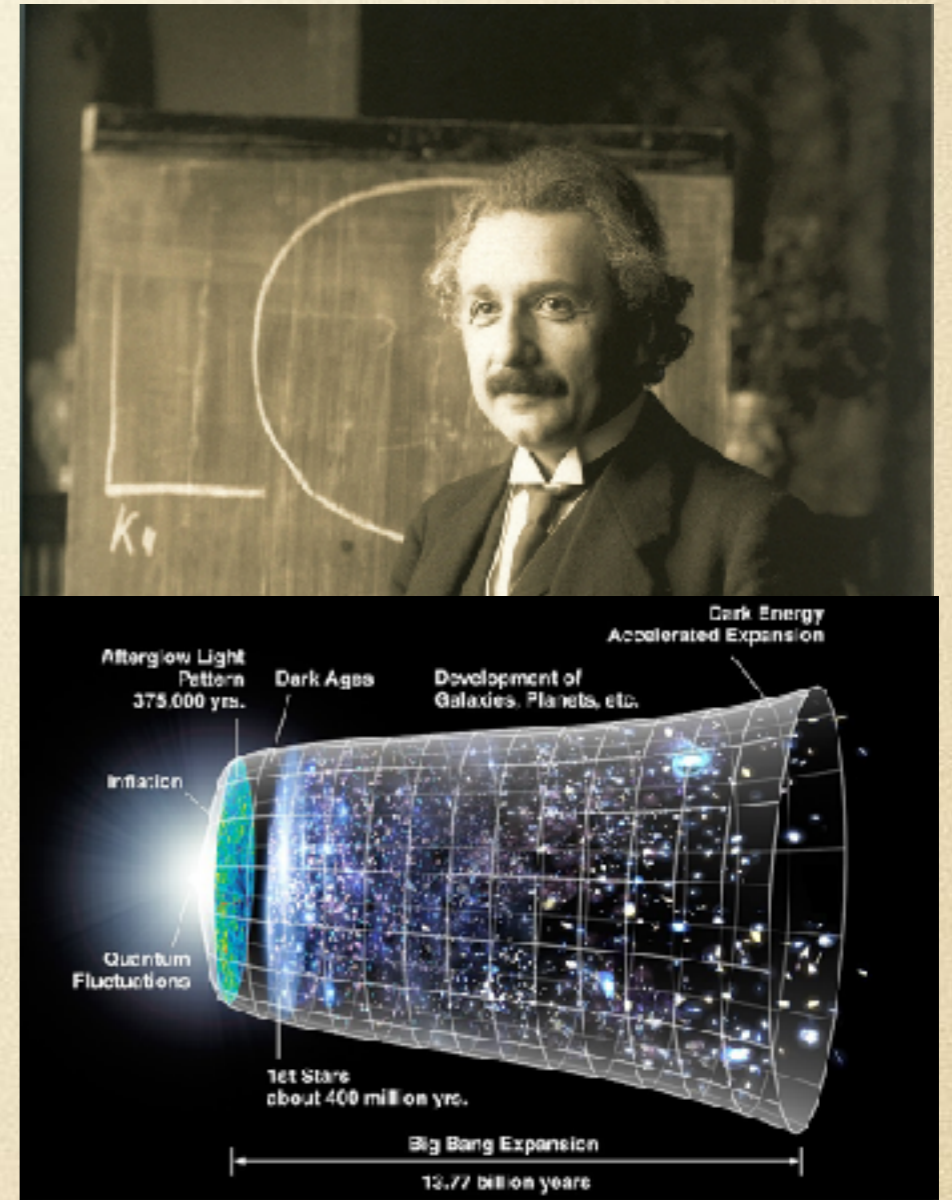
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NP 2011



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measurements:

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NP 2011

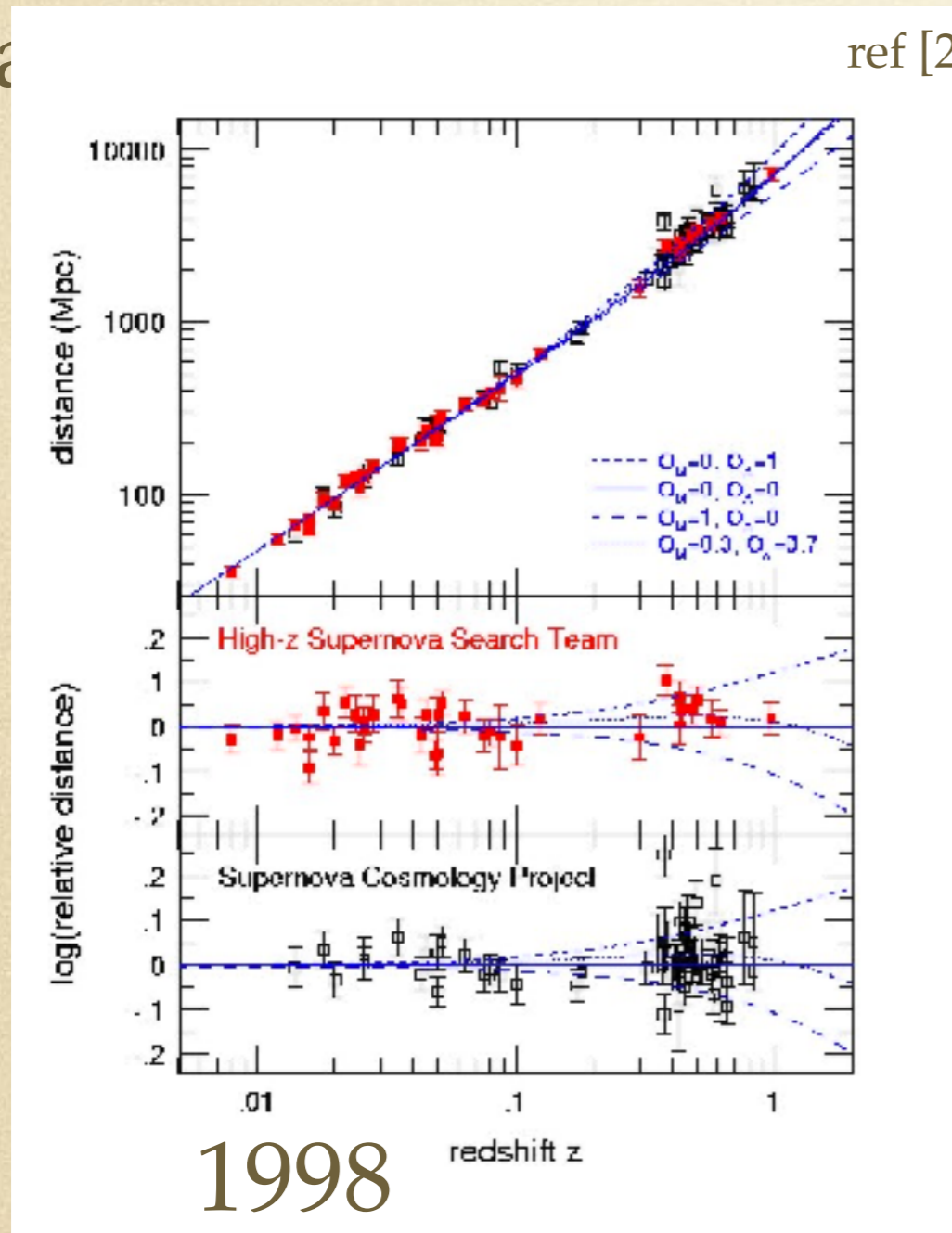


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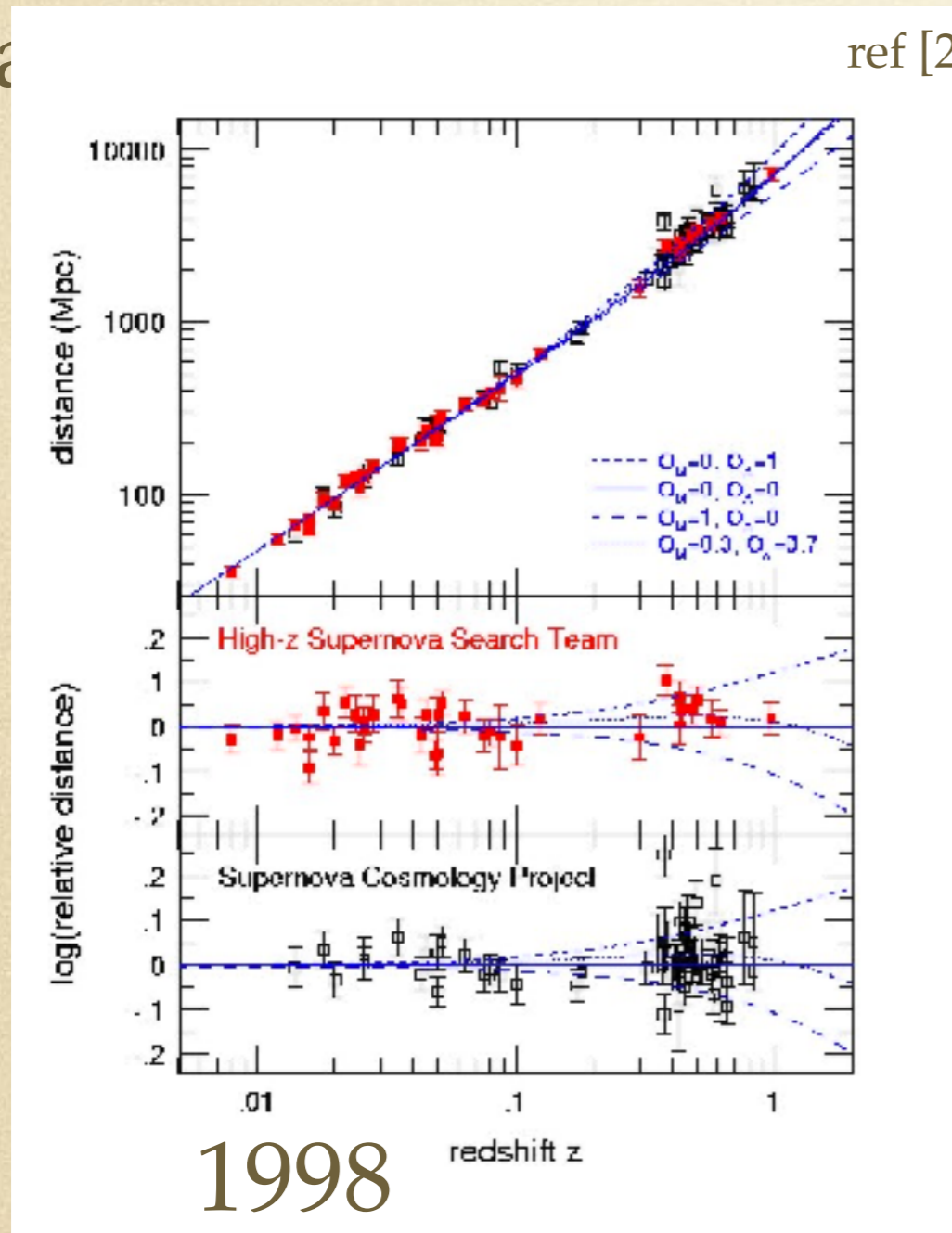


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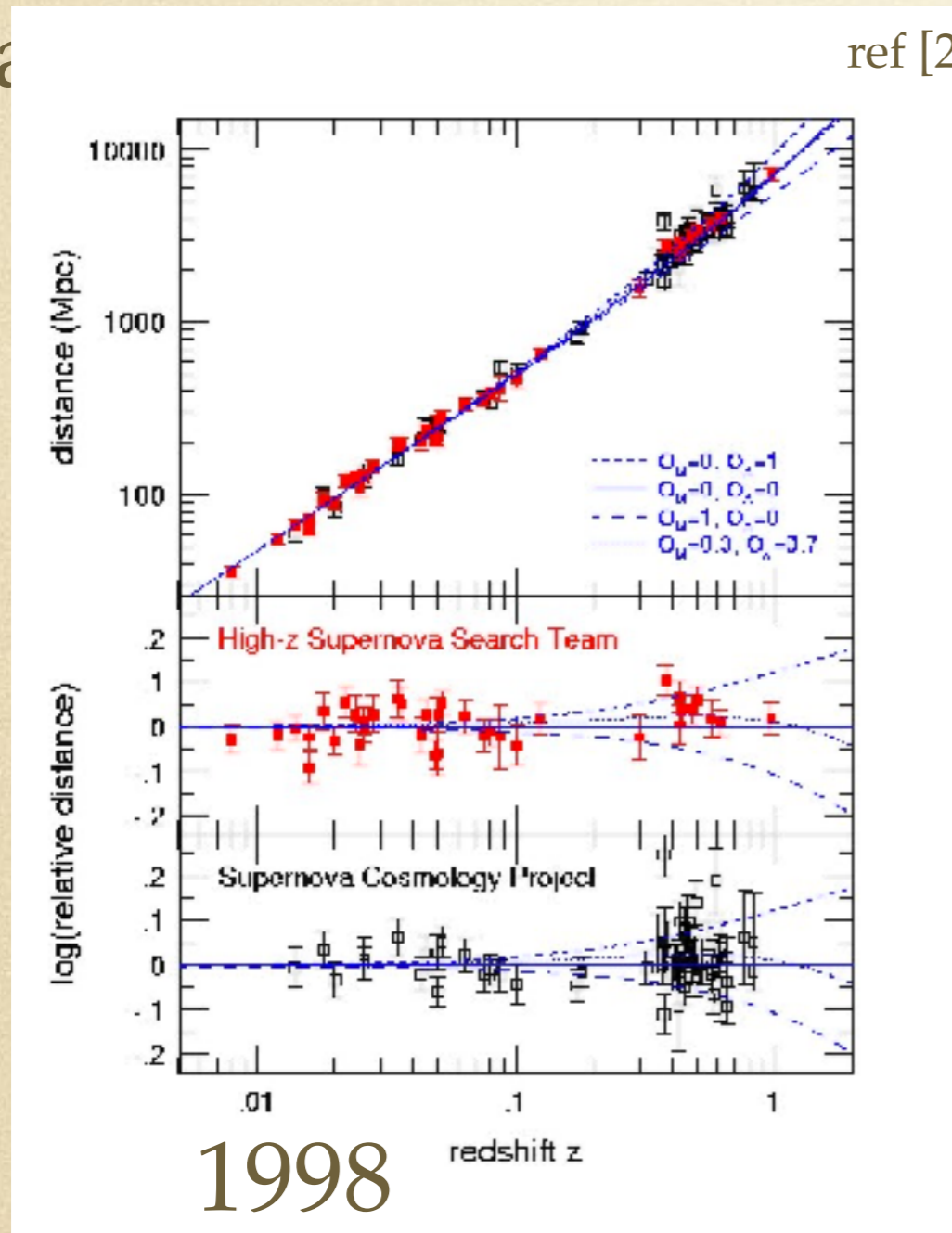


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NP 2011

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$$\ddot{a} > 0 \Rightarrow \Lambda > 0 !$$



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$\Lambda$  as an energy density



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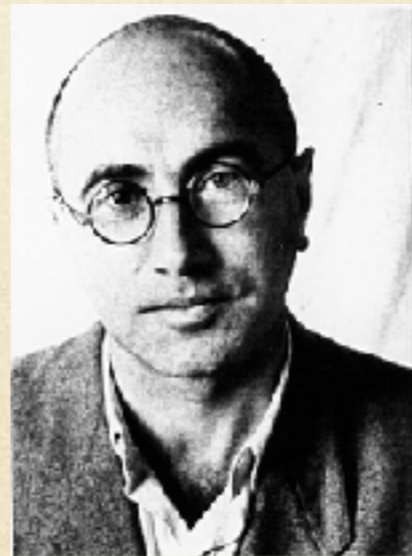
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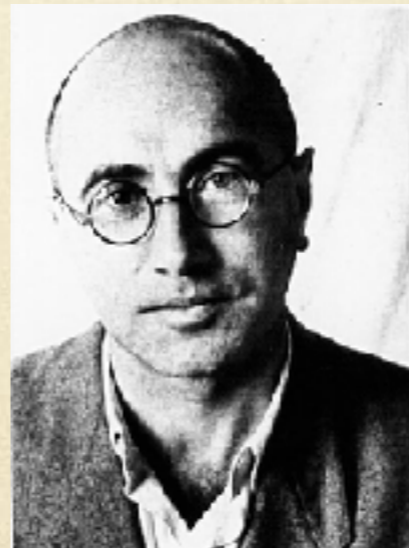
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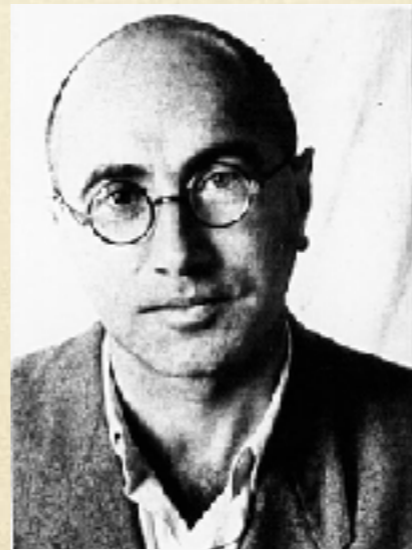
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In short QFT with cutoff  $\rho_Q \sim c\kappa_0^4/\hbar^3$

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$$\Upsilon_0 \equiv \frac{\rho_{\Lambda_0}}{\rho_{Q,0}(\kappa)} = \frac{\Lambda_0 c^3 \hbar^3}{8\pi G_0 \kappa_0^4} = \begin{cases} 10^{-121} & \text{for } \kappa_0 = c \sqrt{\frac{c\hbar}{G_0}} \\ 10^{-55} & \text{for } \kappa_0 = cm_Z. \end{cases}$$



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Many solutions - experimental input needed!



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Seemingly independent, but  
we argue that one leads naturally to the other...



Hypothesis 1:  $H_{Q \leftrightarrow \Lambda}$



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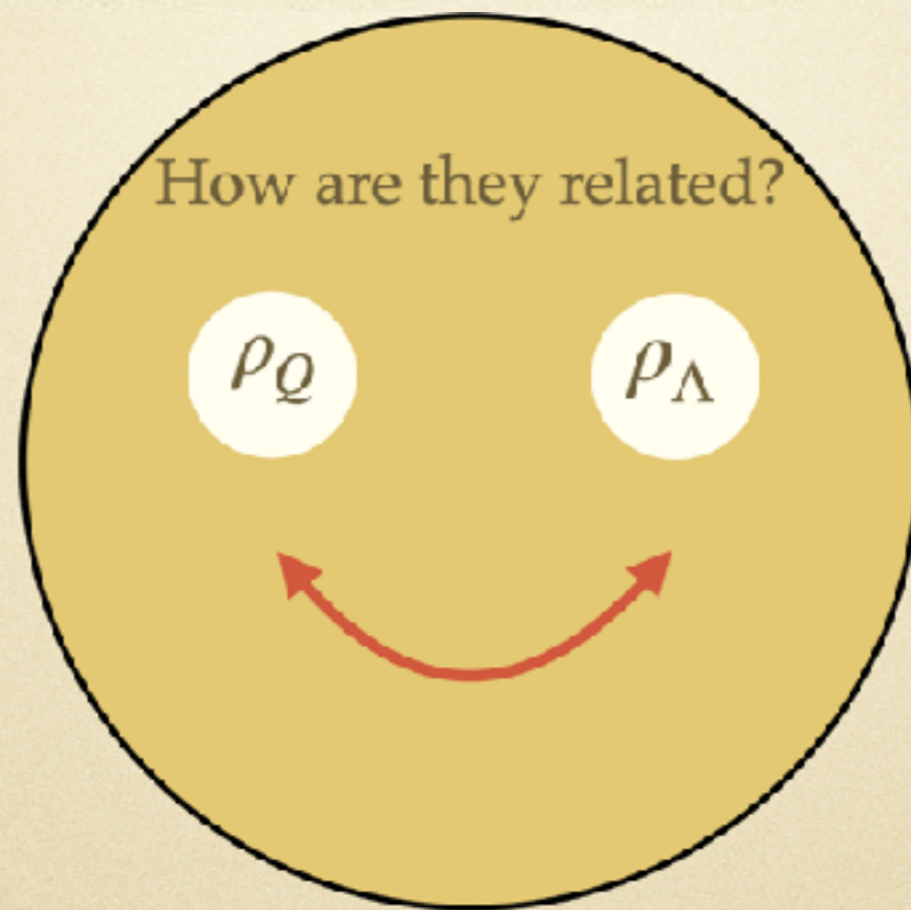


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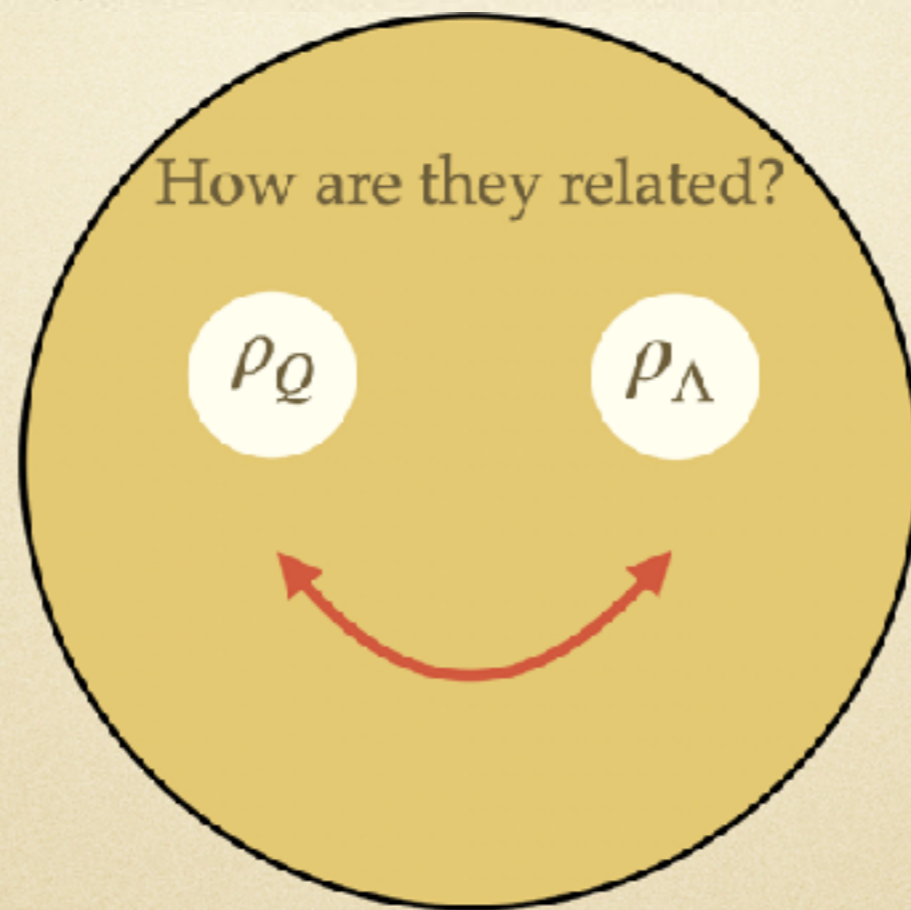




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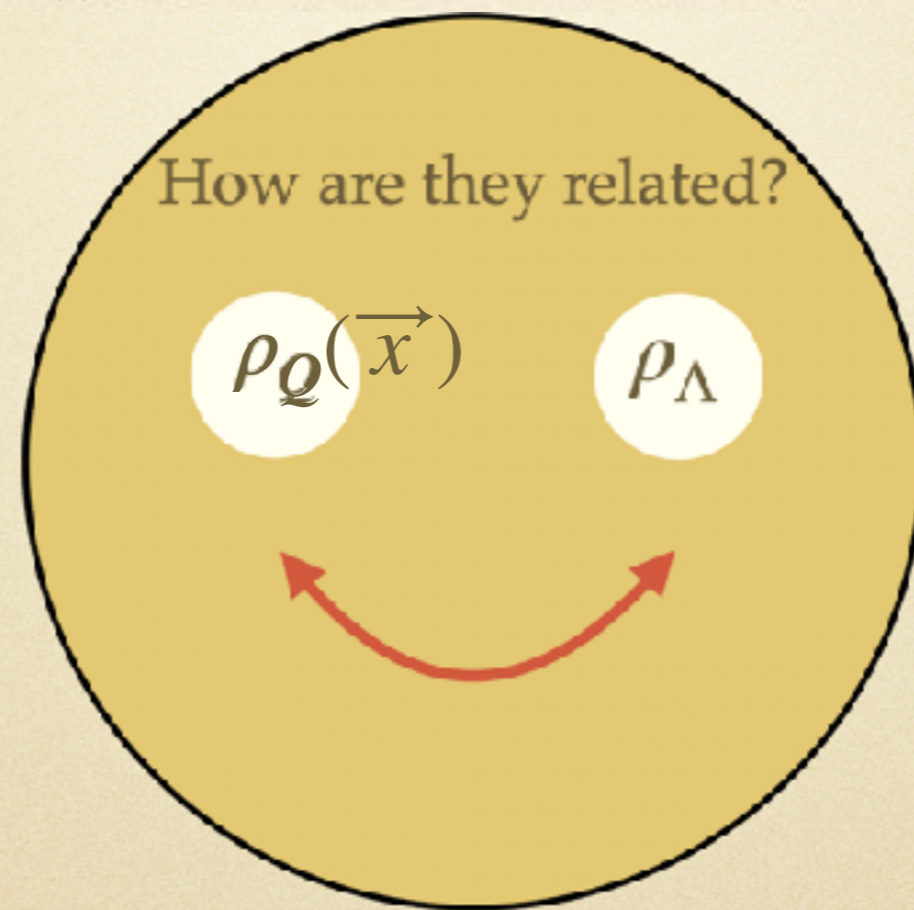


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How are they related?

$\rho_Q(\vec{x})$

$\rho_{\Lambda}$

local variable



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need variable,  
 $\rho_{\Lambda}(\vec{x}), \Lambda(\vec{x}), \dots$



Hypothesis 2:  $H_{\Lambda \leftrightarrow G}$



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- Gravitational couplings connected?



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# Hypothesis 1&2:

$$H_{\Lambda \leftrightarrow G}$$

$$H_{\Lambda \leftrightarrow Q}$$



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Continue with a “minimal version” that

- General covariance
- Small deviation from classical GR
- Local
- 2nd order eom



# SD-Framework



# SD-Framework

Action for both hypothesis



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$$\Gamma_k = \int d^4x \sqrt{-g} \left( c^4 \frac{R - 2\Lambda(k)}{16\pi G(k)} + \mathcal{L}_m(\phi, k) \right)$$



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Covariant!



# SD-Framework



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Only interested in SD small IR modifications



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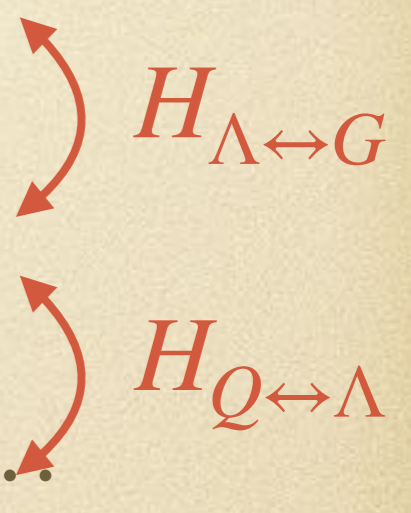
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Experimentalist:  
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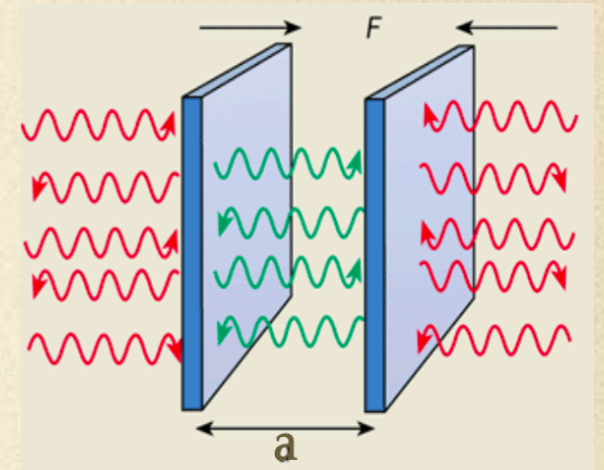


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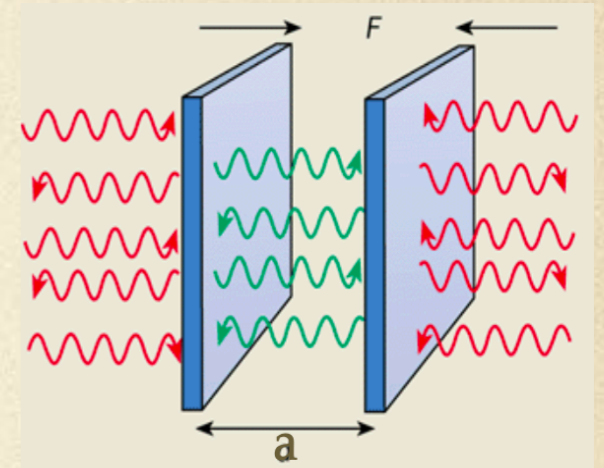




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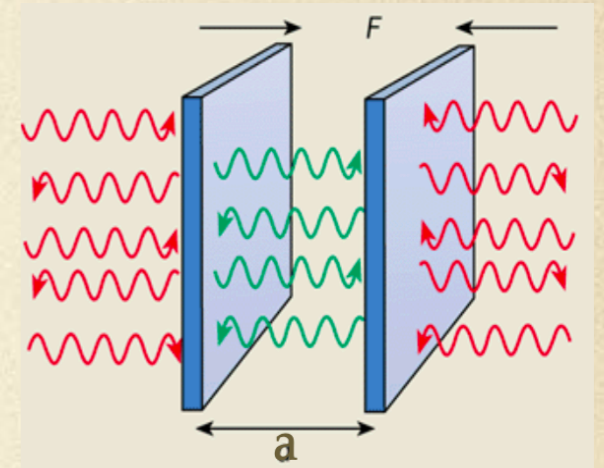
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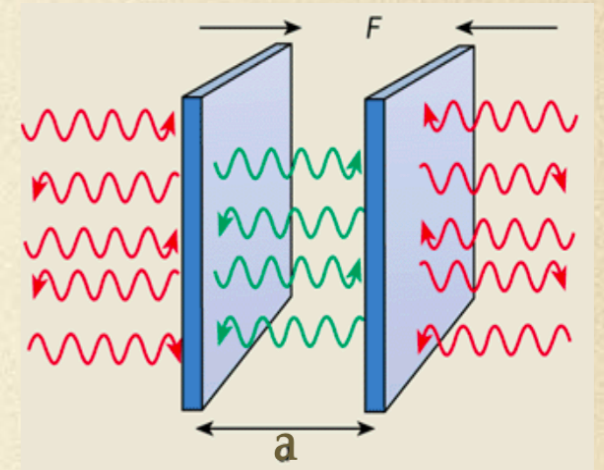


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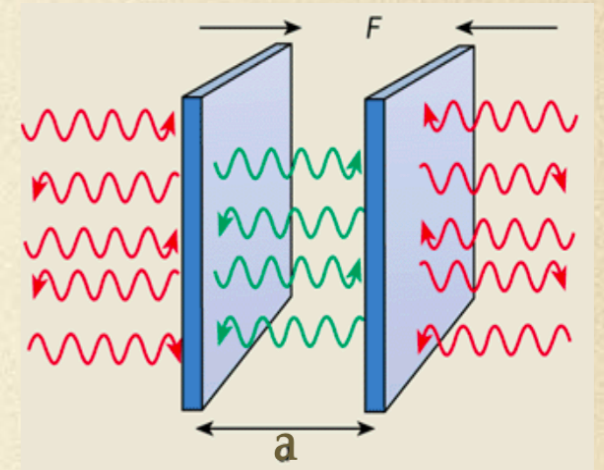
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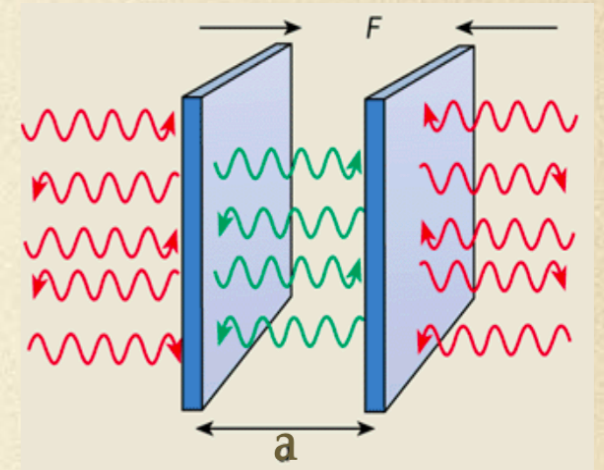
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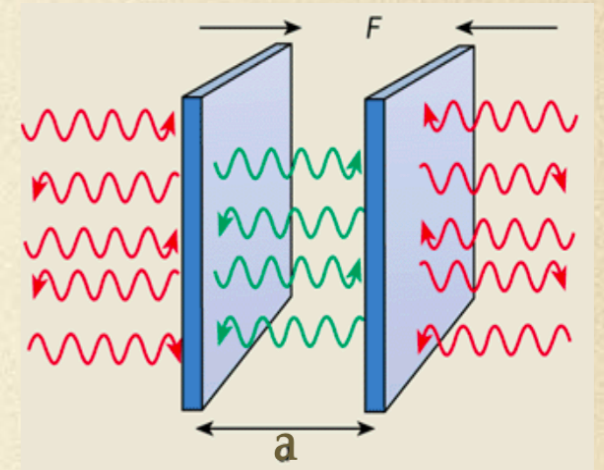
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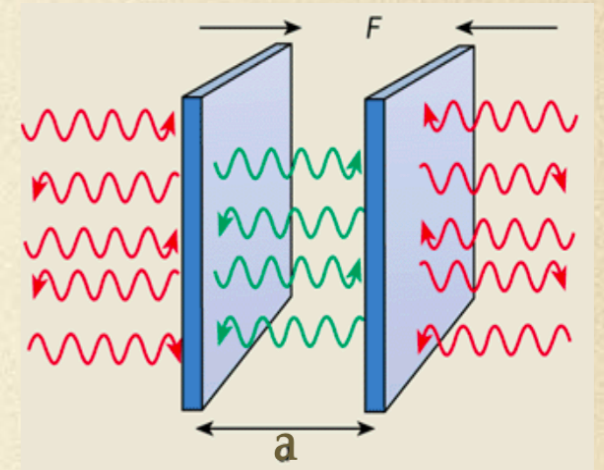
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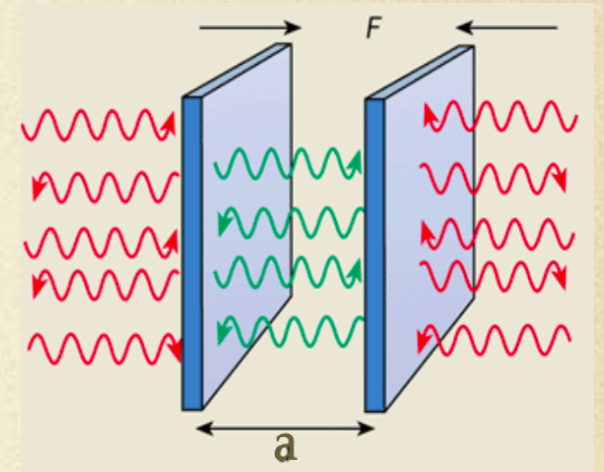
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$\rho_C$



# SD-Casimir

Equation(s)

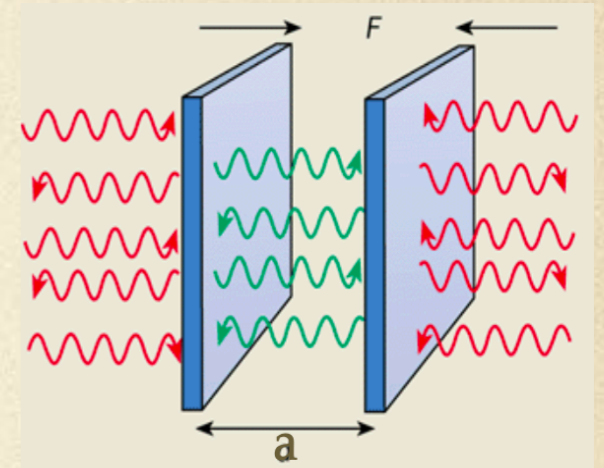




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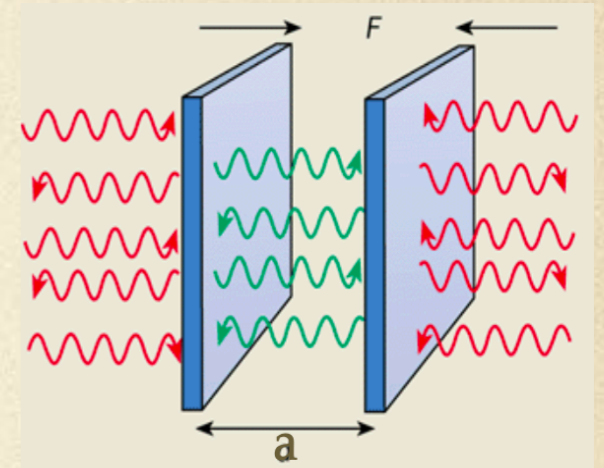




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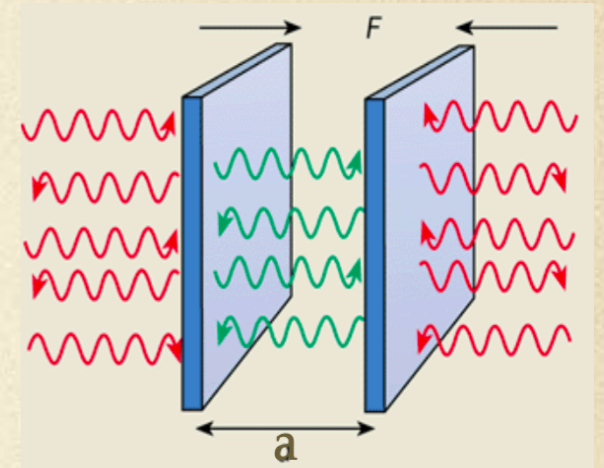
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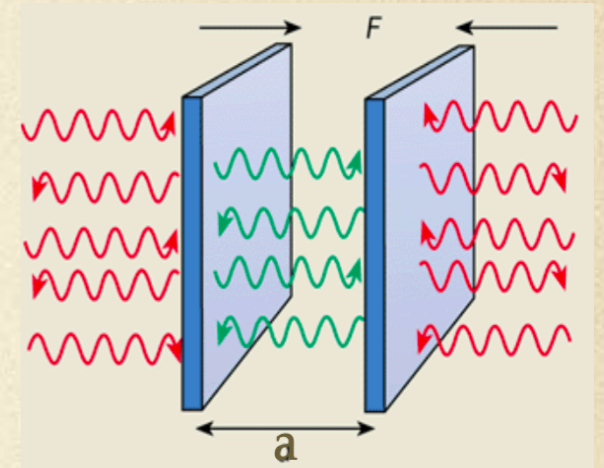
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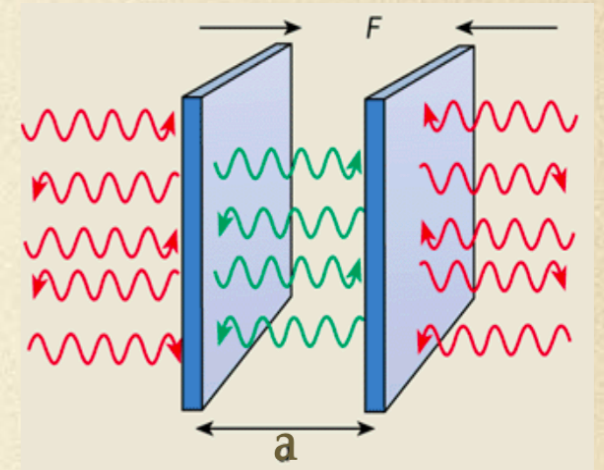
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# SD-Casimir





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⇒ Gravitational attraction between plates changes



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Hypothesis  
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Sensitive to parameters:  $\alpha_1, (C_1 - C_3)$



# Towards experiment



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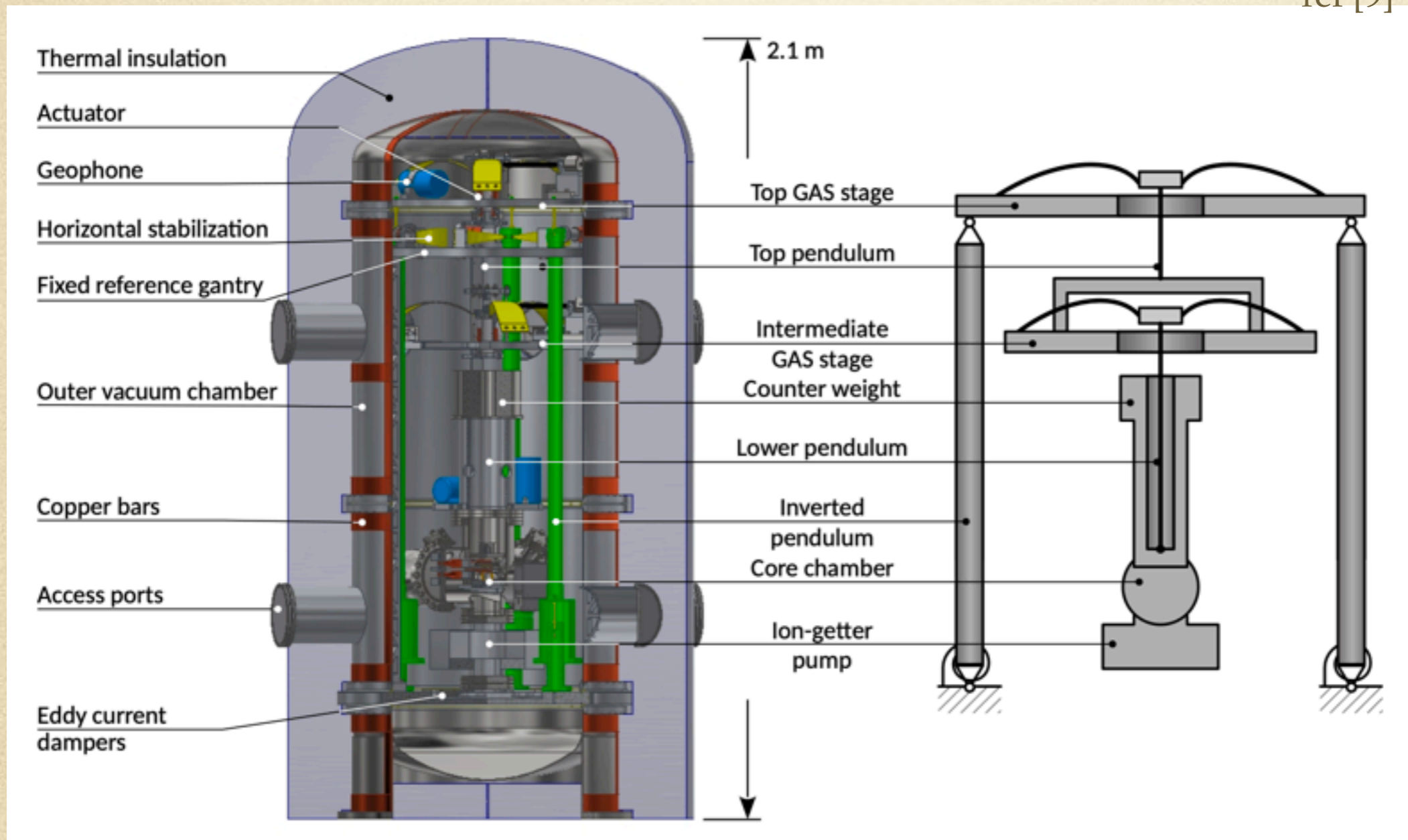
Cannex approved experiment



# Towards experiment

## Cannex approved experiment

ref [9]

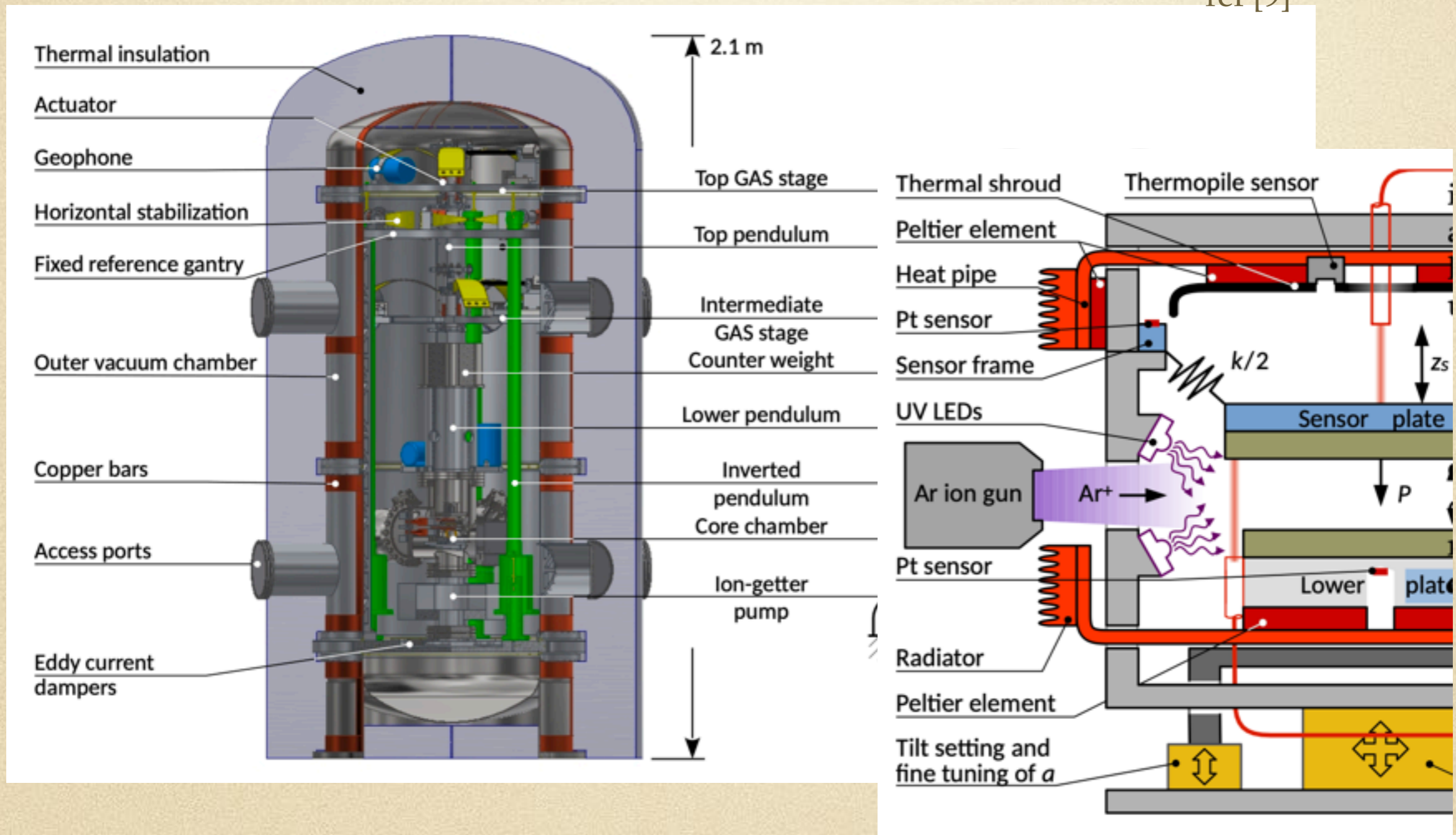




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ref [9]

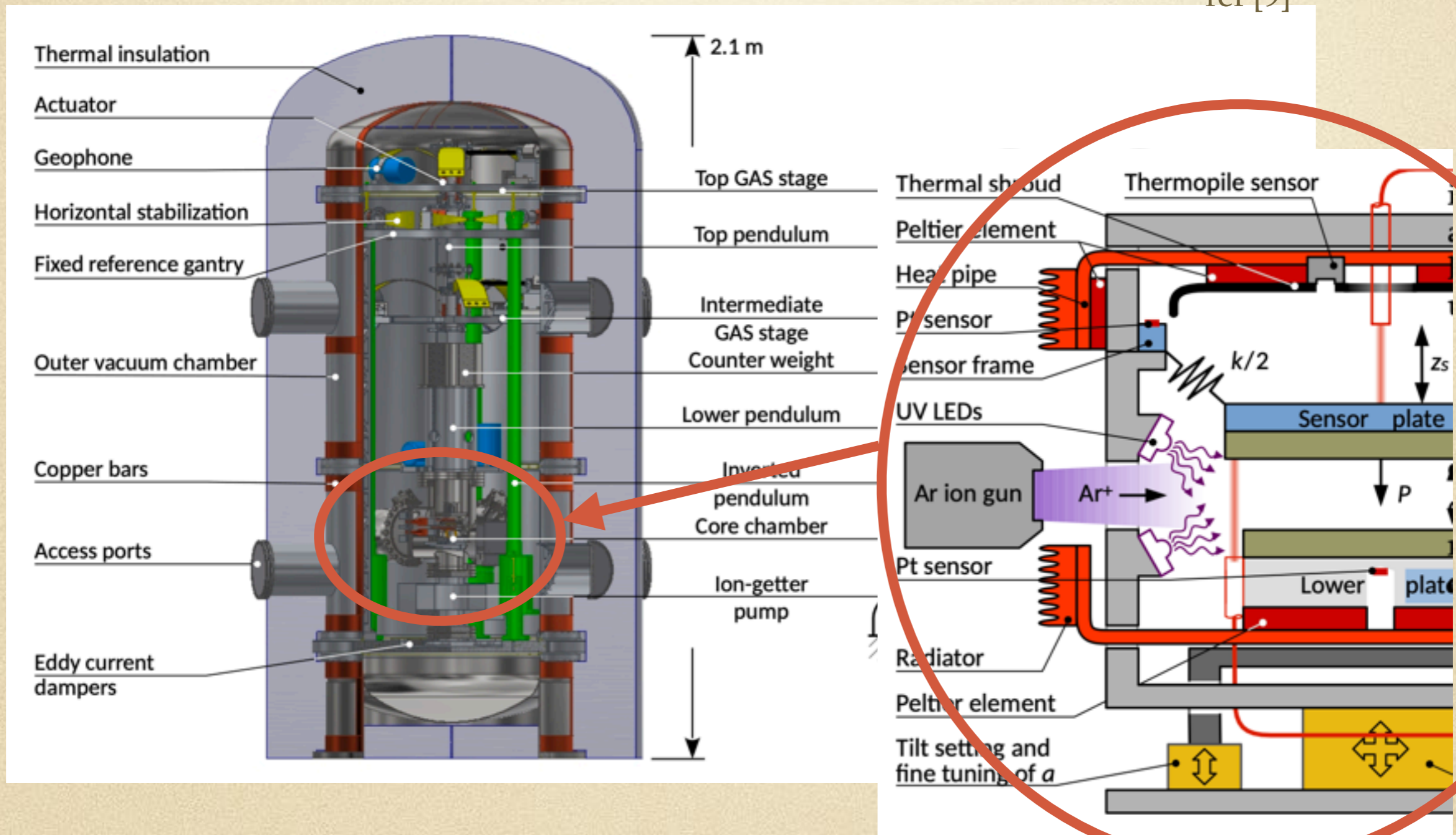




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# Towards experiment

Results (preliminary toy estimate):



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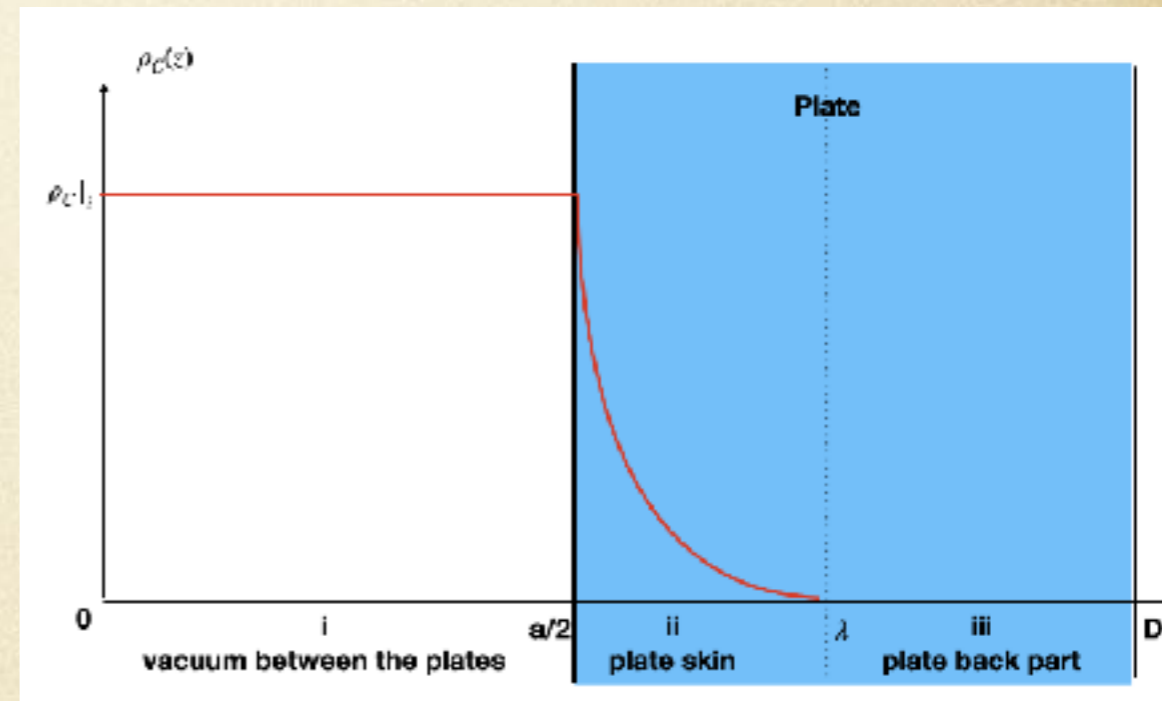
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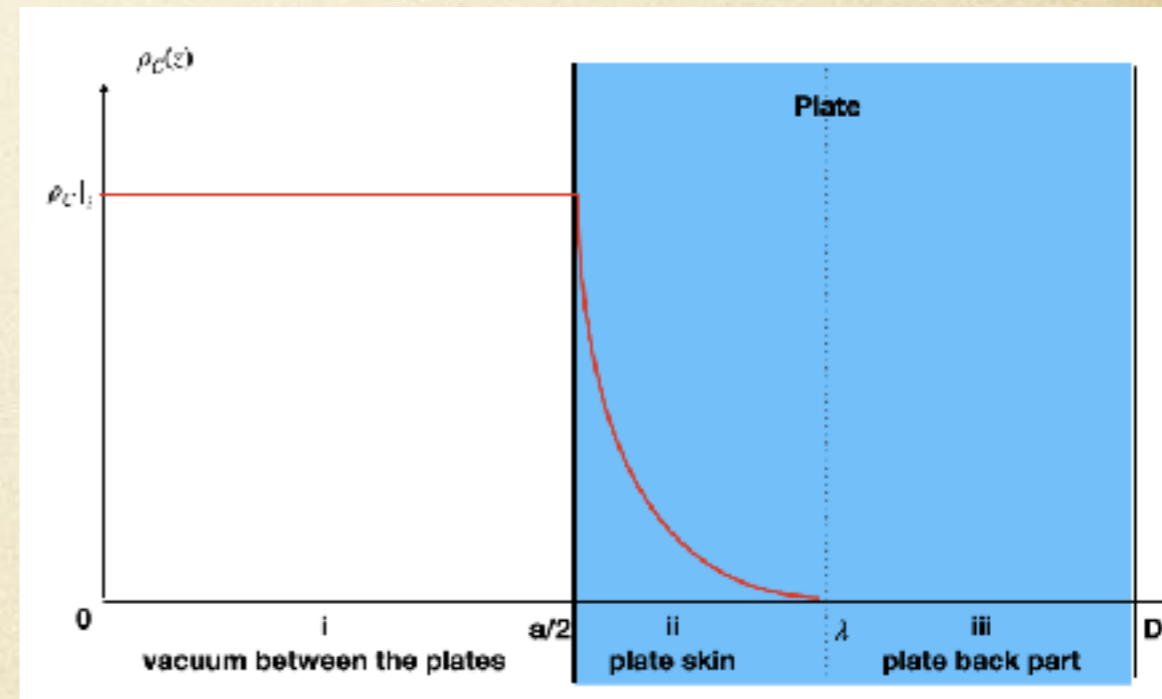




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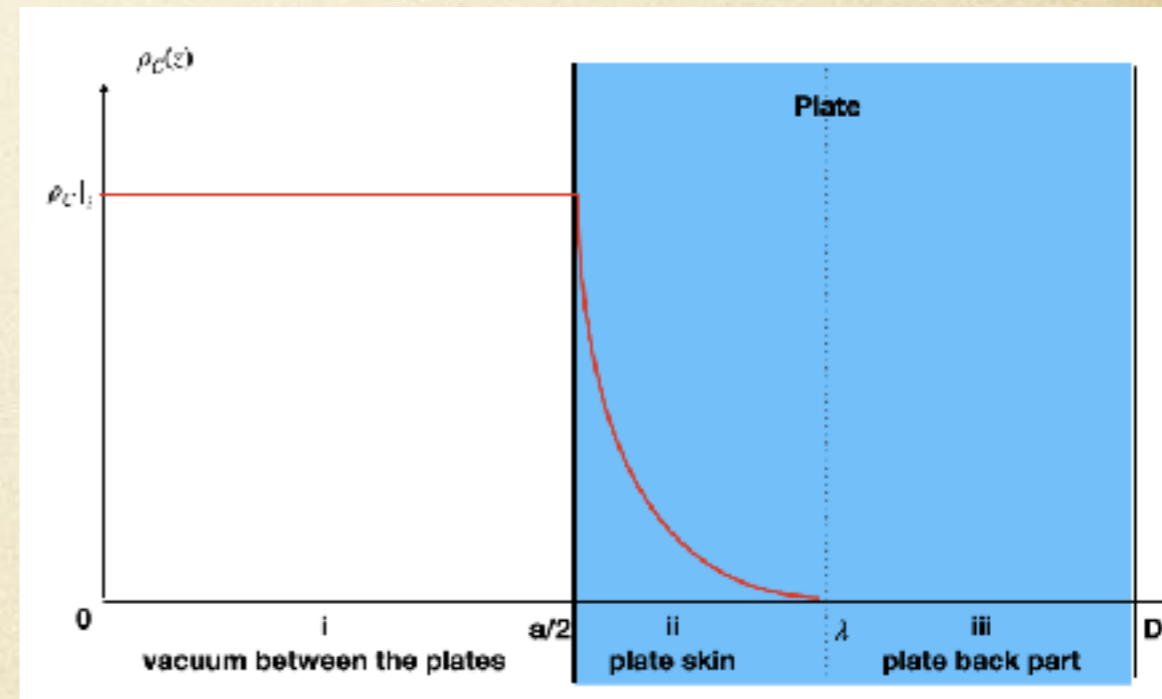
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Corrections tend to be very large, thus coefficient has to be very small

$$\frac{\alpha_1}{C_1 - C_3} \ll 10^{-32}$$



# Take home message I

$$H_{\Lambda \leftrightarrow G}$$

$$H_{Q \leftrightarrow \Lambda}$$



# Take home message I

$$H_{\Lambda \leftrightarrow G} \quad H_{Q \leftrightarrow \Lambda}$$

Covariant implementation  
in SD framework



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*Will be tested!*



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$$H_{\Lambda \leftrightarrow G}$$

$$H_{Q \leftrightarrow \Lambda}$$



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*Will be tested!*



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Expect same, or similar effects,  
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(your model?)



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# Under construction





# Under construction



- Comparison with quantum gravity benchmarks



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- Comparison with quantum gravity benchmarks

	$B_1$	$B_2$	$B_3$
$N_S$	0	0	4
$N_D$	0	1	12
$N_V$	0	1	12
$C_1$	$-15/(16\pi)$	$-4/\pi$	$-11/(2\pi)$
$C_3$	$-15/(16\pi)$	$-3/(2\pi)$	$-3/\pi$
$C_1/(C_1 - C_3)$	$\infty$	1.6	2.2



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- Comparison with quantum gravity benchmarks



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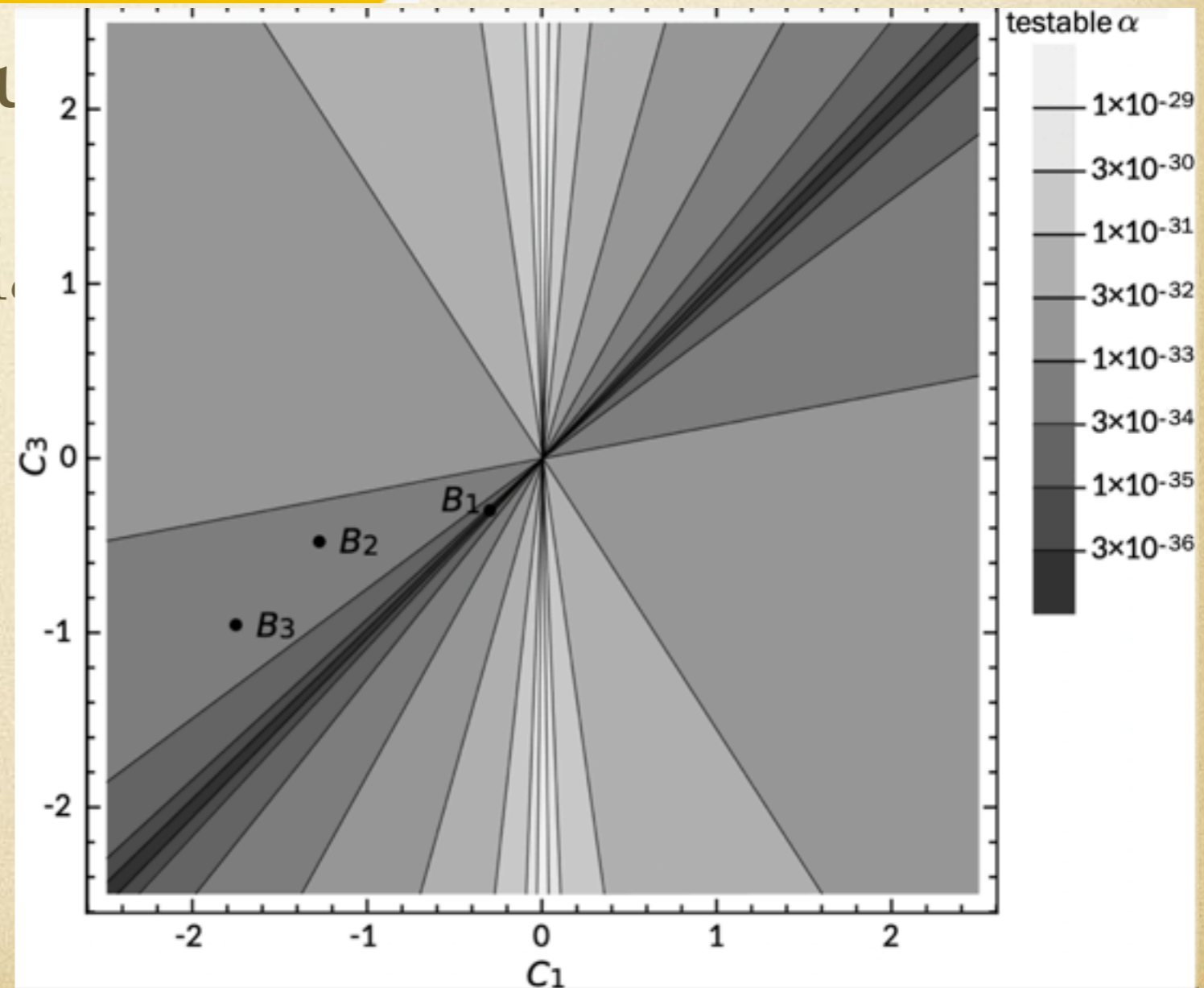
- Comparison with quantum gravity benchmarks
- More realistic simulation for Cannex



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- Comparison with qu
- More realistic simul





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- Comparison with quantum gravity benchmarks
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- Comparison with quantum gravity benchmarks
- More realistic simulation for Cannex
- Simulation for existing experiments



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- Comparison with quantum gravity benchmarks
- More realistic simulation for Cannex
- Simulation for existing experiments
- Implications for the CCP, (more to be said)



# Under construction



- Comparison with quantum gravity benchmarks
- More realistic simulation for Cannex
- Simulation for existing experiments
- Implications for the CCP, (more to be said)
- ...



Thank You!



# Some References

- 0) F. Canales, B. Koch, C. Laporte, A. Rincon JCAP no1 26, 2020
- 1) E. Hubble, Proceedings of the National Academy of Sciences of USA, Volume 15, Issue 3, pp. 168-173
- 2) Supernova Search Team (A. G. Riess (UC, Berkeley, Astron. Dept.) et al.). May 1998. 36 pp  
Published in Astron.J. 116 (1998) 1009-1038
- 3) R. J. Adler, B. Casey and O. C. Jacob, Am. J. Phys. 63, 620 (1995);  
H. Martel, P. R. Shapiro and S. Weinberg, Astrophys. J. 492, 29 (1998);  
S. Weinberg, Rev. Mod. Phys. 61, 1 (1989);  
J. Martin Comptes Rendus Physique 13 (2012) 566-665.
- 4) M. Reuter, F. Saueressig, Phys.Rev. D65 (2002) 065016; and many others
- 5) e.g. M. Niedermaier and M. Reuter, Living Rev. Rel. 9, 5 (2006)
- 6) S.J. Brodsky and R. Shrock, Proc. Nat. Acad.Sci. 108, 45 (2011)
- 7) H. Fritzsch, Nucl. Phys. Proc. Suppl. 203-204, 3 (2010)
- 8) B.K., P. Rioseco, Carlos Contreras, PRD 91 (2015) no2, 025009.
- 9) R. Sedmik, M.Pitschann, Universe, 7, 234, (2021)
- 10) G. Bimonte, B. Spreng, P. A. Maia Neto, G.-L. Ingold, G. L. Klimchitskaya, V. M. Mostepanenko, and R. S. Decca, Universe 7, 93 (2021).  
B. V. Derjaguin, I. I. Abrikosova, and E. M. Lifshitz, Q. Rev. Chem. Soc. 10, 295 (1956).  
S. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997).  
U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998).  
A. Roy, C.-Y. Lin, and U. Mohideen, Phys. Rev. D 60, 111101 (1999).  
G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. 88, 041804 (2002).  
R. S. Decca, D. López, E. Fischbach, and D. E. Krause, Phys. Rev. Lett. 91, 050402 (2003).  
J. M. Obrecht, R. J. Wild, M. Antezza, L. P. Pitaevskii, S. Stringari, and E. A. Cornell, Physical Review Letters 98, 063201 (2007).  
H. B. Chan, Y. Bao, J. Zou, R. A. Cirelli, F. Klemens, W. M. Mansfield, and C. S. Pai, Physical Review Letters 101, 030401 (2008).  
W. J. Kim, A. O. Sushkov, D. A. R. Dalvit, and S. K. Lamoreaux, Phys. Rev. Lett. 103, 060401 (2009).  
P. Zurbier, S. de Man, G. Gruca, K. Heeck, and D. Iannuzzi, New Journal of Physics 13, 023027 (2011), ISSN 1367-2630.  
R. H. Schafer, Ph.D. thesis, UC Riverside (2020).



# Backup



# Interpretation

$$\alpha \frac{C_1}{C_1 - C_3} \ll 10^{-32}$$



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For each interpretation many possible subcategories, e.g.

- B.
  - 1.  $\Lambda$  is not a coupling but a field
  - 2.  $G$  is not a coupling but a field
  - 3. RG group is not universal
  - 4. Hierarchy in QG parameters:  $C_3 \gg C_1$
  - 5. ...



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Implications for the CCP



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$$\Upsilon_0 \equiv \frac{\rho_{\Lambda_0}}{\rho_{Q,0}(\kappa)} = \frac{\Lambda_0 c^3 \hbar^3}{8\pi G_0 \kappa_0^4} = \begin{cases} 10^{-121} & \text{for } \kappa_0 = c \sqrt{\frac{c\hbar}{G_0}} \\ 10^{-55} & \text{for } \kappa_0 = cm_Z. \end{cases}$$



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$$\rho_Q = \rho_{Q,0} + \beta \cdot \rho_C$$

$$\rho_{\Lambda} = \rho_{\Lambda_0} - \alpha \cdot \rho_C$$

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$$\rho_Q = \rho_{Q,0} + \beta \cdot \rho_C$$

$$\rho_{\Lambda} = \rho_{\Lambda_0} - \alpha \cdot \rho_C$$

Should be  $\beta = 1$ , or 0

hypothesis,

but who knows ... 29

$\alpha$



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# Interpretation

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Look at changes of the CCP



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$$Y'_0 \equiv \left. \frac{dY(\rho_Q)}{d\rho_C} \right|_{\rho_C=0}$$



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Look at changes of the CCP

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$$\alpha = \Upsilon'_0 + \beta \Upsilon_0$$



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Look at changes of the CCP

Find

$$Y'_0 \equiv \left. \frac{dY(\rho_Q)}{d\rho_C} \right|_{\rho_C=0}$$

Measuring  $\alpha$

$$\alpha = Y'_0 + \beta Y_0 \quad \Rightarrow$$

Measure changes in CCP



# $\rho_Q$ Puzzle

Problem as a ratio:



# $\rho_Q$ Puzzle

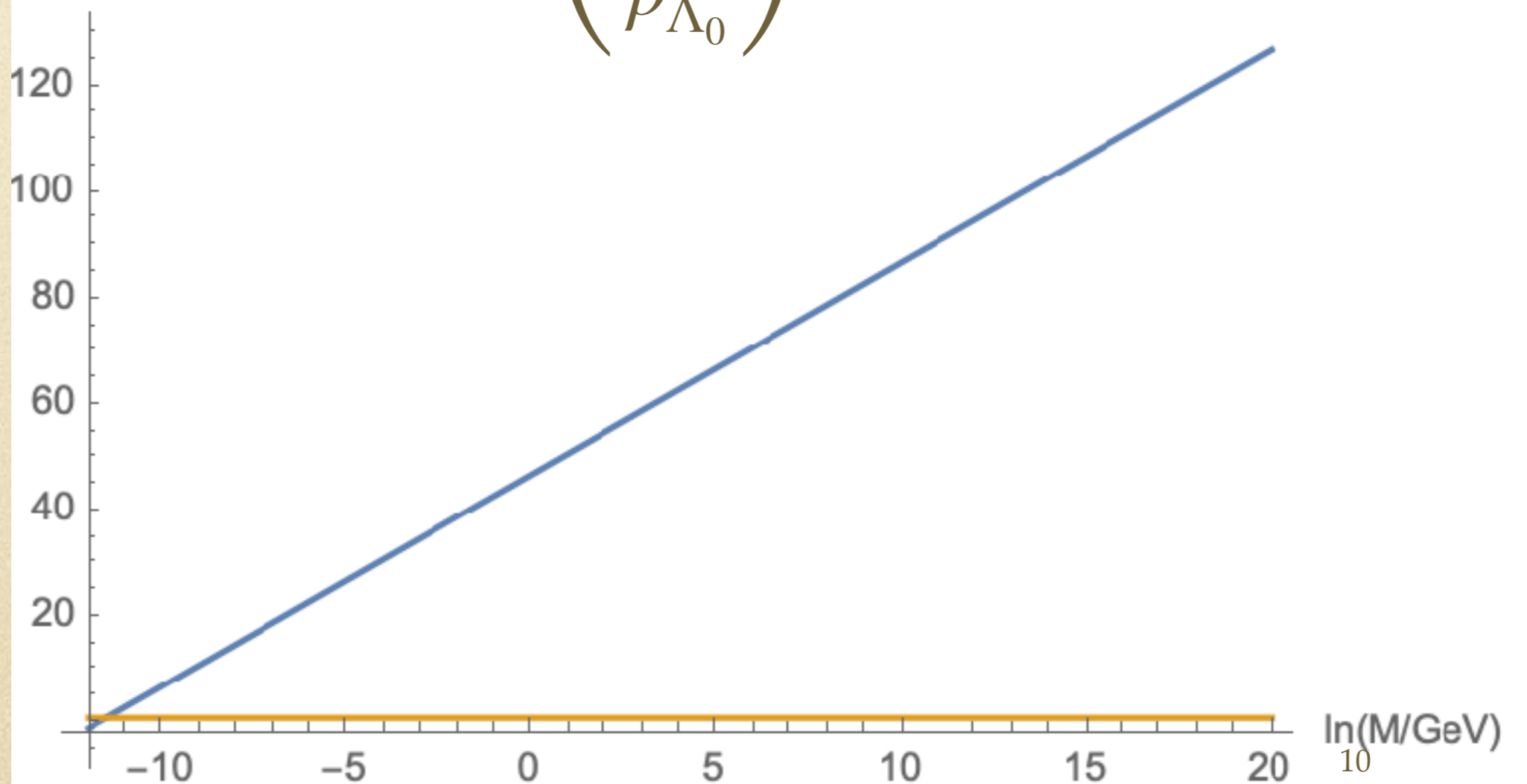
Problem as a ratio:  $\sim \ln \left( \frac{\kappa^4}{\rho_{\Lambda_0}} \right)$



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$\ln_{10}(\text{ratio})$

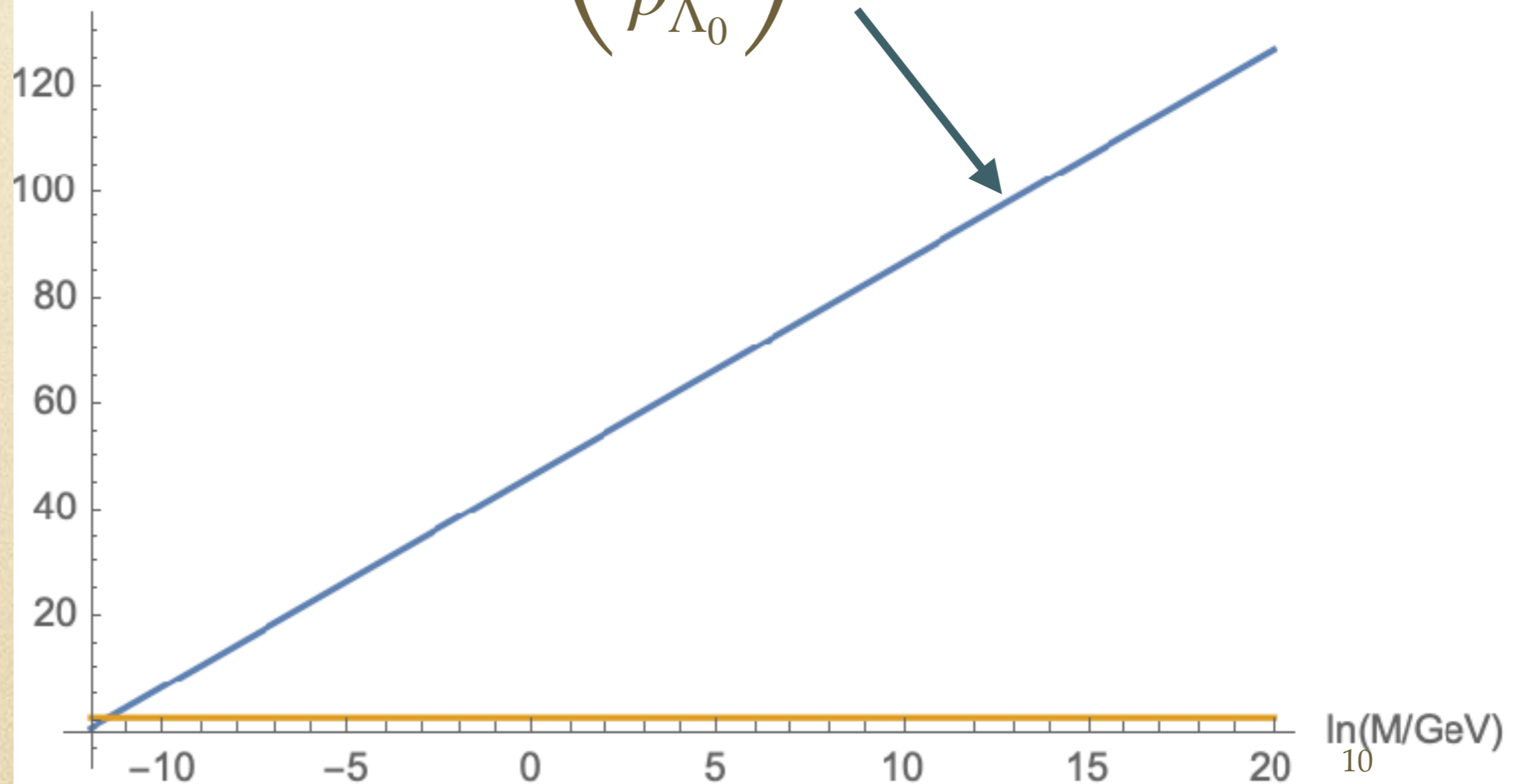




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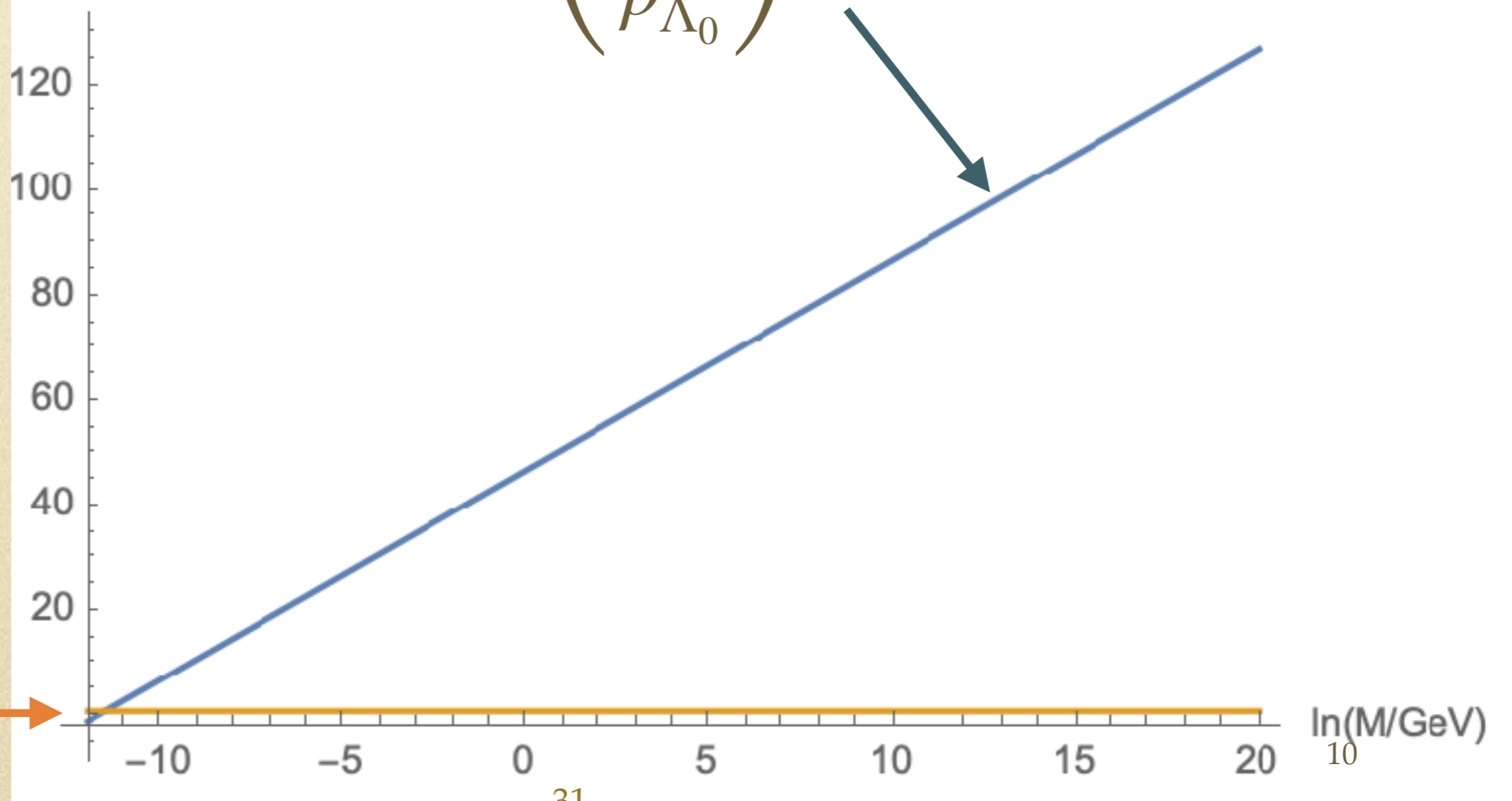




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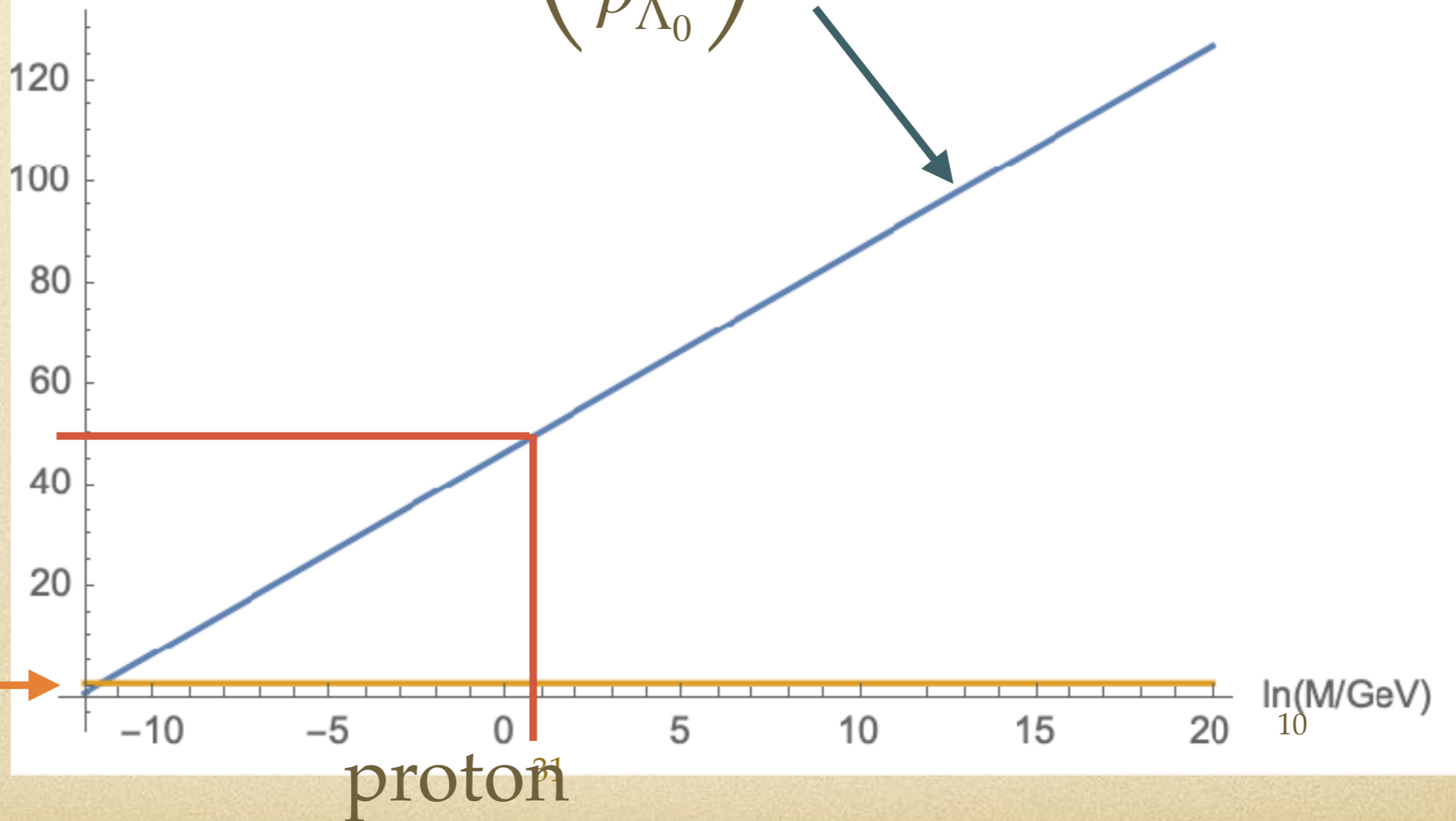
expected  
"1"



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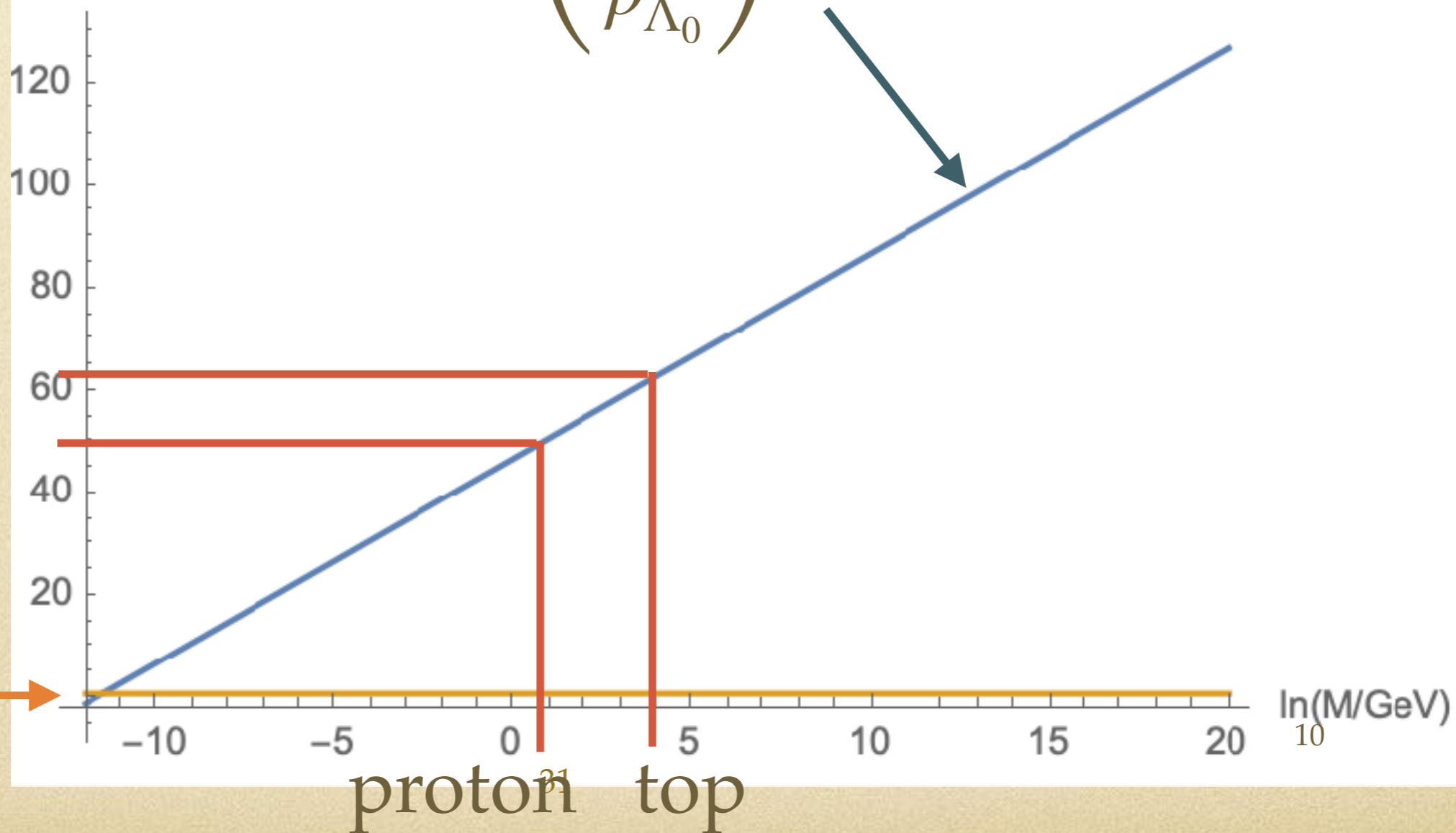




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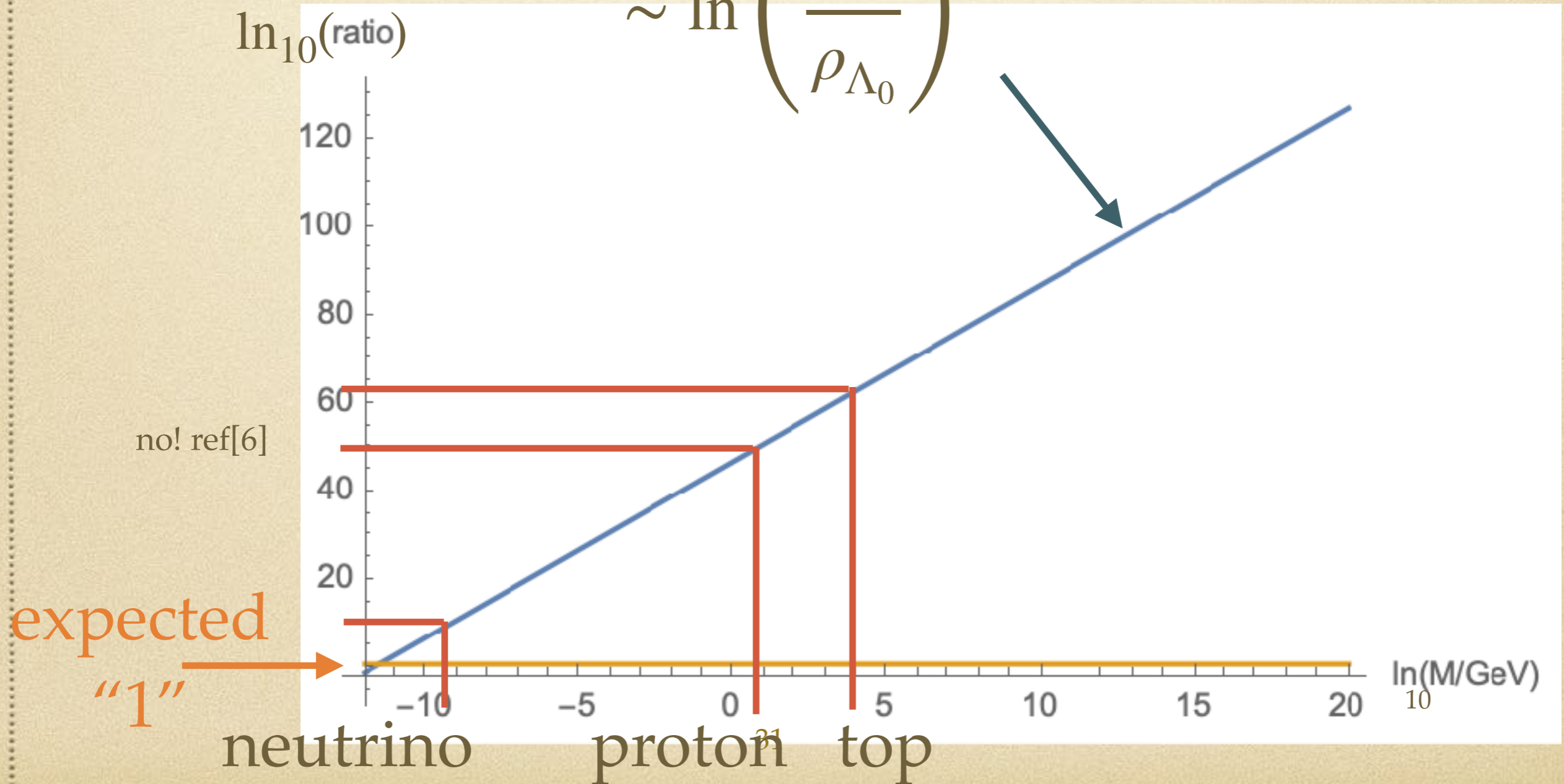
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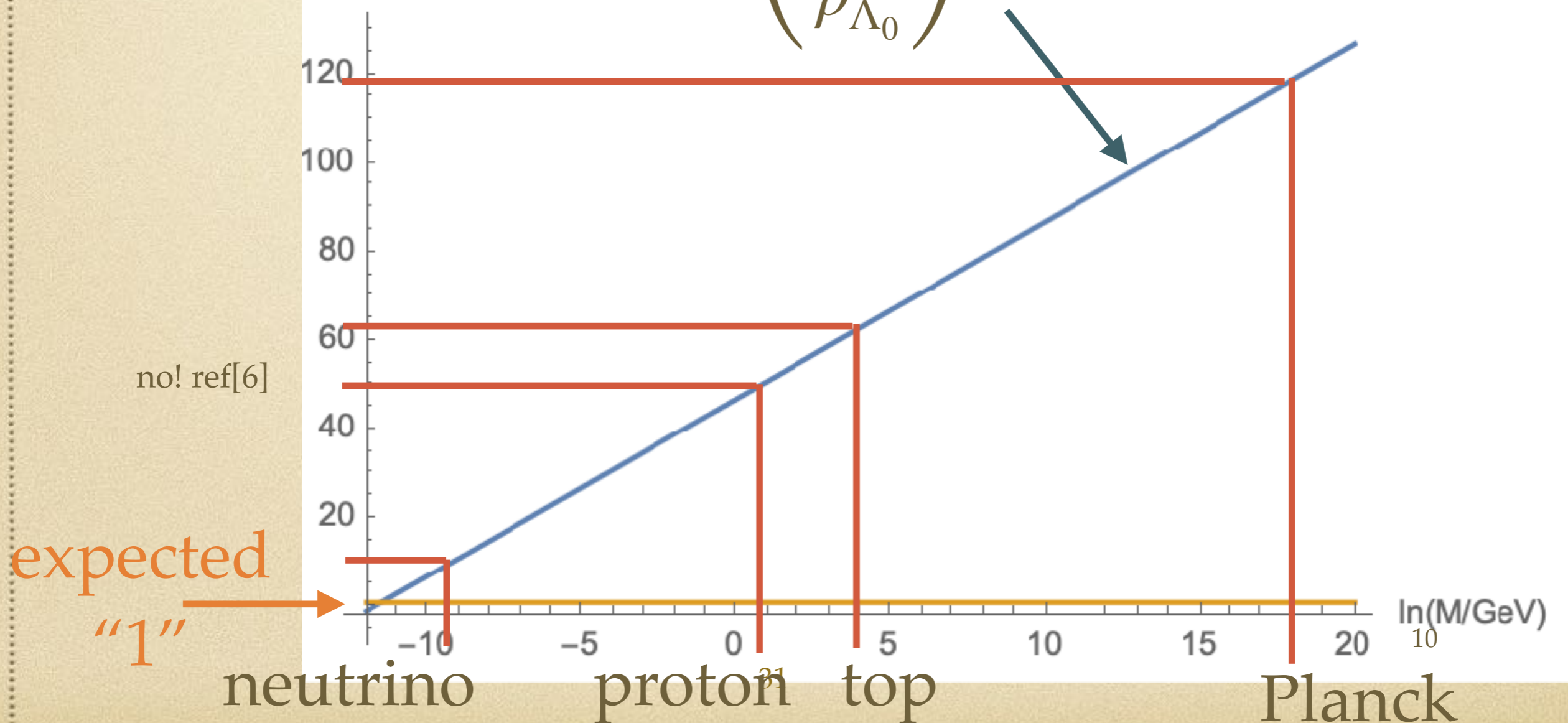




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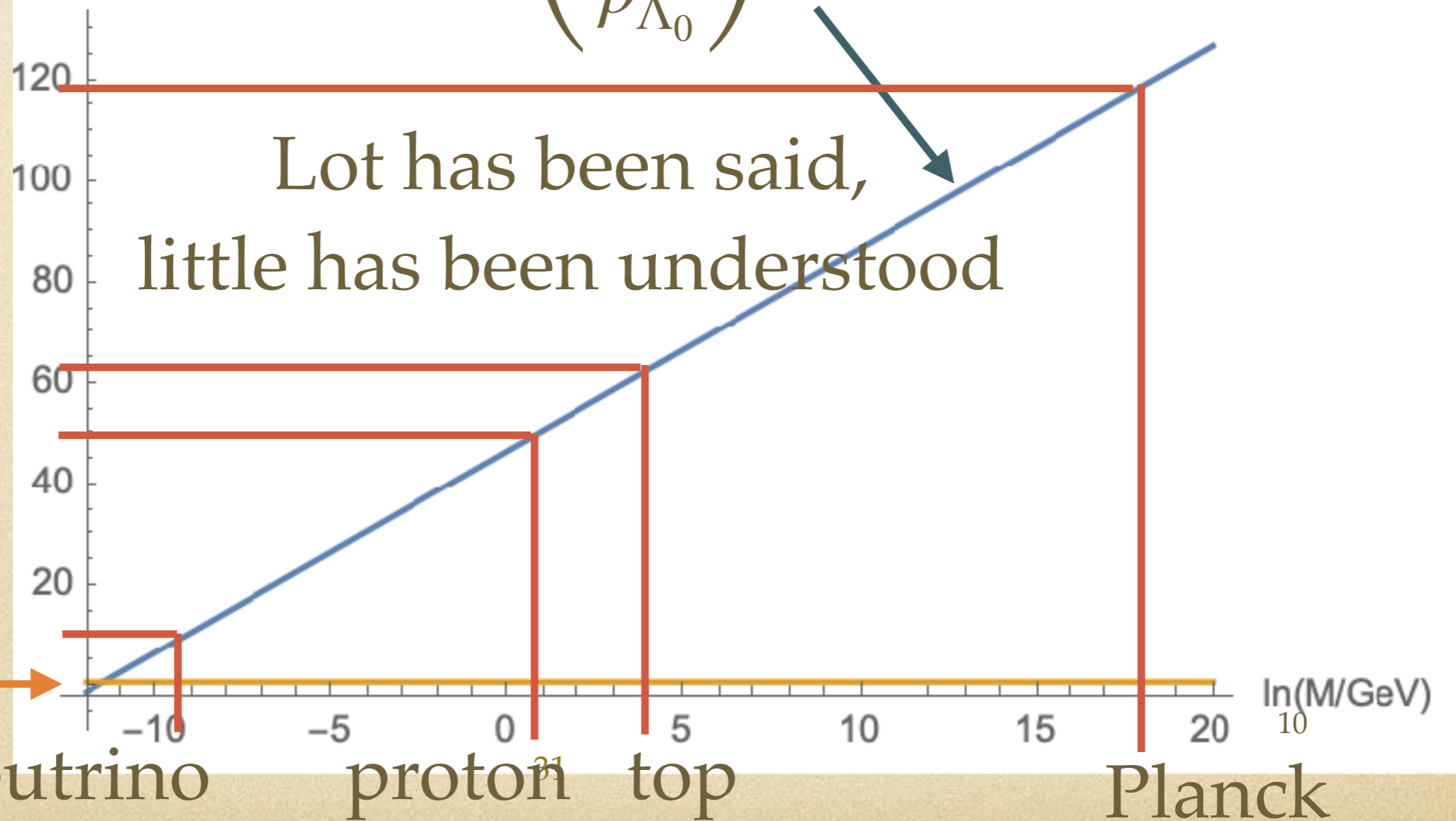
$\ln_{10}(\text{ratio})$





# $\rho_Q$ Puzzle

Problem as a ratio:  $\ln_{10}(\text{ratio}) \sim \ln \left( \frac{\kappa^4}{\rho_{\Lambda_0}} \right)$





# $\rho_Q$ Puzzle

Big theoretical puzzle

Problem as a ratio:  $\sim \ln \left( \frac{\kappa^4}{\rho_{\Lambda_0}} \right)$

$\ln_{10}(\text{ratio})$

