From fluctuating gravitons to spectral functions

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Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}]} \qquad \text{or} \qquad \int \mathcal{D}\hat{g}_{\mu\nu}\, e^{\mathrm{i}S[\hat{g}]}$$

Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral with gauge fixing, sources

$$Z[\bar{g}, J] \sim \int \mathcal{D}\hat{h}_{\mu\nu}\, e^{-S[\bar{g}+\hat{h}]-S_{\mathrm{gf}}[\bar{g},\hat{h}]-S_{\mathrm{gh}}[\bar{g},\hat{h},\hat{c},\hat{\bar{c}}]+\int_x \sqrt{\bar{g}}\, J^{\mu\nu}(x)\hat{h}_{\mu\nu}(x)}$$

- Metric split $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ required by gauge fixing and regulator
- Methods: Perturbation theory, lattice, functional methods, ...

The functional renormalisation group

Non-perturbative renormalisation group equation [Wetterich '93]

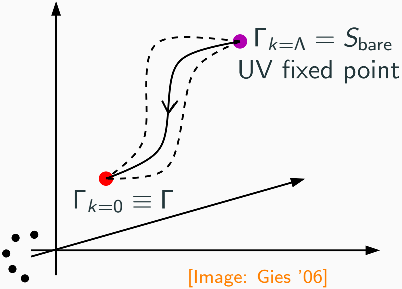
$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k\right]$$

R_k = regulator

Γ_k = scale-dependent effective action

Interpolation between

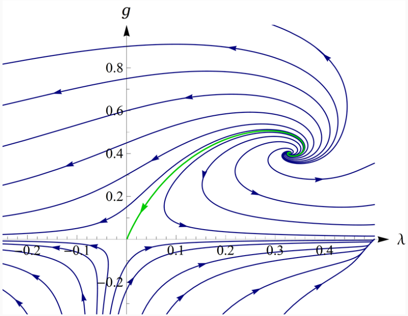
- bare action / UV FP
- quantum effective action Γ
- Wilsonian integrating out of momentum modes



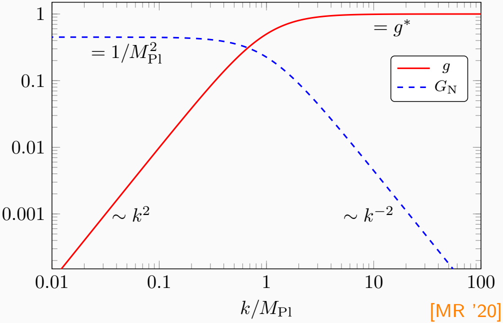
[Image: Gies '06]

Asymptotically safe quantum gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$$



[Reuter '96; Reuter, Saueressig '01]



[MR '20]

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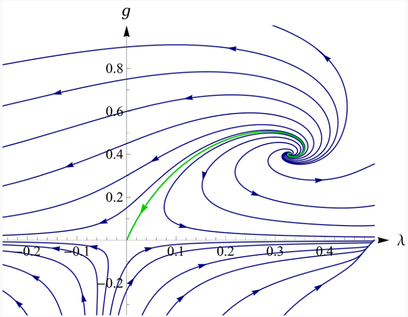

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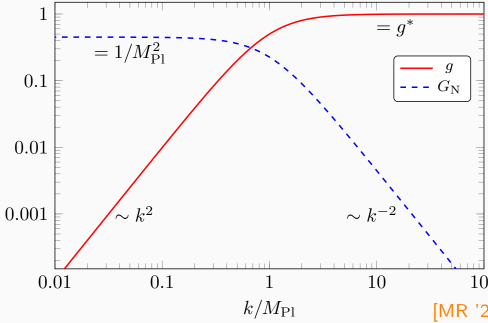


[MR '20]

Predictivity ⇔ UV critical hypersurface is finite dimensional

[Denz, Pawlowski, MR '16; Falls, Ohta, Percacci '20; Kluth, Litim '20; Knorr '21; . . .]

Unitarity ⇔ Properties of the spectral function, scattering amplitudes, ...

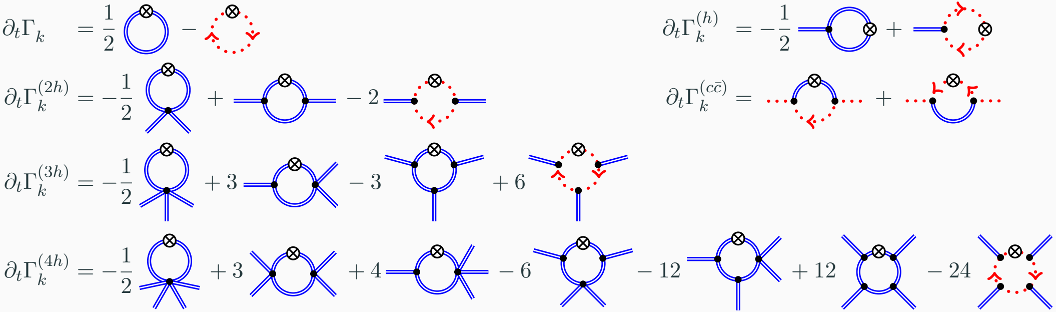
[Bonanno, Denz, Pawlowski, MR '21; Fehre, Litim, Pawlowski, MR '21; . . .]

Fluctuation approach

- Treat $\bar{g}$ and h independently, resolve fluctuation correlation functions

$$\Gamma_k[\bar{g}, h] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Gamma_k^{(0,h_{a_1}\ldots h_{a_n})}[\bar{g}, 0] \cdot h_{a_1} \ldots h_{a_n}$$

- Flat background $\bar{g} = \delta$ allows for momentum-space techniques



[Christiansen, Knorr, Meibohm, Pawlowski, MR '15; Denz, Pawlowski, MR '16; Pawlowski, MR '20, . . .]

Fluctuation approach

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- Flat background $\bar{g} = \delta$ allows for momentum-space techniques
- Extremely large tensor basis [MR (in prep)]

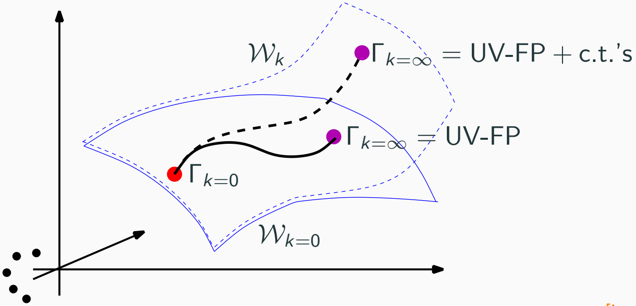
	Basis elements	TT-modes
$\Gamma^{(2h)}$	5	1
$\Gamma^{(3h)}$	33	7
$\Gamma^{(4h)}$	334	69

- Need careful projection on elements for universal one-loop beta functions of Stelle gravity [MR (in prep)]

Controlling the diffeomorphism symmetry

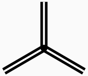
- Background metric $\bar{g}_{\mu\nu}$ and fluctuation field $h_{\mu\nu}$ are treated independently
- Diffeomorphism symmetry is governed by non-trivial Ward identity

$$\mathcal{W}_k = \mathcal{G}\Gamma_k + \mathcal{G}\Delta S_k - \langle \mathcal{G}(S_{\text{gf}} + S_{\text{gh}} + \Delta S_k) \rangle = 0$$

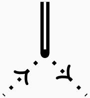


[Image: Gies '06]

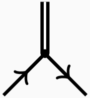
Avatars of couplings



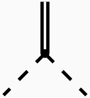
$\longrightarrow G_3(p_1, p_2, p_3)$



$\longrightarrow G_c(p_1, p_2, p_3)$



$\longrightarrow G_\psi(p_1, p_2, p_3)$



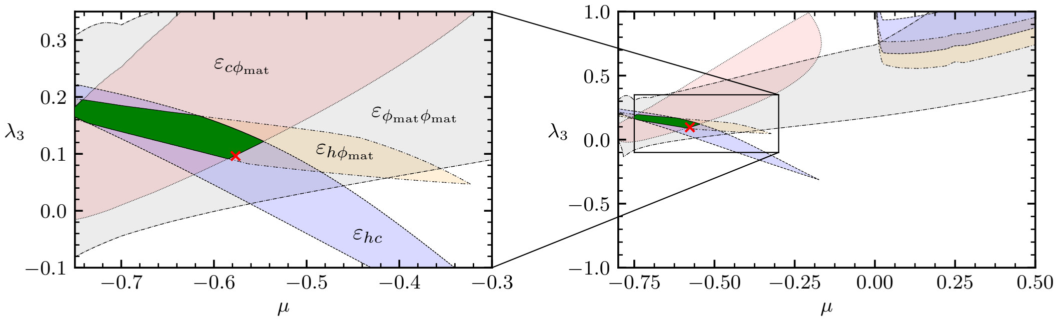
$\longrightarrow G_\varphi(p_1, p_2, p_3)$

$\dots$

- Momentum dependent couplings
- Related by symmetry identities
- Reduce to G_N + higher-order terms for $k \to 0$

Effective universality

Compare avatars of Newton's coupling at UV fixed point

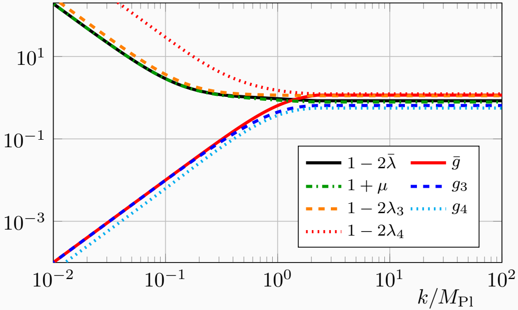


[Eichhorn, Lippoldt, Pawlowski, MR, Schiffer '18]

- In the green region: $G_3 \approx G_c \approx G_\varphi \approx G_\psi \approx G_A$
- Non-trivial effective diffeomorphism invariance at UV fixed point

As non-perturbative as necessary & as perturbative as it gets

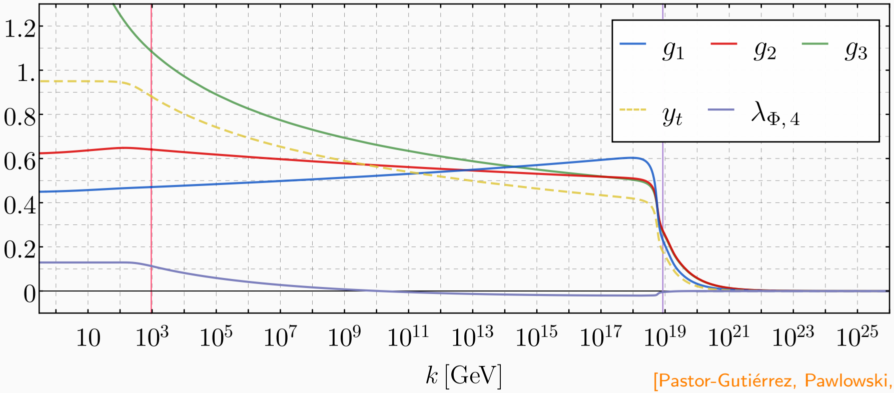
UV-IR trajectory pure gravity



[Denz, Pawlowski, MR '16]

- Includes the running up to the transverse-traceless four-graviton vertex
- (Partial) matching of avatars in the IR
- Three relevant directions: Λ, R, and R^2; $C^2_{\mu\nu\rho\sigma}$ irrelevant

UV-IR trajectory SM

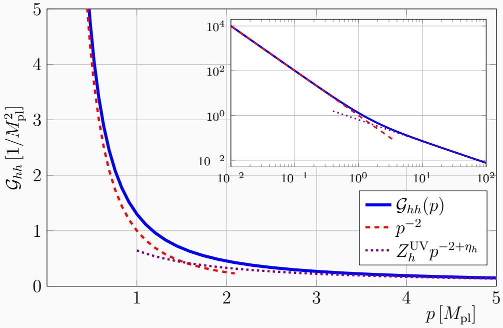


[Pastor-Gutiérrez, Pawlowski, MR '22]

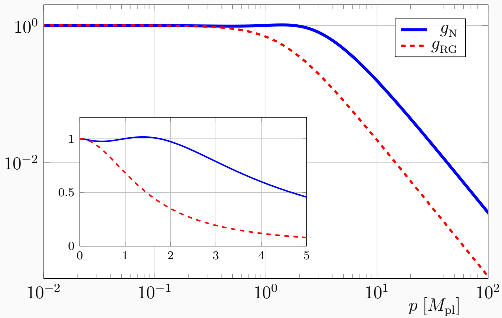
- Full SM flows including all threshold effects
- Gravity sector treated with assumption of effective universality

What is physics?

Propagator



Three-graviton vertex



[Bonanno, Denz, Pawlowski, MR '21]

- Momentum dependent correlation functions integrated to $k = 0$
- RG scale and momentum dependence agree qualitatively

Matching to form factors

Form factor action [Knorr, Ripken, Saueressig '22; ...]

$$\Gamma[\bar{g}, h] = \int_x \left(\frac{2\Lambda - R}{16\pi G_\mathrm{N}} + R f_R(\Box) R + C_{\mu\nu\rho\sigma} f_C(\Box) C^{\mu\nu\rho\sigma} + \dots \right) + S_\mathrm{gf} + S_\mathrm{gh}$$

Fully determined by momentum dependence of propagator ($g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_\mathrm{N}}\, h_{\mu\nu}$)

$$G_{h_{tt}h_{tt}} = \frac{32\pi}{Z_{h_\mathrm{tt}}(p)p^2} = \frac{32\pi}{p^2 + 32\pi G_\mathrm{N} p^4 f_C(p^2)}$$
$$G_{h_s h_s} = \frac{-16\pi}{Z_{h_\mathrm{s}}(p)p^2} = \frac{-16\pi}{p^2 - 96\pi G_\mathrm{N} p^4 f_R(p^2)}$$

Three-point Newton coupling relates to Goroff-Sagnotti form factor

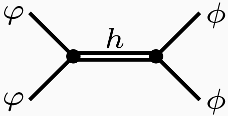
$$\int_x f_{C^3}(\nabla_1, \nabla_2, \nabla_3) C_{\mu\nu\rho\sigma} C^{\rho\sigma\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$$

Lorentzian computations

Towards testing Unitarity

A unitary theory requires

- Well-behaved propagators without ghost or tachyonic instabilities
- Bounds on scattering amplitudes, e.g., violated by GR



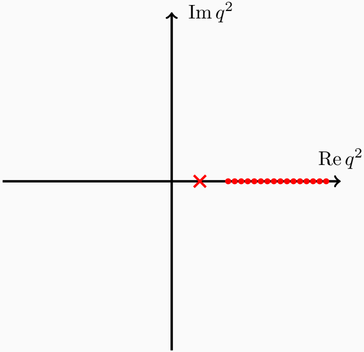
$$\overset{\text{GR}}{\propto} (p_1 + p_2)^2$$

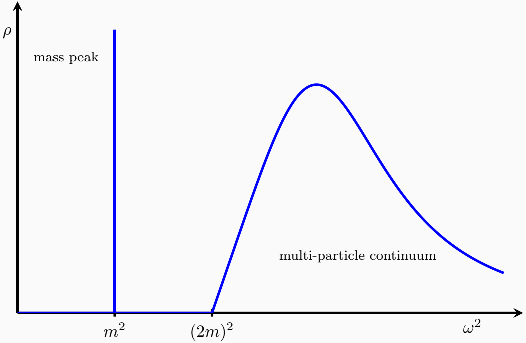
Need access to correlation functions on Lorentzian signature at time-like momenta

Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

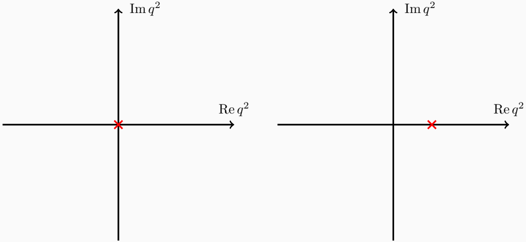
$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon\to 0} \operatorname{Im} G(\omega^2 + i\varepsilon)$$

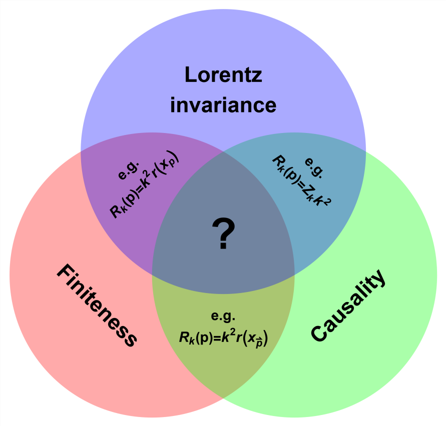




Regulator $R_k = k^2 r_k(x)$

- $r_k = r_k(p^2/k^2)$ breaks causality
- $r_k = r_k(\vec{p}^2/k^2)$ breaks Lorentz invariance
- $r_k = 1$ provides no UV regularisation





Spectral Renormalisation Group

[Fehre, Litim, Pawlowski, MR '21; fQCD, MR '22]

- Callan-Symanzik cutoff uniquely preserves causality and Lorentz invariance

$$R_k = Z_\phi \, k^2$$

- Finite flow equation with counterterms

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \, \mathcal{G}_k \, \partial_t R_k - \partial_t S_{\mathrm{ct},k}$$

- Dimensional regularisation of UV divergences in $d = 4 - \varepsilon$ possible

- Finitely many counter terms if gravity is asymptotically safe

Lorentzian setup

[Fehre, Litim, Pawlowski, MR '21]

- Expansion about flat Minkowski background

- Direct flow of ρ_h with $m_h^2 = k^2(1+\mu)$ and $Z_h = Z_h(p^2 = -m_h^2)$

$$\rho_h = \frac{1}{Z_h}\Big[2\pi\delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda)\Big]$$

- Use ρ_h in flow diagrams

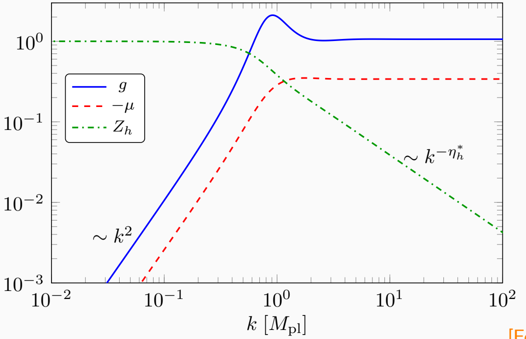
$\partial_t \rho_h \propto$  $+ \dots$ with $\mathcal{G}_h(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{q^2 - \lambda^2}$

- Approximation: neglect feedback from f_h in diagrams
- Flow of $g = G_\mathrm{N} k^2$ from three-graviton vertex at $p = 0$

[Christiansen, Knorr, Meibohm, Pawlowski, MR ' 15; Denz, Pawlowski, MR '16]

Lorentzian UV-IR trajectories

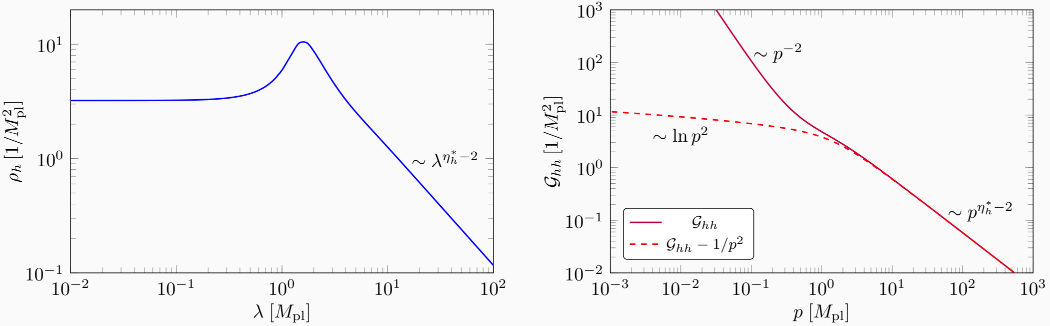
$$(g,\, \eta_h,\, \mu)\big|_* = (1.06,\, 0.96,\, -0.34) \qquad \theta = 2.49 \pm 3.17\, i$$



[Fehre, Litim, Pawlowski, MR '21]

$$G_N(k) = g(k)/k^2 \xrightarrow{k\to 0} G_N \qquad -2\Lambda(k) = k^2\mu(k) \xrightarrow{k\to 0} -2\Lambda = 0$$

Graviton spectral function



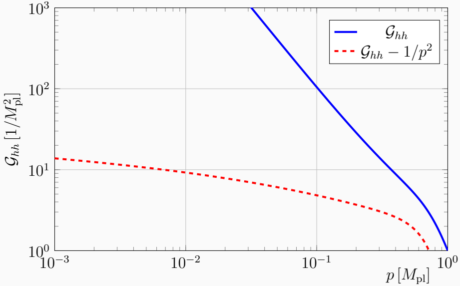
[Fehre, Litim, Pawlowski, MR '21]

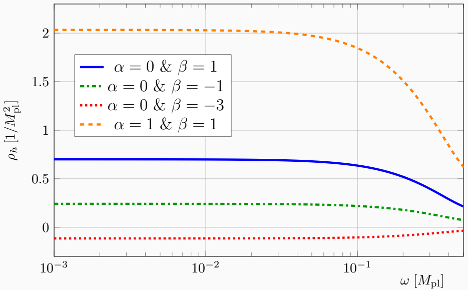
- Massless graviton delta-peak with multi-graviton continuum
- Non-normalisable spectral function $\int \rho_h \mathrm{d}\lambda = \infty$
- No ghosts and no tachyons $\longrightarrow$ no indications for unitarity violation
- Good agreement with reconstruction results [Bonanno, Denz, Pawlowski, MR '21]

Comparison to effective field theory

One-loop effective action: $\Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g}\,(c_1 R \ln(\Box) R + c_2 C_{\mu\nu\rho\sigma} \ln(\Box) C^{\mu\nu\rho\sigma}) + \dots$

Gauge-fixing $\quad S_{\text{gf}} = \frac{1}{\alpha}\int_x F_\mu^2 \quad$ with $\quad F_\mu = \bar\nabla^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar\nabla_\mu h^\nu{}_\nu$





[Pawlowski, MR '23]

Computation matches EFT the IR

Propagator is gauge-dependent but pole structure is typically not

[Kluth, Litim, MR '22]

Summary

- Fluctuation approach to disentangle background and fluctuation field
- Physics at $k = 0$ where symmetry identities are fulfilled
- Direct Lorentzian computation of graviton spectral function with spectral fRG
- Well-behaved spectral function without ghost or tachyonic instabilites
- Key step towards scattering processes and unitarity

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Thank you for your attention!