Towards the phase structure of first-order Lorentzian Palatini quantum gravity via Group Field Theory

Andreas Pithis (LMU, ASC & MCQST)
in collaboration with
A. Jercher, L. Marchetti, D. Oriti and J. Thürigen

mostly **based on** arXiv:
2211.12768 *(PRL 130 (2023) 141501)*,
2209.04297 *(JHEP 02 (2023) 074)*,
2206.15442 *(PRD 106, 066019, 2022)*,
2112.00091 *(JCAP 01 (2022) 01, 050)*,
2110.15336 *(JHEP 2021, 201 (2021))*,
1904.00598 *(PRD 98, 126006 (2018))*
& **wip**

July 14th, 2023

Quantum Gravity 2023 conference,
Radboud University,
Nijmegen, The Netherlands

Funded by DFG Deutsche Forschungsgemeinschaft German Research Foundation

Outline

- Lorentzian Barrett-Crane GFT model
- Landau-Ginzburg mean-field theory applied to the BC GFT model
- Conclusions

What is the Lorentzian Barrett-Crane GFT model?

★ **It is a model for 4d Lorentzian quantum gravity.**

- **Gravitational theory: Lorentzian Palatini gravity in first-order formulation**

- **Quantization technique: Lattice path integral**

What is the Lorentzian Barrett-Crane GFT model?

★ **It is a model for 4d Lorentzian quantum gravity.**

- **Gravitational theory: Lorentzian Palatini gravity in first order formulation**

- **Quantization technique: Lattice path integral**

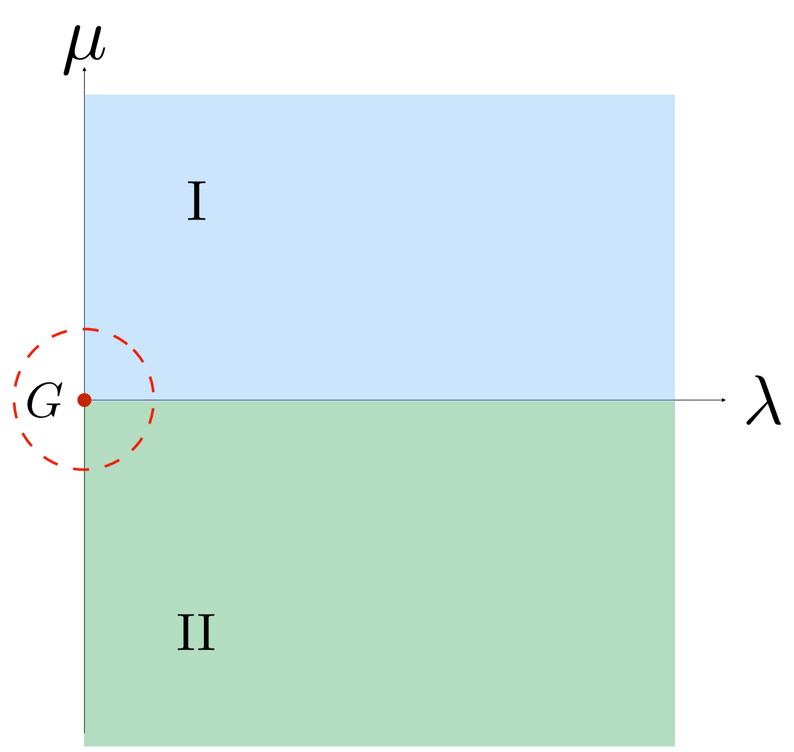
- **Classical level: Palatini gravity in first-order formulation via BF-theory plus constraints**

- **Quantum level: Impose constraints onto GFT quantization of BF-theory**

What is Landau-Ginzburg theory?

★ **It is a method to describe phase transitions at mean-field level.**

- **Approximate evaluation of** $\boxed{Z = \int \mathcal{D}\varphi \mathrm{e}^{-S[\varphi]}}$
- **Provides coarse account of phase structure of field theories**



phase transition

$$\langle \varphi_0 \rangle = 0 \longleftrightarrow \langle \varphi_0 \rangle \neq 0$$

What is Landau-Ginzburg theory?

★ **It is a method to describe phase transitions at mean-field level.**

- **Approximate evaluation of** $\boxed{Z = \int \mathcal{D}\varphi \mathrm{e}^{-S[\varphi]}}$
- **Provides coarse account of phase structure of field theories**

Why apply Landau-Ginzburg theory to GFT?

★ **Substantiate idea of emergence of continuum spacetime ~ control non-perturbative sector**

- **Exploit field theory properties of GFT**
- **Map phases/phase structure of GFT models**
- **Describe transition to condensate phase with non-trivial VEV (non-perturbative vacuum)**

What is Landau-Ginzburg theory?

★ **It is a method to describe phase transitions at mean-field level.**

- **Approximate evaluation of** $\boxed{Z = \int \mathcal{D}\varphi \mathrm{e}^{-S[\varphi]}}$
- **Provides coarse account of phase structure of field theories**

Why apply Landau-Ginzburg theory to GFT?

★ **Substantiate idea of emergence of continuum spacetime ~ control non-perturbative sector**

- **Exploit field theory properties of GFT**
- **Map phases/phase structure of GFT models**
- **Describe transition to condensate phase with non-trivial VEV (non-perturbative vacuum)**
    - **Relevant for problem of the continuum limit in GFT/spin foam models/simplicial gravity**

BF-theory and GFT

BF-theory:

$$S[\omega, B] = \int B_{IJ} \wedge F^{IJ}(\omega)$$

$\mathfrak{g}-$ valued 2-form $\mathrm{Lie}(G) \cong \mathfrak{g}$ field strength: $F^{IJ}(\omega) = \mathrm{d}\omega^{IJ} + \omega^I_J \wedge \omega^{KJ}$ $\mathfrak{g}-$ valued connection 1-form

$$Z = \int \mathcal{D}\omega\mathcal{D}B\mathrm{e}^{iS[\omega,B]} = \int \mathcal{D}\omega\delta(F(\omega))$$

(integral over flat connections, i.e. no local dof)

(but: volume of space of flat connections infinitely large!)

- **ill-defined in the continuum** $\xrightarrow{\text{resort to}}$ **quantization on a regulating lattice structure**

BF-theory and GFT

BF-theory:

$$S[\omega, B] = \int B_{IJ} \wedge F^{IJ}(\omega)$$

$\mathfrak{g}$ – valued 2-form $\mathrm{Lie}(G) \cong \mathfrak{g}$ field strength: $F^{IJ}(\omega) = \mathrm{d}\omega^{IJ} + \omega^I_J \wedge \omega^{KJ}$ $\mathfrak{g}$ – valued connection 1-form

$$Z = \int \mathcal{D}\omega \mathcal{D}B e^{iS[\omega,B]} = \int \mathcal{D}\omega \delta(F(\omega))$$

(integral over flat connections, i.e. no local dof)

(but: volume of space of flat connections infinitely large!)

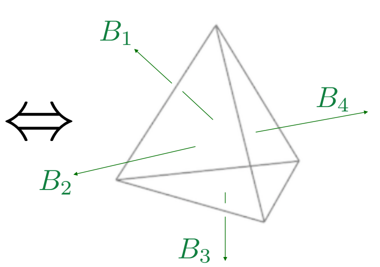
▸ **ill-defined in the continuum** $\xrightarrow{\text{resort to}}$ **quantization on a regulating lattice structure**

via Ooguri GFT model:

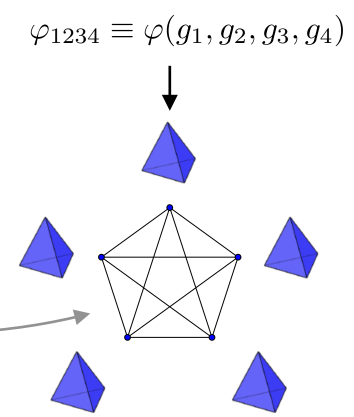
[Ooguri 9205090]

$$\varphi(g_1, g_2, g_3, g_4) = \varphi(g_1 h, g_2 h, g_3 h, g_4 h), \quad g_i, h \in G$$

▸ **invariance of field corresponds to closure of bi-vectors** $B_i \in \mathfrak{g}$ **to form tetrahedron:** $\sum_{i=1}^{4} B_i = 0 \Leftrightarrow$



$$S = \int (\mathrm{d}g)^4 |\varphi_{1234}|^2 + \frac{\lambda}{5!} \int (\mathrm{d}g)^{10} \varphi_{1234}\varphi_{4567}\varphi_{7389}\varphi_{962(10)}\varphi_{(10)851} + \text{c.c.}$$



combinatorics of a 4-simplex

BF-theory and GFT

BF-theory:

$$S[\omega, B] = \int B_{IJ} \wedge F^{IJ}(\omega)$$

$\mathfrak{g}-$ valued 2-form
$\mathrm{Lie}(G) \cong \mathfrak{g}$

field strength: $F^{IJ}(\omega) = \mathrm{d}\omega^{IJ} + \omega^I_J \wedge \omega^{KJ}$

$\mathfrak{g}-$ valued connection 1-form

$$Z = \int \mathcal{D}\omega\mathcal{D}B e^{iS[\omega,B]} = \int \mathcal{D}\omega\delta(F(\omega))$$

(integral over flat connections, i.e. no local dof)

(but: volume of space of flat connections infinitely large!)

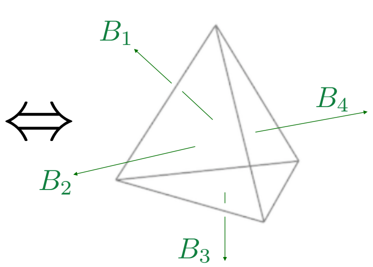
- **ill-defined in the continuum** $\xrightarrow{\text{resort to}}$ **quantization on a regulating lattice structure**

via Ooguri GFT model:

[Ooguri 9205090]

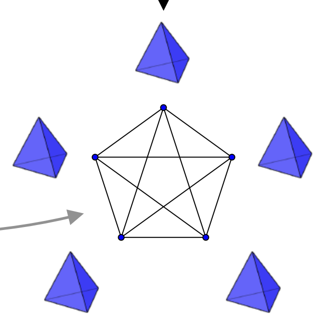
$$\varphi(g_1, g_2, g_3, g_4) = \varphi(g_1 h, g_2 h, g_3 h, g_4 h), \quad g_i, h \in G$$

- **invariance of field corresponds to closure of bi-vectors** $B_i \in \mathfrak{g}$ **to form tetrahedron:** $\sum_{i=1}^{4} B_i = 0 \Leftrightarrow$



$\varphi_{1234} \equiv \varphi(g_1, g_2, g_3, g_4)$

$$S = \int (\mathrm{d}g)^4 |\varphi_{1234}|^2 + \frac{\lambda}{5!} \int (\mathrm{d}g)^{10} \varphi_{1234}\varphi_{4567}\varphi_{7389}\varphi_{962(10)}\varphi_{(10)851} + \mathrm{c.c.}$$



combinatorics of a 4-simplex

# of vertices

$$Z_{\mathrm{GFT}} = \int \mathcal{D}\varphi\mathcal{D}\bar{\varphi}e^{-S_{\mathrm{GFT}}[\varphi,\bar{\varphi}]} = \sum_{\Gamma} \frac{\lambda^{V_\Gamma}}{\mathrm{Sym}(\Gamma)} \mathcal{A}_\Gamma$$

GFT Feynman amplitude $\mathcal{A}_\Gamma$

Γ graph dual to triangulation (lattice)

BF-theory and GFT

BF-theory:

$$S[\omega, B] = \int B_{IJ} \wedge F^{IJ}(\omega)$$

$\mathfrak{g}$ – valued 2-form, $\mathrm{Lie}(G) \cong \mathfrak{g}$

field strength: $F^{IJ}(\omega) = \mathrm{d}\omega^{IJ} + \omega^I_J \wedge \omega^{KJ}$

$\mathfrak{g}$ – valued connection 1-form

$$Z = \int \mathcal{D}\omega\mathcal{D}B e^{iS[\omega,B]} = \int \mathcal{D}\omega\delta(F(\omega))$$

(integral over flat connections, i.e. no local dof)

(but: volume of space of flat connections infinitely large!)

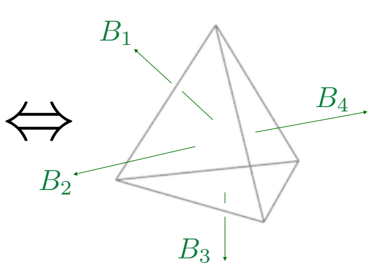
- **ill-defined in the continuum** $\xrightarrow{\text{resort to}}$ **quantization on a regulating lattice structure**

via Ooguri GFT model:

[Ooguri 9205090]

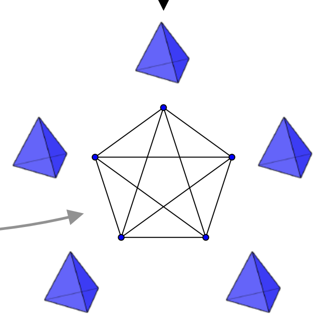
$$\varphi(g_1, g_2, g_3, g_4) = \varphi(g_1 h, g_2 h, g_3 h, g_4 h), \quad g_i, h \in G$$

- **invariance of field corresponds to closure of bi-vectors** $B_i \in \mathfrak{g}$ **to form tetrahedron:** $\sum_{i=1}^{4} B_i = 0 \Leftrightarrow$



$$S = \int (\mathrm{d}g)^4 |\varphi_{1234}|^2 + \frac{\lambda}{5!} \int (\mathrm{d}g)^{10} \varphi_{1234}\varphi_{4567}\varphi_{7389}\varphi_{962(10)}\varphi_{(10)851} + \mathrm{c.c.}$$

$\varphi_{1234} \equiv \varphi(g_1, g_2, g_3, g_4)$



combinatorics of a 4-simplex

$$Z_{\mathrm{GFT}} = \int \mathcal{D}\varphi\mathcal{D}\bar{\varphi} e^{-S_{\mathrm{GFT}}[\varphi,\bar{\varphi}]} = \sum_{\Gamma} \frac{\lambda^{V_\Gamma}}{\mathrm{Sym}(\Gamma)} \mathcal{A}_\Gamma$$

# of vertices

GFT Feynman amplitude $\mathcal{A}_\Gamma$

Γ graph dual to triangulation (lattice)

- **improvement of expansion by using coloured GFTs: no topological pathologies** [Gurau 0907.2582, 0911.1945, 1006.0714,...]

Bring in gravity (classical level)

- **via constrained BF-theory:**

$$S[\omega, B, \mu] = \int \left[B_{IJ} \wedge F^{IJ}(\omega) + \frac{1}{2} \underset{\substack{\downarrow \\ \text{Lagrange multiplier}}}{\mu_{IJKL}} B^{IJ} \wedge B^{KL} \right]$$

- $G = \mathrm{SL}(2, \mathbb{C})$
- variation wrt μ $\longrightarrow$ "simplicity constraint" on B:

$$B^{IJ} \wedge B^{KL} = e\epsilon^{IJKL}, \quad e = \frac{1}{4!} \epsilon_{IJKL} B^{IJ} \wedge B^{KL}$$

- solve for B $\longrightarrow$ solutions in two sectors: (1) topological sector vs.

  (2) gravitational sector (Palatini)

  first-order formulation

Bring in gravity (classical level)

- **via constrained BF-theory:**

$$S[\omega, B, \mu] = \int \left[B_{IJ} \wedge F^{IJ}(\omega) + \frac{1}{2}\underset{\substack{\downarrow \\ \text{Lagrange multiplier}}}{\mu_{IJKL}} B^{IJ} \wedge B^{KL} \right]$$

- $G = \mathrm{SL}(2, \mathbb{C})$
- variation wrt μ $\longrightarrow$ "simplicity constraint" on B:

$$B^{IJ} \wedge B^{KL} = e\epsilon^{IJKL}, \quad e = \frac{1}{4!}\epsilon_{IJKL}B^{IJ} \wedge B^{KL}$$

- solve for B $\longrightarrow$ solutions in two sectors: (1) topological sector vs.
  (2) gravitational sector (Palatini)
  first-order formulation

- **imposition of simplicity constraint:**

$$\longrightarrow \; S_{\text{Palatini}}[e, \omega] = \frac{1}{2}\int \epsilon_{IJKL} e^I \wedge \underset{\searrow\,\text{tetrad field}}{e^J} \wedge F^{KL}$$

$$\delta_e S = 0 \rightarrow \text{Einstein field eqns.} \qquad \delta_\omega S = 0 \rightarrow \text{1st Cartan: } \mathrm{d}_\omega e^I + \underset{\searrow\,\text{spin connection}}{\omega^I_J} \wedge e^J = 0$$

Bring in gravity (Barrett-Crane GFT model)

[Barrett, Crane 9904025; Perez, Rovelli 0009021 & 0011037;
Oriti, Baratin 1108.1178;
Jercher, Oriti, Pithis 2112.00091 & 2206.15442]

- **add to domain non-dynamical timelike, spacelike or lightlike normal vector** X_α

$$\varphi(g_1, ..., g_4; X_\alpha) : \mathrm{SL}(2,\mathbb{C})^4 \times \mathrm{SL}(2,\mathbb{C})/U^{(\alpha)} \to \mathbb{C} \quad \text{with} \quad \alpha \in \{+, 0, -\}$$

$U^{(+)} = \mathrm{SU}(2),\ U^{(0)} = \mathrm{ISO}(2),\ U^{(-)} = \mathrm{SU}(1,1)$ **stabilizers of normal vectors** $X_+ = (1,0,0,0)$, (timelike) $X_0 = \frac{1}{\sqrt{2}}(1,0,0,1)$, (lightlike) $X_- = (0,0,0,1)$ (spacelike)

homogeneous spaces:

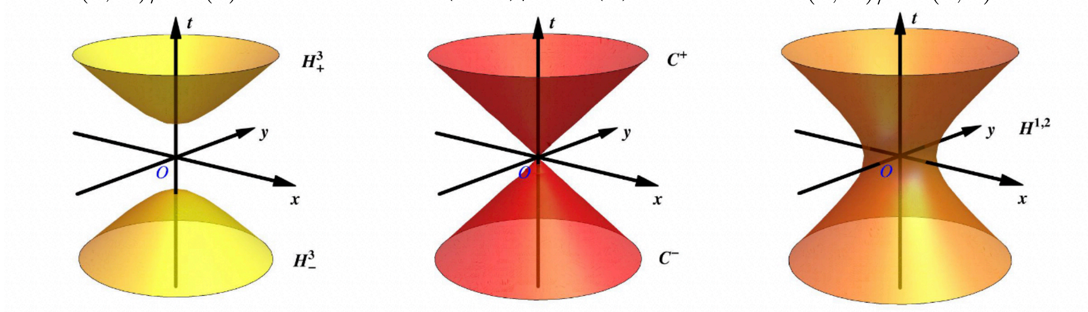
distinguished hypersurfaces in Minkowski space

$y \in \mathbb{R}^{1,3}$

$a:$ skirt radius

z-axis suppressed

$\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2) \cong \mathbb{H}^3$ $\quad$ $\mathrm{SL}(2,\mathbb{C})/\mathrm{ISO}(2) \cong C$ $\quad$ $\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(1,1) \cong \mathbb{H}^{1,2}$



$y^\mu y_\mu = a^2$ $\quad$ $y^\mu y_\mu = 0$ $\quad$ $y^\mu y_\mu = -a^2$

$X_+ = (1,0,0,0) \in \mathbb{H}^3$ $\quad$ $X_0 = \frac{1}{\sqrt{2}}(1,0,0,1) \in C$ $\quad$ $X_- = (0,0,0,1) \in \mathbb{H}^{1,2}$

Bring in gravity (Barrett-Crane GFT model)

[Barrett, Crane 9904025; Perez, Rovelli 0009021 & 0011037; Oriti, Baratin 1108.1178; Jercher, Oriti, Pithis 2112.00091 & 2206.15442]

- **add to domain non-dynamical timelike, spacelike or lightlike normal vector X_α**

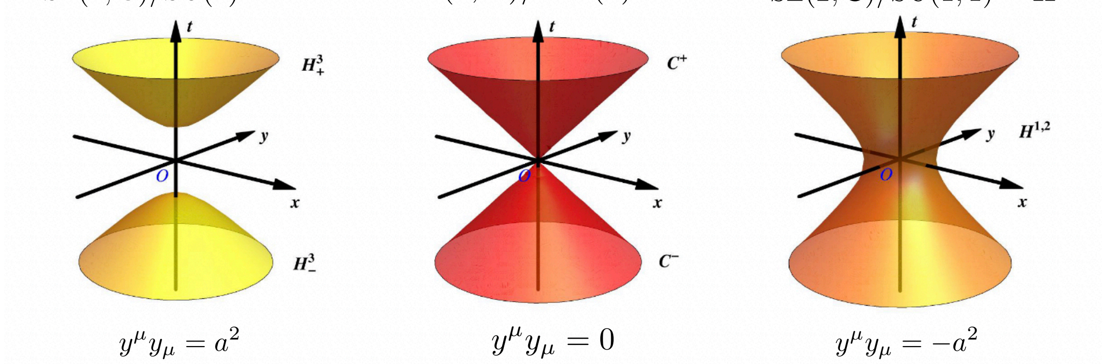
$$\varphi(g_1, ..., g_4; X_\alpha) : \mathrm{SL}(2,\mathbb{C})^4 \times \mathrm{SL}(2,\mathbb{C})/U^{(\alpha)} \to \mathbb{C} \quad \text{with} \quad \alpha \in \{+, 0, -\}$$

$U^{(+)} = \mathrm{SU}(2),\ U^{(0)} = \mathrm{ISO}(2),\ U^{(-)} = \mathrm{SU}(1,1)$ **stabilizers of normal vectors** $X_+ = (1,0,0,0)$, $X_0 = \frac{1}{\sqrt{2}}(1,0,0,1)$, $X_- = (0,0,0,1)$
timelike, lightlike, spacelike

homogeneous spaces:

distinguished hypersurfaces in Minkowski space
$y \in \mathbb{R}^{1,3}$
$a :$ skirt radius
z-axis suppressed

$\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2) \cong \mathbb{H}^3$ $\quad$ $\mathrm{SL}(2,\mathbb{C})/\mathrm{ISO}(2) \cong C$ $\quad$ $\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(1,1) \cong \mathbb{H}^{1,2}$



$y^\mu y_\mu = a^2$ $\quad$ $y^\mu y_\mu = 0$ $\quad$ $y^\mu y_\mu = -a^2$

$X_+ = (1,0,0,0) \in \mathbb{H}^3$ $\quad$ $X_0 = \frac{1}{\sqrt{2}}(1,0,0,1) \in C$ $\quad$ $X_- = (0,0,0,1) \in \mathbb{H}^{1,2}$

<u>properties of the field</u>:

$$1)\ \varphi(g_1, ..., g_4; X_\alpha) = \varphi(g_1 h^{-1}, ..., g_4 h^{-1}; h \cdot X_\alpha), \quad \forall h \in \mathrm{SL}(2,\mathbb{C})$$ **(closure, already needed in Ooguri model)**

$$2)\ \varphi(g_1, ..., g_4; X_\alpha) = \varphi(g_1 u_1, ..., g_4 u_4; X_\alpha), \quad \forall u_1, ..., u_4 \in \mathrm{U}_{X_\alpha}$$ **(simplicity constraint)**

<u>geometric interpretation:</u>

➡Go to bi-vector representation

$$2) \Leftrightarrow \quad X_A (*B)^{AB} = 0 \longrightarrow$$

Lorentz index $A \in \{0,1,2,3\}$

- **fields correspond to spacelike, timelike and lightlike tetrahedra**
- **turns the model into one for 4d Lorentzian quantum gravity (first-order Palatini gravity)**

Bring in gravity (Barrett-Crane GFT model)

[Barrett, Crane 9904025; Perez, Rovelli 0009021 & 0011037; Oriti, Baratin 1108.1178; Jercher, Oriti, Pithis 2112.00091 & 2206.15442]

- **add to domain non-dynamical timelike, spacelike or lightlike normal vector X_α**

$$\varphi(g_1,...,g_4;X_\alpha):\mathrm{SL}(2,\mathbb{C})^4\times \mathrm{SL}(2,\mathbb{C})/U^{(\alpha)}\to\mathbb{C} \quad \text{with} \quad \alpha\in\{+,0,-\}$$

$U^{(+)}=\mathrm{SU}(2),\ U^{(0)}=\mathrm{ISO}(2),\ U^{(-)}=\mathrm{SU}(1,1)$ **stabilizers of normal vectors** $X_+=(1,0,0,0)$ (timelike), $X_0=\frac{1}{\sqrt{2}}(1,0,0,1)$ (lightlike), $X_-=(0,0,0,1)$ (spacelike)

homogeneous spaces:

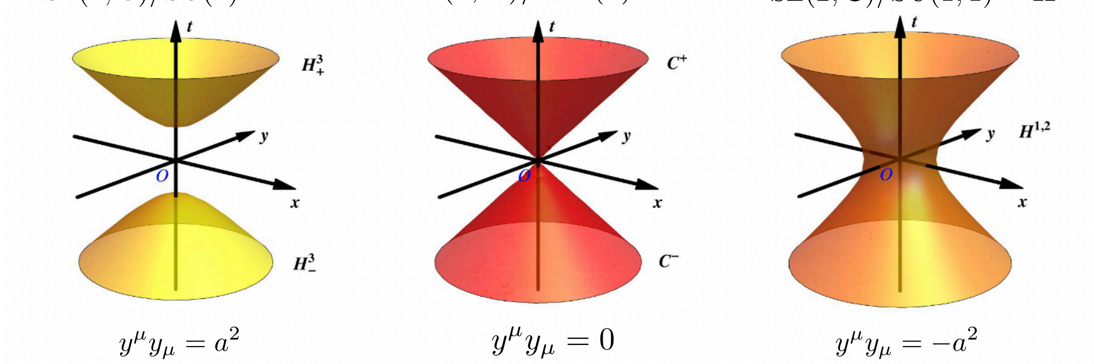
distinguished hypersurfaces in Minkowski space

$y\in\mathbb{R}^{1,3}$

$a:$ skirt radius

z-axis suppressed

$\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2)\cong\mathbb{H}^3$ $\quad$ $\mathrm{SL}(2,\mathbb{C})/\mathrm{ISO}(2)\cong C$ $\quad$ $\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(1,1)\cong\mathbb{H}^{1,2}$



$y^\mu y_\mu=a^2$ $\quad$ $y^\mu y_\mu=0$ $\quad$ $y^\mu y_\mu=-a^2$

$X_+=(1,0,0,0)\in\mathbb{H}^3$ $\quad$ $X_0=\frac{1}{\sqrt{2}}(1,0,0,1)\in C$ $\quad$ $X_-=(0,0,0,1)\in\mathbb{H}^{1,2}$

properties of the field:

$$1)\ \varphi(g_1,...,g_4;X_\alpha)=\varphi(g_1h^{-1},...,g_4h^{-1};h\cdot X_\alpha),\quad \forall h\in\mathrm{SL}(2,\mathbb{C})$$ **(closure, already needed in Ooguri model)**

$$2)\ \varphi(g_1,...,g_4;X_\alpha)=\varphi(g_1u_1,...,g_4u_4;X_\alpha),\quad \forall u_1,...,u_4\in\mathrm{U}_{X_\alpha}$$ **(simplicity constraint)**

geometric interpretation:

➡Go to bi-vector representation

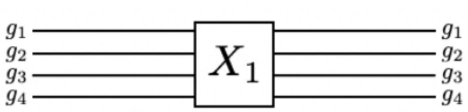
$$2)\ \Leftrightarrow\ X_A(*B)^{AB}=0 \longrightarrow$$

Lorentz index $A\in\{0,1,2,3\}$

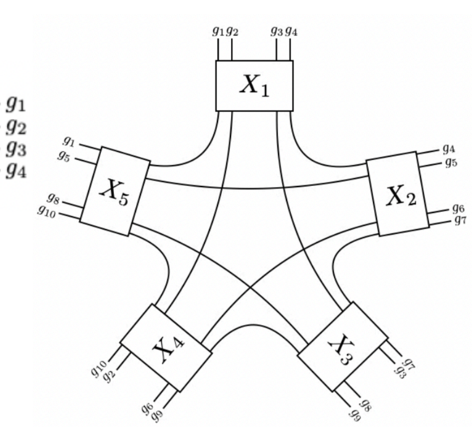
- **fields correspond to spacelike, timelike and lightlike tetrahedra**
- **turns it into a model for 4d Lorentzian quantum gravity (first-order Palatini gravity)**

action: $S[\varphi,\bar\varphi]=K[\varphi,\bar\varphi]+V[\varphi,\bar\varphi]$

$$K[\varphi,\bar\varphi]=\sum_{\substack{\alpha\\ \alpha\in\{+,0,-\}}}\int_{\mathrm{SL}(2,\mathbb{C})^4}(\mathrm{d}g)^4\int_{\mathrm{SL}(2,\mathbb{C}/\mathrm{U}^{(\alpha)}}\mathrm{d}X_\alpha\bar\varphi(g_1,...g_4;X_\alpha)\varphi(g_1,...g_4;X_\alpha)$$



$$V[\varphi,\bar\varphi]=\int(\mathrm{d}g)^{10}\sum_{\alpha_1...\alpha_5}\int\mathrm{d}X_{\alpha_1}...\int\mathrm{d}X_{\alpha_5}\varphi_{1234}(X_{\alpha_1})\varphi_{4567}(X_{\alpha_2})\varphi_{7389}(X_{\alpha_3})\varphi_{962(10)}(X_{\alpha_4})\varphi_{(10)851}(X_{\alpha_5})+\mathrm{c.c}$$



Bring in gravity (Barrett-Crane GFT model)

[Barrett, Crane 9904025; Perez, Rovelli 0009021 & 0011037; Oriti, Baratin 1108.1178; Jercher, Oriti, Pithis 2112.00091 & 2206.15442]

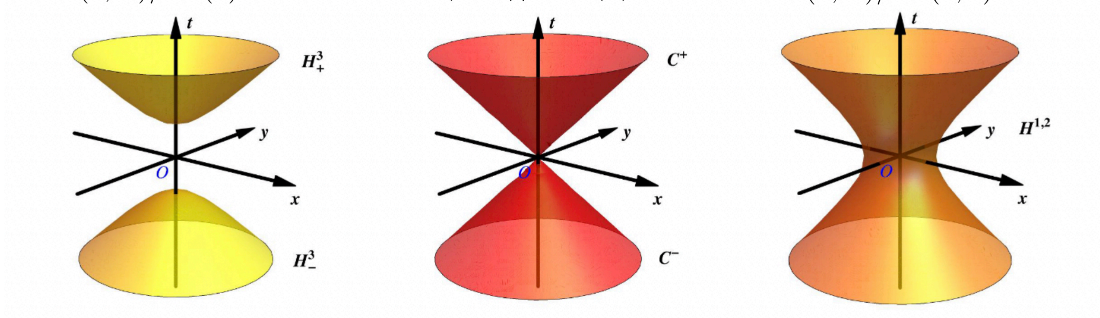
- **add to domain non-dynamical timelike, spacelike or lightlike normal vector X_α**

$$\varphi(g_1, ..., g_4; X_\alpha) : \mathrm{SL}(2,\mathbb{C})^4 \times \mathrm{SL}(2,\mathbb{C})/U^{(\alpha)} \to \mathbb{C} \quad \text{with} \quad \alpha \in \{+,0,-\}$$

$U^{(+)} = \mathrm{SU}(2),\ U^{(0)} = \mathrm{ISO}(2),\ U^{(-)} = \mathrm{SU}(1,1)$ **stabilizers of normal vectors** $X_+ = (1,0,0,0),$ timelike $\quad X_0 = \frac{1}{\sqrt{2}}(1,0,0,1),$ lightlike $\quad X_- = (0,0,0,1)$ spacelike

homogeneous spaces:

$\mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(2) \cong \mathbb{H}^3 \qquad \mathrm{SL}(2,\mathbb{C})/\mathrm{ISO}(2) \cong C \qquad \mathrm{SL}(2,\mathbb{C})/\mathrm{SU}(1,1) \cong \mathbb{H}^{1,2}$



$y^\mu y_\mu = a^2 \qquad y^\mu y_\mu = 0 \qquad y^\mu y_\mu = -a^2$

$X_+ = (1,0,0,0) \in \mathbb{H}^3 \qquad X_0 = \frac{1}{\sqrt{2}}(1,0,0,1) \in C \qquad X_- = (0,0,0,1) \in \mathbb{H}^{1,2}$

distinguished hypersurfaces in Minkowski space

$y \in \mathbb{R}^{1,3}$

$a:$ skirt radius

z-axis suppressed

<u>properties of the field</u>:

$$1)\ \varphi(g_1, ..., g_4; X_\alpha) = \varphi(g_1 h^{-1}, ..., g_4 h^{-1}; h \cdot X_\alpha), \quad \forall h \in \mathrm{SL}(2,\mathbb{C})$$ **(closure, already needed in Ooguri model)**

$$2)\ \varphi(g_1, ..., g_4; X_\alpha) = \varphi(g_1 u_1, ..., g_4 u_4; X_\alpha), \quad \forall u_1, ..., u_4 \in \mathrm{U}_{X_\alpha}$$ **(simplicity constraint)**

<u>geometric interpretation:</u>

➡ **Go to bi-vector representation**

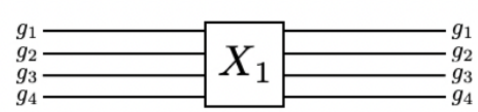
$$2) \Leftrightarrow X_A (*B)^{AB} = 0 \longrightarrow$$

Lorentz index $A \in \{0,1,2,3\}$

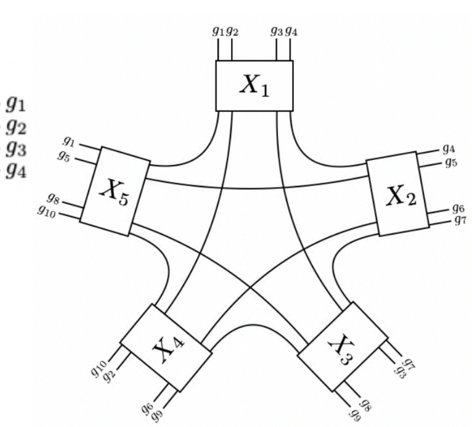
- **fields correspond to spacelike, timelike and lightlike tetrahedra**
- **turns it into a model for 4d Lorentzian quantum gravity (first-order Palatini gravity)**

<u>action:</u> $S[\varphi, \bar\varphi] = K[\varphi, \bar\varphi] + V[\varphi, \bar\varphi]$

$$K[\varphi, \bar\varphi] = \sum_{\substack{\alpha \\ \alpha \in \{+,0,-\}}} \int_{\mathrm{SL}(2,\mathbb{C})^4} (\mathrm{d}g)^4 \int_{\mathrm{SL}(2,\mathbb{C}/\mathrm{U}^{(\alpha)}} \mathrm{d}X_\alpha \bar\varphi(g_1, ...g_4; X_\alpha) \varphi(g_1, ...g_4; X_\alpha)$$



$$V[\varphi, \bar\varphi] = \int (\mathrm{d}g)^{10} \sum_{\alpha_1 \ldots \alpha_5} \int \mathrm{d}X_{\alpha_1} \ldots \int \mathrm{d}X_{\alpha_5} \varphi_{1234}(X_{\alpha_1}) \varphi_{4567}(X_{\alpha_2}) \varphi_{7389}(X_{\alpha_3}) \varphi_{962(10)}(X_{\alpha_4}) \varphi_{(10)851}(X_{\alpha_5}) + \mathrm{c.c}$$



extensions:

- coloured setting
- tensor-invariant interactions

Application of LG theory to BC model

[Marchetti, Oriti, Thürigen, Pithis 2209.04297, 2211.12768]

How to:

0.1) add into the action Laplace-Beltrami operator on $\mathrm{SL}(2,\mathbb{C})^4 \longrightarrow$ introduces coarse-graining scale ξ

0.2) restrict Barrett-Crane model to case with spacelike tetrahedra/timelike normals: $\varphi(g_1, g_2, g_3, g_4, X_+) \quad \text{with} \quad g_i, X_+ \in \mathbb{H}^3$

Application of LG theory to BC model

[Marchetti, Oriti, Thürigen, Pithis 2209.04297, 2211.12768]

How to:

0.1) add into the action Laplace-Beltrami operator on $\mathrm{SL}(2,\mathbb{C})^4 \longrightarrow$ introduces coarse-graining scale ξ

0.2) restrict Barrett-Crane model to case with spacelike tetrahedra/timelike normals: $\varphi(g_1, g_2, g_3, g_4, X_+)$ with $g_i, X_+ \in \mathbb{H}^3$

1) determine **uniform field configurations** which are minimizers of the BC action functional

2) study **correlations of fluctuations** around this uniform background (aka Gaussian approximation)

2. a) **linearize classical equations of motion** using fluctuations over the background $\varphi(g_1, g_2, g_3, g_4) \rightarrow \underbrace{\varphi_0}_{\text{background}} + \underbrace{\delta\varphi(g_1, g_2, g_3, g_4)}_{\text{fluctuations}}$

2. b) solve for **correlation function** $\boxed{C(g_1, g_2, g_3, g_4)}$

2. c) correlator is exponentially decaying function $\longrightarrow$ determine **correlation length** ξ

Application of LG theory to BC model

[Marchetti, Oriti, Thürigen, Pithis 2209.04297, 2211.12768]

How to:

0.1) add into the action Laplace-Beltrami operator on $\mathrm{SL}(2,\mathbb{C})^4$ ⟶ introduces coarse-graining scale ξ

0.2) restrict Barrett-Crane model to case with spacelike tetrahedra/timelike normals: $\varphi(g_1, g_2, g_3, g_4, X_+)$ with $g_i, X_+ \in \mathbb{H}^3$

1) determine **uniform field configurations** which are minimizers of the BC action functional

2) study **correlations of fluctuations** around this uniform background (aka Gaussian approximation)

2. a) **linearize classical equations of motion** using fluctuations over the background $\varphi(g_1, g_2, g_3, g_4) \rightarrow \underbrace{\varphi_0}_{\text{background}} + \underbrace{\delta\varphi(g_1, g_2, g_3, g_4)}_{\text{fluctuations}}$

2. b) solve for **correlation function** $\boxed{C(g_1, g_2, g_3, g_4)}$

2. c) correlator is exponentially decaying function ⟶ determine **correlation length** ξ

3) determine **domain of validity of LG MFT:** **Ginzburg parameter** (measures strength of fluctuations)

$$Q = \frac{\int_\xi (\mathrm{d}g)^4 C(g_1, g_2, g_3, g_4)}{\int_\xi (\mathrm{d}g)^4 \varphi_0^2}$$

▸ **Fluctuations and coupling should remain small then mean-field theory self-consistent**

- **coarse picture of phase diagram**

Results: Ginzburg Q

- results for quartic local scalar field theory on one 3-hyperboloid:

finite skirt radius a : $\quad Q \sim \underset{\substack{\uparrow \\ \text{coupling}}}{\lambda_\gamma} \xi^2 \boxed{\mathrm{e}^{-2\cdot 1\frac{\xi}{a}}}$

[Benedetti 1403.6712]

exponential suppression due to hyperbolicity of the domain

Results: Ginzburg Q

- results for quartic local scalar field theory on one 3-hyperboloid:

finite skirt radius a : $$Q \sim \lambda_\gamma \xi^2 \boxed{\mathrm{e}^{-2\cdot 1 \frac{\xi}{a}}}$$

coupling [Benedetti 1403.6712]

exponential suppression due to hyperbolicity of the domain

- results for quartic restricted BC model:

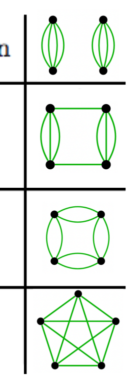
rank of the group field

domain derived from Lorentz group

finite skirt radius a : $$Q \sim \lambda_\gamma \xi^2 \boxed{\mathrm{e}^{-2(4-s_0)\frac{\xi}{a}}}$$

specifies non-local combinatorics of interaction

		graph γ	
$s_0 = 0$	double-trace melon		tensorial
$s_0 = 1$	quartic melonic		tensorial
$s_0 = 2$	quartic necklace		tensorial
$s_0 = 3$	simplicial		



$(s_0 < 4)$

valency of interaction

- can be generalized to arbitrary interactions: $$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{V_\gamma}{V_\gamma - 2}} \mathrm{e}^{-2(4-s_0)\frac{\xi}{a}}$$

flat limit$_{a \to \infty}$: $$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{2V_\gamma}{V_\gamma - 2} - 3(4-s_0)}$$

dim. of 3-hyperboloid

[agrees with our results 2110.15336]

Results: Ginzburg Q

- results for quartic local scalar field theory on one 3-hyperboloid:

finite skirt radius a : $$Q \sim \lambda_\gamma \xi^2 \boxed{e^{-2\cdot 1 \frac{\xi}{a}}}$$

coupling [Benedetti 1403.6712]

exponential suppression due to hyperbolicity of the domain

- results for quartic restricted BC model:

rank of the group field

domain derived from Lorentz group

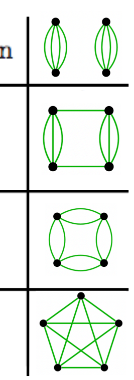
finite skirt radius a : $$Q \sim \lambda_\gamma \xi^2 \boxed{e^{-2(4-s_0)\frac{\xi}{a}}}$$

specifies non-local combinatorics of interaction

		graph γ
$s_0 = 0$	double-trace melon	
$s_0 = 1$	quartic melonic	
$s_0 = 2$	quartic necklace	
$s_0 = 3$	simplicial	

tensorial

$(s_0 < 4)$



valency of interaction

- can be generalized to arbitrary interactions: $$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{V_\gamma}{V_\gamma - 2}} e^{-2(4-s_0)\frac{\xi}{a}}$$

flat limit$_{a\to\infty}$: $$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{2V_\gamma}{V_\gamma - 2} - 3(4-s_0)}$$

dim. of 3-hyperboloid

[agrees with our results 2110.15336]

→ **finite skirt radius : Ginzburg Q always very small**

→ **Self-consistent account of phase structure and transition**

Results: Ginzburg Q

- results for quartic local scalar field theory on one 3-hyperboloid:

finite skirt radius a : $$Q \sim \lambda_\gamma \xi^2 \boxed{\mathrm{e}^{-2\cdot 1 \frac{\xi}{a}}}$$

coupling [Benedetti 1403.6712]

exponential suppression due to hyperbolicity of the domain

- results for quartic restricted BC model:

rank of the group field

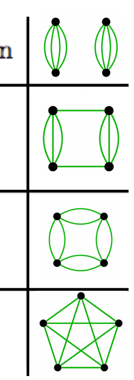
domain derived from Lorentz group

finite skirt radius a : $$Q \sim \lambda_\gamma \xi^2 \mathrm{e}^{\boxed{-2(4-s_0)\frac{\xi}{a}}}$$

specifies non-local combinatorics of interaction

graph γ

$s_0 = 0$	double-trace melon	tensorial
$s_0 = 1$	quartic melonic	tensorial
$s_0 = 2$	quartic necklace	tensorial
$s_0 = 3$	simplicial	



$(s_0 < 4)$

valency of interaction

- can be generalized to arbitrary interactions: $$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{V_\gamma}{V_\gamma - 2}} \mathrm{e}^{-2(4-s_0)\frac{\xi}{a}}$$

flat limit $_{a \to \infty}$: $$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{2V_\gamma}{V_\gamma - 2} - 3(4-s_0)}$$

dim. of 3-hyperboloid

[agrees with our results 2110.15336]

→ **finite skirt radius : Ginzburg Q always very small**

→ **Self-consistent account of phase structure and transition**

- Extension: Include free massless scalar matter on lattice ✓
- Why is tree-level theory good enough?
- Interpretation of flat limit?

Summary & Conclusions

◉ formulation of BC model for Lorentzian quantum gravity with all causal building blocks

◉ LG MFT theory also applicable to GFT in spite of its non-local interactions

- applied to BC model restricted to spacelike tetrahedra only
- gives accurate account of its phase structure
- transition to non-perturbative vacuum/condensate phase
- state highly occupied by geometric quanta ~ continuum limit

◉ GFT condensate cosmology

- dynamics of condensate states leads to Friedmann-like relational evolution

→ **Corroborates evidence for existence of meaningful continuum gravitational regime in GFT.**

Beyond mountains, more mountains…

- impact on phase structure of all causal building blocks [Jercher, Pithis wip]
    - local causality conditions?

- apply method to EPRL model [Dekhil, Jercher, Oriti, Pithis wip]

- complement by full-fledged **functional RG analyses in search for NGFPs** [Jercher, Pithis, Thürigen wip]

- devise **observables & tools** to characterize different phases wrt their geometric properties

- connect with recent FRG analysis for first-order Palatini gravity in the continuum [Gies, Sabor Salek 2209.10435]

Thank you for your attention!

Backup slides

Disambiguation: Causal structure

classical level: Lorentzian structure/causality plays an important role in continuum spacetime physics

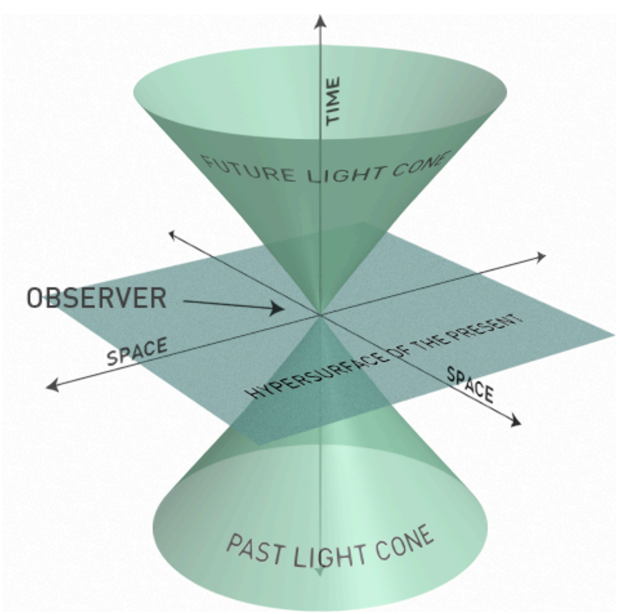
What do we mean **here** by causal structure?

➡ bare causality + time orientation

e.g. [Livine, Oriti 0210064; Bianchi, Martin-Dussaud 2109.00986; Jercher, Oriti, Pithis 2206.15442]

- Encoded by the Lorentz group $G = \mathrm{SL}(2, \mathbb{C})$

Causal structure	**bare causality**	**time orientation**
local level	tangent vectors: timelike, lightlike spacelike	timelike tangent vectors either past or future pointing
global level	two points (events) either have spacelike, timelike or lightlike separation	causal ordering of timelike separated points



[wikipedia]

Criticism against the BC model and alleviations

- BC vertex does not yield tensorial structure of lattice graviton propagator [Alesci, Rovelli 0708.0883]
    - Obvious mismatch of LQG boundary states and BC boundary states. [Baratin, Oriti 1108.1178]
- Area-length constraints are missing [Alexandrov 0802.3389]
    - Recently it was shown (on a hypercubical lattice) that the BC model is still viable and potentially lies in the same universality class as the EPRL model in an effective continuum limit. [Dittrich 2105.10808]
- What is the role of degenerate geometries in the BC model? [Barrett, Steele 0209023]
    - Need further analysis including timelike and lightlike configurations.
- Constraints are "too strongly" imposed [Engle, Pereira, Rovelli 0705.2388]
- Closure and simplicity are imposed in a non-covariant and non-commuting manner [Baratin, Oriti 1002.4723]
    - Both problems resolved in extended BC model. [Baratin, Oriti 1108.1178]
- EPRL model favored since boundary states are closer to canonical LQG, the Barbero-Immirzi parameter is incorporated
    - Absence of BI parameter does not rule out the BC model. At the same time, questions wrt the precise value and running of the BI parameter and parity violation issues of the EPRL model should be addressed. [Charles 1705.10984; Benedetti, Speziale 1111.0884]

For now, criticisms are not conclusive and the BC model deserves further attention.