A new perspective in the construction of spin foam models to quantum gravity

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- To try to give sense to

$$\int D[g_{\mu\nu}]e^{iS_{GR}[g]}$$

- Quantization of discrete BF theory$+$ simplicity constraints$=$spin foam model.
- EPRL model.
  J. Engle, E. Livine, R. Pereira and C. Rovelli, Nucl. Phys. B **799**, 136-149 (2008) [arXiv:0711.0146 [gr-qc]].
  - Most widely accepted in the loop quantum gravity community
    - Provides well defined expressions for transition amplitudes.
    - It is very successful to compute quantum effects in some physical situations.
    - It has a well defined discrete semi-classical limit.

Why a new perspective?

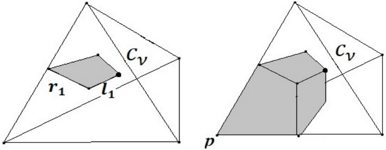
- What is the role of a tetrad field in a quantum theory of gravity?
- What is the role of the curvature in a quantum theory of gravity?
- Some concerns raised about the imposition of the simplicity constraints at the quantum level
  S. Alexandrov, Phys. Rev. D **82** (2010), 024024,
  . A possible answer was provided
  A. Perez, Papers Phys. **4**, 040004 (2012) doi:10.4279/PIP.040004 [arXiv:1205.0911 [gr-qc]].,
  is there an alternative construction avoiding these issues?

Discretization.

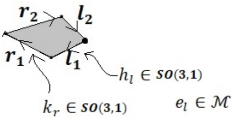
The derived complex; used in Reisenberger M. 1997 Class. Quantum Grav. 14 1753

-Oriented simplicial complex $\triangle$ in 4 dimensions and its dual $\triangle^*$.

- Wedges $s(\sigma\nu)$ and corner cells c_p



- Edges $l(\nu\tau)$ y $r(\tau\sigma)$.



Discretization of the curvature:

$$g_{\partial s} = h_{l_2}^{-1} k_{r_2}^{-1} k_{r_1} h_{l_1}$$

Action for the classical model.

- The action for the classical discrete model is:

$$S_\triangle(e,h,k) = \sum_{\nu<\triangle} \sum_{l,l',s<\nu} \mathrm{sgn}(l,l',s) e_l^K e_{l'}^L (T_i)^{JI} P_{IJKL} \theta_s^i.$$

with $\theta_s^i := \mathrm{tr}\left[T^i g_{\partial s}\right]$,
$\theta_s^i (T_i)^I{}_J$ a discretization of $F^I{}_J$ and
$P_{IJKL} := \epsilon_{IJKL} - \frac{1}{2\gamma}\epsilon_{IJMN}\epsilon^{MN}{}_{KL}$

- $S_\triangle$ converges to

$$S_H(e,\omega) = \int_M \Sigma_i(e) \wedge F^i(\omega)$$

which is a way to rewrite the Holst action

$$S_H(e,\omega) = \frac{1}{2}\int_M \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}(\omega) + \frac{1}{\gamma}\int_M e_I \wedge e_J \wedge F^{IJ}(\omega)$$

- This is the $e-$model.

Characteristics of the e-model

- Basic fields: e_l, h_l, k_r
- A discrete tetrad field for each corner cell c_P.
- A discrete metric for c_P.

$$g_{l_1 l_2}(c_P) = e^I_{l_1} e^J_{l_2} \eta_{IJ} \text{ for any } l_1, l_2 < c_P$$

- The corners have pairwise compatible metrics.
- No simplicity constrains.
- We can write functions of the space of histories; geometric quantities like areas, volumes, 4-volumes in the bulk.

Formulation as a constrained BF theory

- Action for the model

$$S_\Delta(B,h,k) = \sum_{\nu<\Delta} \sum_{s,s'<\nu} \mathrm{sgn}(s,s') \left(B_{si} - \frac{1}{\gamma} * B_{si} \right) \theta^i_{s'}$$

- If we impose the conditions:
  - Given ν, for each line $l < \nu$, there is a non-zero vector n, such that

$$n_{II} B^{IJ}_s = 0 \text{ for each s with } l < s$$

  -

$$\mathrm{sgn}(s,s')\epsilon_{IJKL} B^{IJ}_s \wedge B^{KL}_{s'} = \mathrm{sgn}(s'',s''')\epsilon_{IJKL} B^{IJ}_{s''} \wedge B^{KL}_{s'''}$$

  -

$$g_{QR}(c_P) = \frac{1}{12V(c_P)} f_{ijk}\epsilon^{ABCDP} B^i_{QA} B^j_{BC} B^k_{DR}$$

    with the same signature than η,

  Then the exist a tetrad field for each corner cell, such that $B_{PQ} = *(e_P \wedge e_Q)$ (in the non-degenerate sector).
- This defines the B-model.
- Both the e-model and the B-model are equivalent.

Characteristics of the B-model.

- Basic fields: B_s, h_l, k_r
- A metric density for each corner (following Urbantke)

$$\tilde{g}_{QR}(c_P) = \frac{1}{12} f_{ijk} \epsilon^{ABCDP} B^i_{QA} B^j_{BC} B^k_{DR}$$

- Constrained BF theory $\rightarrow$ standard spinfoam quantization.

Shared characteristics

- Boundary data: Boundary connection k_r and its momenta $u_\sigma := \frac{\delta S}{\delta k_r}$
- Well defined bulk and boundary geometry$\rightarrow$ Study correlation of geometric observables.
- There is a natural way to regularize matter couplings.
- Each 4-simplex is curved;
  - Standard Regge discretization$\rightarrow$Every simplex if flat
  - Studies with more simplices are very recent
    P. Donà, F. Gozzini and G. Sarno, Phys. Rev. D **102**, no.10, 106003 (2020) doi:10.1103 [arXiv:2004.12911 [gr-qc]].
  - In the quantization of our models one 4-simplex includes curvature.

Spin foam quantization of the B-model.

- Partition function

$$\mathcal{Z}(\Delta) = \int \prod_{r<\Delta} dk_r \prod_{\nu<\Delta} A_\nu(k_r)$$

with

$$A_\nu(k_r) := \int \prod_{l<\nu} dh_l \prod_{s,\bar{s}} dB_s \exp\left[i \operatorname{sgn}(s,\bar{s}) B_{si} \theta^i_{\bar{s}} \right]$$

the vertex amplitude.

Spin foam quantization of the B-model.

- For the BF part que have:

$$A_\nu(k_r) = \int \prod_{l<\nu} dh_l \prod_{s<\nu} \sum_\rho d_\rho \mathrm{tr} D^\rho \left[\prod_{\bar{s}<\nu} g_{\partial \bar{s}}^{\mathrm{sgn}(s,\bar{s})} \right],$$

  with $\rho \to$ irreducible representation of G.

- For example, for $G = SO(4)$ we can recover the standard quantization of BF theory.
- Implementation of the constrains$\to$ requires to identify a "corner amplitude": A_{cP}, and glue them to recover $A_\nu(k_r)$
  - Classical constraints$\to$ the u are simple only on each corner.

Future perspectives

- How to implement the constraints of the B-model in the quantum regime?
- Quantization of the e-model.
- What is the relation between the quantum e and B-model?
- The role of the matter coupling using the metric.
- What are the effects of the curvature in the quantum regime?
- There is another B-model not mentioned here, but potentially useful for (even more efficient) numerical simulations.
  S. K. Asante, B. Dittrich and J. Padua-Arguelles, Class. Quant. Grav. **38**, no.19, 195002 (2021) [arXiv:2104.00485 [gr-qc]].

For more details of the classical part:

Beltrán, C.E., Zapata, J.A. A discretization of Holst's action for general relativity. Gen Relativ Gravit 55, 77 (2023) arXiv:2208.13808 [gr-qc]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