Renormalization and scattering in a shift-invariant scalar model

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Outline

Various flavors of RG running

Physical running. Define the coupling in terms of the scattering amplitude at some particular momentum $p = \mu_R$. Changing μ_R changes the value of the coupling.

$\mu-$**running.** In perturbation theory using dimreg or cutoff regularization one has to introduce a parameter μ to preserve dimensions, e.g. in $\log(p^2/\mu^2)$. Taking the derivative of the coupling with respect to μ defines another kind of RG.

Non-perturbative RG. One studies the dependence of the couplings in the quantum effective action on a UV cutoff (Wilsonian RG) or IR cutoff (FRG).

When do they give the same results?

Stelle gravity

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\Big[-2Z_N\Lambda + Z_N R - \frac{1}{2\lambda}C^2 - \frac{1}{\xi}R^2 + \frac{1}{\rho}E\Big], \\
&= \int d^4x\,\sqrt{-g}\left[-2Z_N\Lambda + Z_N R - \frac{1}{2\lambda}\left(C^2 - \frac{2\omega}{3}R^2 + 2\theta E\right)\right]
\end{aligned}
$$

$$
Z_N = \frac{1}{16\pi G}\,,
$$

Note: $S_E = -S_L$

Perturbative beta functions from dimreg

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25 + 1098\omega + 200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56 - 171\theta)}{90}\lambda$$

[I.G. Avramidi, A.O. Barvinski, Phys. Lett. **159 B**, 269 (1985).]

Gravity/QCD analogy

- weakly coupled in IR limit
- AF in the UV limit
- strongly coupled in intermediate regime

B. Holdom and J. Ren, “QCD analogy for quantum gravity,” Phys. Rev. D **93** (2016) no.12, 124030 [arXiv:1512.05305 [hep-th]].

A. Salvio and A. Strumia, Agravity, JHEP 06 (2014) 080, arXiv: 1403.4226 [hep-ph]

4DG and AS

S_{4DG} used as truncation Ansatz for the coarse-grained EA.
Beta functions extracted from FRGE

One loop FRGE reproduces perturbative beta functions for the marginal couplings, plus nontrivial beta functions for Λ and G

[A. Codello, R. P., Phys.Rev.Lett. **97** 22 (2006).]

[M. Niedermaier, Nucl. Phys. B833, 226-270 (2010).]

[K. Groh, S. Rechenberger, F. Saueressig, O. Zanusso, PoS EPS **-HEP2011** (2011) 124 [arXiv:1111.1743 [hep-th]].]
[N. Ohta, R.Percacci, Class. Quant. Grav. **31** 015024 (2014); arXiv:1308.3398]

FRG beyond 1 loop

Truncated FRGE on Einstein space gives nontrivial FP

[D. Benedetti, P. F. Machado, F. Saueressig, Mod. Phys. Lett. A **24** (2009) 2233 [arXiv:0901.2984 [hep-th]]]

Einstein background not enough because can read off beta function of only two combinations of λ, ξ and ρ.

- Truncate FRGE to 4DG action on arbitrary background
- use higher-derivative gauge fixing
- Expand to first order in Z_N.

[K. Falls , N. Ohta, R.Percacci, arXiv:2004.04126 [hep-th]]

Also more recently without expanding in Z_N:

[S. Sen, C. Wetterich, M. Yamada, JHEP 03 (2022) 130, arXiv:2111.04696 [hep-th]]

Beta functions (without cosmological term)

$$\beta_\lambda = -\frac{133}{160\pi^2}\lambda^2 + \tilde{Z}_N \lambda^3 \frac{251\xi - 58\lambda}{120\pi^2 \xi}$$

$$\beta_\xi = -\frac{5(72\lambda^2 - 36\lambda\xi + \xi^2)}{576\pi^2} + \tilde{Z}_N \frac{9720\lambda^3 - 1980\lambda^2\xi + 489\lambda\xi^2 - 14\xi^3}{6480\pi^2}$$

$$\beta_\rho = -\frac{49}{180\pi^2}\rho^2 + \tilde{Z}_N \lambda\rho^2 \frac{233\xi - 58\lambda}{240\pi^2\xi}$$

$$\beta_{\tilde{Z}_N} = \left(-2 + \frac{(30\lambda - \xi)(4\lambda + \xi)}{192\pi^2\xi}\right)\tilde{Z}_N + \frac{-3168\lambda^2 + 654\lambda\xi + 35\xi^2}{1152\pi^2\xi(6\lambda + \xi)}$$

$$-\frac{72\lambda^2 - 84\lambda\xi + 65\xi^2}{192\pi^2(6\lambda + \xi)^2}\log\left(\frac{2}{3} - \frac{2\lambda}{\xi}\right) .$$

Fixed points

	λ_*	ξ_*	ω_*	$\tilde{Z}_{N*}$	$\tilde{\mathcal{V}}_*$	$\tilde{G}_*$	$\tilde{\Lambda}_*$
FP_1	0	0	-0.02286	0.00833	0.00649	2.388	0.3894
FP_2	24.91	-287	0.2603	0.01635	0.00457	1.217	0.1399
FP_3	28.24	175	-0.4825	0.01499	0.00693	1.327	0.2310
FP_4	0	-312	0	0.009222	0.00609	2.157	0.3303

all with $\rho_* = 0$
plus several others with $\lambda = 0$, $\xi \neq 0$
FP_2 has right signs for absence of tachyons

Scaling exponents

FP_1	4	2	0	0	0
FP_2	$2.352 + 1.677i$	$2.352 - 1.677i$	1.767	0	-3.200
FP_3	$2.327 + 1.521i$	$2.327 - 1.521i$	1.237	0	-5.277

Einstein–Hilbert GFP

Expanding $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$
\begin{aligned}
S &= \frac{1}{G}\int d^d x \left[(\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2 + \dots\right] \\
&\quad + \frac{1}{\lambda}\int d^d x \left[(\Box h)^2 + h(\Box h)^2 + h^2(\Box h)^2 + \dots\right]
\end{aligned}
$$

then rescaling $h \to \sqrt{G}\, h$

$$
\begin{aligned}
S &= \int d^d x \left[(\partial h)^2 + \sqrt{G}\, h(\partial h)^2 + G\, h^2(\partial h)^2 + \dots\right] \\
&\quad + \frac{G}{\lambda}\int d^d x \left[(\Box h)^2 + \sqrt{G}\, h(\Box h)^2 + G\, h^2(\Box h)^2 + \dots\right]
\end{aligned}
$$

GFP for $\lambda \neq 0$ or $\lambda \to \infty$

Stelle GFP

Expanding $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\begin{aligned} S &= \frac{1}{G}\int d^d x \left[(\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2 + \ldots\right] \\ &\quad + \frac{1}{\lambda}\int d^d x \left[(\Box h)^2 + h(\Box h)^2 + h^2(\Box h)^2 + \ldots\right] \end{aligned}$$

rescaling $h \to \sqrt{\lambda}\, h$

$$\begin{aligned} S &= \frac{\lambda}{G}\int d^d x \left[(\partial h)^2 + \sqrt{G}\, h(\partial h)^2 + G\, h^2(\partial h)^2 + \ldots\right] \\ &\quad + \int d^d x \left[(\Box h)^2 + \sqrt{\lambda}\, h(\Box h)^2 + \lambda\, h^2(\Box h)^2 + \ldots\right] \end{aligned}$$

GFP for $G \neq 0$ or even $G \to \infty$

Summary

- EH FP describes gravity in the IR
- Stelle FP (FP_1) possible UV completion
- There seem to be other (nontrivial) FP's supporting AS.

Important questions:

- can we trust the nontrivial FPs?
- what are their physical signatures?
- can we flow from FP_1 to the nontrivial FP or vice-versa?
- can we flow from the UV fixed point to the EH FP?

Shift-invariant scalar

$$S[\phi] = \int d^4x \left[-\frac{1}{2} Z_1 (\partial\phi)^2 - \frac{1}{2} Z_2 (\Box\phi)^2 - \frac{1}{4} g((\partial\phi)^2)^2 \right]$$

FRG analysis:
D. B. and R.Percacci, Renormalization group flows between Gaussian fixed points, JHEP 10 (2022) 113, e-Print: 2207.10596 [hep-th]

Has two Gaussian fixed points

$$g = Z_2 = 0\,, \quad S[\phi] = \frac{1}{2}\int d^4x(\partial\phi)^2\,, \quad [\phi] = 1\,, \qquad FP_1$$

$$g = Z_1 = 0\,, \quad S[\phi] = \frac{1}{2}\int d^4x(\Box\phi)^2\,, \quad [\phi] = 0\,, \qquad FP_2$$

Give rise to two separate perturbation theories.

Classical running of free theories

$$S[\phi] = \int d^4x \left[\frac{1}{2}Z_1(\partial\phi)^2 + \frac{1}{2}Z_2(\Box\phi)^2\right]$$

Choosing $[\phi] = 1$, $Z_1 = 1$ and defining $\tilde{Z}_2 = Z_2 p^2$,
$\tilde{Z}_2$ goes from 0 in the IR to ∞ in the UV.

O.J. Rosten, arXiv:1106.2544

D. Benedetti, R. Gurau, S. Harribey and D. Lettera, JHEP 02 (2022) 147 [arXiv:2111.11792] .

Beta functions of dimful couplings

With field of dimension 1, the FRG gives

$$
\begin{aligned}
\partial_t Z_1 &= -\frac{(8-\eta_1)Z_1 + 16k^2 Z_2}{128\pi^2 (Z_1 + k^2 Z_2)^2}\, g k^4 \ , \\
\partial_t Z_2 &= 0 \ , \\
\partial_t g &= \frac{(10-\eta_1)Z_1 + 20k^2 Z_2}{64\pi^2 (Z_1 + k^2 Z_2)^3}\, g^2 k^4 \ .
\end{aligned}
$$

where $\eta_1 = -\partial_t Z_1/Z_1$.
With dimensionless field the beta functions are the same, except for factors of k.

Parametrization 1

$$\tilde{g} = \frac{gk^4}{Z_1^2}\,, \qquad \tilde{Z}_2 = \frac{Z_2 k^2}{Z_1}\,.$$

$$\eta_1 = \frac{8\tilde{g}\left(1+2\tilde{Z}_2\right)}{\tilde{g}+128\pi^2(1+\tilde{Z}_2)^2}$$

$$\beta_{\tilde{g}} \equiv \partial_t \tilde{g} = (4+2\eta_1)\tilde{g} + \frac{10+20\tilde{Z}_2-\eta_1}{64\pi^2(1+\tilde{Z}_2)^3}\tilde{g}^2$$

$$\partial_t \tilde{Z}_2 = (2+\eta_1)\tilde{Z}_2\,.$$

FPs in chart 1

FP	$\tilde{Z}_{2*}$	$\tilde{g}_*$	η_{1*}	θ_1	θ_2
GFP_1	0	0	0	-4	-2
$NGFP_1$	0	-127.6	-0.90	4.40	-1.10
$NGFP_2$	0	-12505	8.90	43.60	-10.90
$NGFP_3$	-0.6	-1011	-2	-13.84	10.84

Also seen in

G. P. de Brito, A. Eichhorn and R. R. L. d. Santos, JHEP 11 (2021), 110 [arXiv:2107.03839 [gr-qc]].
C. Laporte, A. D. Pereira, F. Saueressig and J. Wang, [arXiv:2110.09566 [hep-th]].
C. Laporte, N. Locht, A. D. Pereira and F. Saueressig, [arXiv:2207.06749 [hep-th]].
G. P. de Brito, B. Knorr and M. Schiffer, Phys. Rev. D 108 (2023), 2 [arXiv:2302.10989 [hep-th]].

Parametrization 2

For dimensionless field

$$S[\varphi] = \int d^4x \left[\frac{1}{2}\zeta_1(\partial\varphi)^2 + \frac{1}{2}\zeta_2(\Box\varphi)^2 + \frac{1}{4}\gamma((\partial\varphi)^2)^2 \right]$$

comes from redefining the couplings

$$\phi = k\varphi\ , \quad Z_1 = k^{-2}\zeta_1\ , \quad Z_2 = k^{-2}\zeta_2\ , \quad g = k^{-4}\gamma\ .$$

The power counting is that of a renormalizable theory, with ζ_1 having the meaning of a mass. The natural variables for the parametrization of theory space are

$$\hat{\zeta}_1 = \frac{\zeta_1}{\zeta_2 k^2}\ , \qquad \hat{\gamma} = \frac{g}{\zeta_2^2}\ .$$

Beta functions in chart 2

$$\eta_2 = 0 \, ,$$

while the beta functions are

$$\beta_{\hat{\zeta}_1} = -2\hat{\zeta}_1 - \frac{8\hat{\gamma}(2+\hat{\zeta}_1)}{\hat{\gamma} + 128\pi^2(1+\hat{\zeta}_1)^2}$$

and

$$\beta_{\hat{\gamma}} = \frac{(2+\hat{\zeta}_1)(\hat{\gamma} + 640\pi^2(1+\hat{\zeta}_1)^2)}{32\pi^2(1+\hat{\zeta}_1)^3\left(\hat{\gamma} + 128\pi^2(1+\hat{\zeta}_1)^2\right)}\hat{\gamma}^2 \, .$$

FPs in chart 2

FP	$\hat{\zeta}_{1*}$	$\hat{\gamma}_*$	η_{2*}	θ_1	θ_2
GFP_2	0	0	0	2	0
$NGFP_3$	-1.67	-2807	0	-13.84	10.84

Coordinate transformation

$$\hat{\zeta}_1 = \frac{1}{\tilde{Z}_2}\,, \qquad \hat{\gamma} = \frac{\tilde{g}}{\tilde{Z}_2^2} \qquad \textit{or conversely} \quad \tilde{g} = \frac{\hat{\gamma}}{\hat{\zeta}_1^2}\,.$$

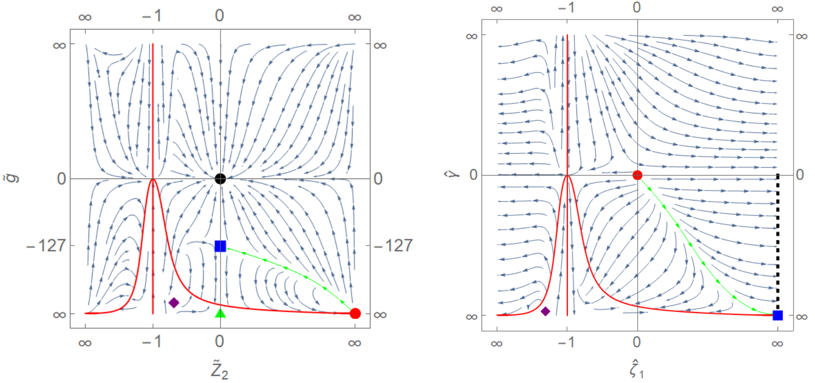


Figure: Left: flow in chart 1. Right: flow in chart 2.

Properties

1) Recall that in chart 1, $\partial_t \tilde{Z}_2 = (2 + \eta_1)\tilde{Z}_2$.
For $k \to \infty$ we have $\eta_1 \to -2$, which can be reabsorbed in the definition of the field. Then the dimension of the field is

$$1 + \frac{1}{2}\eta_1 \to 0$$

The FRG correcty interpolates the dimension of the fields in the flow between the two FP's.

2) There are trajectories that remain perturbative for all k.
The separatrix lies at the other extreme.

3) Mass of the ghost increases when trajectory becomes less perturbative. Goes to infinity for the AS trajectory.

Generalizations

$$\phi\,\Box^k\phi \qquad\qquad GFP_k$$

with $0 < k < \infty$, have been studied perturbatively as CFT's

[M. Safari, A. Stergiou, G. P. Vacca and O. Zanusso, JHEP 02 (2022), 034 [arXiv:2112.01084 [hep-th]].]

- Every GFP defines a perturbative expansion and a chart.
- There is only one GFP in each chart.
- The other GFP's are in the closure of the domain of the chart.
- There seem to be flows between all these, lowering k.

The simplest case ($1 \to 0$)

Flow from GFP$_1$ to GFP$_0$:

$$S[\phi] = \int d^4x \left[-\frac{1}{2} Z_1 (\partial\phi)^2 - \frac{1}{2} Z_0 \phi^2 - \frac{1}{4} \lambda \phi^4 \right]$$

$$\beta_\lambda \sim \lambda^2$$

AF for $\lambda < 0$.

Symanzik's asymptotically free theory *ante litteram*

Lessons for gravity

- There are trajectories joining GFP_2 to GFP_1
- Asymptotic safety may be a limiting case of asymptotic freedom
- The physics?

Calculate scattering amplitudes
perturbative analysis:

D. B., J. Donoghue, R.Percacci, Amplitudes and Renormalization Group Techniques: A Case Study, arXiv: 2307.00055 [hep-th]

Amplitudes

Compute scattering amplitudes and physical running at one loop and compare with the solutions of the FRG flow.

Comparison limited to perturbative regime

Downgrade FRG to one loop. Not much difference.

Shift-invariant scalar

Reparametrize

$$\mathcal{L} = -\frac{Z_1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{Z_1}{2m^2}\Box\phi\Box\phi - \frac{Z_1^2 g}{4m^4}(\partial_\mu\phi\partial^\mu\phi)(\partial_\nu\phi\partial^\nu\phi)$$

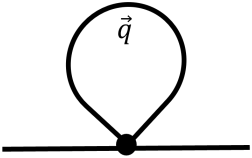
Characteristic scales:
- ghost mass m
- interaction scale: $m/\sqrt[4]{g}$

In order for ghosts to be propagating and weakly coupled need $g \ll 1$

Energy domains

- $E < m$ low energy regime: only massless particles propagate and are weakly coupled; massive ghosts do not propagate
- $m < E < m/\sqrt[4]{g}$: intermediate energy regime; also ghosts propagate and are weakly coupled
- $m/\sqrt[4]{g} < E$ high energy regime; apparently strongly interacting

2-point function



$$i\frac{3}{2}\frac{1}{Z_1}p^2\frac{1}{(4\pi)^2}\left(\frac{1}{\epsilon}+\log 4\pi-\gamma-\log\frac{m^2}{\mu^2}+\frac{7}{6}+O(\epsilon)\right)$$

No renormalization of Z_2

There is μ-running of Z_1 but no physical running.

Low energy EFT

At low energy, putting $Z_2 = 0$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{g}{m^4}(\partial_\mu\phi\partial^\mu\phi)(\partial_\nu\phi\partial^\nu\phi) + \mathcal{L}_6 + \mathcal{L}_8 + \dots$$

$$\begin{aligned}\mathcal{L}_6 &= \frac{g_6}{4m^6}\partial_\mu\phi\partial^\mu\phi\Box\partial_\nu\phi\partial^\nu\phi + \frac{g_6'}{4m^6}\partial_\mu\phi\partial_\nu\phi\Box\partial^\mu\phi\partial^\nu\phi \\ \mathcal{L}_8 &= -\frac{g_8}{4m^8}\partial_\mu\phi\partial^\mu\phi\Box^2\partial_\nu\phi\partial^\nu\phi - \frac{g_8'}{4m^8}\partial_\mu\phi\partial_\nu\phi\Box^2\partial^\mu\phi\partial^\nu\phi\end{aligned}$$

EFT amplitude

In terms of

$$s = -(p_1 + p_2)^2 \qquad t = -(p_1 + p_3)^2 \qquad u = -(p_1 + p_4)^2$$

$$\begin{aligned}
&-\frac{g}{2m^4}(s^2 + t^2 + u^2) \\
&+\frac{g_6}{2m^6}(s^3 + t^3 + u^3) + \frac{g_6'}{4m^6}(s^2 t + s^2 u + t^2 u + t^2 s + u^2 s + u^2 t) \\
&+\frac{g_8(\mu_R)}{m^8}(s^4 + t^4 + u^4) + \frac{g_8'(\mu_R)}{2m^8}(s^2 t^2 + s^2 u^2 + t^2 u^2) \\
&+\frac{g^2}{1920\pi^2 m^8}\left[41 s^4 \log\left(\frac{-s}{\mu_R^2}\right) + 41 t^4 \log\left(\frac{-t}{\mu_R^2}\right) + 41 u^4 \log\left(\frac{-u}{\mu_R^2}\right)\right. \\
&+ s^2(t^2 + u^2)\log\left(\frac{-s}{\mu_R^2}\right) + t^2(s^2 + u^2)\log\left(\frac{-t}{\mu_R^2}\right) + u^2(t^2 + s^2)\log\left(\frac{-u}{\mu_R^2}\right)
\end{aligned}$$

EFT physical beta functions

$$
\begin{aligned}
\beta_g &= 0 \\
\beta_{g_6} &= 0 \\
\beta_{g_6'} &= 0 \\
\beta_{g_8} &= \frac{41 g^2}{480\pi^2} \\
\beta_{g_8'} &= \frac{g^2}{240\pi^2}
\end{aligned}
$$

General amplitude

$$\frac{5g^2\left(s^2+t^2+u^2\right)}{64\pi^2 m^4\epsilon}+\frac{g^2}{5760\pi^2 m^8}\left\{\frac{m^4}{s^2}\left[-6m^4\left(s^2+t^2+u^2\right)+3sm^2\left(-31s^2+9\left(t^2+u^2\right)\right)\right.\right.$$

$$\left.+2s^2\left((352-195\gamma_E)s^2-(15\gamma_E-37)\left(t^2+u^2\right)\right)\right]$$

$$+6s^{-1/2}m^4\sqrt{4m^2-s}[16m^4(6s^2+t^2+u^2)-8sm^2(16s^2+t^2+u^2)+s^2(41s^2+t^2+u^2)]\,\mathrm{arccot}\sqrt{\frac{4m^2}{s}-1}$$

$$+3s^2\left(41s^2+t^2+u^2\right)\log\left(-\frac{m^2}{s}\right)$$

$$+\frac{6(s-m^2)^3}{s^3}\log\left(\frac{m^2}{m^2-s}\right)\left[m^4\left(s^2+t^2+u^2\right)-2sm^2\left(-9s^2+t^2+u^2\right)+s^2\left(41s^2+t^2+u^2\right)\right]$$

$$+(\text{same with } u\to s\to t)+(\text{samewith } t\to u\to s)$$

$$\left.+450m^4\left(s^2+t^2+u^2\right)\log\left(\frac{4\pi\mu^2}{m^2}\right)\right\}$$

Low energy amplitude

EFT amplitude with renormalized coupling

$$g(\mu) = g_B - \frac{5g^2 m^4}{32\pi^2 M^4}\left[\frac{1}{\epsilon} - \gamma_E - \log\left(\frac{4\pi\mu^2}{m^2}\right) + \frac{11}{30}\right]$$

There is μ-running of g but no physical running.

Low energy beta function

the μ-beta function

$$\mu \frac{\partial g(\mu)}{\partial \mu} = \frac{5g^2}{16\pi^2}$$

the EFT beta function

$$\beta_g = 0$$

and the low energy FRG

$$\beta_g = \frac{5(Z_1 + 2k^2/m^2)}{32\pi^2 (Z_1 + k^2/m^2)^3} \frac{g^2 k^4}{M^4} \to \frac{5g^2}{32\pi^2} \frac{k^4}{m^4}$$

that indeed goes to zero in the limit $k \to 0$

High energy amplitude

$$\bar{g}(\mu_R) = g + \frac{5g^2}{32\pi^2}\left[\log\left(\frac{\mu_R^2}{m^2}\right) - \frac{17}{30}\right]$$

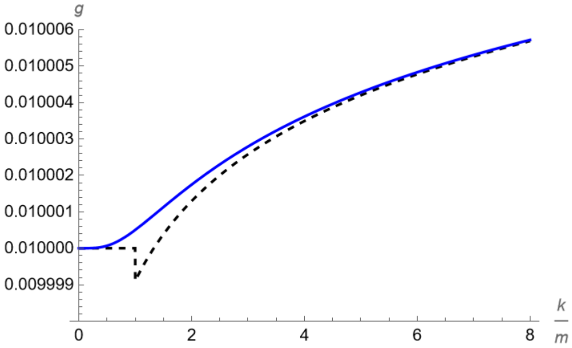
$$\begin{aligned}
&-\frac{\bar{g}(\mu_R)}{2m^4}(s^2+t^2+u^2)\\
&+\frac{\bar{g}^2}{192\pi^2 m^4}\left[\log\left(\frac{-s}{\mu_R^2}\right)(13s^2+t^2+u^2)\right.\\
&\qquad\qquad +\log\left(\frac{-t}{\mu_R^2}\right)(s^2+13t^2+u^2)\\
&\qquad\qquad \left.+\log\left(\frac{-u}{\mu_R^2}\right)(s^2+t^2+13u^2)\right]
\end{aligned}$$

High energy physical beta function

$$\beta_{\bar{g}} = \frac{5\bar{g}^2}{16\pi^2}$$

agrees with the μ-beta function and with the FRG

The offset



$$k = \sqrt{s}/\xi \quad \text{with} \quad \xi = \exp\left(\frac{25}{40} - \frac{17}{60}\right) \approx 1.4$$

High energy puzzles

Theory is asymptotically free for $g < 0$

Still it seems to become strongly coupled for $E > m/\sqrt[4]{g}$

Theory is perturbative: also the 4-derivative kinetic term $\sim E^4$

What is the meaning of asymptotic freedom in this case?

Does the theory have propagating d.o.f. at all?

Cross sections

Cancellations at tree level between the contributions of massless particles and ghosts in the inclusive cross sections

B. Holdom, [arXiv:2303.06723 [hep-th]]

Verified also at one loop (D. Buccio)

But why can one not consider exclusive cross sections?

Lessons - 1

- physical running only defined in asymptotic regions.
  FRG running agrees with physical running in these regions.
  μ-running agrees with physical running only in the UV limit.

Lessons - 2

- in the low energy EFT at one loop there are higher order operators with 6 and 8 derivatives;
  the dimensionful coupling has no physical running
- cancellation of higher dimension operators above the mass threshold is a new and partly unexpected phenomenon, joining the EFT regime to a AF (and possibly AS) regime

Lessons - 3

- power law running seen in the FRG is an aspect of threshold behavior with no clear physical counterpart in scattering amplitudes
- however, it is crucial for the adjustment of the dimension of the field in the FRG
- μ-running is not always physical. Well established results such as the universal beta functions of HDNLSM seem to contain unphysical terms of this type

  J. Donoghue and G. Menezes, e-Print: 2308.13704 [hep-th]