Second law from the Noether current on null hypersurfaces

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Motivations

Main motivations

The Noether charge is black hole entropy for stationary Killing horizons (Wald93, IyerWald94). What about the dynamical case ?

- Why the Noether charge is a good candidate for entropy ?

  Wald93, IyerWald94

- Does the Noether charge procedure give us good candidates for the entropy of dynamical black holes ?
- What about more general null hypersurfaces at finite distance ?

Based on

- Rignon-Bret, *Second law from the Noether current on null hypersurfaces*, arxiv:2303.07262, *under referee*

Balance law from the Noether current

Equations of motion

$$\delta L = \frac{\delta L}{\delta \phi}\delta\phi + d\Theta \quad (1)$$

Definition 1

The Noether current is given by $j_\xi = I_\xi \Theta - i_\xi L$.

Conservation of the Noether current

$$dj_\xi = -\frac{\delta L}{\delta \phi}\mathscr{L}_\xi \phi - \frac{\partial L}{\partial \chi}\mathscr{L}_\xi \chi \quad (2)$$

where χ is some background field (ex : the metric in flat spacetime).
The Noether current is conserved iff $\mathscr{L}_\xi \chi = 0$ and $\frac{\delta L}{\delta \phi} = 0$.

Balance law from the Noether current

Balance law from the Noether current

For ξ tangent to the boundary $\mathscr{N}$, we get

$$\Delta Q_\xi = Q_\xi^{\Sigma_2} - Q_\xi^{\Sigma_1} = \int_{\mathscr{N}} I_\xi \Theta + \int_{\mathscr{M}} \frac{\delta L}{\delta \phi} \mathscr{L}_\xi \phi \tag{3}$$

Examples

- For e.m in a background spacetime with Killing vector $\xi = \frac{\partial}{\partial t}$: $\Delta e = -\int_{\mathscr{N}} \pi_i n^i \varepsilon_{\mathscr{N}} + \int_{\mathscr{M}} J^i E_i \varepsilon_{\mathscr{M}}$ with e : e.m energy, π_i : Poynting vector, J_i : charged current, E_i : electric field.
- No background field χ, any ξ is symmetry. From Bianchi identities : $\Delta q_\xi = \int_{\mathscr{N}} I_\xi \Theta + \int_{\mathscr{N}} T_{\mu\nu} \xi^\mu n^\nu \varepsilon_{\mathscr{N}}$. If $\mathscr{N}$ null, NEC : $T_{\mu\nu} n^\mu n^\nu \geq 0$

Entropy balance law

If NEC satisfied, analogous to $\Delta S = S_{ex} + S_c = \frac{\mathscr{Q}}{T_{ext}} + S_c$ with $S_c \geq 0$

Dirichlet boundary conditions

Charges and flux Chandrasekaran-Flanagan-Prabhu,18 and Ashtekar-Khera-Kolanowski-Lewandoski,22

- Dirichlet symplectic potential : $\Theta^D = \frac{1}{16\pi G}[\sigma_n^{\mu\nu}\delta\gamma_{\mu\nu} - 2\frac{D-3}{D-2}\theta_n \delta\varepsilon_{\mathcal{N}}]$
- Weyl ST, $\xi = Wv\partial_v$, $Q^D_{Wv\partial_v}(S) = \frac{1}{8\pi G}\int_S W[1-\theta_{v\partial_v}]\varepsilon_S$

Dirichlet balance equation

The charge variation is given by $\Delta Q^D_\xi(S) = \int_{\mathcal{N}} I_\xi \Theta^D + \int_{\mathcal{N}} T_{\mu\nu}\xi^\mu n^\nu \varepsilon_{\mathcal{N}}$.

Perturbation around a Killing horizon see also Wald-Zhang,in prep and Visser-Yan,in prep

For a perturbed Killing horizon $\xi = \kappa v\partial_v$, $I_\xi \theta^D$ is of second order. At first order we find

$$\Delta Q^D_\xi = \int_{\mathcal{N}} T_{\mu\nu}\xi^\mu n^\nu \varepsilon_{\mathcal{N}} = \Delta M - \Omega_H \Delta J - \Phi_H \Delta Q = \frac{\kappa}{2\pi}\Delta S \tag{4}$$

$$S = \frac{1}{4G}\left(A - v\frac{dA}{dv}\right) \tag{5}$$

From Dirichlet flux to York flux

Issues with the Dirichlet dynamical entropy

- The usual thermodynamic relation $\delta E = T\delta S - P\delta V + ...$, analogous to the black hole first law relating the area to the ADM asymptotic charges, is properly speaking an identity that holds if we study *phase space* variations of thermodynamic functions at equilibrium. It is not a dynamical formula.
- However, we expect that a local and dynamical entropy increases in time. Furthermore, it should be positive. This is the case for the Dirichlet dynamical entropy at first and second order in perturbation around a Killing horizon, this is not true in general. For instance, it decreases and is negative at the beginning of the spherical symmetric collapse.

York boundary conditions

Charges and flux see also Hopfmuller-Freidel,18, Odak-Rignon-Bret-Speziale,in prep

Legendre transformation

- York symplectic potential : $\Theta^Y = \frac{1}{16\pi G}[\sigma_n^{\mu\nu}\delta\gamma_{\mu\nu} + 2\frac{D-3}{D-2}\varepsilon_{\mathcal{N}}\delta\theta_n]$
- Weyl ST, $\xi = Wv\partial_v$ $Q_\xi^Y(S) = \frac{1}{8\pi G}\int_S W(1 - \frac{1}{D-2}\theta_{v\partial_v})\varepsilon_S$

Master equation

Take normal $n = v\partial_v$ and take $\xi = n$ a normalized Weyl ST.

$$\Delta Q_n^Y = \frac{1}{8\pi}\int_{\Delta\mathcal{N}} \varepsilon_{\mathcal{N}}\{(D-3)[\frac{\theta_n}{D-2} - (\frac{\theta_n}{D-2})^2] + \frac{1}{D-2}\sigma_n^2\} \quad (6)$$

$$+ \frac{1}{D-2}\int_{\Delta\mathcal{N}} T_{\mu\nu}n^\mu n^\nu \varepsilon_{\mathcal{N}} \quad (7)$$

Positive flux if NEC and $0 \leq \theta_n \leq D-2$

Positive flux theorems for the York charge

Theorem 2

If $\mathscr{N}$ is a (portion of) null hypersurface with topology $B \times \mathbb{R}$ such that the geodesic congruence is future complete, if ξ is a future pointing Weyl supertranslation which vanishes on some compact (spacelike) cross sections S, and if S_1 and S_2 are two compact spacelike cross sections such that $S \prec S_1 \prec S_2$ then $\Delta_{S_2 - S_1} Q^Y_\xi \geq 0$ and $Q^Y_\xi \geq 0$

Example 3

For $\xi = 2\pi n$, we have $Q^Y_n = \frac{1}{4G}(A - \frac{v}{D-2}\frac{dA}{dv}) = S^Y$

Theorem 4

During a spherical symmetric collapse, the York charge $Q^Y_{2\pi n}$ increases on the event horizon. $Q^Y_{2\pi n} = \frac{A}{4G}$ on the stationary event horizon and $Q^Y_{2\pi n} = 0$ as long as matter and shear have not entered the BH.

Spherical symmetric collapse

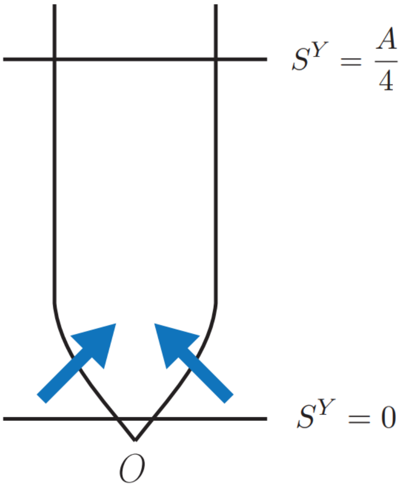

Figure 1: We set $S^Y = Q^Y_{2\pi n}$. O is a caustic located at $v = 0$.

Phase transition of the event horizon

Spherical symmetric collapse analogous to a phase transition

- θ_n is the order parameter : $\theta_n = D - 2$ on the flat light cone phase (low entropy) $\longrightarrow$ 0 on the stationary phase (high entropy)
- Enlargement of the symmetry group preserving the pullback metric $\mathscr{L}_\xi q_{\mu\nu} = 0$ in the phase of large symmetry. $SO(D-1)$ (low symmetry) $\Rightarrow SO(D-1) \times \mathbb{R}^S_W \times \mathbb{R}^S_T$ (high symmetry)
- Discontinuity of the order parameter θ_n :
$D - 2 = \lim_{u \to 0} \lim_{v \to +\infty} \theta_n \neq \lim_{v \to +\infty} \lim_{u \to 0} \theta_n = 0$

Microscopical details related to Hawking-Perry-Strominger,16

Collapse spherically symmetric on a macroscopic scale, but not in a microscopical scale $\Rightarrow$ On NEH, Weyl ST charges capture details of the collapse

$$Q^Y_{Wn} = \int_S W(x^A) \varepsilon_S \tag{8}$$

Conclusions

Summary

- The Noether charge is a good candidate for Dynamical Entropy (DE) because it satisfies a balance law similar to the entropy balance law if the NEC are imposed.
- Ambiguities in polarization give us two candidates for DE of black holes, both reduce to BH entropy in the stationary case.
- The first one satisfies the first law locally.
- The second one is positive and increases on any cross section of a null hypersurface with topology $B \times \mathbb{R}$ that is future complete. Furthermore it vanishes on Minkowski light cone.
- For a Spherical Symmetric Collapse (SSC), the entropy vanishes and does not increase before matter or shear enters the horizon.
- SSC analogous to a phase transition. In the stationary phase, supertranslations are symmetries and the charge aspect is the local area.

Thank You