Towards a quantitative phenomenology of quantum gravity

Aaron Held

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Jena University

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Quantum Spacetime and the Renormalization Group 2023, Oct 02 – 06 2023.

Towards a quantitative phenomenology of quantum gravity in black-hole astrophysics

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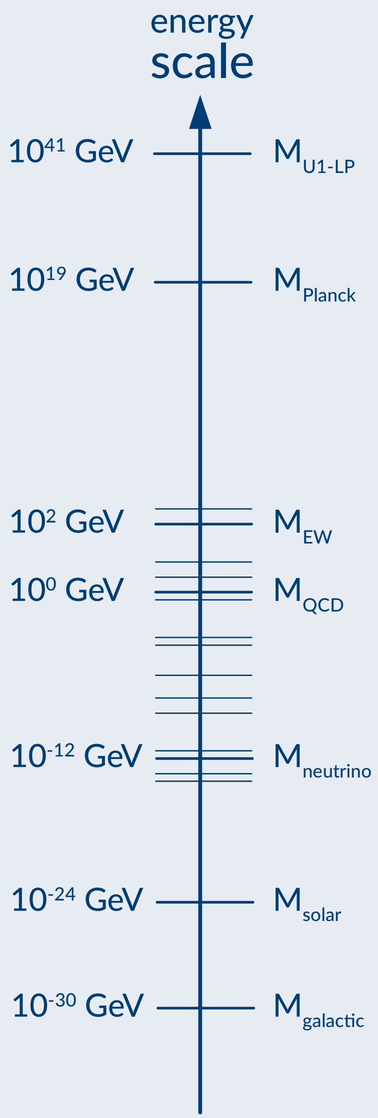
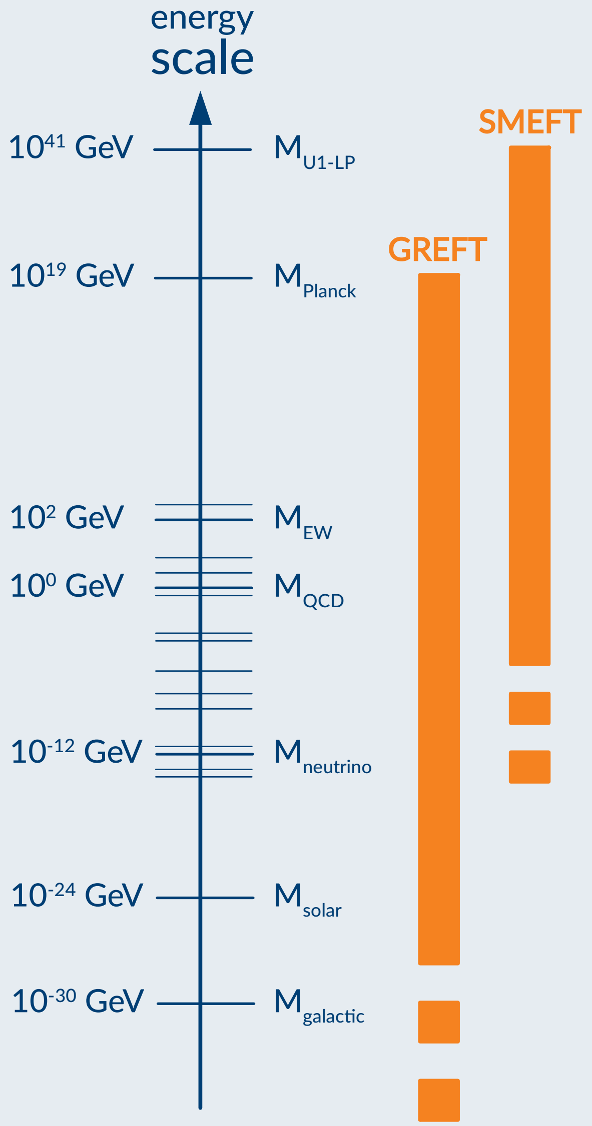
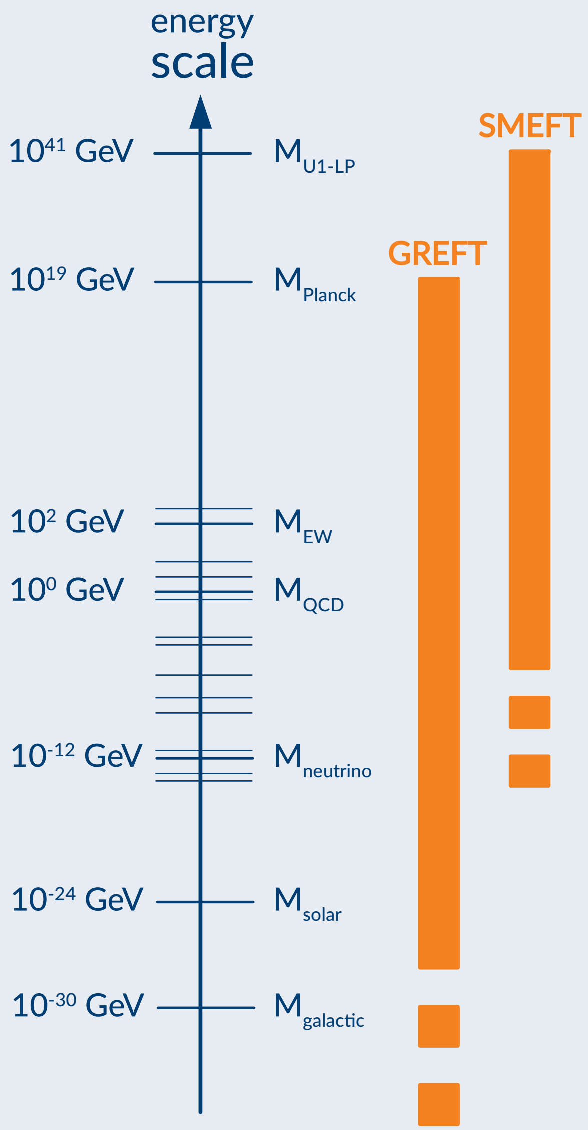

energy
scale
10^41 GeV
M_U1-LP
10^19 GeV
M_Planck
10^2 GeV
M_EW
10^0 GeV
M_QCD
10^-12 GeV
M_neutrino
10^-24 GeV
M_solar
10^-30 GeV
M_galactic



- SMEFT + GREFT works …



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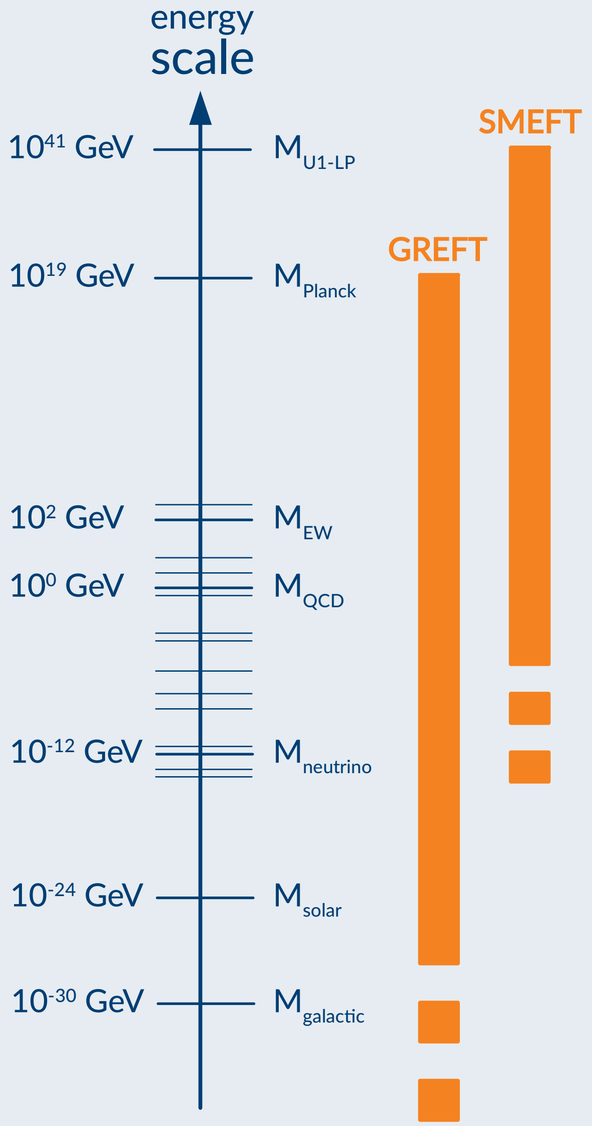
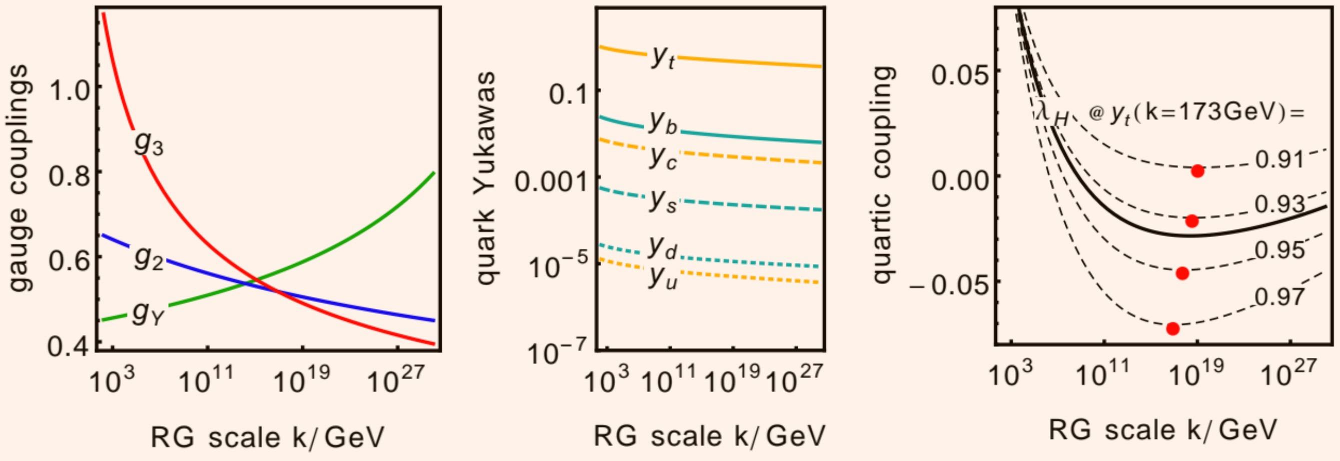


Held, PhD thesis, '19

1-loop running of **gauge couplings**:
~ unification at 10^{15} GeV
consistent beyond 10^{19} GeV
U(1)-Landau pole at 10^{41} GeV

1-loop running of
quark-Yukawas
Consistent >>10^{19} GeV
peculiar hierarchy pattern

1-loop running of the Higgs-quartic for different values of the top-Yukawa coupling: experimental values are consistent with exact **scale symmetry** at 10^{19} GeV



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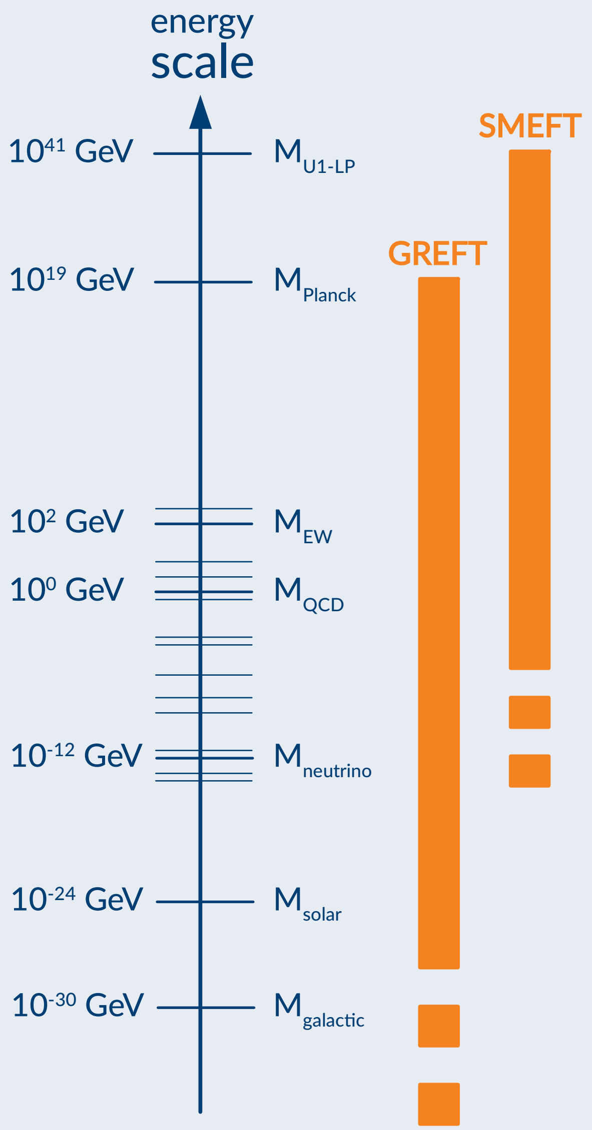

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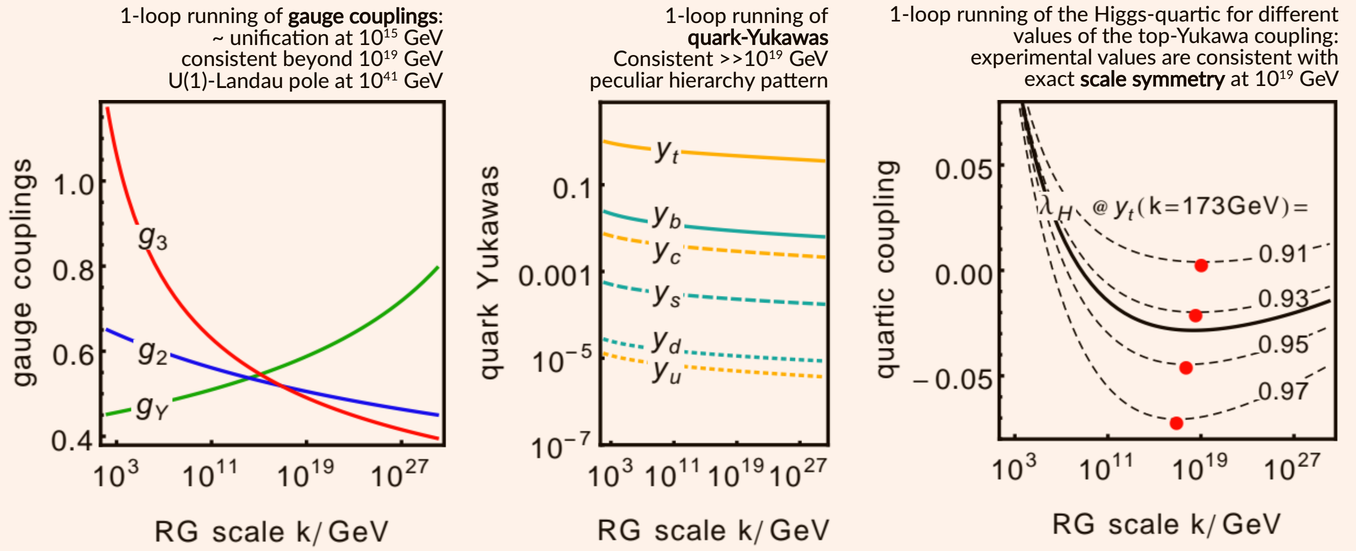
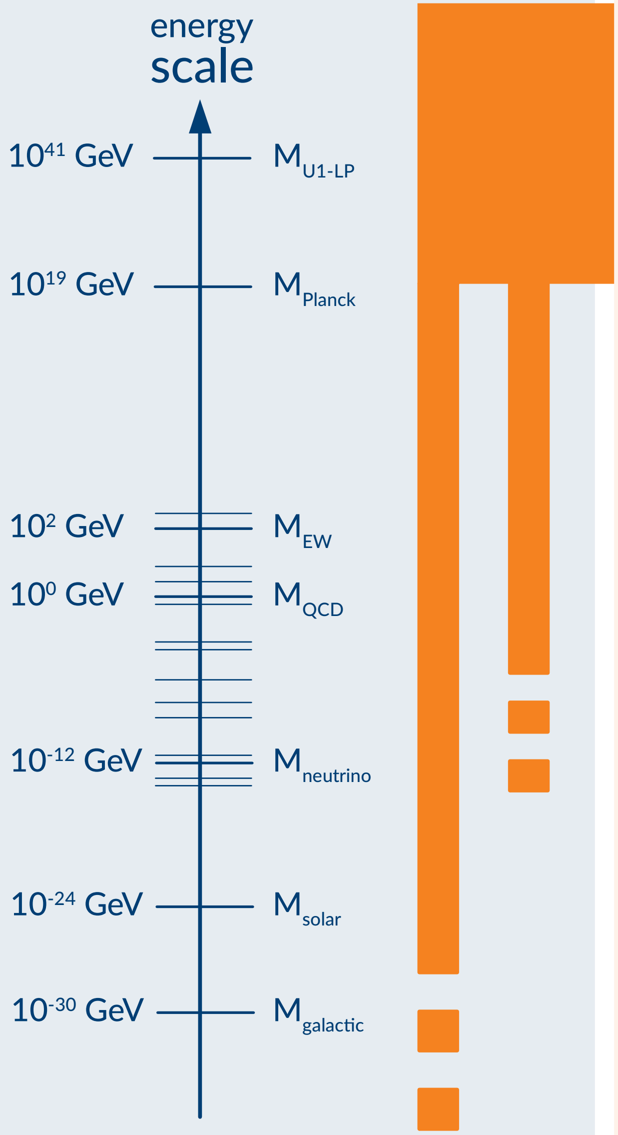


- SMEFT + GREFT works … let's make use of it



ANY PROPOSAL FOR A QUANTITATIVE AND PREDICTIVE UV-COMPLETION

Held, PhD thesis, '19

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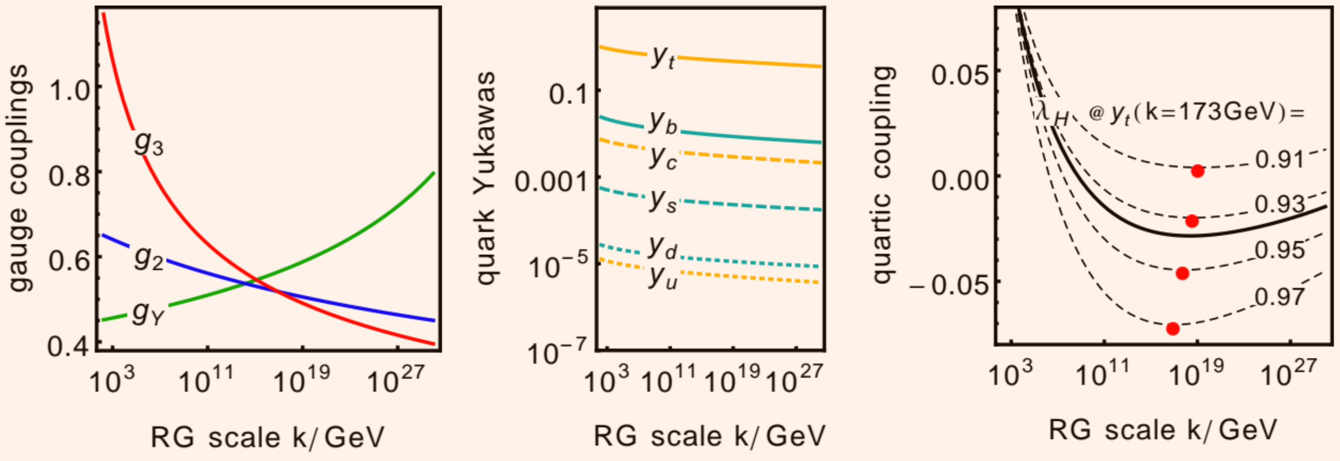

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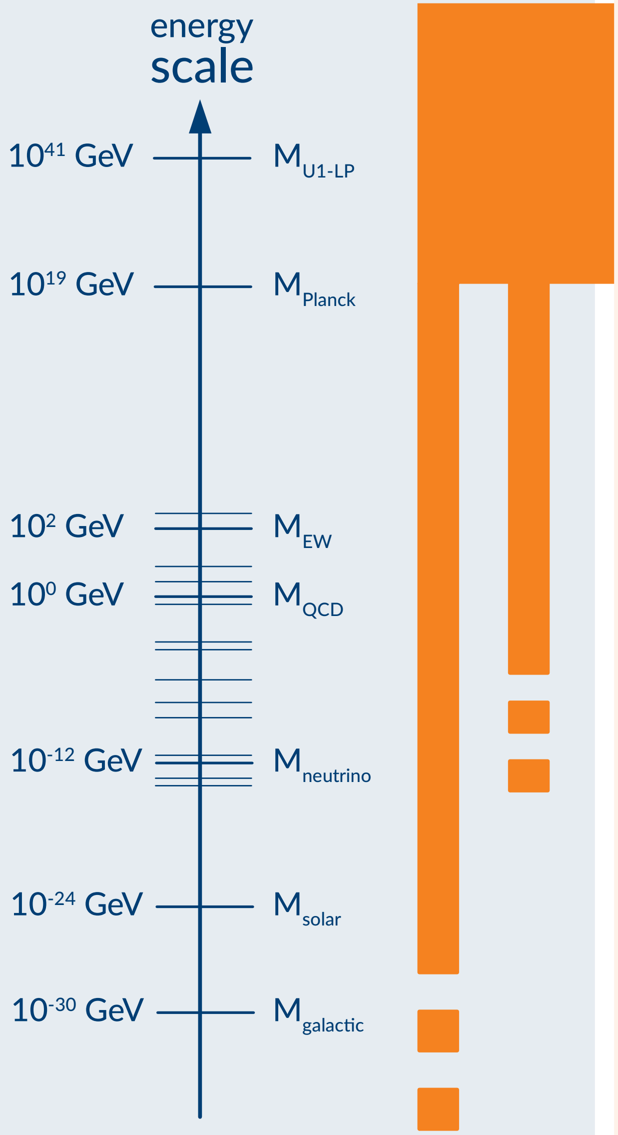
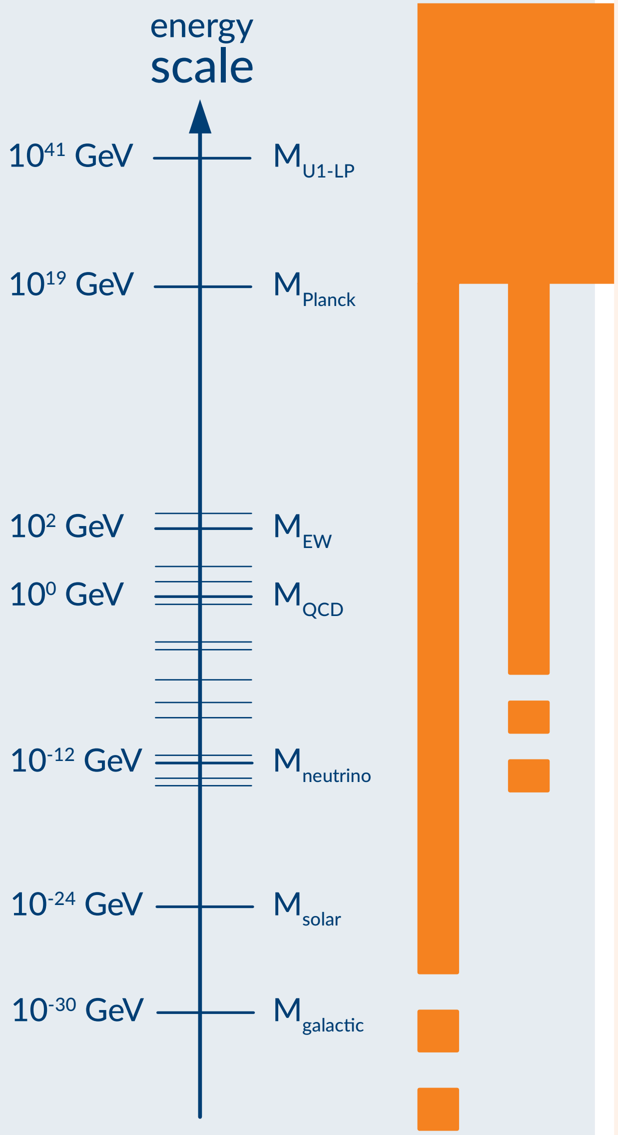


asymptotically safe effective action

only a few free parameters!



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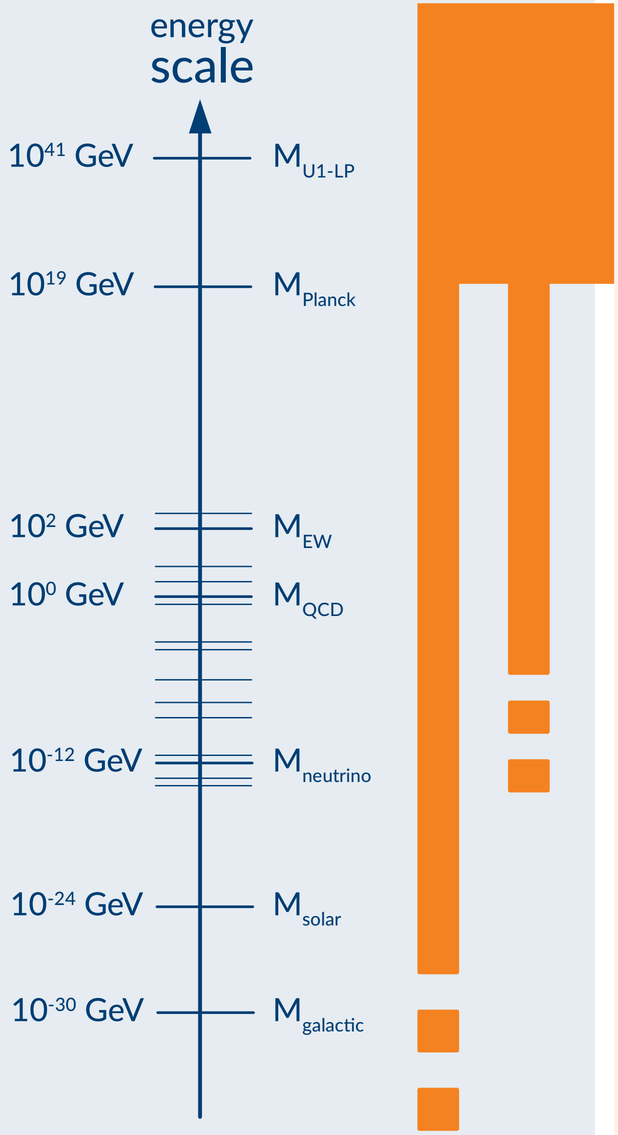

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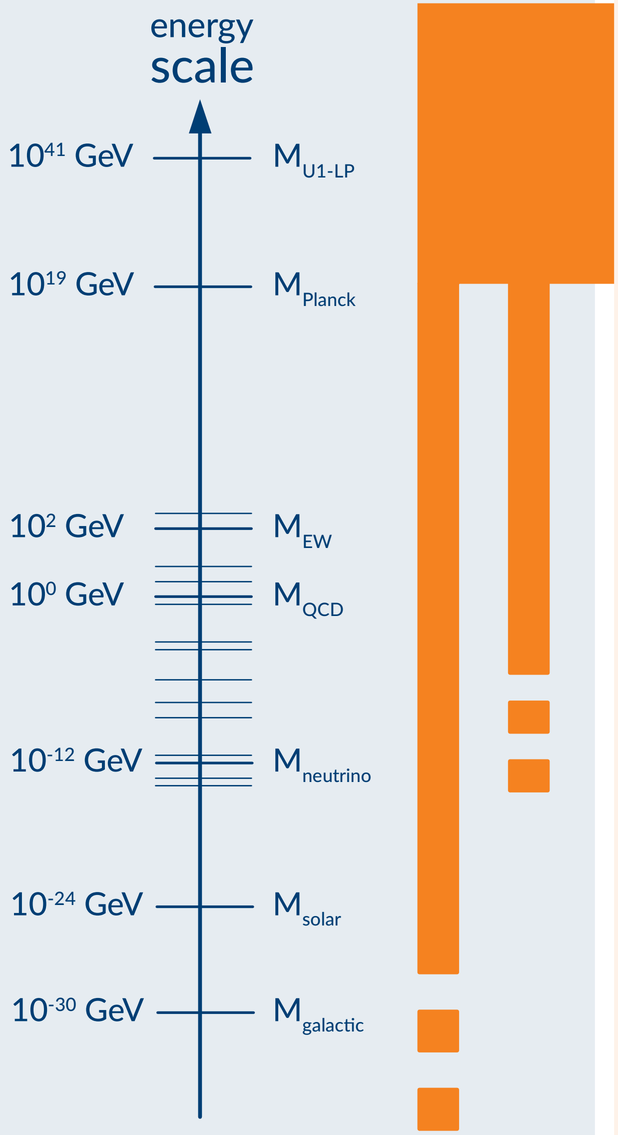

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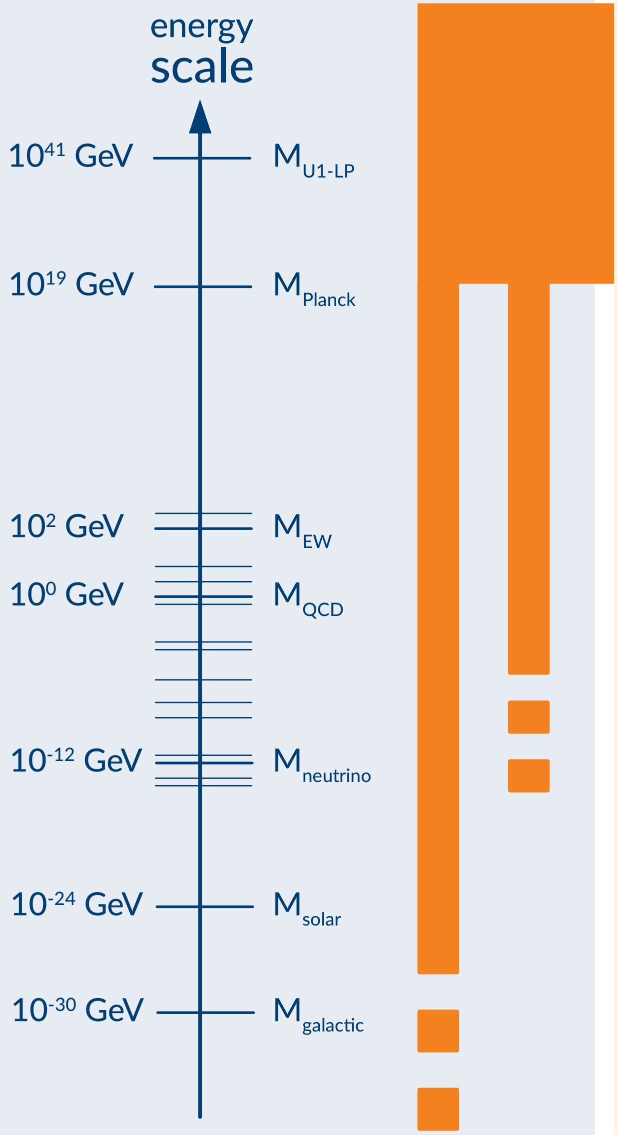

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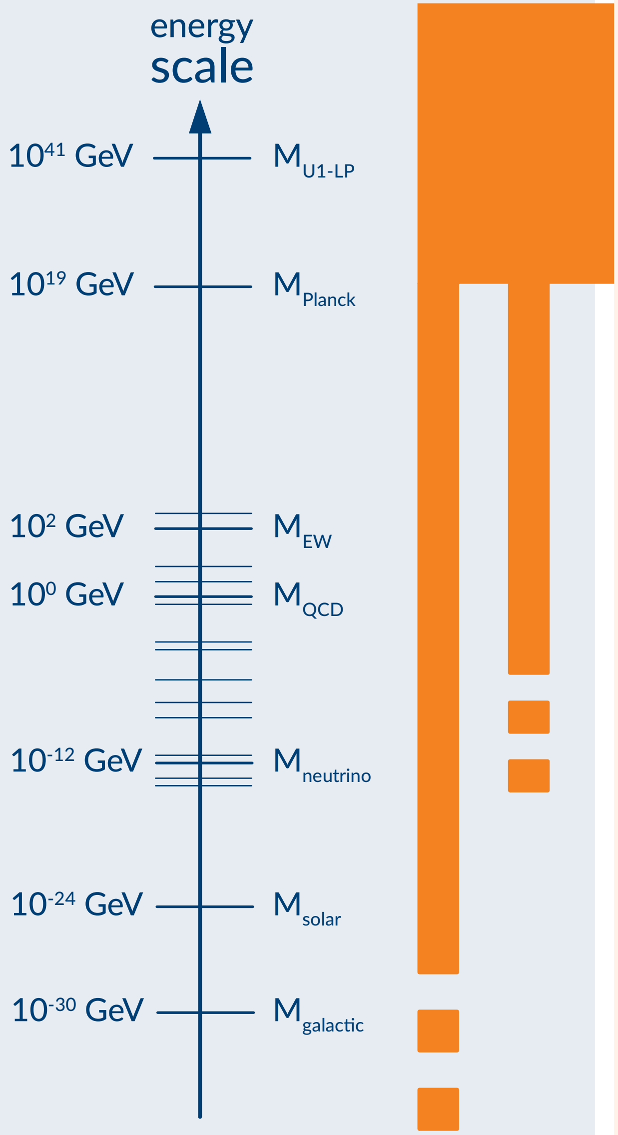

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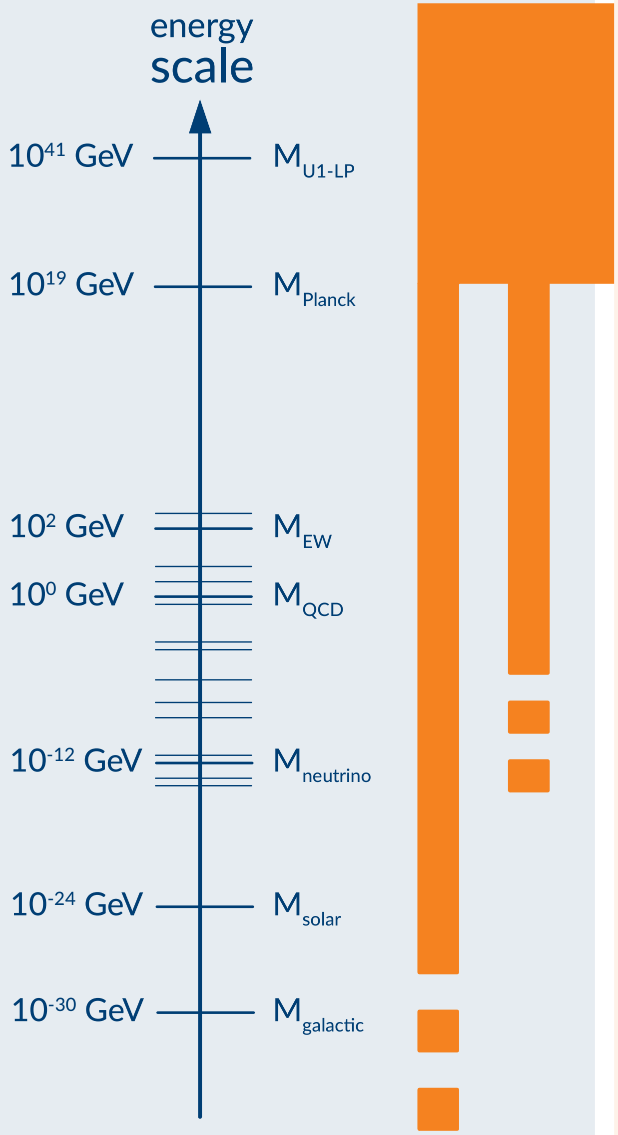
singularity theorems



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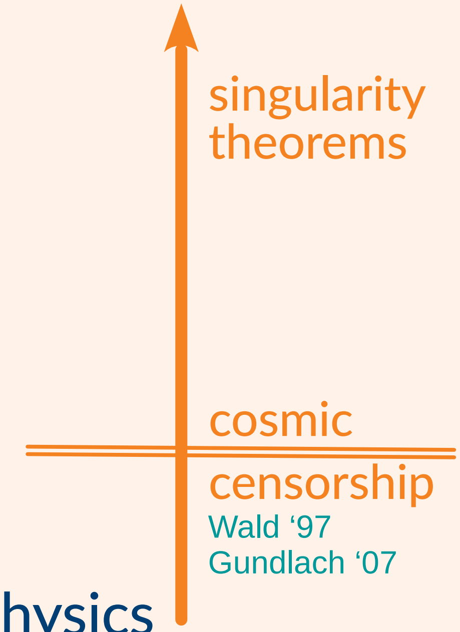

- cosmology again [late universe]

Maintaining

cosmic censorship in gravitational EFTs

seems to be very nontrivial!

Ripley, Pretorius ‘19
Figueras, France ‘20
Hegade, Ripley, Yunes ‘22
...

Maintaining

cosmic censorship in gravitational EFTs

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Violations … either suggest that the **EFT is incomplete**
or suggest an opportunity for **smoking-gun signatures.**

Part I:
global
"solutions"

Interlude:
The quantum
effective action

Part II:
linear
dynamics

Part III:
nonlinear
dynamics

A comment
on ghosts?

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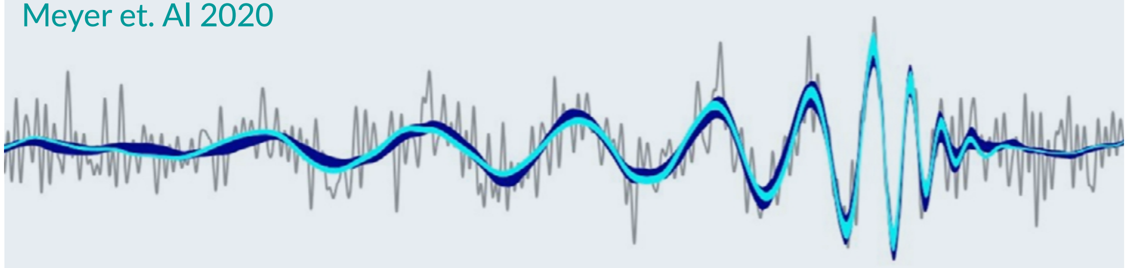
EHT collaboration

adapted from
Meyer et. Al 2020



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Part I: global (horizonless) solutions

by overspinning regular black holes: Eichhorn, Held, JCAP 01 (2023) 032, …
by distinct branches of global solutions: Daas et.Al ‘22, …

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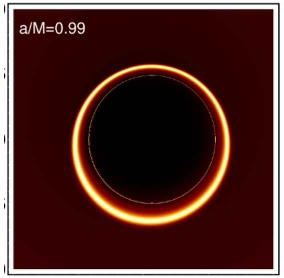
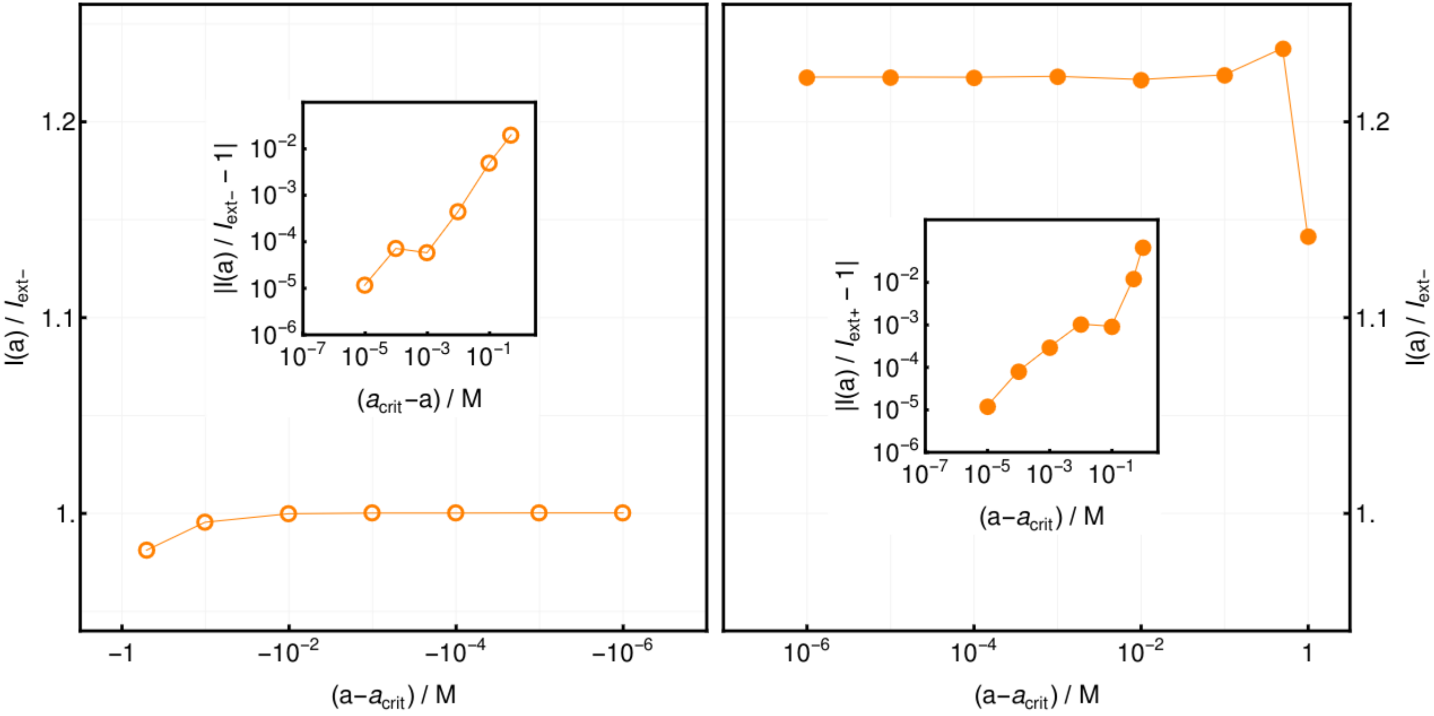
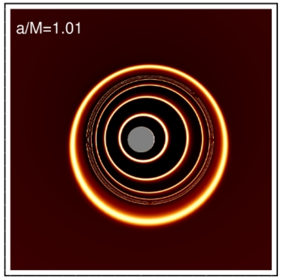
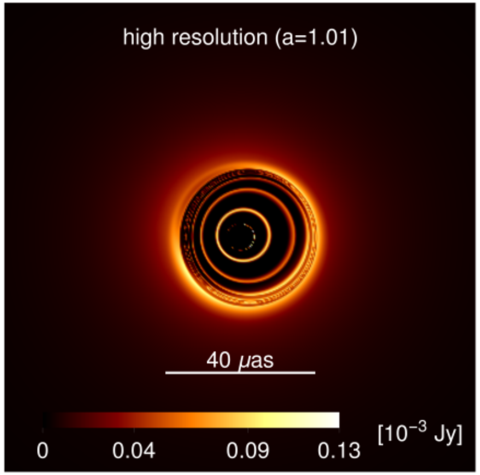


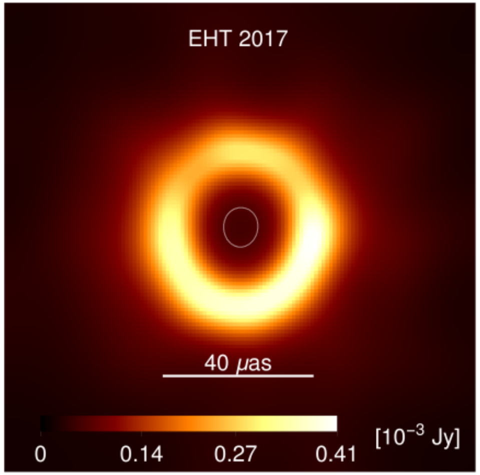




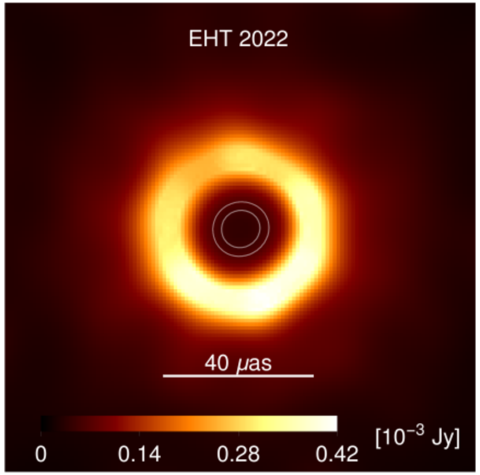
Can we exclude this?

Eichhorn, Held, Gold, ApJ. 950 (2023) 2, 117





ehtim reconstruction
EHT array 2017
8 telescopes
230 GHz



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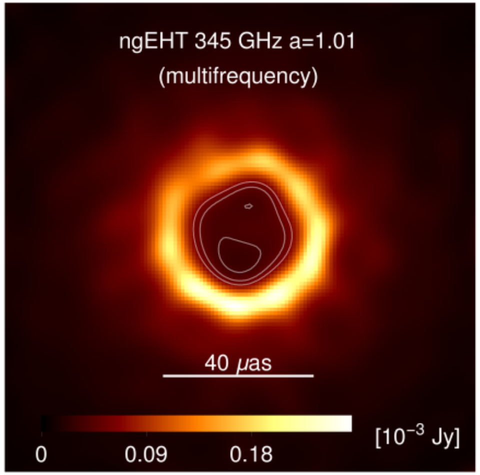

ehtim reconstruction
proposed ngEHT array
21 telescopes
230 | 345 GHz
(multifrequency)

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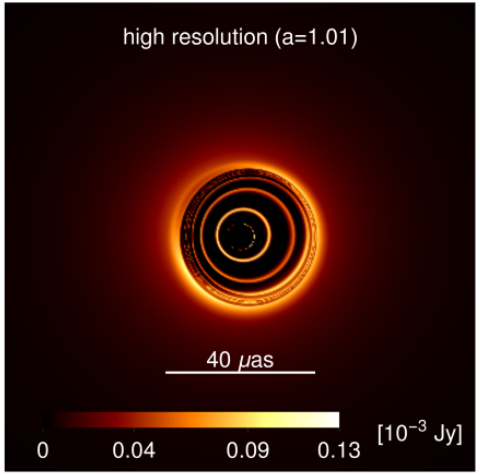

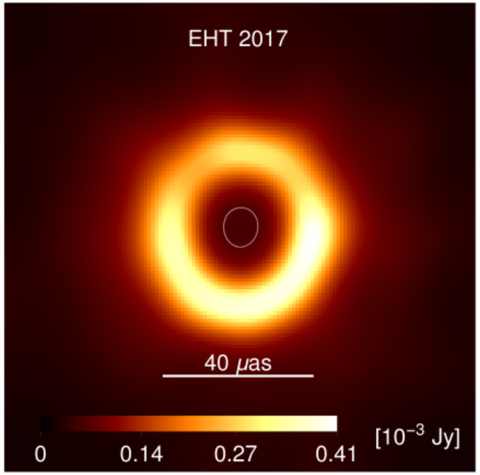

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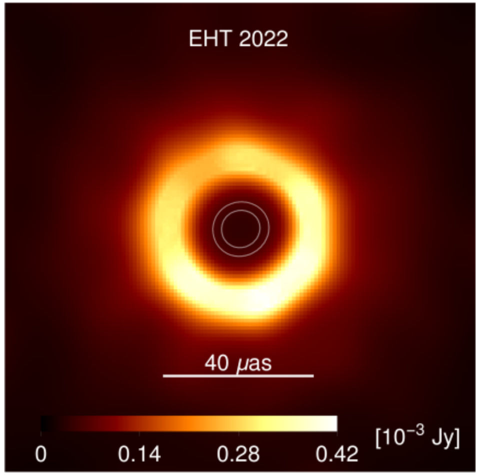

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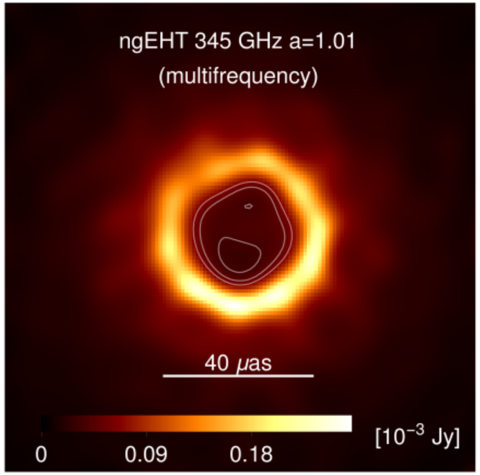

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Do we really trust the RG improvement? Held 2105.11458

Interlude: The quantum effective action

How to expand?

- might be **nontrivial to resolve curvature & momentum**

$$N_R \;:\; \text{power of curvature}$$
$$N_P \;:\; \text{power of momenta}$$

How to expand?

- might be **nontrivial to resolve curvature & momentum**

$$N_R[\text{Riem}] = 1 \quad N_P[\text{Riem}] = 2$$

$$N_R[\Box] = 0 \quad N_P[\Box] = 2$$

N_R : power of curvature

N_P : power of momenta

How to expand?

- might be **nontrivial to resolve curvature & momentum**

$$\mathrm{N_R[Riem]} = 1 \quad \tilde{\mathrm{N}}_\mathrm{P}[\mathrm{Riem}] = 0$$

$$\mathrm{N_R}[\Box] = 0 \qquad \tilde{\mathrm{N}}_\mathrm{P}[\Box] = 1$$

$\mathrm{N_R}$: power of curvature

$\tilde{\mathrm{N}}_\mathrm{P} = \mathrm{N_P}/2 - \mathrm{N_R}$

How to expand?

- might be **nontrivial to resolve curvature & momentum**

N_R : power of curvature

$\tilde{N}_P = N_P/2 - N_R$

General Relativity

$$-\frac{\mathcal{L}_{\mathrm{GR}}}{\sqrt{-g}} = M_{Pl}^2\left[\lambda + \frac{1}{2}R\right]$$

$\tilde{N}_P = 0$

$N_R = 1$

... propagates **2 DoF**

How to expand?

- might be **nontrivial to resolve curvature & momentum**

N_R : power of curvature

$\tilde{N}_P = N_P/2 - N_R$

Quadratic Gravity

$$-\frac{\mathcal{L}_{\text{quadratic}}}{\sqrt{-g}} = M_{Pl}^2 \left[\lambda + \frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

$\tilde{N}_P = 0$

$N_R = 2$

... propagates **2 + 1 + 5 DoF**

How to expand?

- might be **nontrivial to resolve curvature & momentum**

N_R : power of curvature

$\tilde{N}_P = N_P/2 - N_R$

Cubic Gravity

$$-\frac{\mathcal{L}_{\text{cubic}}}{\sqrt{-g}} = M_{Pl}^2 \left[\cdots + \frac{d_1}{\Lambda^4} R_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta\rho\sigma} + \frac{d_1}{\Lambda^4} \tilde{R}_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta\rho\sigma} \right]$$

$\tilde{N}_P = 0$

$N_R = 3$

... propagates **2 + 1 + 5 DoF**

How to expand?

- might be **nontrivial to resolve curvature & momentum**

N_R : power of curvature

$\tilde{N}_P = N_P/2 - N_R$

Quartic Gravity

$$-\frac{\mathcal{L}_{\text{quartic}}}{\sqrt{-g}} = M_{Pl}^2 \left[\cdots + \frac{e_1}{\Lambda^6}\,\mathcal{C}^2 + \frac{e_2}{\Lambda^6}\,\tilde{\mathcal{C}}^2 + \frac{e_3}{\Lambda^6}\,\tilde{\mathcal{C}}\mathcal{C} \right]$$

$\tilde{N}_P = 0$

$N_R = 4$

... propagates **2 + 1 + 5 DoF**

How to expand?

- might be **nontrivial to resolve curvature & momentum**

N_R : power of curvature

$\tilde{N}_P = N_P/2 - N_R$

f(Riemann) Gravity

$$-\frac{\mathcal{L}_\infty}{\sqrt{-g}} = \mathcal{F}(\text{Riem})$$

$\tilde{N}_P = 0$

$N_R = \infty$

Linear: Hindawi, Ovrut, Waldram, PRD 53 (1996)
Nonlinear: ongoing work with Pau Figueras

... propagates **2 + 1 + 5 DoF**

How to expand?

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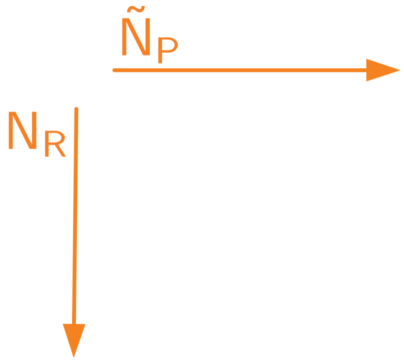
N_R : power of curvature

$\tilde{N}_P = N_P/2 - N_R$



fiducial DoF

Barnaby, Kamran, JHEP 02 (2008) 008

DoF saturate to

2 + 1 + 5

Hindawi, Ovrut, Waldram, PRD 53 (1996)

How to expand?

- might be **nontrivial to resolve curvature & momentum**

N_R : power of curvature

$\tilde{N}_P = N_P/2 - N_R$

Form factors

$$-\frac{\mathcal{L}_\infty}{\sqrt{-g}} = R\,\mathcal{F}_R(\Box)R + R_{\mu\nu}\,\mathcal{F}_{Ric}(\Box)R^{\mu\nu} + C_{\mu\nu\rho\sigma}\,\mathcal{F}_{Weyl}(\Box)C^{\mu\nu\rho\sigma}$$

$\tilde{N}_P = \infty$

$N_R = 2$

... propagating **DoF depend on poles**

Quadratic Gravity

Stelle, PRD 16 (1977) 953-969

$$S = \int d^4x \sqrt{|g|}\, M^2_{\mathrm{Planck}} \left[\frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2

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massless spin-2 massive spin-0 massive spin-2

as a fundamental theory

[perturbatively renormalizable; asymtotically free; ghost]

Stelle, PRD 16 (1977) 953-969

see also talk by Luca Buoninfante

Avramidi, Barvinsky, PLB 159 (1985) 269-274

Bender, Mannheim, PRL 100 (2008)

Donoghue, Menezes, PRD 104 (2021) 4

Quadratic Gravity

Stelle, PRD 16 (1977) 953-969

$$S = \int d^4x \sqrt{|g|}\, M^2_{\mathrm{Planck}} \left[\frac{1}{2} R \;+\; \frac{1}{12 m_0^2} R^2 \;+\; \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2

as the marginal terms
in the effective action

modulo:

essential scheme / field redefinitions

Baldazzi, Falls ‘21
Knorr ‘22

Burgess, Living Rev. Rel. 7:5,2004
Endlich et. Al, JHEP 09 (2017) 122

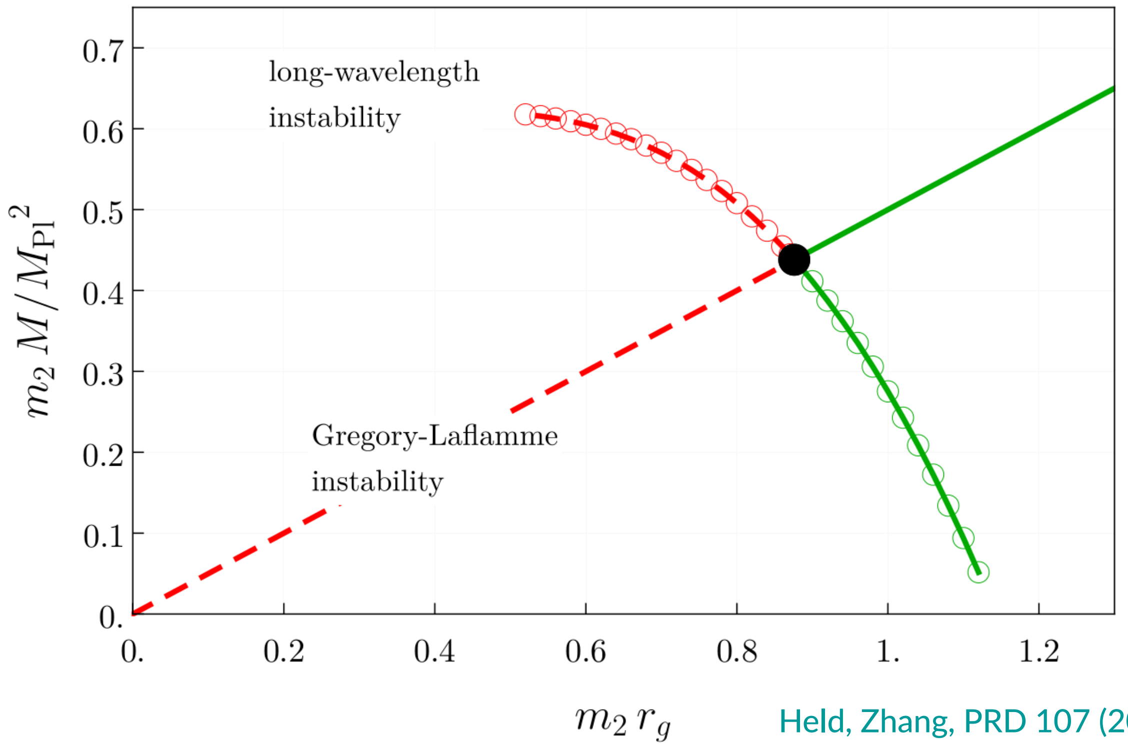
see also talk by Kevin Falls

Part II: Linear dynamics in Quadratic Gravity

Held, Zhang, PRD 107 (2023) 6

Linear stability: Spherically-symmetric BHs

Held, Zhang, PRD 107 (2023) 6



Held, Zhang, PRD 107 (2023) 6

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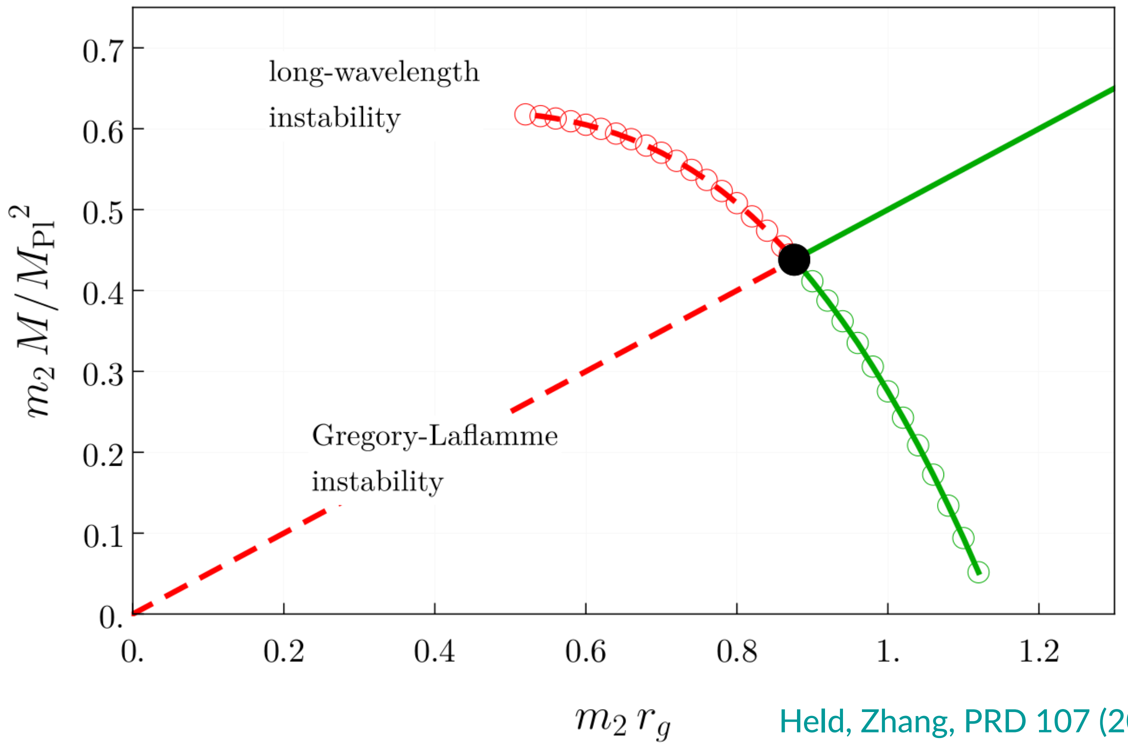
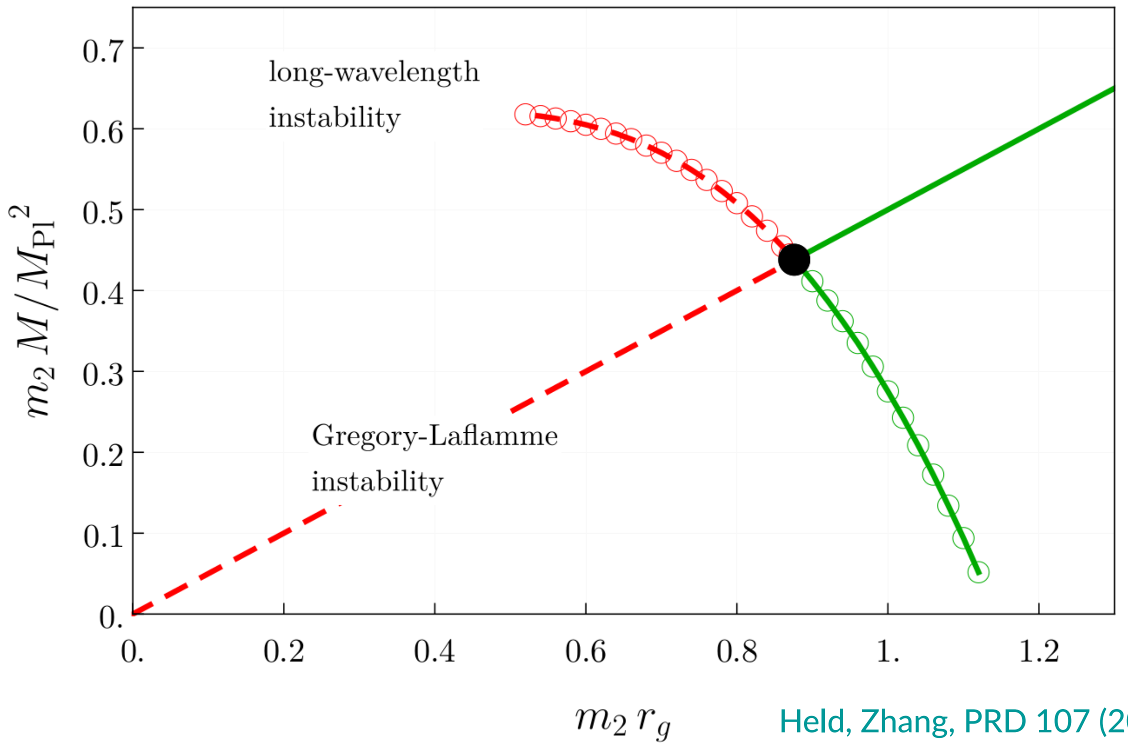


Held, Zhang, PRD 107 (2023) 6

- **Black-hole uniqueness is broken** (in the fundamental theory)
  Svarc et. Al, 2209.15089

Linear stability: Spherically-symmetric BHs

Held, Zhang, PRD 107 (2023) 6



Held, Zhang, PRD 107 (2023) 6

- **Black-hole uniqueness is broken** (in the fundamental theory)
  Svarc et. Al, 2209.15089

- **Large BHs are stable**
- **Small BHs are unstable**

Dynamics: linear DoF

Background: decomposition

Boundary conditions:

Dynamics: linear DoF

$$\mathcal{L}_{QG} = M_{Pl}^2 \left[\frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

- massless spin-2 (graviton) h_{ab}
- massive spin-0 ϕ
- massive spin-2 ψ_{ab}

Background: decomposition

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Background: decomposition

- spherical harmonics $Y_{\ell m}(\theta,\phi)$

$$h_{ab}^{(polar)} = e^{-i\omega t}\, h_{ab}^{(polar)\ell m}(r)\, Y^{\ell m}(\theta,\phi)$$
$$h_{ab}^{(axial)} = e^{-i\omega t}\, h_{ab}^{(axial)\ell m}(r)\, Y^{\ell m}(\theta,\phi)$$
$$\psi_{ab}^{(polar)} = e^{-i\omega t}\, \psi_{ab}^{(polar)\ell m}(r)\, Y^{\ell m}(\theta,\phi)$$
$$\psi_{ab}^{(axial)} = e^{-i\omega t}\, \psi_{ab}^{(axial)\ell m}(r)\, Y^{\ell m}(\theta,\phi)$$

Boundary conditions:

Dynamics: linear DoF

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- massless spin-2 (graviton) h_{ab}
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- massive spin-2 ψ_{ab}

Background: decomposition

- spherical harmonics $Y_{\ell m}(\theta, \phi)$
- axisymmetric perturbations $m = 0$

$$h_{ab}^{(polar)} = e^{-i\omega t}\begin{pmatrix} AH_0 & H_1 & 0 & 0 \\ H_1 & H_2/B & 0 & 0 \\ 0 & 0 & r^2\mathcal{K} & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta\,\mathcal{K} \end{pmatrix} Y^\ell(\theta)$$

$$\psi_{ab}^{(polar)} = e^{-i\omega t}\begin{pmatrix} AF_0 & F_1 & \mathcal{F}_0\,\partial_\theta & 0 \\ F_1 & F_2/B & \mathcal{F}_1\,\partial_\theta & 0 \\ \mathcal{F}_0\,\partial_\theta & \mathcal{F}_1\,\partial_\theta & \mathcal{M}+\mathcal{N}\,\partial_\theta^2 & 0 \\ 0 & 0 & 0 & \sin^2\theta\,\mathcal{M} \end{pmatrix} Y^\ell(\theta)$$

Boundary conditions:

Dynamics: linear DoF

$$\mathcal{L}_{QG} = M_{Pl}^2 \left[\frac{1}{2} R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

- ➢ massless spin-2 (graviton) h_{ab}
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$$\frac{d^2}{dr_*^2} \psi(r) + \psi(r) \left[\omega^2 - V(r) \right] = 0$$

GR-background: Brito, Cardoso, Pani '13
non-GR: **Held**, Zhang, PRD 107 (2023) 6

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non-GR: **Held**, Zhang, PRD 107 (2023) 6

Boundary conditions:

- purely ingoing waves at the horizon
- outgoing waves at asymptotic infinity define QNMs
- ingoing waves at asymptotic infinity define bound states

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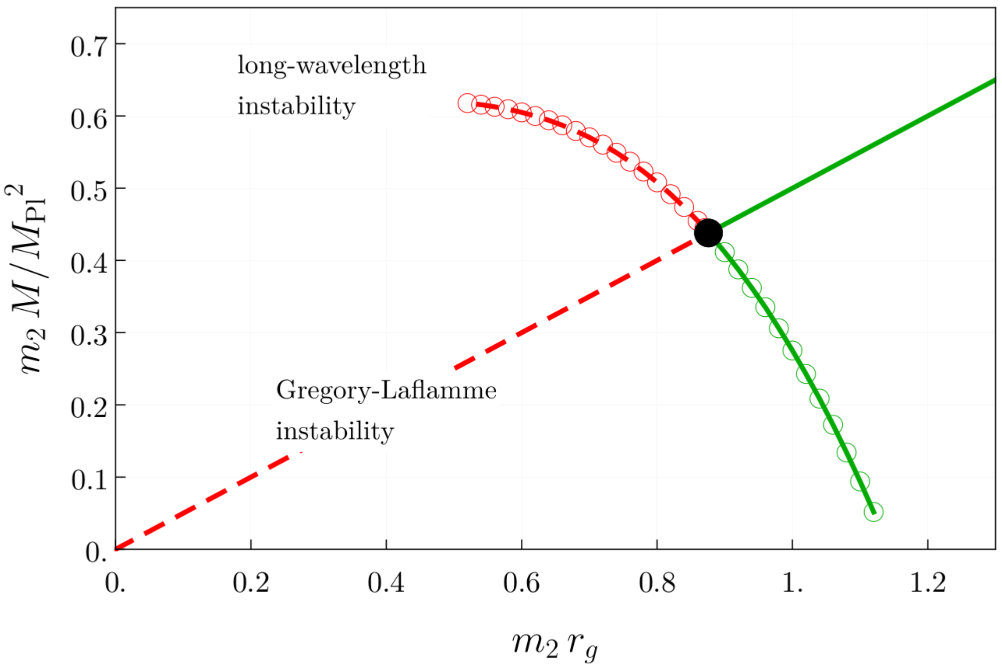
- positive imaginary part signals instability
- negative imaginary part signals stability

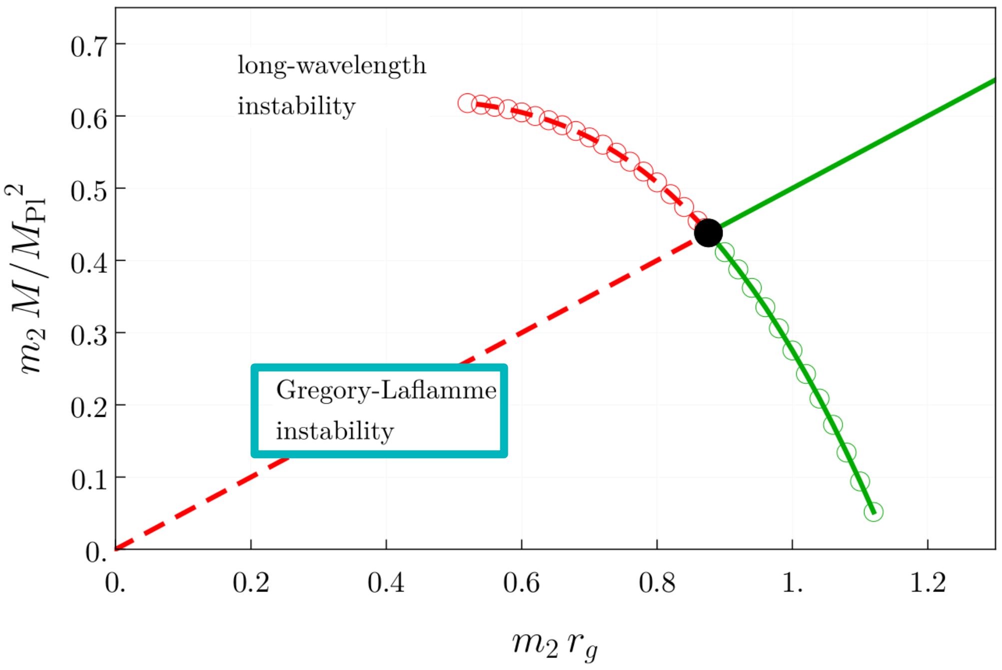
Linear stability of BHs in Quadratic Gravity

Held, Zhang, PRD 107 (2023) 6



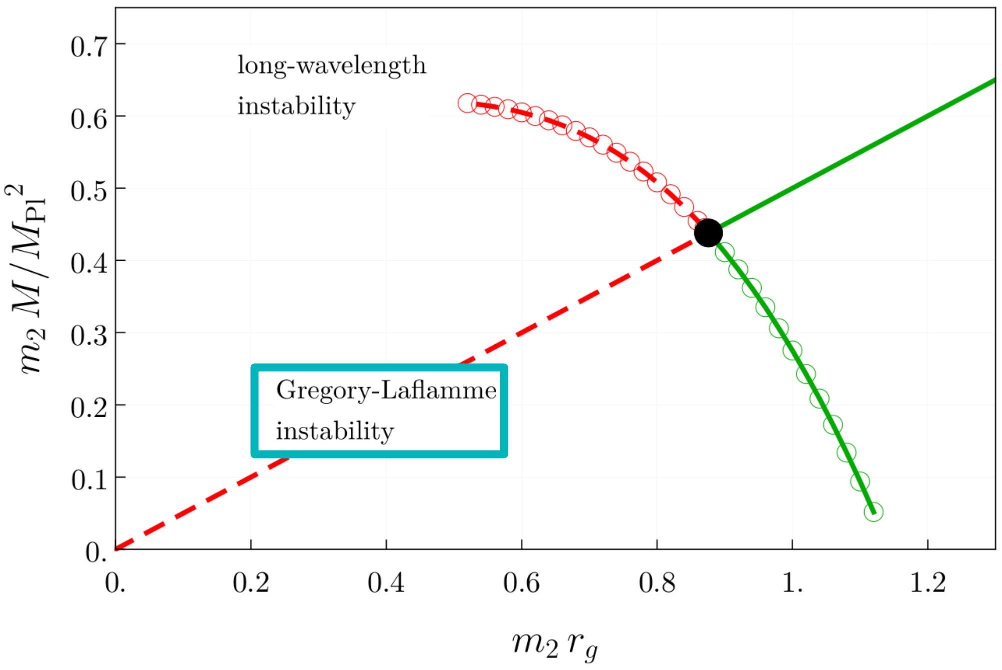
Linear stability of BHs in Quadratic Gravity

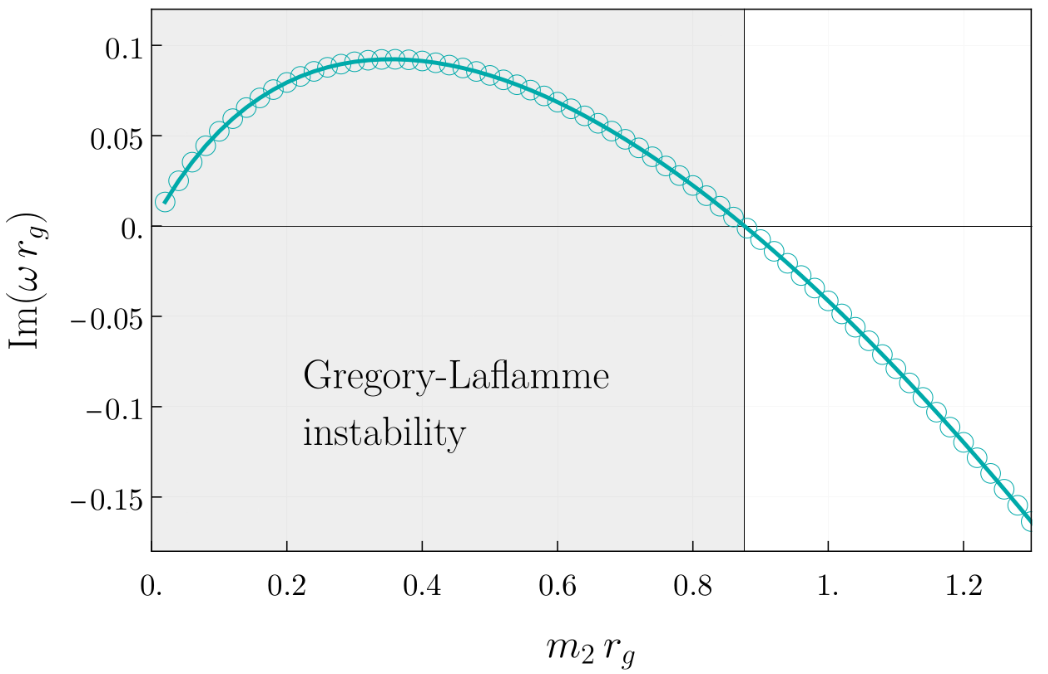
Held, Zhang, PRD 107 (2023) 6



Linear stability of BHs in Quadratic Gravity

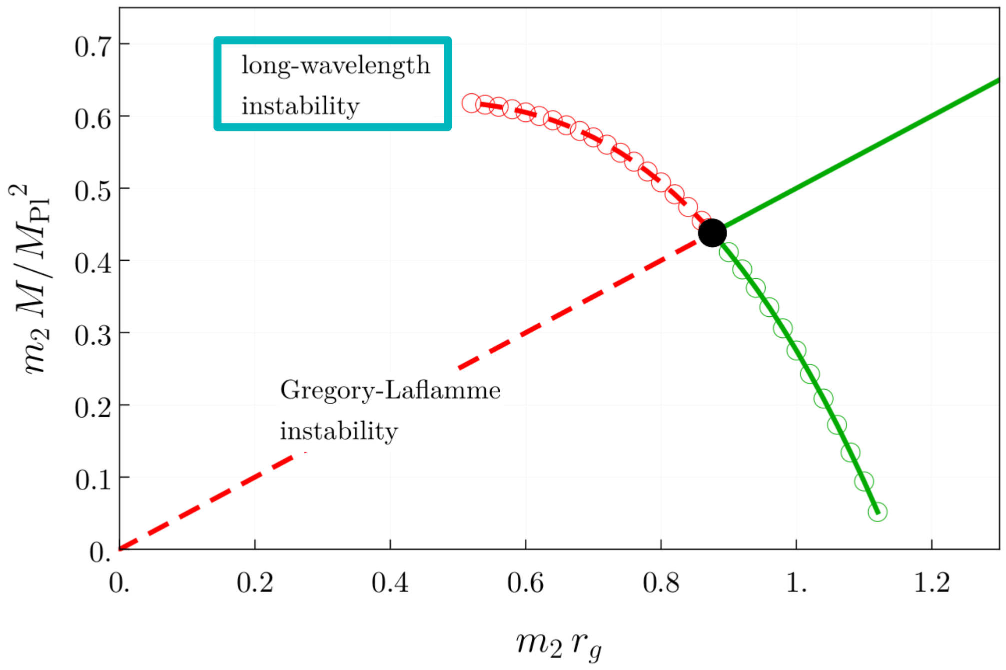
Held, Zhang, PRD 107 (2023) 6





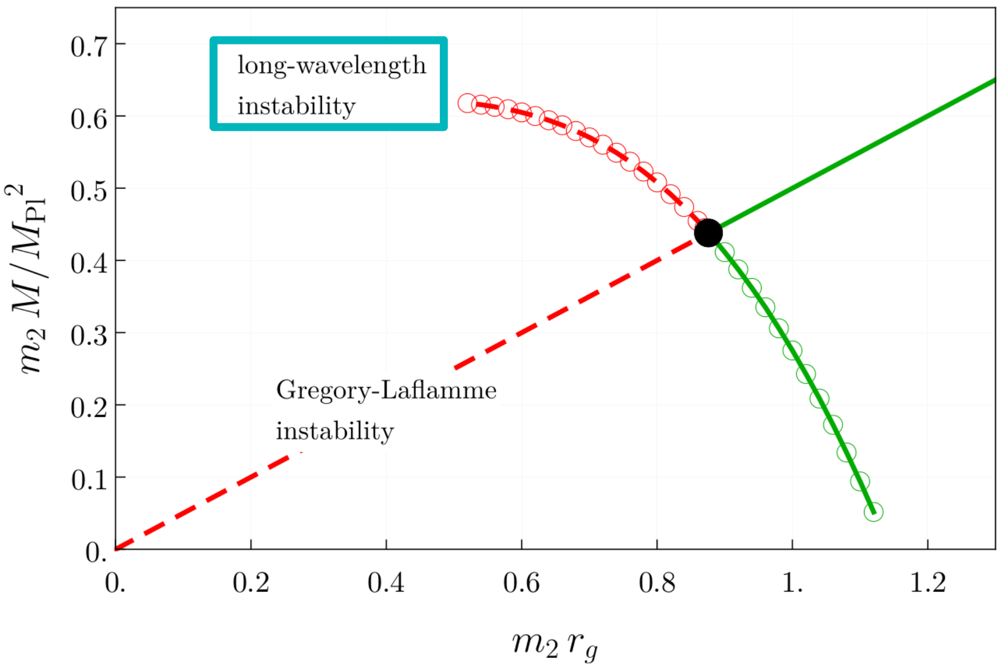
Linear stability of BHs in Quadratic Gravity

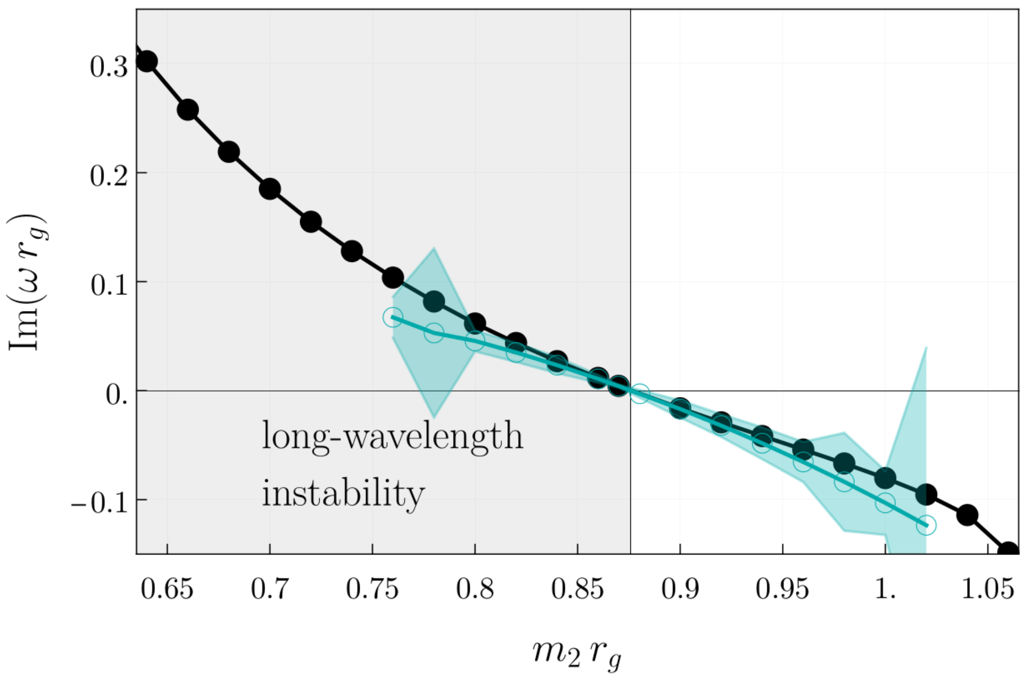
Held, Zhang, PRD 107 (2023) 6



Linear stability of BHs in Quadratic Gravity

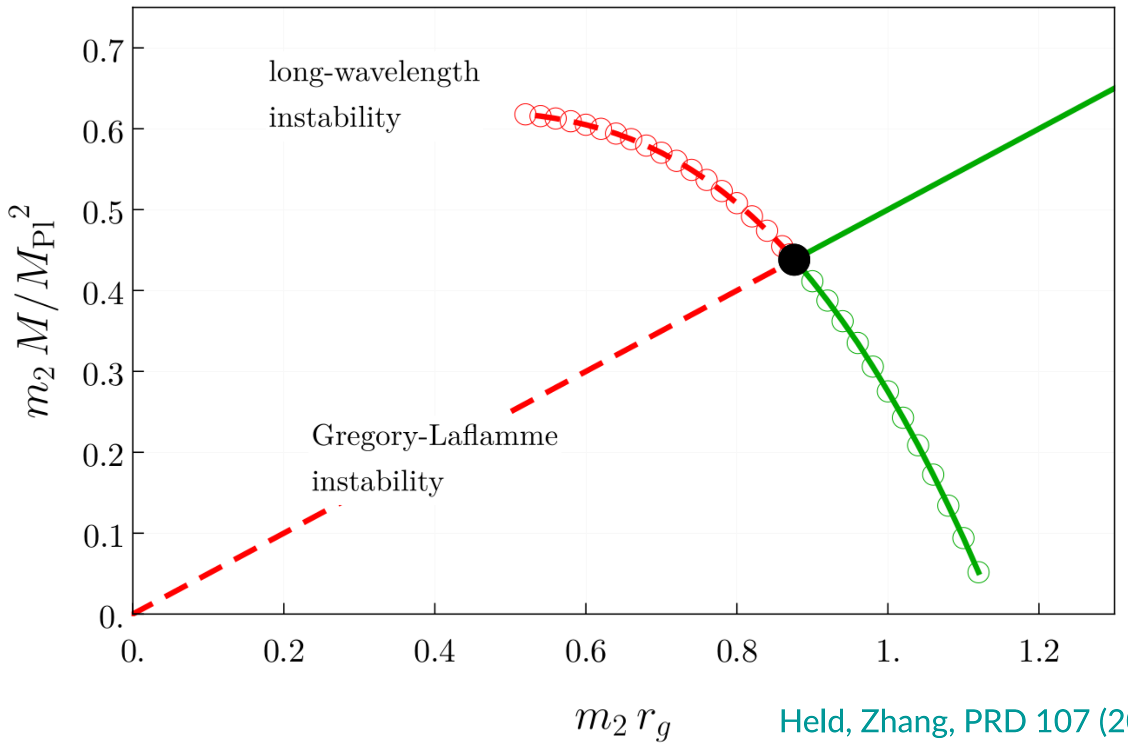
Held, Zhang, PRD 107 (2023) 6





Linear stability: Spherically-symmetric BHs

Held, Zhang, PRD 107 (2023) 6



Held, Zhang, PRD 107 (2023) 6

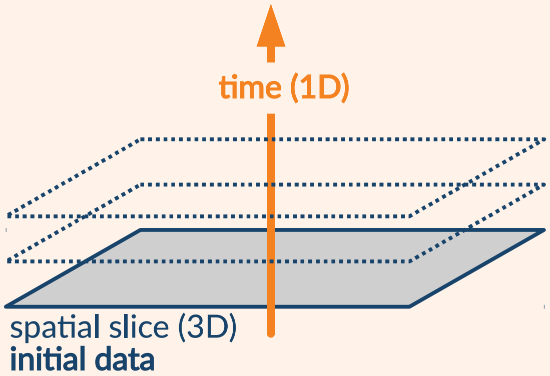
- **Black-hole uniqueness is broken** (in the fundamental theory)
  Svarc et. Al, 2209.15089

- **Large BHs are stable**
- **Small BHs are unstable**

Part III: Nonlinear dynamics in Quadratic Gravity

Held, Lim, PRD 104 (2021) 8
Held, Lim, 2306.04725

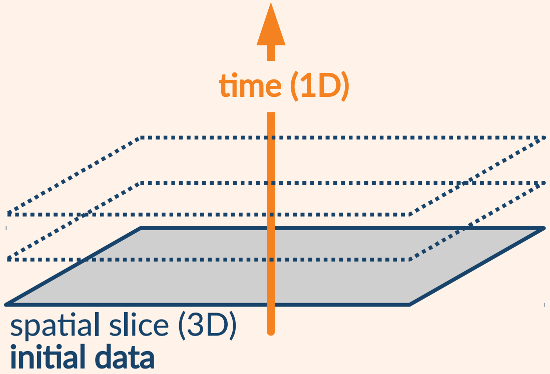
A well-posed initial value problem (IVP) ...



"An initial value problem is well-posed if a solution

- **exists** for all future time
- is **unique**
- and depends **continuous**ly on the initial data"

A well-posed initial value problem (IVP) ...



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... for General Relativity

Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat '52

(3+1) numerical evolution
Frans Pretorius '05
Baumgarte, Shapiro, Shibata, Nakamura '87-'99
Sarbach et.Al '02-'04

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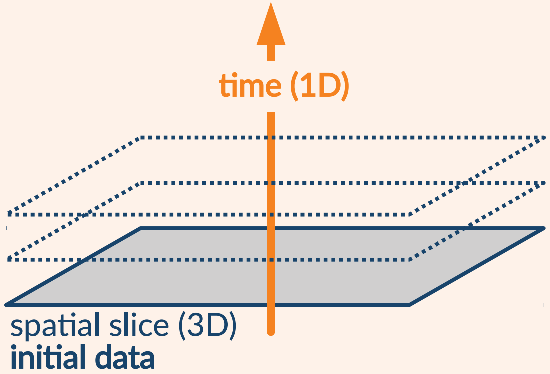

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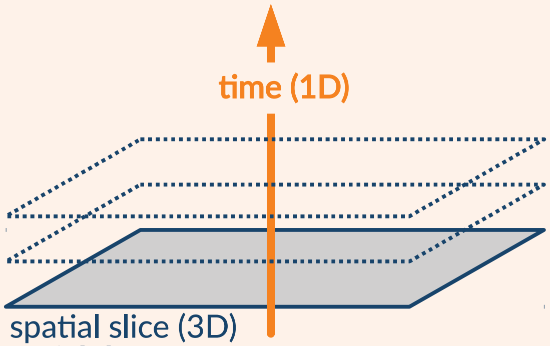
… and for Quadratic Gravity

Formal proof of existence and uniqueness
Noakes '83

spherical symmetry: Held, Lim, PRD 104 (2021) 8
(3+1): Held, Lim, 2306.04725

A well-posed initial value problem (IVP) …



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… and for Quadratic Gravity

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spherical symmetry: Held, Lim, PRD 104 (2021) 8
(3+1): Held, Lim, 2306.04725
Cayuso, 2307.15163
East, Siemonsen, 2309.05096

Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim 2306.04725

2nd-order variables

$$R_{ab}(\Box g) = \widetilde{\mathcal{R}}_{ab} + \frac{1}{4} g_{ab}\mathcal{R} \equiv \widetilde{T}_{ab}$$

massless spin-2 (graviton)

$$\Box \mathcal{R} = m_0^2 \mathcal{R} + 2T^c{}_c$$

massive spin-0 (scalar)

$$\Box \widetilde{\mathcal{R}}_{ab} = -\frac{1}{3}\left(\frac{m_2^2}{m_0^2} - 1\right)(\nabla_a \nabla_b \mathcal{R}) - 2\widetilde{\mathcal{R}}^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$

massive spin-2 (ghost)

Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim 2306.04725

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massive spin-2 (ghost)

1st-order variables

$$\tilde{V}_{ab} \equiv -n^c \nabla_c \tilde{\mathcal{R}}_{ab}$$

$$\hat{\mathcal{R}} \equiv -n^c \nabla_c \mathcal{R}$$

2nd order quasilinear diagonal

+ constraints

(in harmonic gauge)

Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim 2306.04725

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Leray's theorem guarantees well-posed IVP for $\mathcal{C}^\infty$ initial data

Leray '53
Choquet-Bruhat & DeWitt-Morette '77

Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim 2306.04725

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(3+1) decomposition

$$g_{ab} = \gamma_{ab} + n_a n_b$$

$$\tilde{\mathcal{R}}_{ab} = \mathcal{A}_{ab} + \frac{1}{3}\gamma_{ab}\,\mathcal{A} - 2\,n_{(a}\mathcal{C}_{b)} + n_a n_b\,\mathcal{A}$$

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Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim 2306.04725

2nd-order variables

massless spin-2 (ADM / BSSN)

$$R_{ab}(\Box g) = \tilde{\mathcal{R}}_{ab} + \frac{1}{4} g_{ab}\mathcal{R} \equiv \tilde{T}_{ab}$$

massless spin-2 (graviton)

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Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
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spin-0

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Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
Held, Lim 2306.04725

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$$\Box\mathcal{R} = m_0^2\mathcal{R} + 2T^c{}_c$$

massive spin-0 (scalar)

massive spin-2

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Well-posed evolution in Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
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massless spin-2

$$(n^c\nabla_c\gamma_{ij}) = -2\,D_{(i}n_{j)} + \mathcal{O}_{ij}$$

$$(n^c\nabla_c K_{ij}) = -(n^c\nabla_c n_i)(n^c\nabla_c n_j) - 2\,D_{(i}n^c\nabla_c n_{j)} - 2\,K_{m(i}D_{j)}n^m + {}^{(3)}R_{ij} + \mathcal{O}_{ij}$$

massive spin-0

$$n^a\nabla_a\mathcal{R} = \mathcal{O}$$

$$n^a\nabla_a\hat{\mathcal{R}} = -D_iD^i\mathcal{R} + \mathcal{O}$$

constraints

$$0 = D_jK^j_i - D_iK + \mathcal{C}_i$$

$$0 = {}^{(3)}R - K_{ij}K^{ij} + K^2 - \frac{1}{2}\mathcal{R}$$

$$\mathcal{E}_a = -K^b_a\mathcal{C}_b - K\mathcal{C}_a - D^b\mathcal{A}_{ab} - \frac{1}{3}D_a\mathcal{A} + \frac{1}{4}D_a\mathcal{R}$$

$$\hat{\mathcal{R}} = -4\,D^b\mathcal{C}_b$$

constraint evolution

$$n^c\nabla_c\mathcal{C}_i = -\mathcal{E}_i + \mathcal{O}_i \qquad n^c\nabla_c\mathcal{E}_i = \ \dots$$

massive spin-2

$$n^c\nabla_c\mathcal{A} = \mathcal{O}$$

$$n^c\nabla_c\mathcal{A}_{ij} = \frac{2}{3}\mathcal{A}\,D_{(i}n_{j)} + \mathcal{O}_{ij}$$

$$n^c\nabla_c\mathcal{B} = +2\left(\mathcal{A}^{ij} + \frac{1}{3}\gamma^{ij}\mathcal{A}\right){}^{(3)}R_{ij} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2} + 1\right)D_iD^i\mathcal{R} - D_iD^i\mathcal{A} + 2\,a^k\mathcal{E}_k - a_iD^i\mathcal{A} + 4\,\mathcal{C}^j\left(D^iK_{ij} - D_jK\right) + \mathcal{O}$$

$$n^c\nabla_c\mathcal{B}_{ij} = +2\left(\mathcal{A}^{kl} + \frac{1}{3}\gamma^{kl}\mathcal{A}\right){}^{(3)}R_{ikjl} - \frac{1}{3}\left(\frac{m_2^2}{m_0^2} + 1\right)D_iD_j\mathcal{R} - \left(D_kD^k + a_kD^k\right)\left(\mathcal{A}_{ij} + \frac{1}{3}\gamma_{ij}\mathcal{A}\right) + \frac{2}{3}\mathcal{B}\,D_{(i}n_{j)} + 2\,a^c\gamma_{c(i}\mathcal{E}_{j)} - \frac{1}{3}\gamma_{ij}\left(n^c\nabla_c\mathcal{B}\right) + 4\,\mathcal{C}^k\left(D_{[i}K_{k]j} + D_{[j}K_{k]i}\right) + \mathcal{O}_{ij}$$

- massive spin-0/spin-2 do **not impact** the massless spin-2 **principal part**
- amenable to **1^{st}-order strong-hyperbolicity analysis**

Sarbach et.Al '02-'04 (for GR)

Numerical Evolution of Quadratic Gravity …

Held, Lim, 2306.04725

Dendro-GR [adapted]

- parallelized adaptive mesh refinement
- wavelet adaptive multiresolution
- 4^{th} order finite differencing
- 4^{th} order Runge-Kutta

Fernando et.Al. 2018

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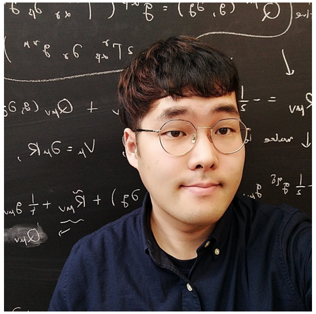

Hyun Lim
Los Alamos

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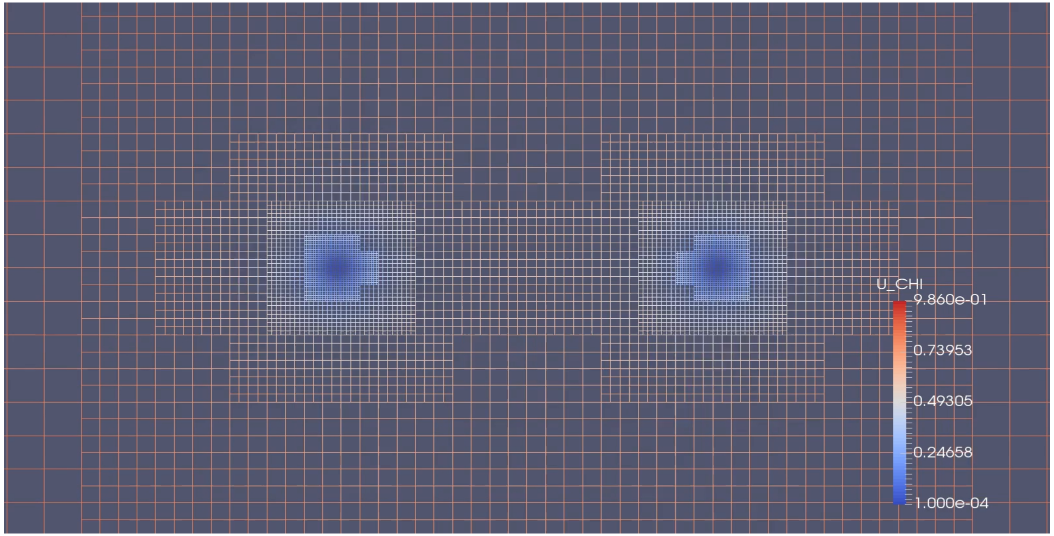

Dendro-GR (Fernando et.Al. 2018), https://github.com/paralab/Dendro-GR

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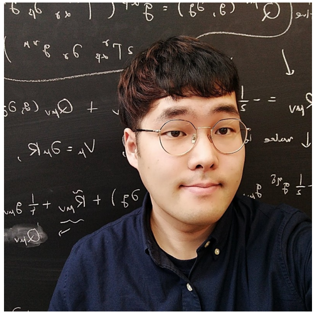

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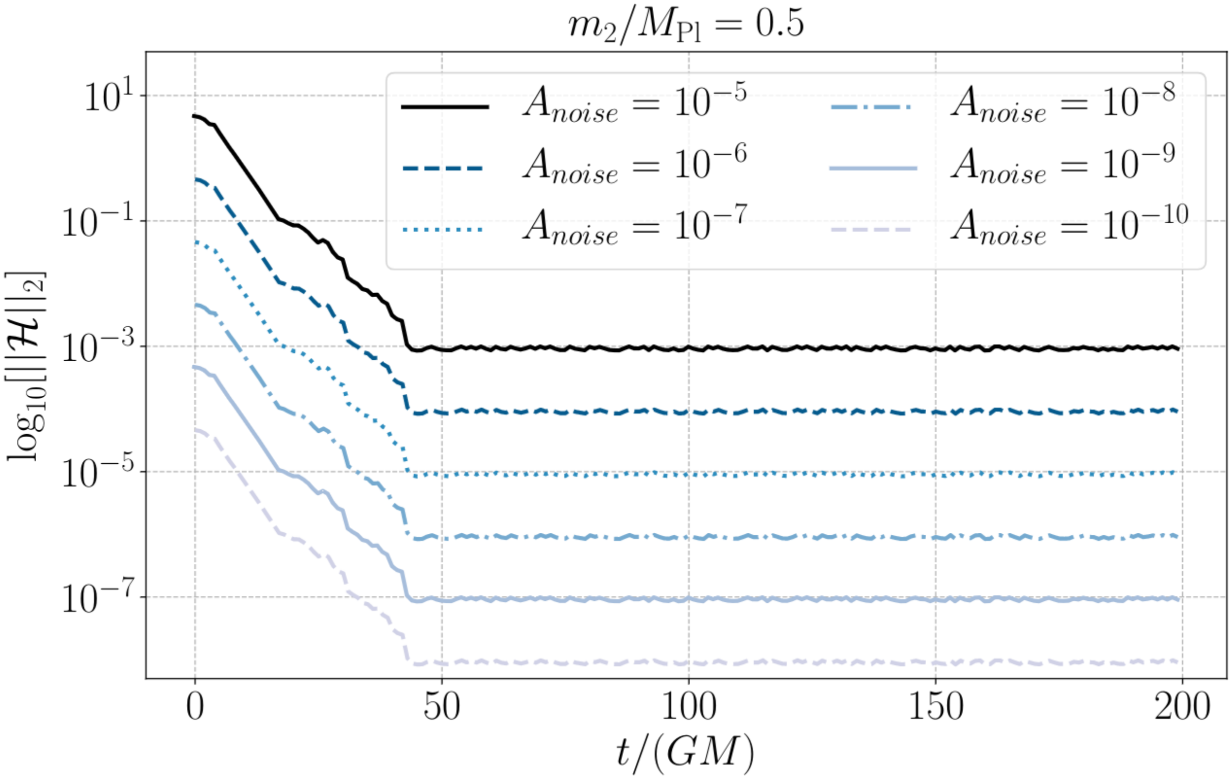

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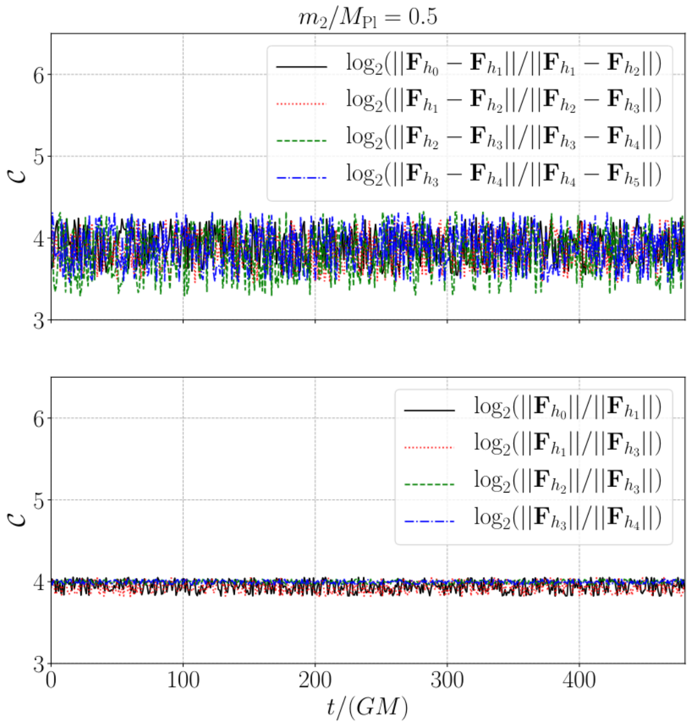

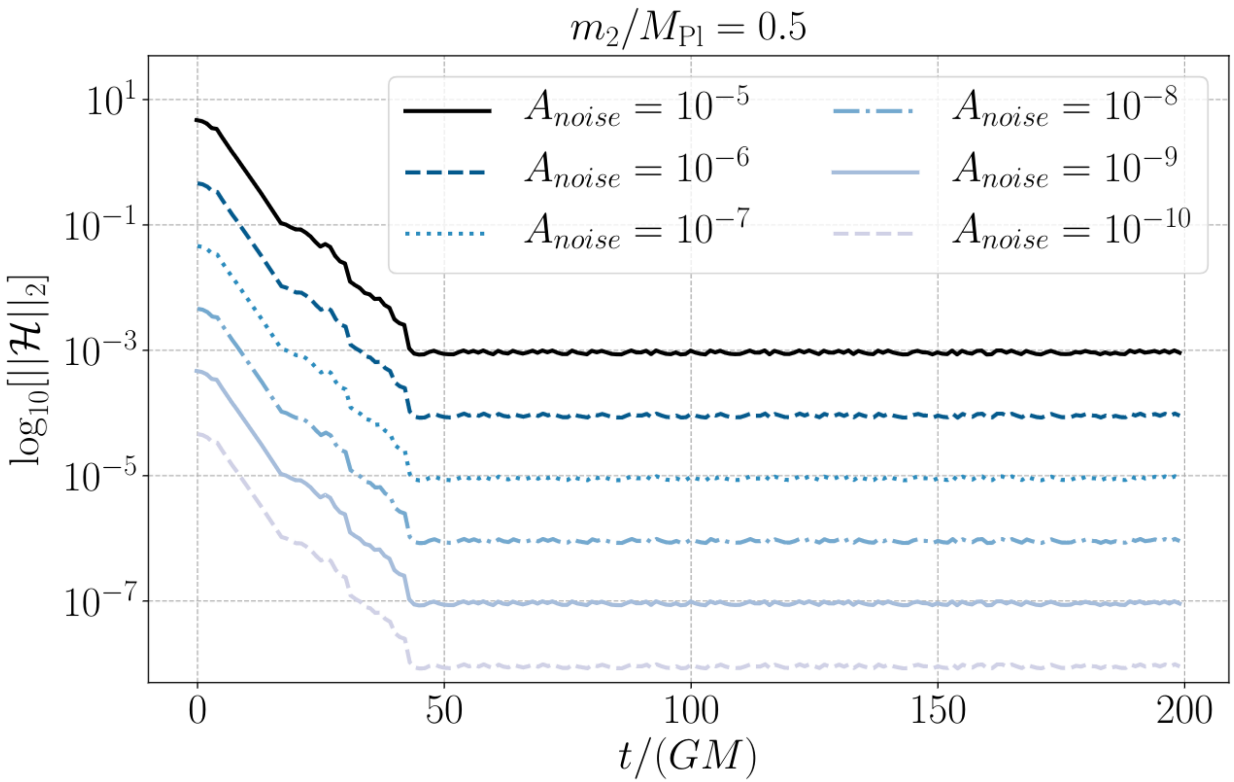

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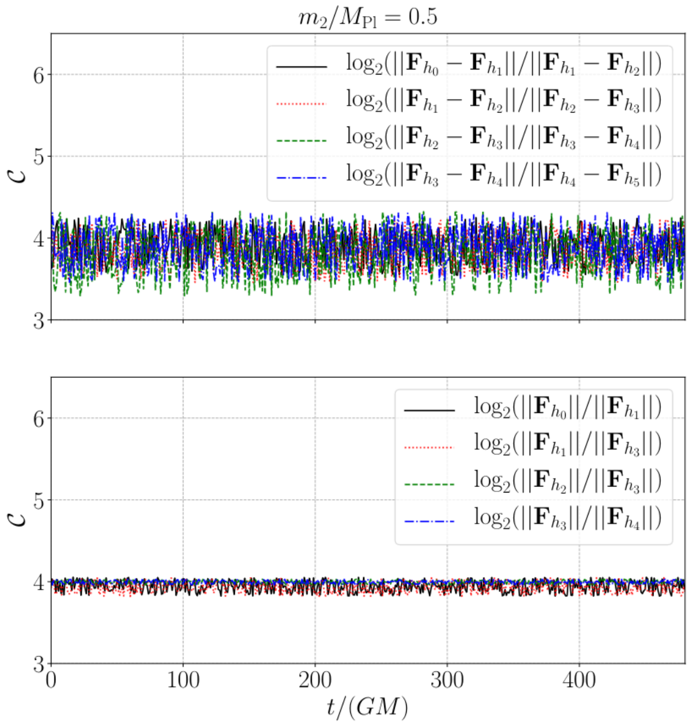

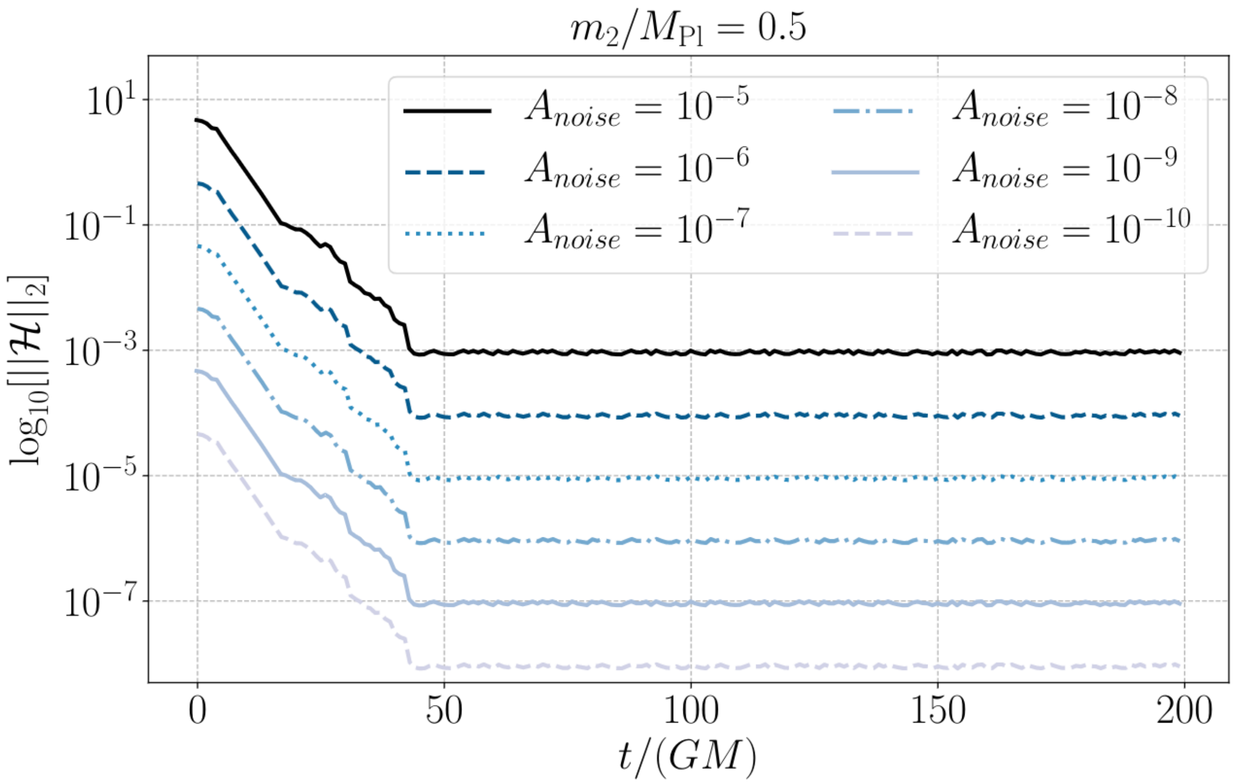

… is numerically stable.

Results (vacuum)

Held, Lim, 2306.04725

Recover the linear instability ...

Held, Lim, 2306.04725

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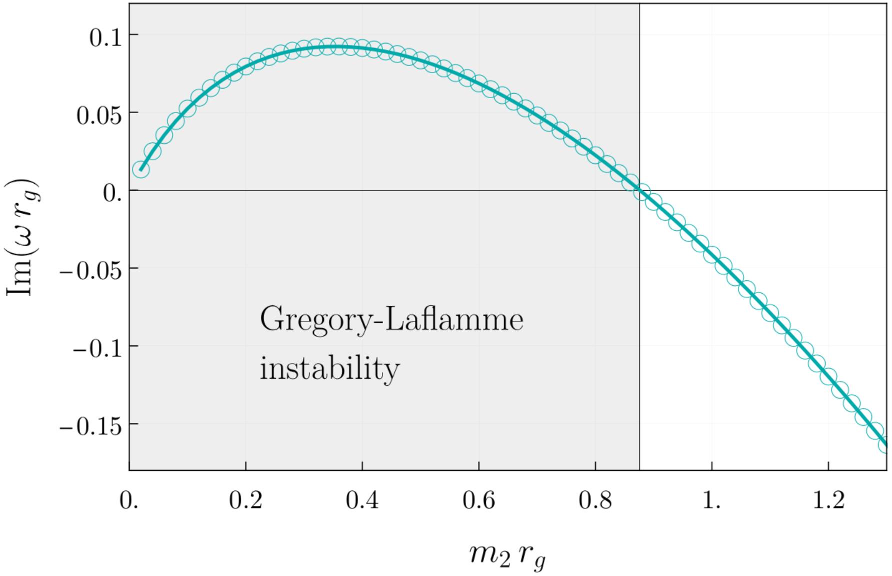
Held, Lim, 2306.04725

Held, Zhang, PRD 107 (2023) 6



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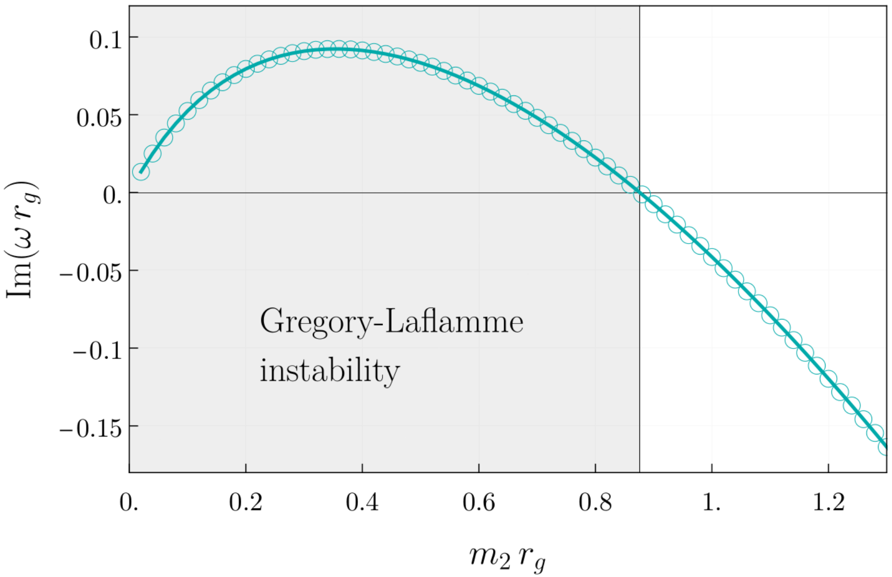

$$\frac{1}{4\pi}\frac{m_2}{M_{Pl}}\frac{M}{M_{Pl}} < 0.87$$

instability condition

$$\frac{t_{GL}}{GM} \sim \frac{1}{GM\,\mathrm{Im}(\omega)}$$

growth rate

Recover the linear instability ...

Held, Lim, 2306.04725

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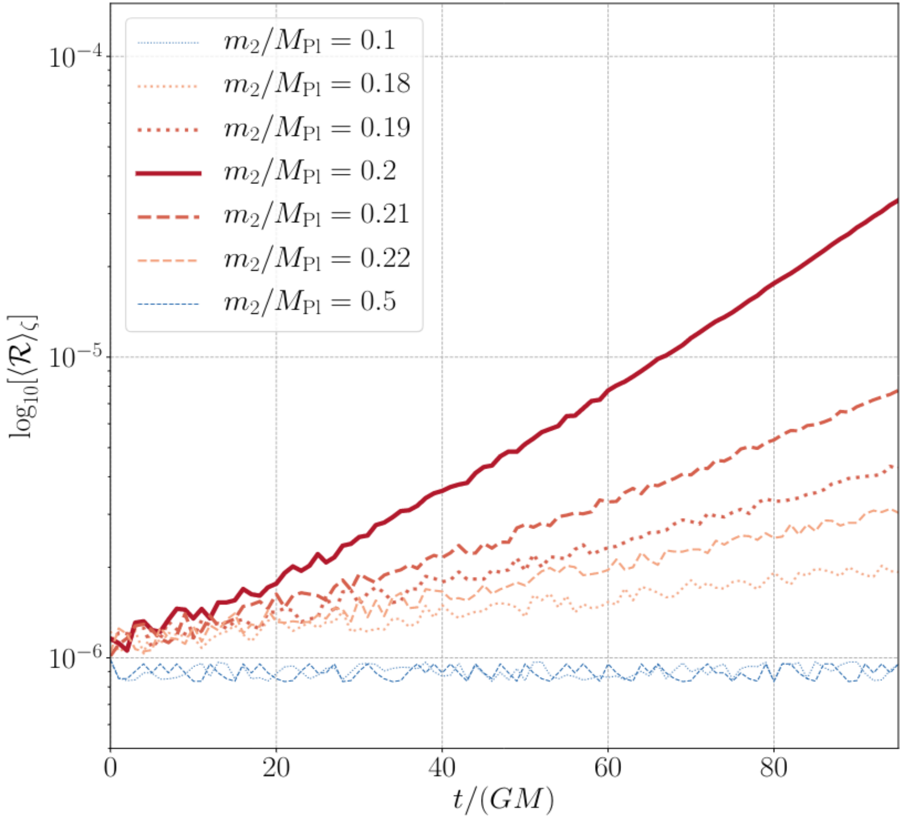

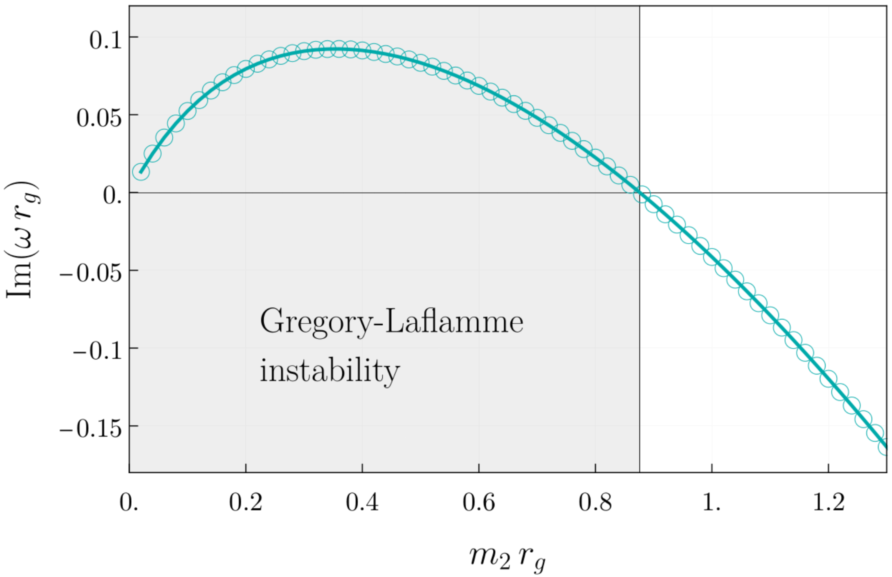

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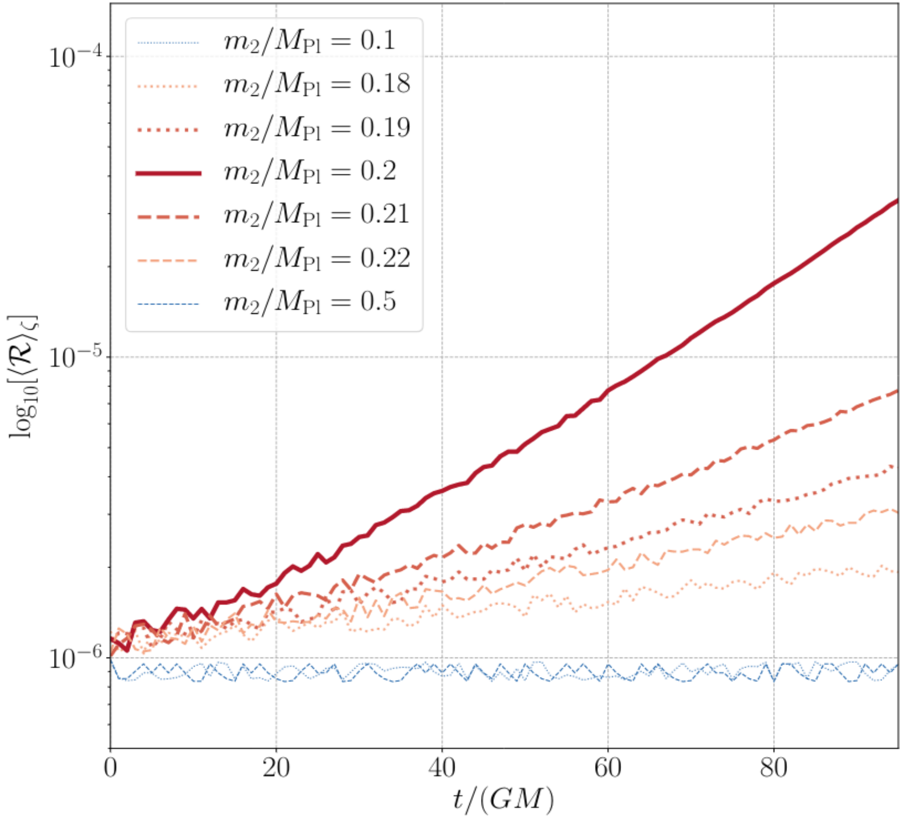

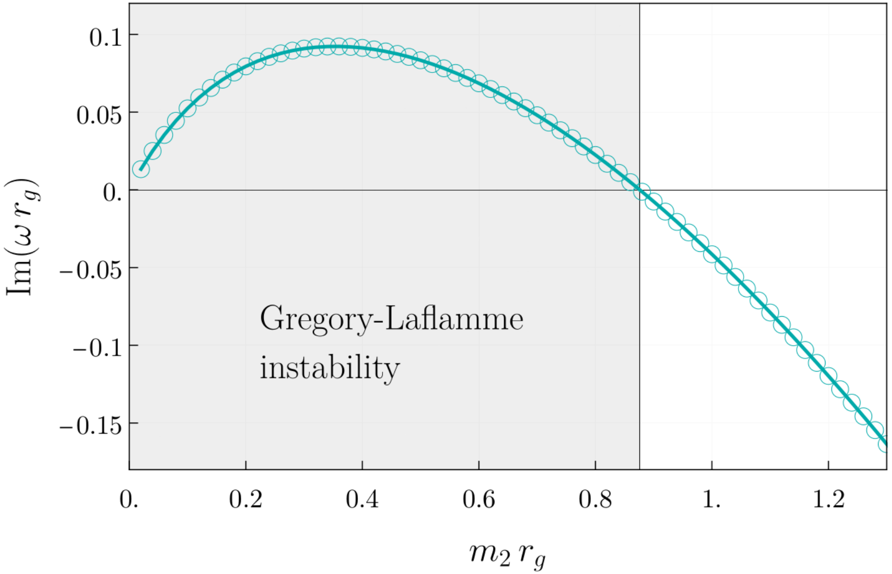

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… but otherwise we find a physically stable subsector.

Physical stability of the Ricci-flat subsector …

Held, Lim, 2306.04725

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- apparent **stability** of a single black hole perturbed by **Teukolsky waves**

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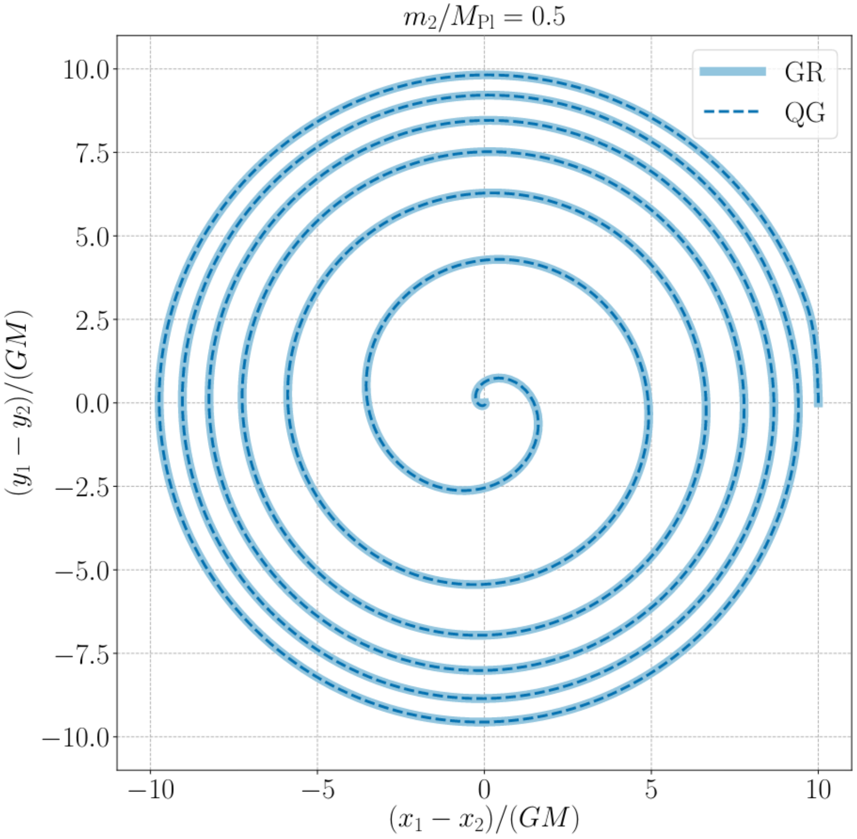

GW150914 initial data [EinsteinToolkit]

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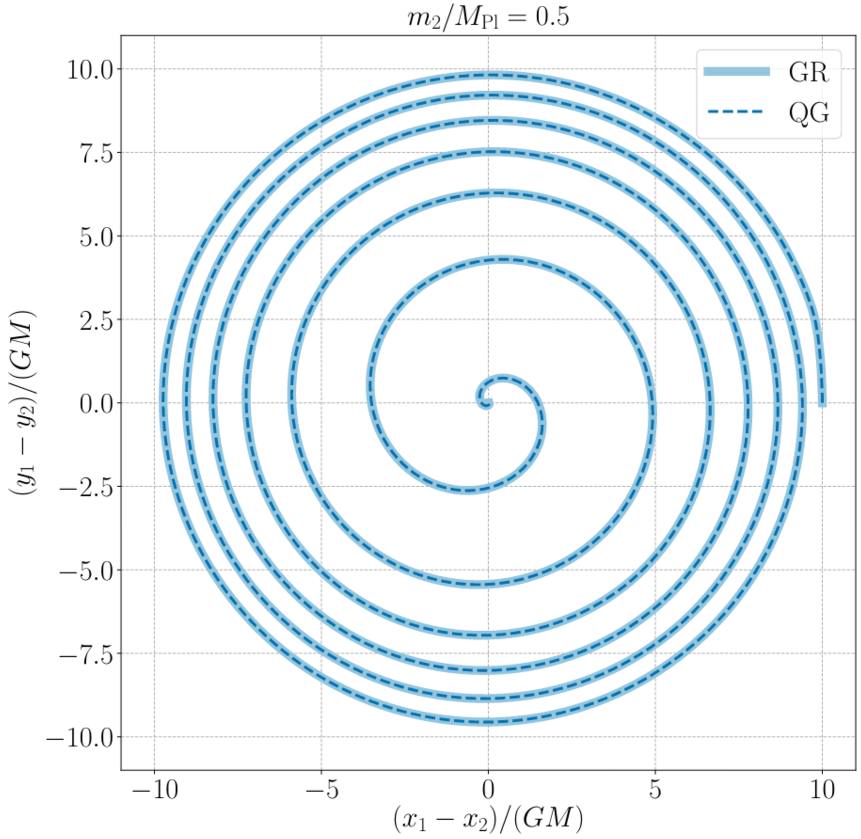

GW150914 initial data [EinsteinToolkit]

… suggests Quadratic Gravity can mimic vacuum GR.

Where to go next?

- **Ricci-flat (GR vacuum) subsector**
  - nonlinear endpoint of the linear instability

see also
Lehner, Pretorius, 1106.5184
Figueras et.Al., Phys.Rev.D 107 (2023) 4

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Figueras et.Al., Phys.Rev.D 107 (2023) 4

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Cayuso, 2307.15163

- **comparison**
  - with the fixing approach
  - on both sides of the field redefinition

Cayuso, Lehner PRD 102 (2020)
Cayuso et.Al 2303.07246

… so what about ghosts?

Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4
Deffayet, Held, Mukohyama, Vikman, 2305.09631

What about the Ostrogradski theorem?

Quadratic Gravity

$$-\frac{\mathcal{L}_{\text{quadratic}}}{\sqrt{-g}} = M_{Pl}^2\left[\lambda + \frac{1}{2}R + \frac{1}{12m_0^2}R^2 + \frac{1}{4m_2^2}C_{abcd}C^{abcd}\right]$$

... propagates **2 + 1 + 5 DoF**

What about the Ostrogradski theorem?

"All higher-derivative theories are unstable"

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Point-particle systems w/ opposite-sign kinetic terms …

~~“All higher-derivative theories are unstable”~~

“All **non-degenerate** higher-derivative **classical point-particle** theories exhibit **runaway** solutions”

Point-particle systems w/ opposite-sign kinetic terms …

"The **Hamiltonian** of all higher-derivative classical point-particle theories is **unbounded from above and below**"

Ostrogradski 1857

[french, thus not 100% sure that this is the actual content]

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Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4

… can nevertheless be globally (Lagrange) stable.

Point-particle systems w/ opposite-sign kinetic terms …

Integrable Liouville models …

$$H_{LV} = \frac{p_x^2}{2} + \sigma \frac{p_y^2}{2} + V_{LV}(x, y)$$

$$V_{LV} = \frac{f(u) - g(v)}{u^2 - v^2}$$

$$u^2 = 1/2 \left(r^2 + c + \sqrt{(r^2 + c)^2 - 4\,c\,x^2} \right)$$

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Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4
Deffayet, Held, Mukohyama, Vikman, JCAP, to appear

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... are Lagrange stable if ...

(i) ... f(u) and g(v) are bounded below, i.e.,

$$f(u) \geqslant f_0 \quad \& \quad g(v) \geqslant g_0$$

(ii) ... at large |u| and |v|, these bounds sharpen to

$$f(u) \geqslant 4F_0\,|u|^{\zeta} > 0 \quad \& \quad g(v) \geqslant 4G_0\,|v|^{\eta} > 0$$

with $f_0, g_0 \in \mathbb{R}\,, \quad F_0, G_0 \in \mathbb{R}^+\,, \quad \zeta > 2\,, \; > 2$

Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4
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... contain a polynomial subclass ...

$$f(u) = \sum_{n=1}^{N} \mathcal{C}_n \left(u^2\right)^n$$

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$$
\begin{aligned}
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& + \frac{1}{\tilde{c}}\left(\frac{\omega_x^2}{2} - \frac{\omega_y^2}{2}\right)(x^2 - y^2)^2 \\
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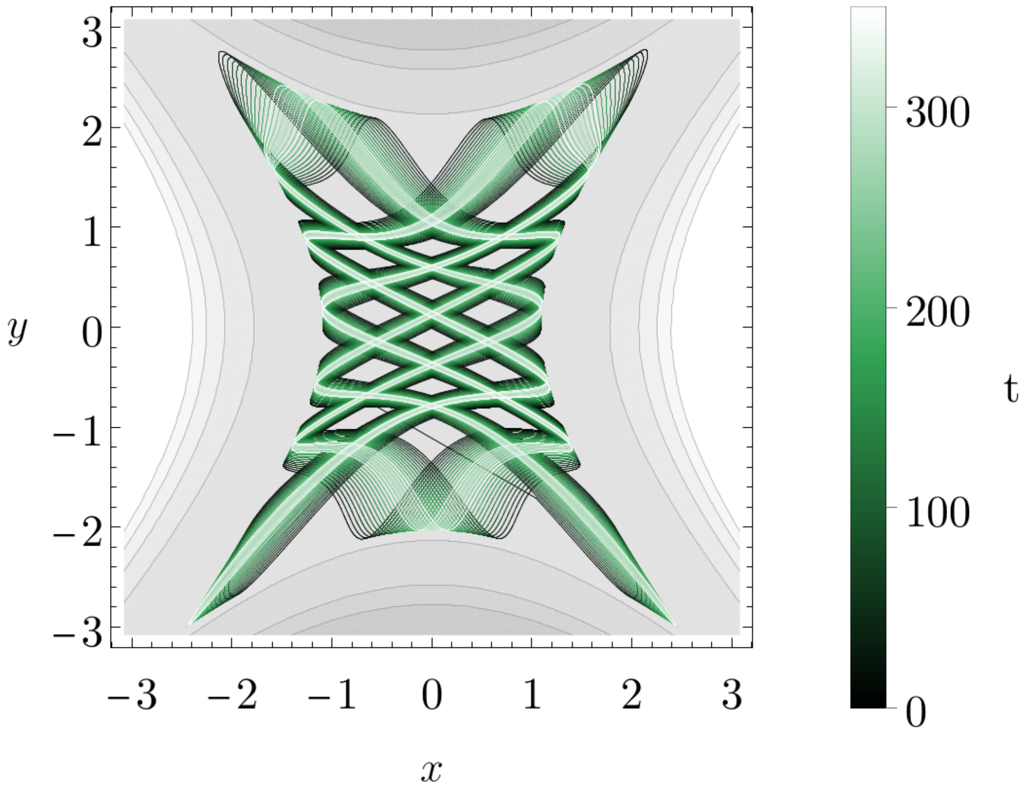

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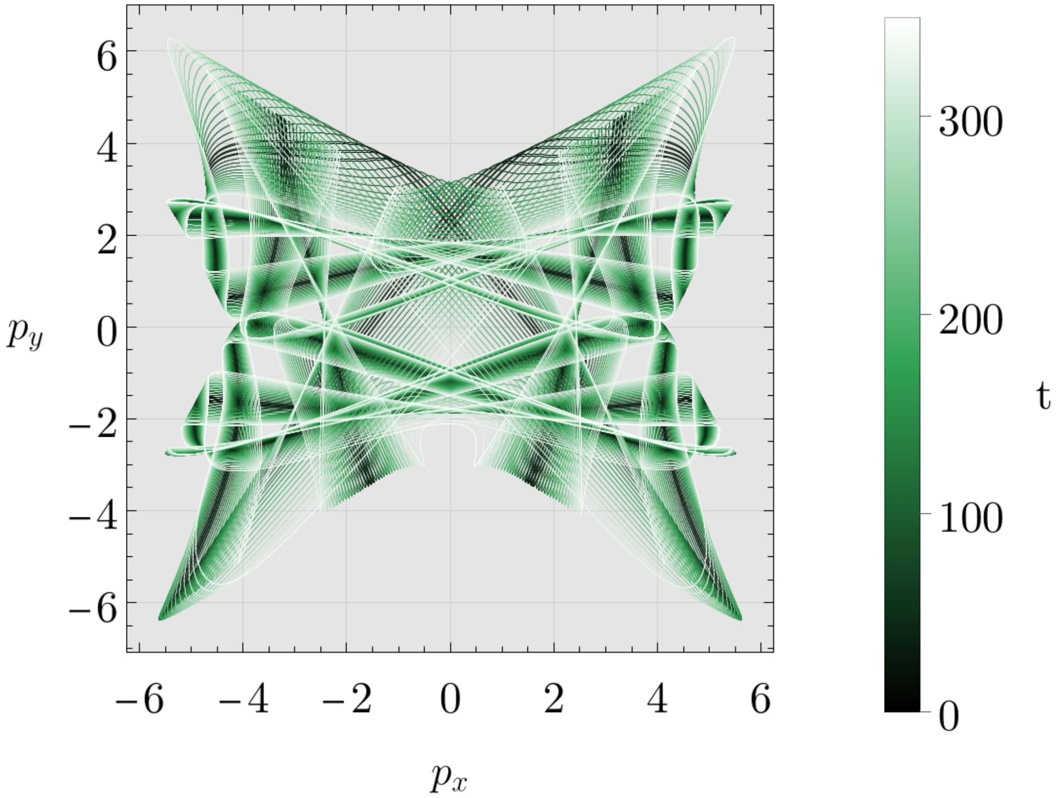

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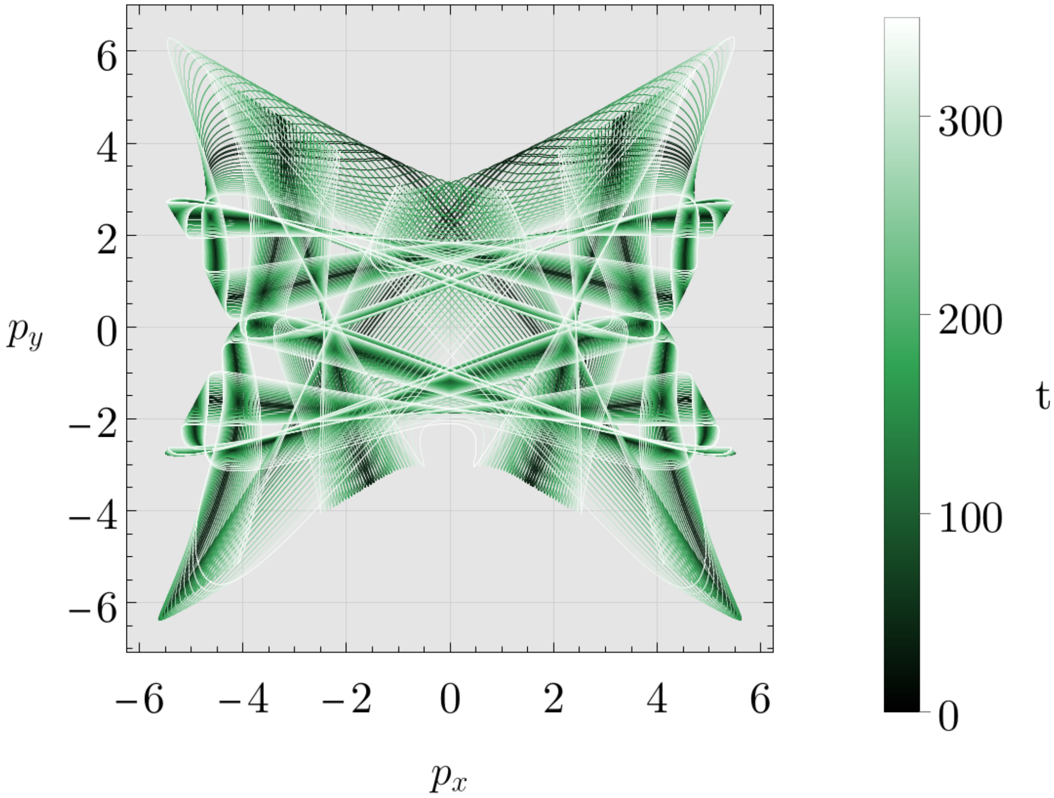
Deffayet, Held, Mukohyama, Vikman, JCAP, to appear



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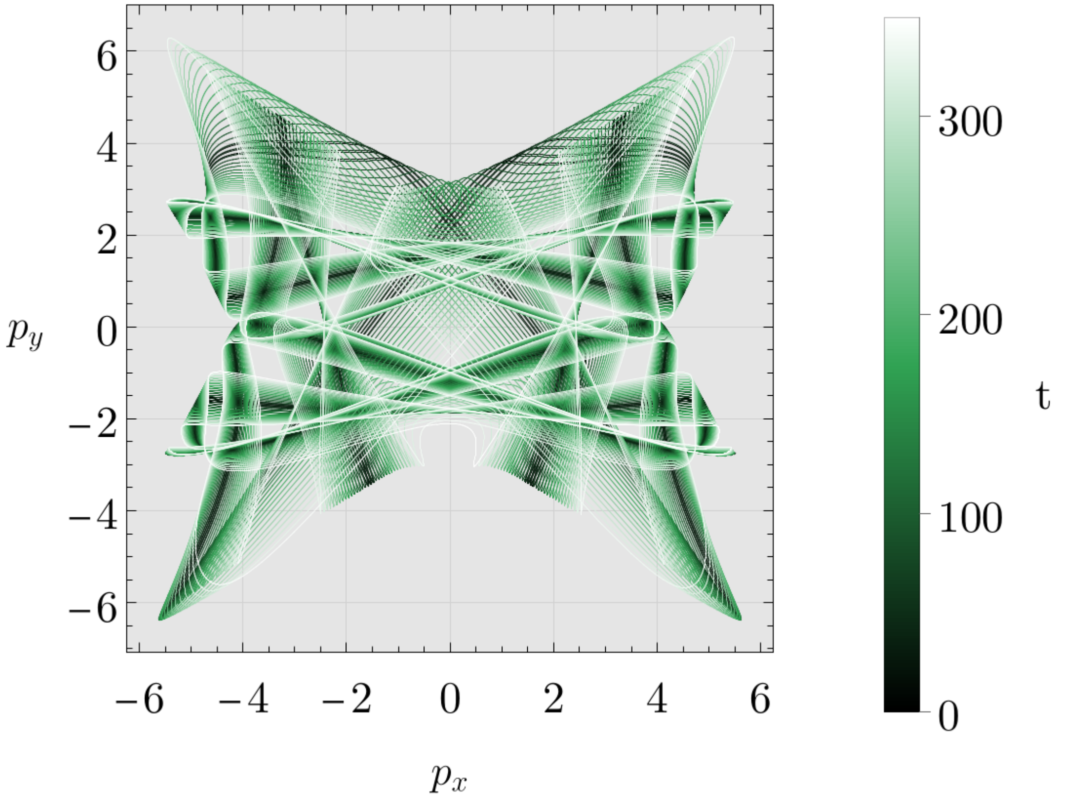
Deffayet, Held, Mukohyama, Vikman, JCAP, to appear



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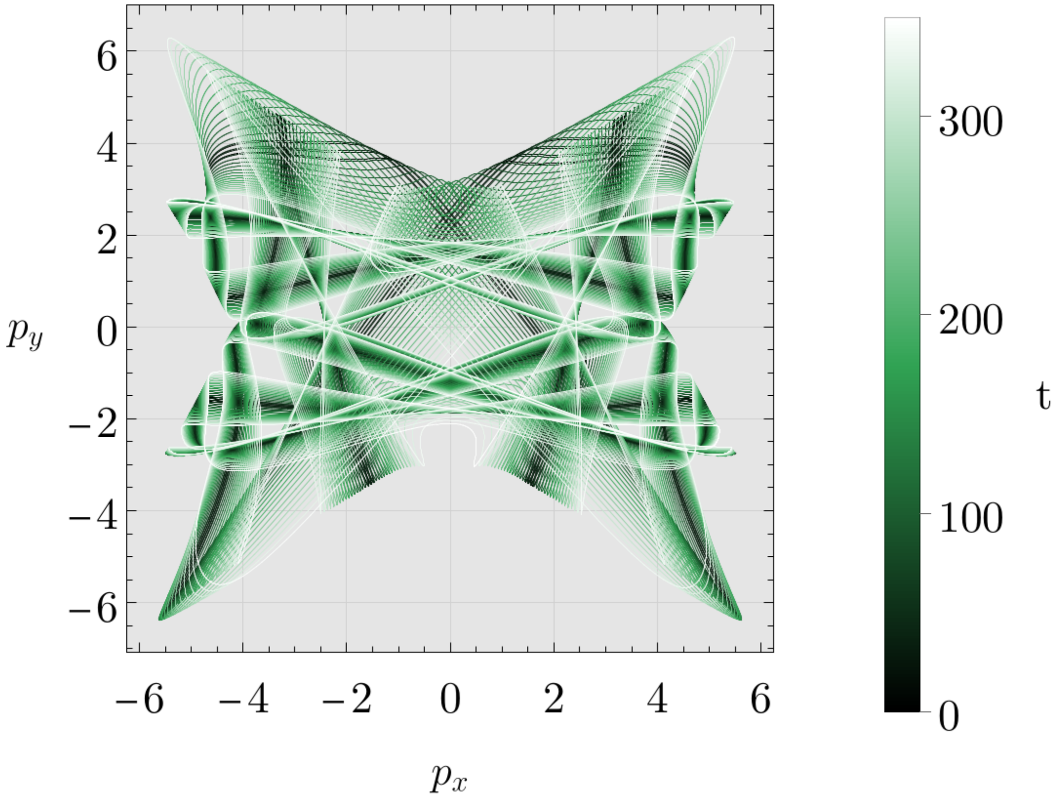
- **quantization** not yet explored

Deffayet, Held, Mukohyama, Vikman, JCAP, to appear



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joint phenomenology of cosmology, particle physics, and gravitational astronomy.

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