QUANTUM COMPUTATIONS IN LOOP QUANTUM GRAVITY

GRZEGORZ CZELUSTA

arXiv:2304.03559   arXiv: 2003.13124 (with Jakub Mielczarek)

Quantum Cosmos Lab

JAGIELLONIAN UNIVERSITY IN KRAKÓW

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QUANTUM COMPUTERS

- qubits $\equiv$ 2 level quantum systems

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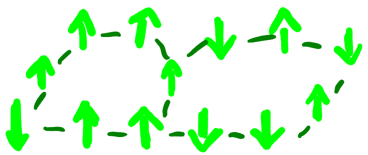
- Hilbert space dimension $= 2^{\#\text{qubits}}$



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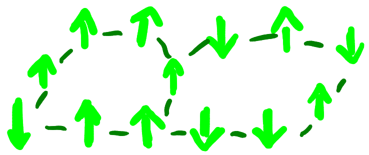
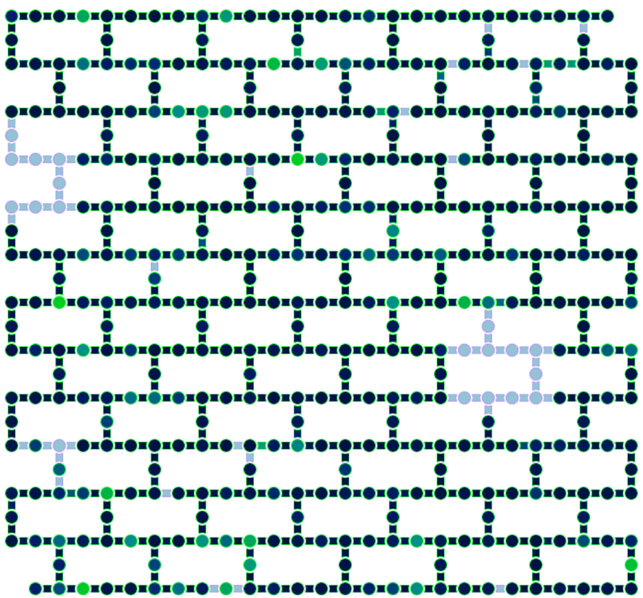


- e.g.: IBM SEATTLE, 433 qubits



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- there are known quantum algorithms that are faster than classical

- e.g.: Shor's algorithm factorizes integer N with complexity
  $\mathcal{O}((\log N)^2(\log\log N))$ vs fastest classical algorithm
  $\mathcal{O}(e^{1.9(\log N)^{1/3}(\log\log N)^{2/3}})$

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→ fault tolerant quantum computers

QUANTUM COMPUTERS

NISQ era $\equiv$ noisy intermediate-scale quantum era

- short circuits
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PROBLEM:

- transition between quantum $\longleftrightarrow$ classical data

SOLUTION:

- work as long as possible only on quantum data

simulations of quantum systems in physics

LOOP QUANTUM GRAVITY

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- canonical quantization of gravity

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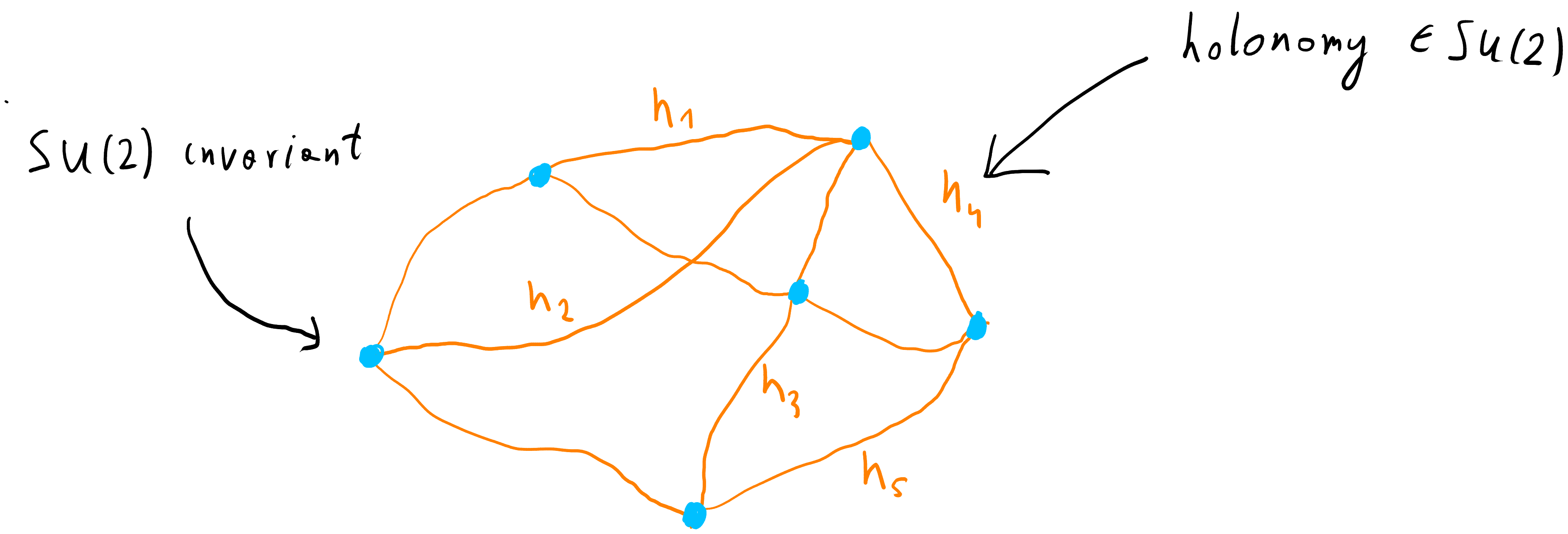
- Ashtekar variables

$$\begin{cases} E^a_i = \frac{1}{2}\varepsilon_{ijk}\varepsilon^{abc}e^j_b e^k_c \\ A^i_a = \gamma\omega^{0i}_a + \frac{1}{2}\varepsilon^i_{jk}\omega^{jk}_a \end{cases} \qquad \{A^i_a(x), E^b_j(y)\} = 8\pi G\gamma\delta^b_a\delta^i_j\delta(x-y)$$

- gravity as SU(2) gauge theory

SPIN NETWORKS

- in LQG represent quantum geometry of space



$$\Psi_\Gamma : SU(2)^{\times E} \to \mathbb{C}$$

$$\{g_e\}_{e\in\Gamma} \longmapsto \Psi_\Gamma(\{g_e\}_{e\in\Gamma}) = \Psi_\Gamma(\{h_{t(e)}\, g_e\, h_{s(e)}^{-1}\})$$

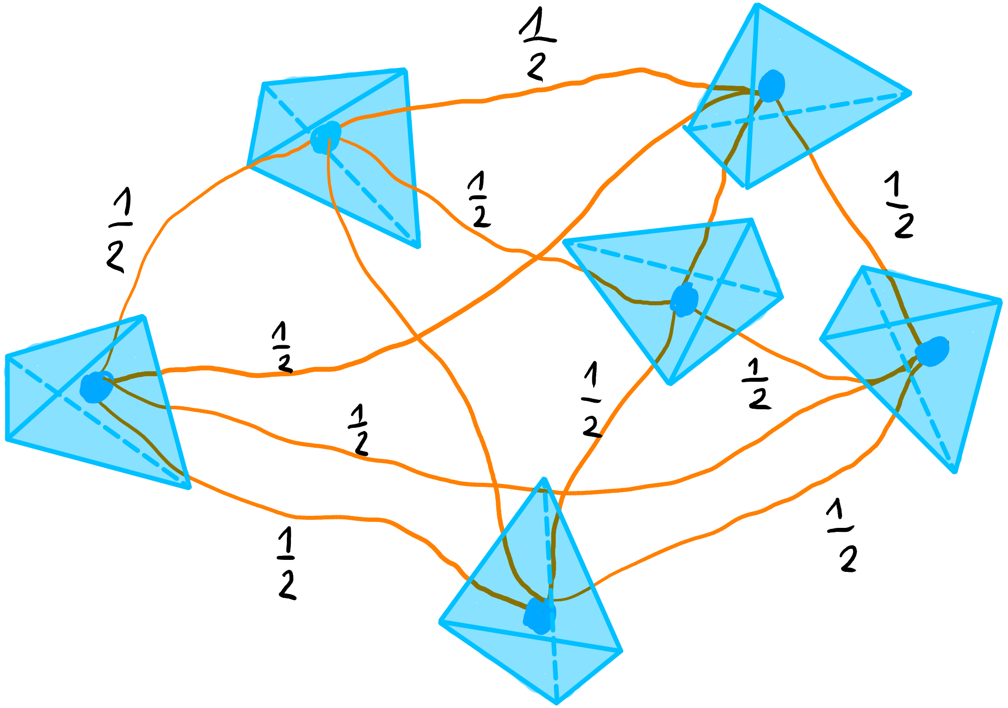
SPIN NETWORKS

- nodes are associated with quanta of volume
- links are associated with areas around given volume
- set of all spin networks (up to diffeomorphisms) constitutes basis of kinematical Hilbert space
- spin networks can be also used for general gauge theories with a compact Lie group and a connection form

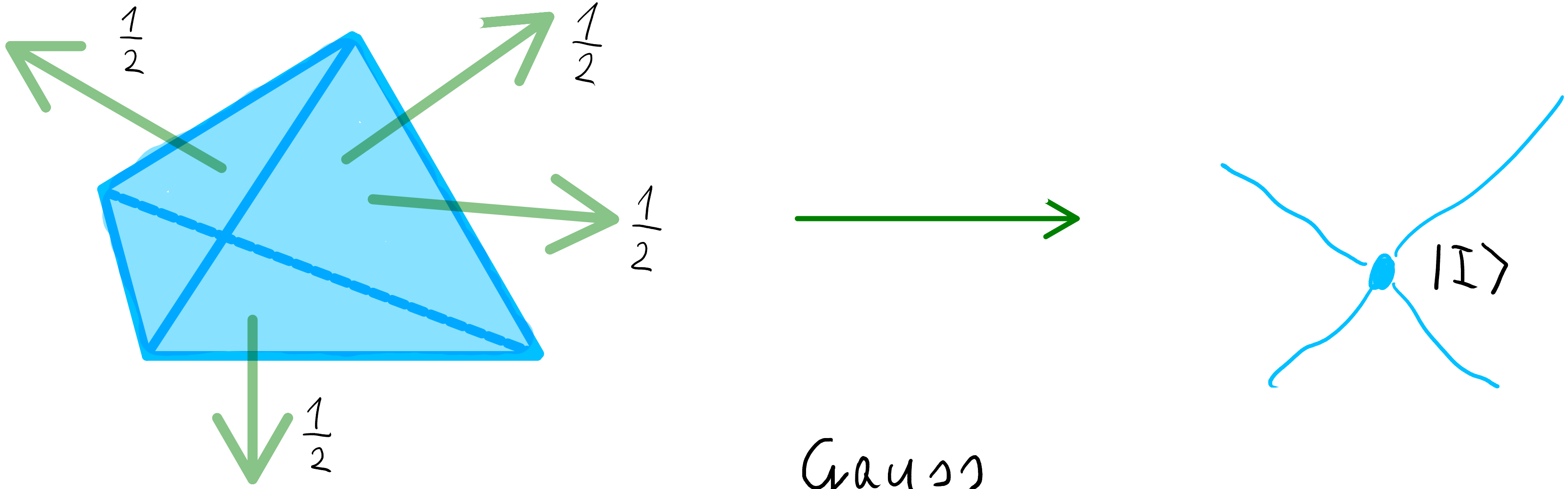
SPIN NETWORKS IN OUR CASE

- 4-valent

- all holonomies in fundamental SU(2) representation

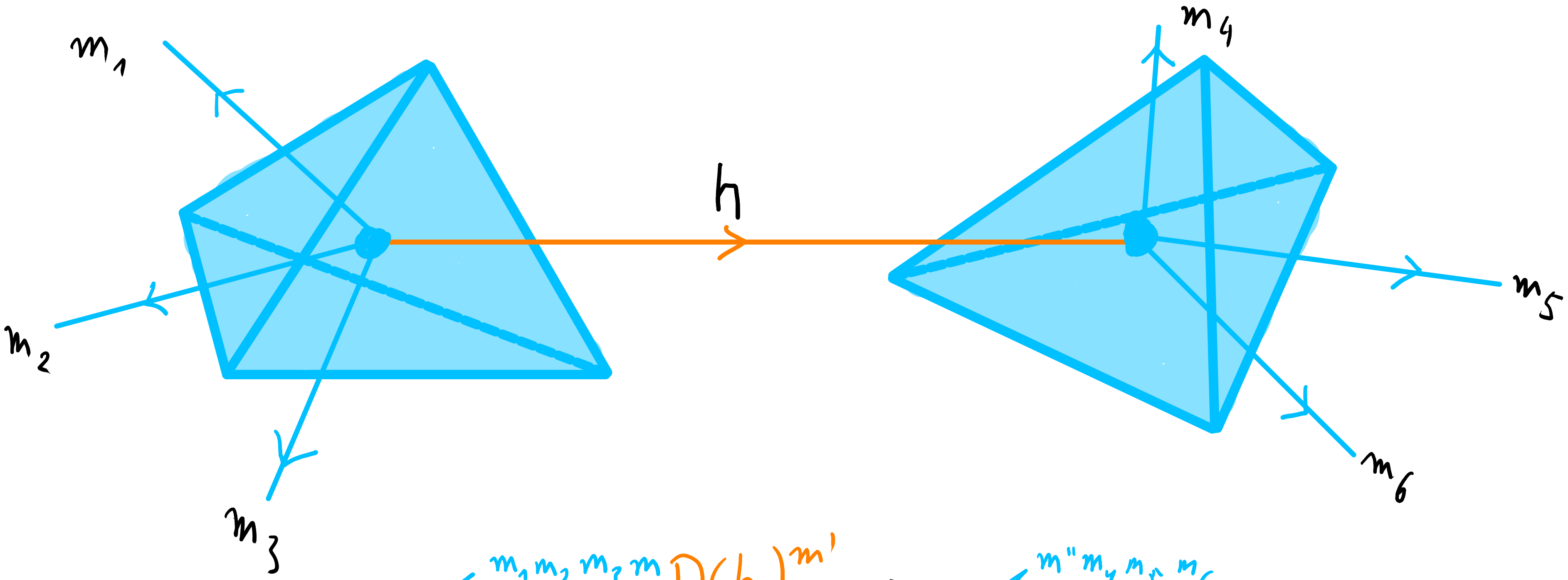


SPIN NETWORKS IN OUR CASE



$$\mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \xrightarrow[\text{SU(2) invariance}]{\text{Gauss constraint}} |I\rangle = \alpha|0\rangle + \beta|1\rangle$$

STATE OF SPIN NETWORK



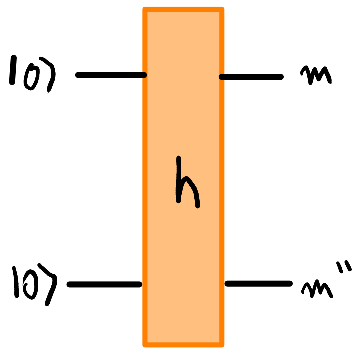
$$\iota^{m_1 m_2 m_3 m}\, D(h)^{m'}{}_{m}\, \epsilon_{m'm''}\, \iota^{m'' m_4 m_5 m_6}$$

$$\epsilon_{mm'} = \delta_{m,-m'}(-1)^{j-m}$$

Wigner 4j-symbol

Wigner matrix

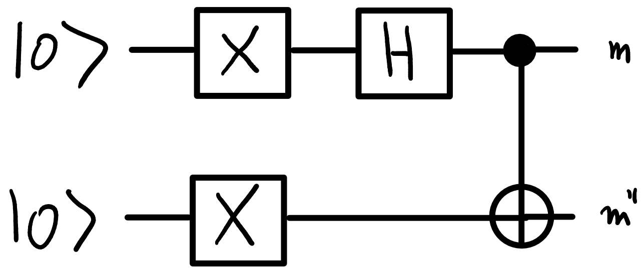
LINKS AND GAUSS CONSTRAINT AS QUANTUM CIRCUITS



e.g.: $h = \mathbb{1}$

$$D(h)^{m'}{}_{m}\, f_{m'm''} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\rightsquigarrow \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$



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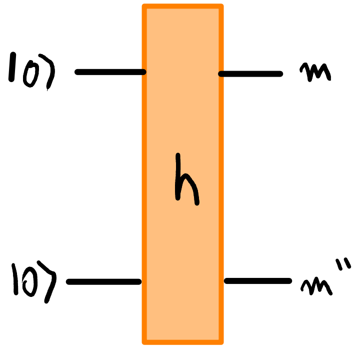

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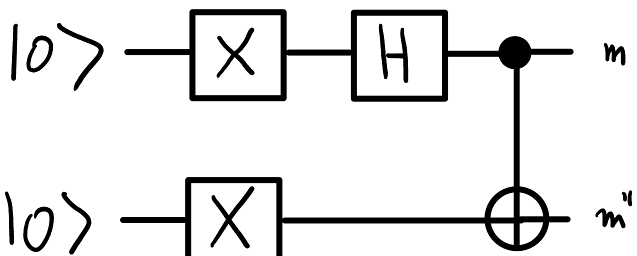
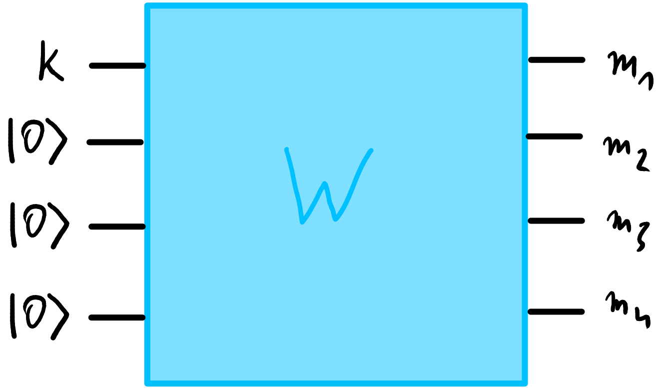
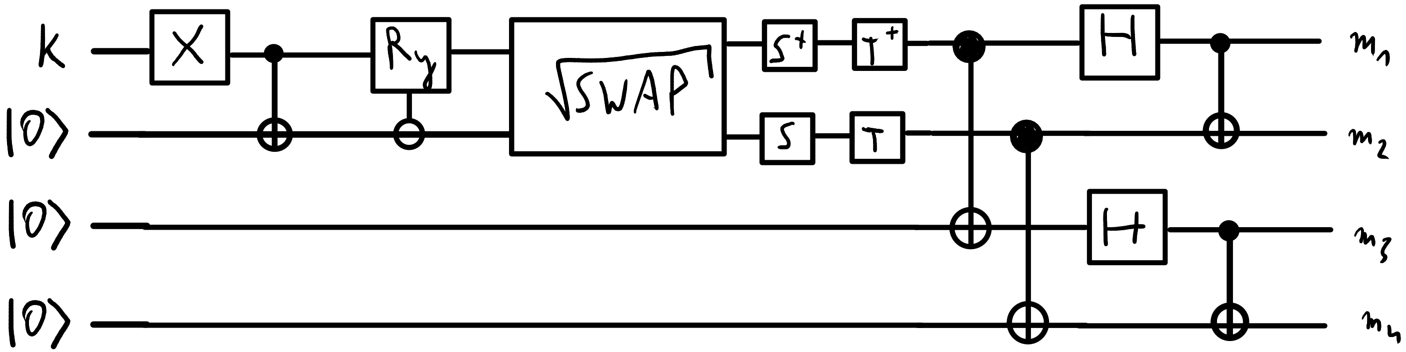
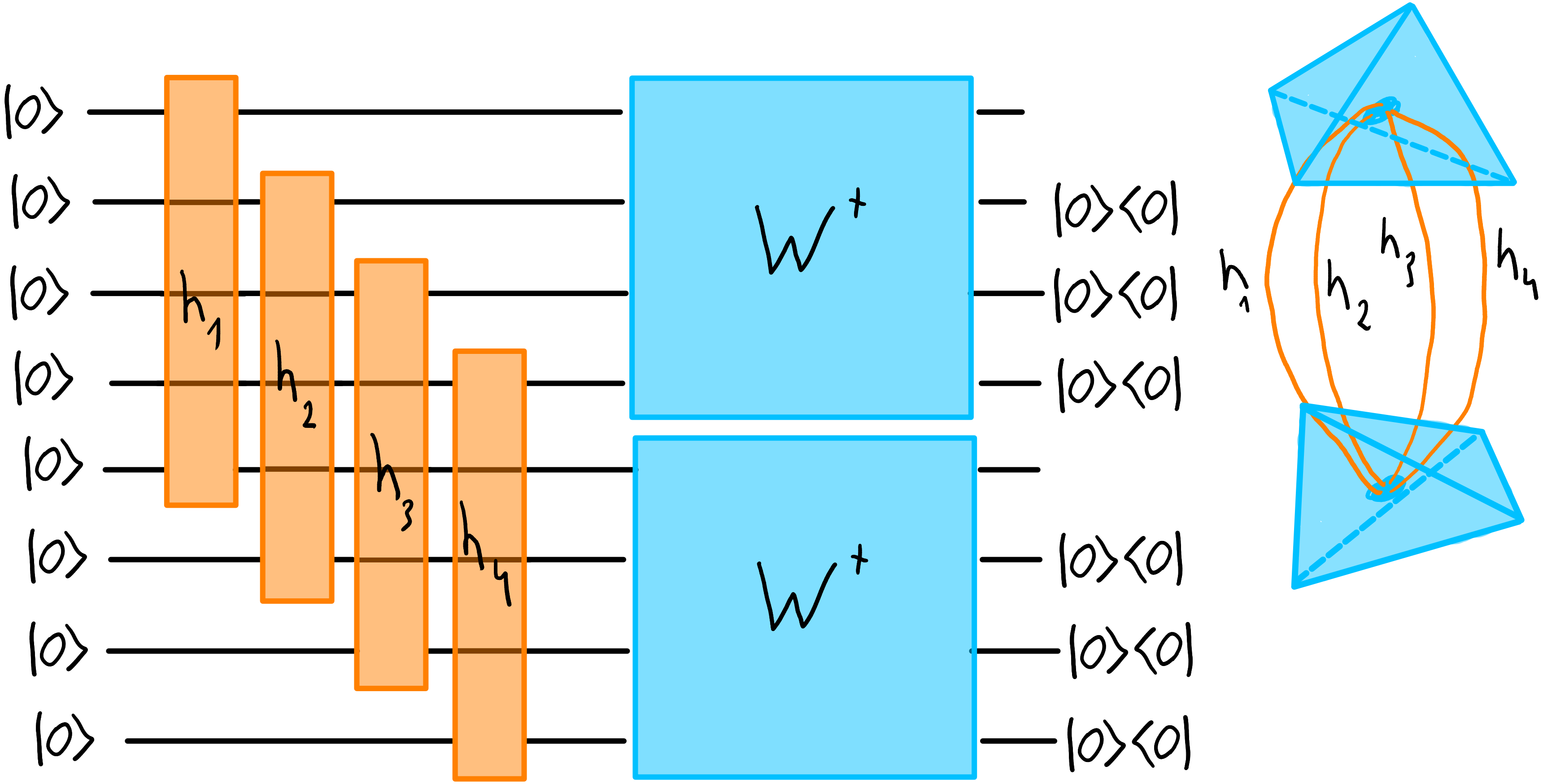






$$W|k\rangle|000\rangle = \sum_{m_i} c_{(k)}^{m_1 m_2 m_3 m_4}\, |m_1 m_2 m_3 m_4\rangle$$

1ST METHOD: PROJECTION



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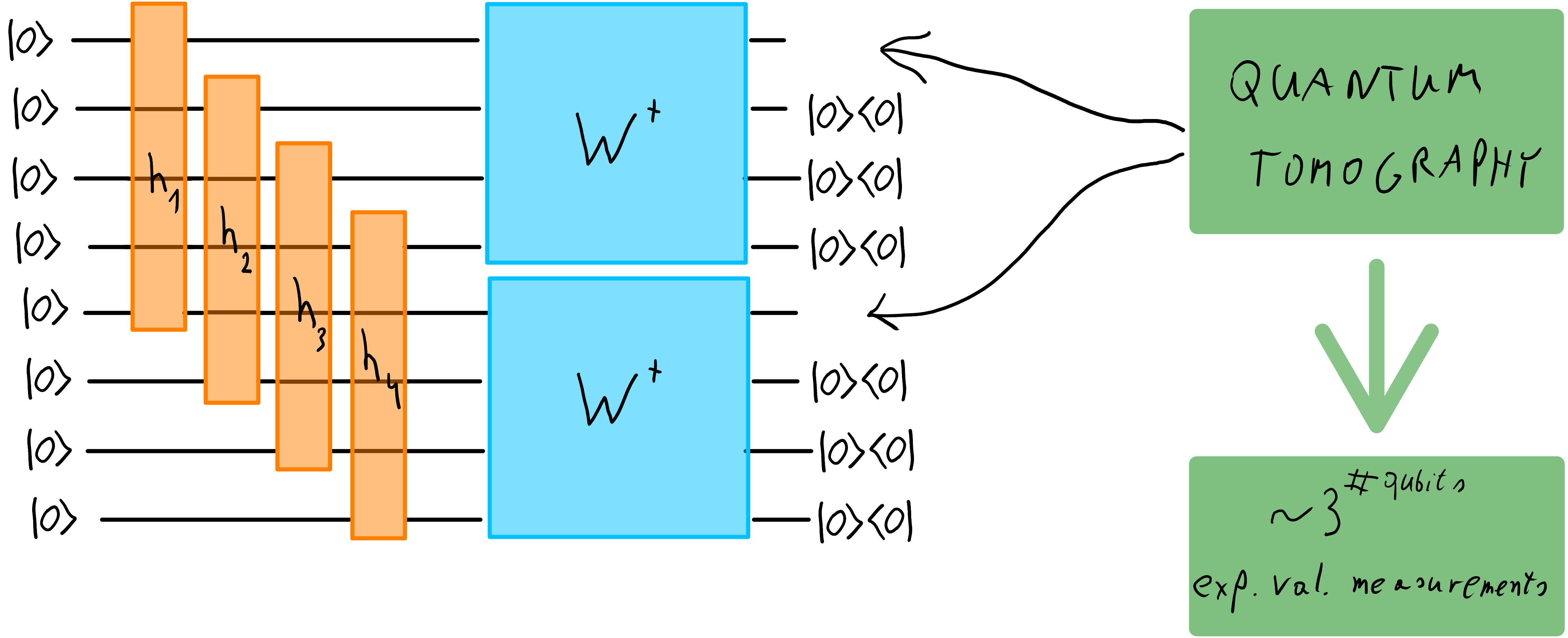

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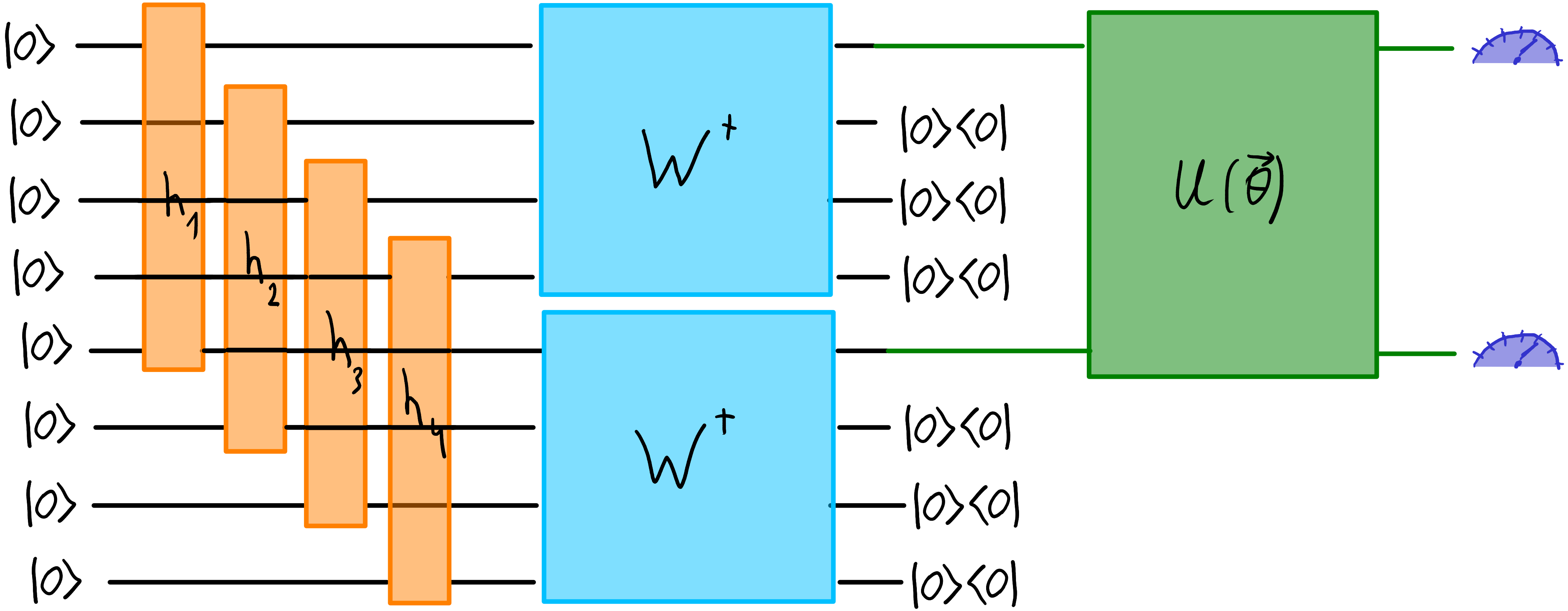

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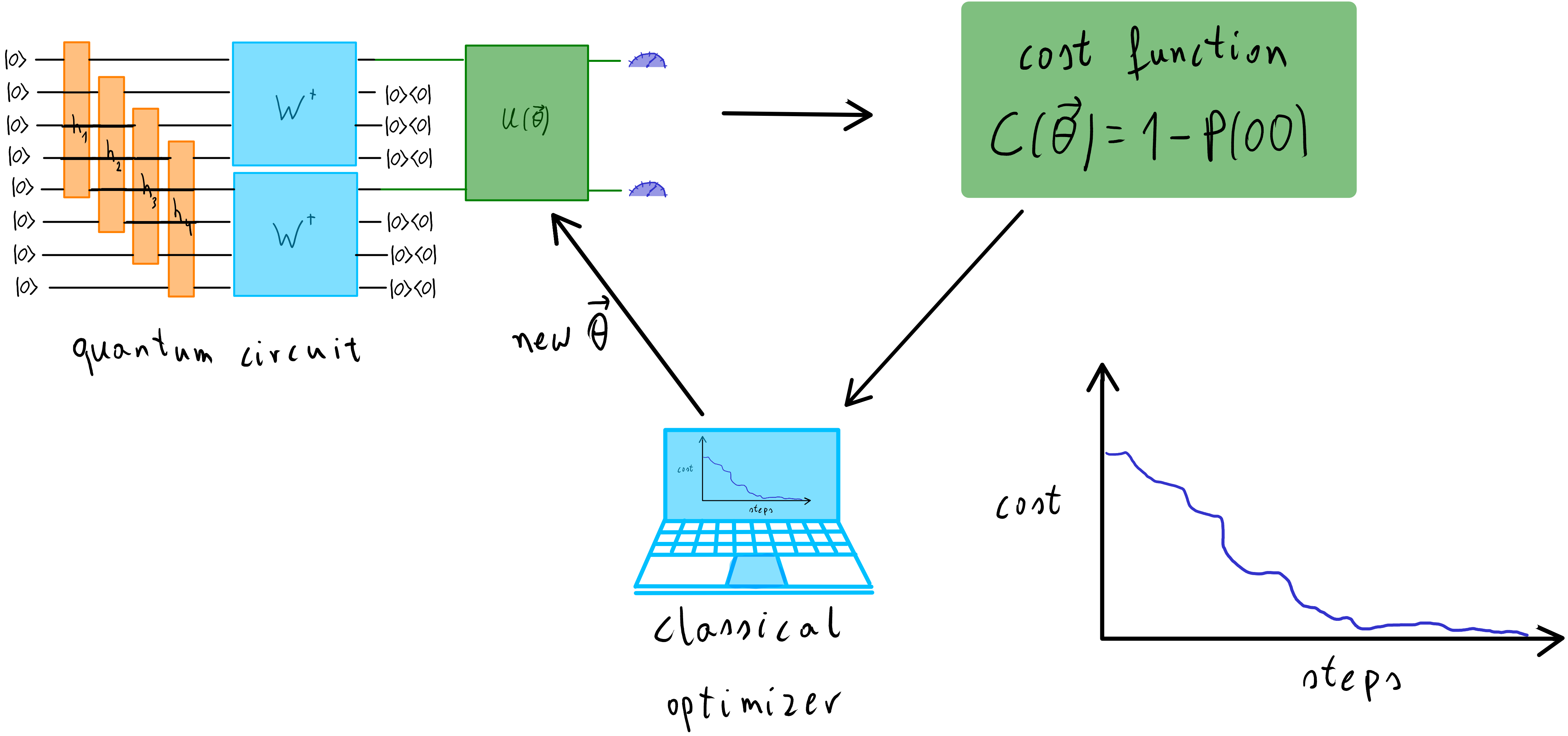

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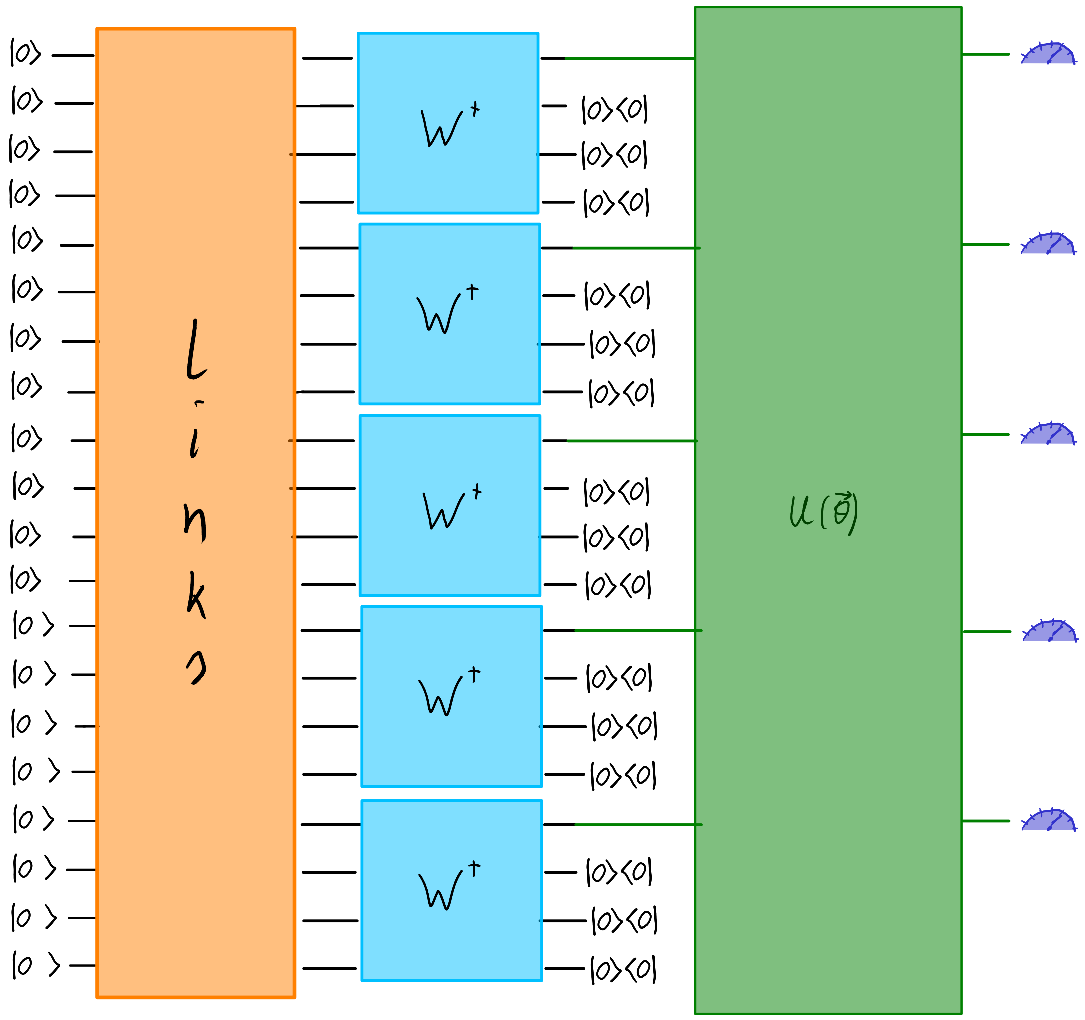
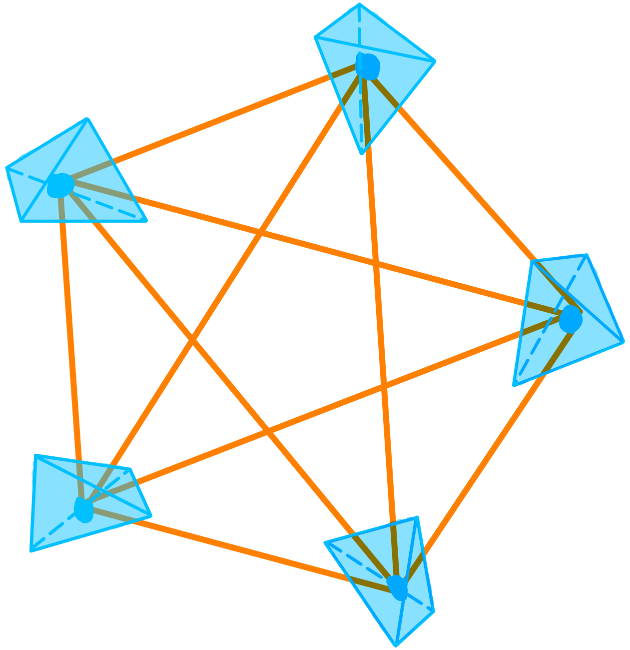
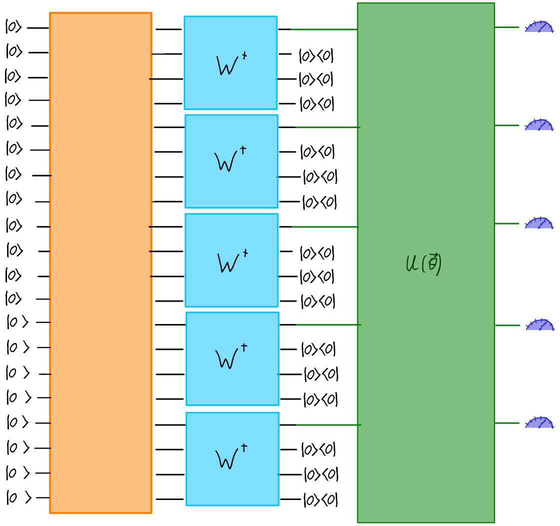




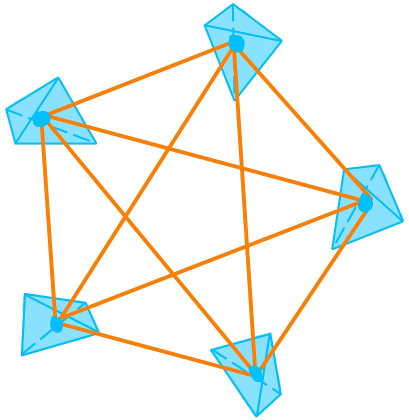
1st METHOD: PROJECTION

\# qubits



=

4 · # nodes

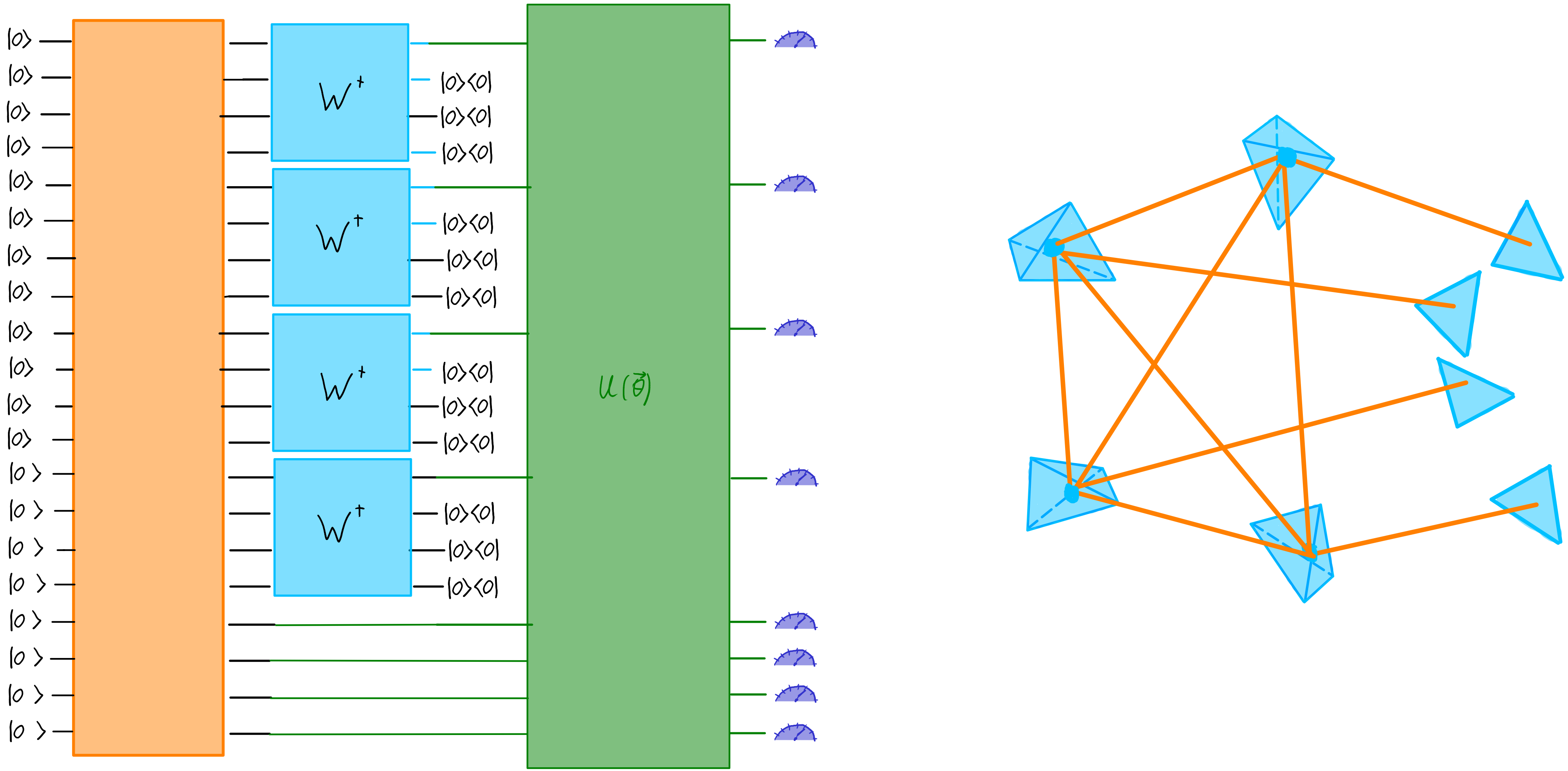


projection on $|0\rangle\langle 0|^{\otimes 3}$ by postselection of outcomes

exponentially many results rejected

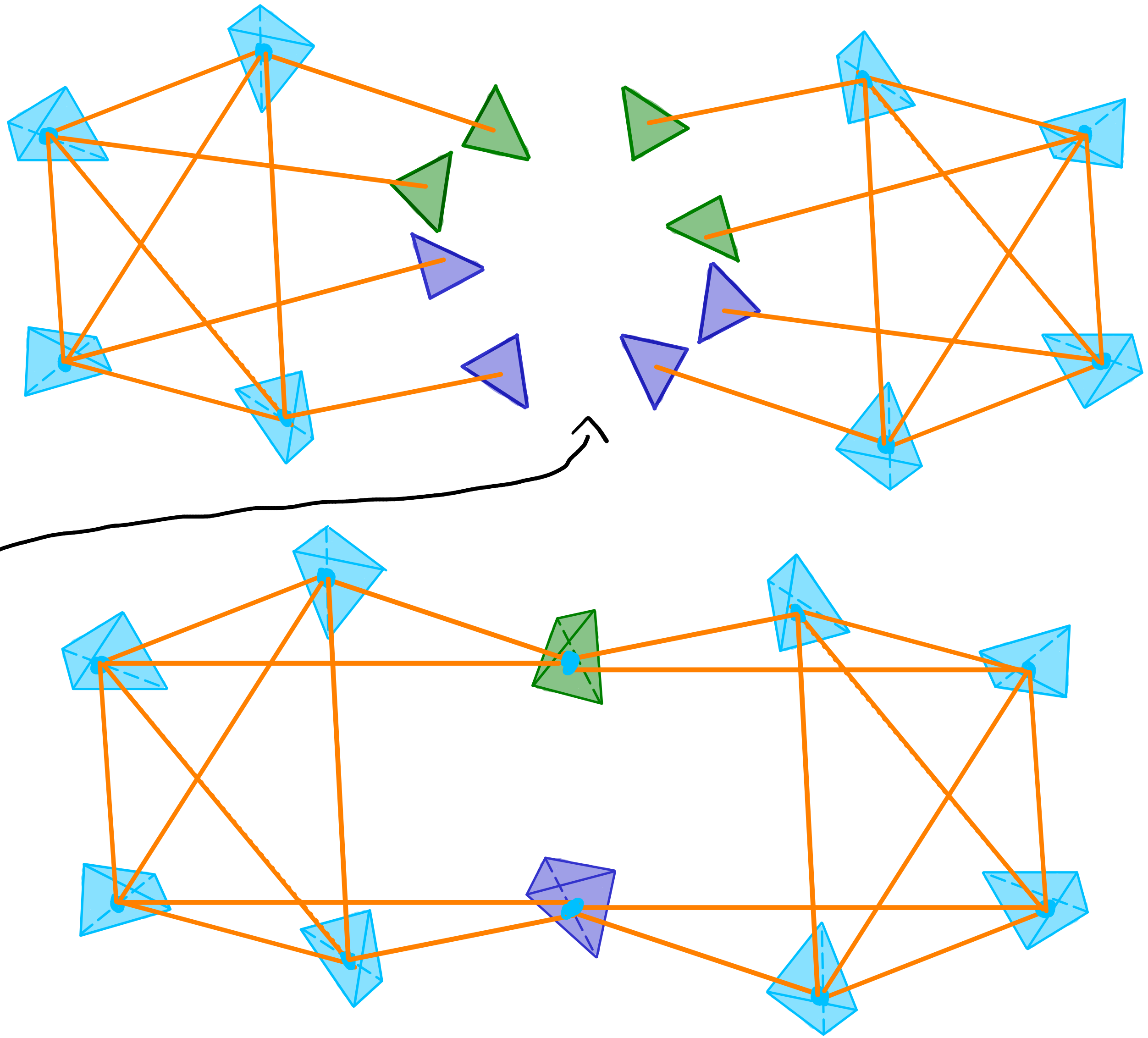
exponentially many measurements required to estimate cost function

1ST METHOD: PROJECTION. PARTIAL PROJECTION

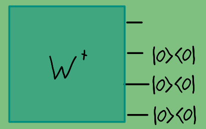


1ST METHOD: PROJECTION. GLUING.

partially projected network

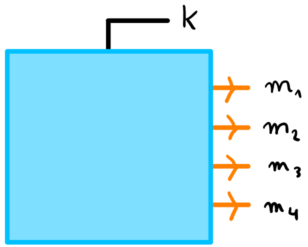


gluing by applying

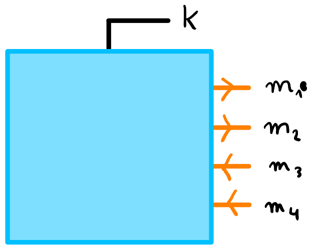


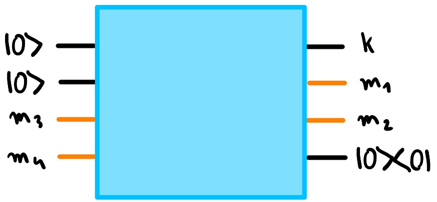
2ND METHOD: TENSOR NETWORK

$$\tau^{m_1 m_2 m_3 m_4}_{(k)}$$



$$\tau^{m_1 m_2}_{(k) m_3 m_4}$$





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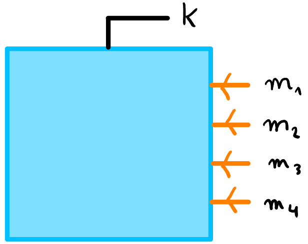
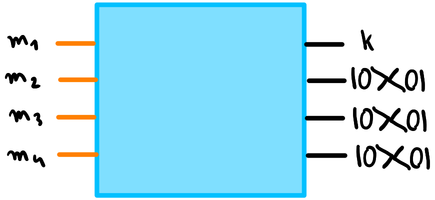
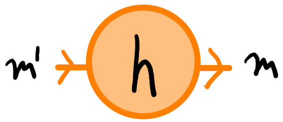
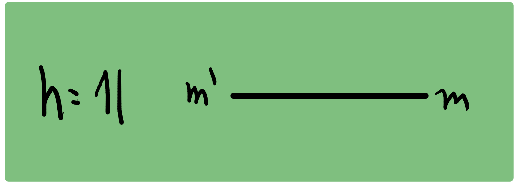


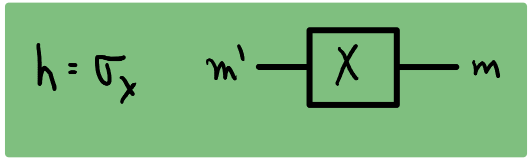


$$D(h)^{m}{}_{m'}$$

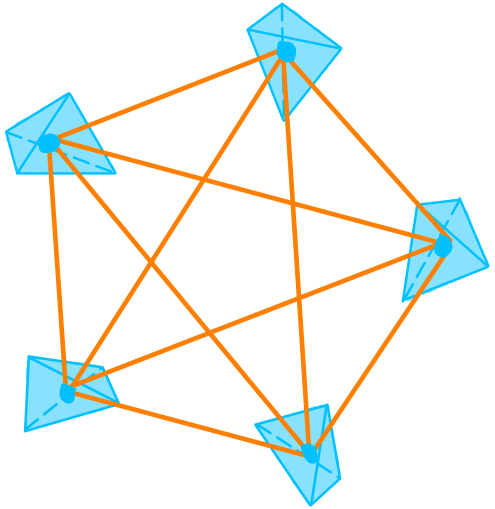


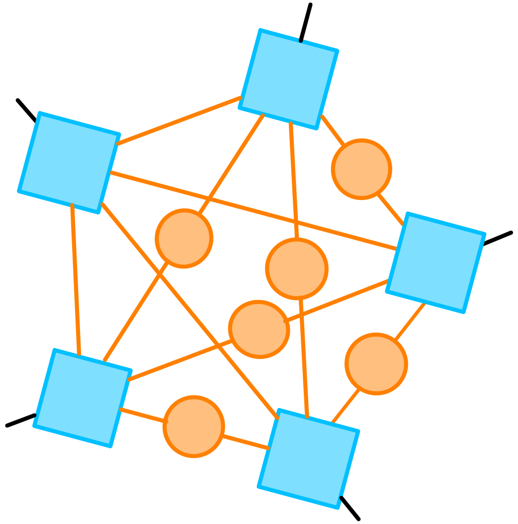
e.g.

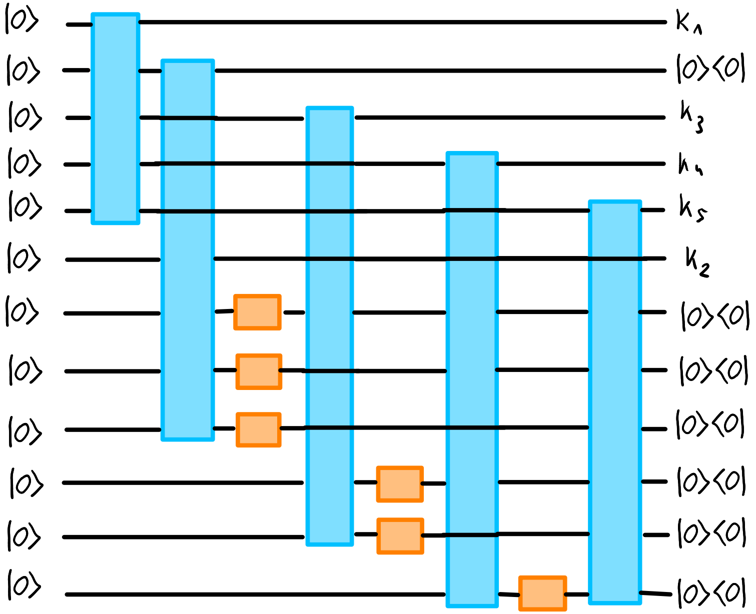




2ND METHOD: TENSOR NETWORKS







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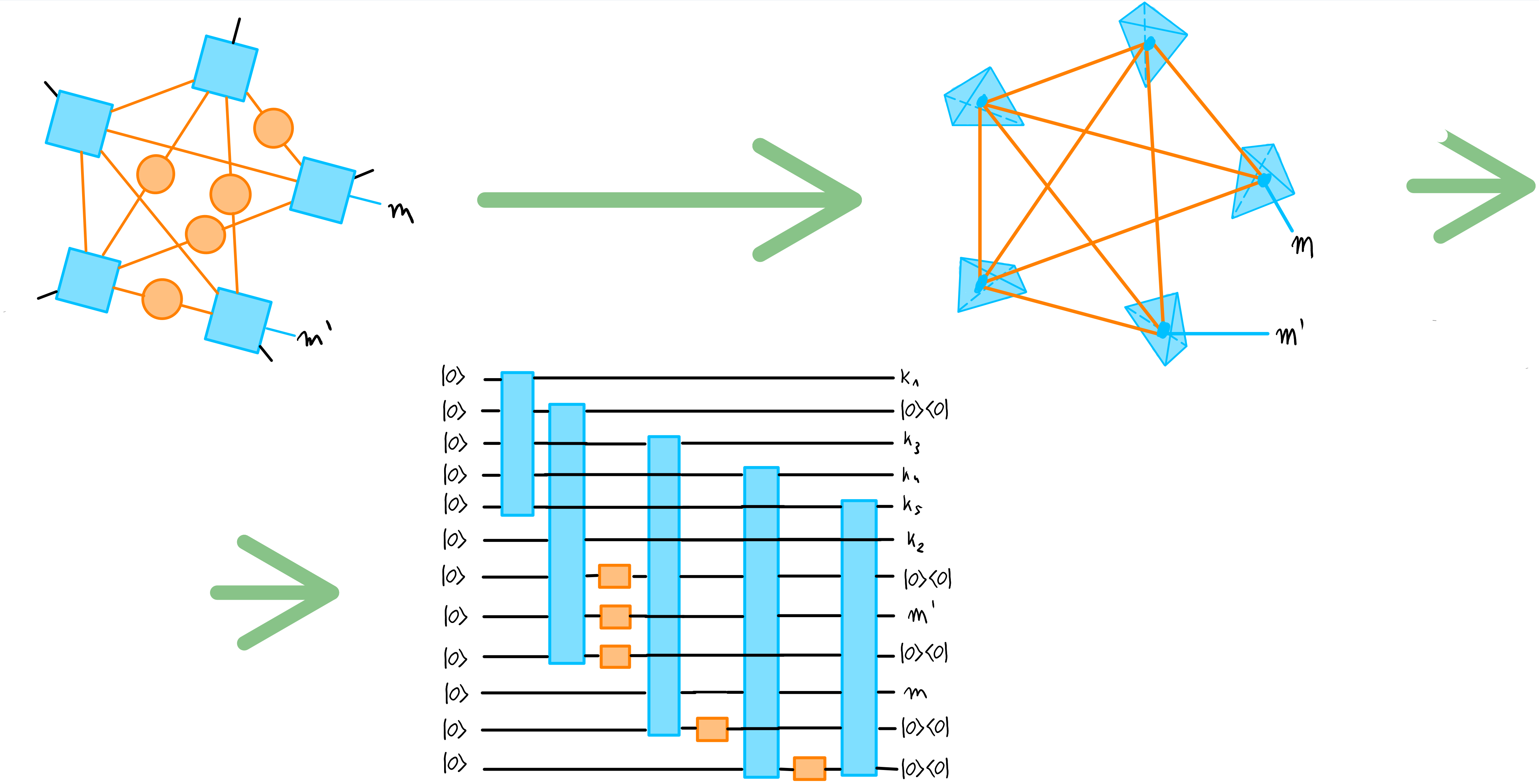
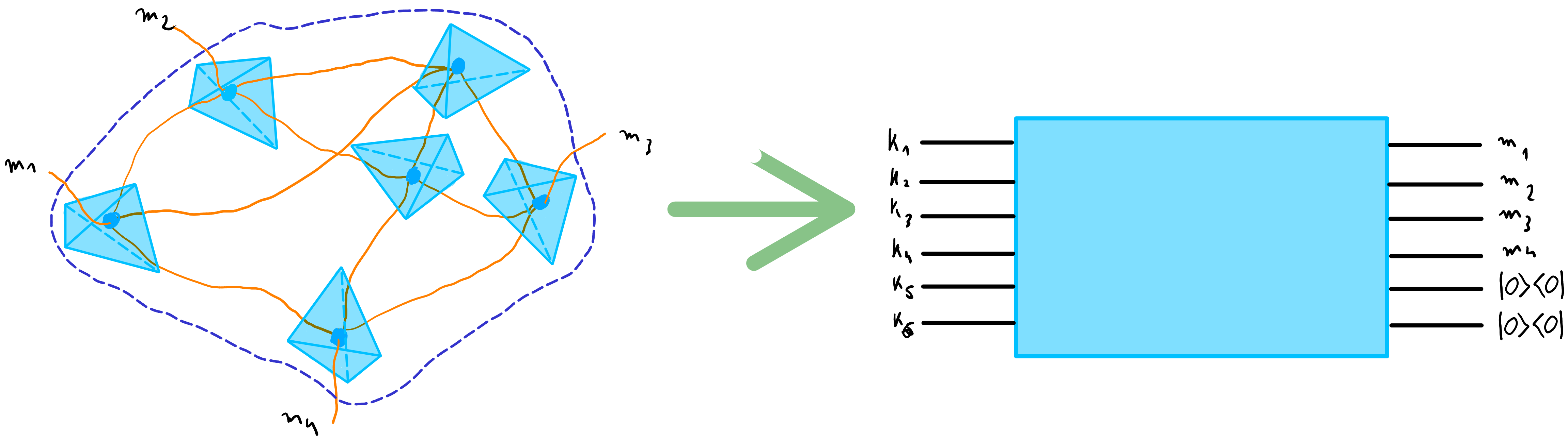


BULK-BOUNDARY MAP



SUMMARY

- we can construct explicite quantum circuit for arbitrary 4-valent spin network with all spins $\frac{1}{2}$
- it can be used as a tool for computations that are classically intractable
- and as a quantum information characteristics of quantum gravity states, e.g.: their complexity