Observables and Observers in Quantum Spacetime

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Let us suppose that we have an asymptotically safe theory of quantum gravity.

How can we characterize the quantum geometry of spacetime?

How can we make contact with observables or make predictions?

The problem: Lessons from Hamiltonian formalism

1 Quantum waves and the spectral flow

Spectral flow methods

Euclidean vs. Lorentzian

to tackle the experimental ambiguities of integrating out modes

2 Running relational observables

Relational formalism

Composite operators and critical exponents

to study universal properties of diffeomorphism-invariant observables

Lessons from Hamiltonian formalism

The notion of **absolute time and space** has to be corrected.

The metric become dynamical, **geometry is no longer just an observer**.

The laws of physics are **Background Independent**, mathematically expressed by the classical Einstein equations which are **diffeomorphism invariant**.

Problem: interpretation of quantum mechanics in cosmological circumstances when the **observer is part of the system**. There is no outside of the universe by definition.

Lessons from Hamiltonian formalism

The time parameter does **not** have the status of **a gauge parameter.**

GR is a totally constrained theory with vanishing canonical Hamiltonian.

It is the time parameter defined by the Hamiltonian which corresponds to the **notion of time of a physical observer**.

E.g.: in GR in the asymptotically flat case the time parameter corresponding to the ADM Hamiltonian is the time parameter used by an observer in an asymptotic inertial system in Minkowski space.

Lessons from Hamiltonian formalism

1. By using techniques of group theory: find out an orthonormal basis of Hilbert space and define the fundamental **quantum observables of area and volume operators**.

   Analysis of the **spectra of these observables** shows that **the texture of the spacetime at the ultramicroscopic scale is discrete** and composed of minimal quanta of area and volume, proportional to the Planck area and volume, respectively.

2. Definition of observables within the framework of complete observables, that encode **relations between dynamical fields**.

   Perturbative calculations have been performed and within deparameterizable toy models it was possible for the first time to construct **a full set of gauge invariant observables for a background independent field theory**.

Quantum waves: spectral flow

Spacetime properties from the spectral flow

We explore the microstructure of spacetime and its effective quantum geometry by sending **waves** represented by a **scalar field**.

We construct the **Laplacian operator** and consider the **eigenvalue problem**.

The idea is do this at all points of the generalized RG trajectory.

Macroscopic metric

$(\bar{g}^{sc}_{k=0})_{\mu\nu}$

classical, smooth manifolds

$g^{\text{Minkowski}}_{\mu\nu}, \; g^{\text{de Sitter}}_{\mu\nu}, \dots$

$\longrightarrow$

$(\bar{g}^{sc}_{k})_{\mu\nu}$

Zoom into spacetime's microstructure

$\longrightarrow$

?

Gravitational Effective Average Action
A Background-Independent and diffeomorphism-covariant continuum approach to quantum gravity

Pick a solution of the FRGE:

Running effective action	$\Gamma_k[h_{\mu\nu}, \cdots; \bar{g}_{\mu\nu}]$
↓	
Running self-consistent metrics	$(\bar{g}_k^{sc})_{\mu\nu}$
↓	
Running kinetic operators	$\mathcal{K}_k = -\Box_g + \cdots \vert_{g=\bar{g}_k^{sc}}$
↓	
Running spectral problem	$\mathcal{K}_k \; \chi_n(k) = \mathcal{F}_n(k) \; \chi_n(k)$
↓	

Running spectra $\{\mathcal{F}_n(k)\}_{n=1,2,\ldots}$ and eigenfunctions $\{\chi_n(k)\}_{n=1,2,\ldots}$

Spectral flow

Running Gravitational Effective Average Action

↓

Running effective field equations

Running self-consistent metrics

↓

Running spectra

Background Independence:
$h_{\mu\nu}$-dynamics on all backgrounds

$$\langle \hat{g}_{\mu\nu} \rangle_{\bar{g}} \equiv \bar{g}_{\mu\nu} + \langle \hat{h}_{\mu\nu} \rangle_{\bar{g}}$$

$$\langle \hat{h}_{\mu\nu} \rangle_{\bar{g}} = 0 \iff \langle \hat{g}_{\mu\nu} \rangle_{\bar{g}} = \bar{g}_{\mu\nu} \quad \text{for} \quad \bar{g} = \bar{g}^{sc}_k$$

Self-consistent geometries

$$\frac{\delta}{\delta h_{\mu\nu}(x)} \Gamma_k \left[h; \bar{g}\right]\Big|_{h=0,\ \bar{g}=\bar{g}^{sc}_k} = 0$$

tadpole condition or effective Einstein equation

Generic solutions $(\bar{g}^{sc}_k)_{\mu\nu}$ will depend on the RG scale k: $k \mapsto \left(\Gamma_k,\ (\bar{g}^{sc}_k)_{\mu\nu}\right)$

Remark all the expectation values have a nontrivial (indirect) dependence on the background, which is kept completely arbitrary

the dynamics determines the expectation value of the metric s.t. the fluctuations are "as content as possible" about it

① Quantum waves: spectral flow

Running Gravitational Effective Average Action

↓

Running effective field equations

Running self-consistent metrics

↓

Running spectra

Concretely:

$$\Gamma_k[h;\bar{g}] = \frac{1}{16\pi\, G(k)} \int d^4x\, \sqrt{-g}\, \Big(R(g) - 2\Lambda(k)\Big)\Big|_{g=\bar{g}+h} + \cdots$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda(k) g_{\mu\nu} = 0$$

$$\text{e.g.} \quad g_{\mu\nu} \sim \mathrm{S}^4(L^{sc}(k))$$

$$-\Box_{\bar{g}_k^{sc}}\, \chi_{nm}(x;k) \;=\; \mathcal{F}_n(k)\, \chi_{nm}(x;k)$$

eigenvalue problem

$$k \mapsto \{\mathcal{F}_n(k)\}$$

spectral flow

Cutoff modes (COMs): $\chi_{n_{\mathrm{COM}(k)}}(x)$ with

$$\mathcal{F}_n(k)\big|_{n=n_{\mathrm{COM}}(k)} = k^2$$

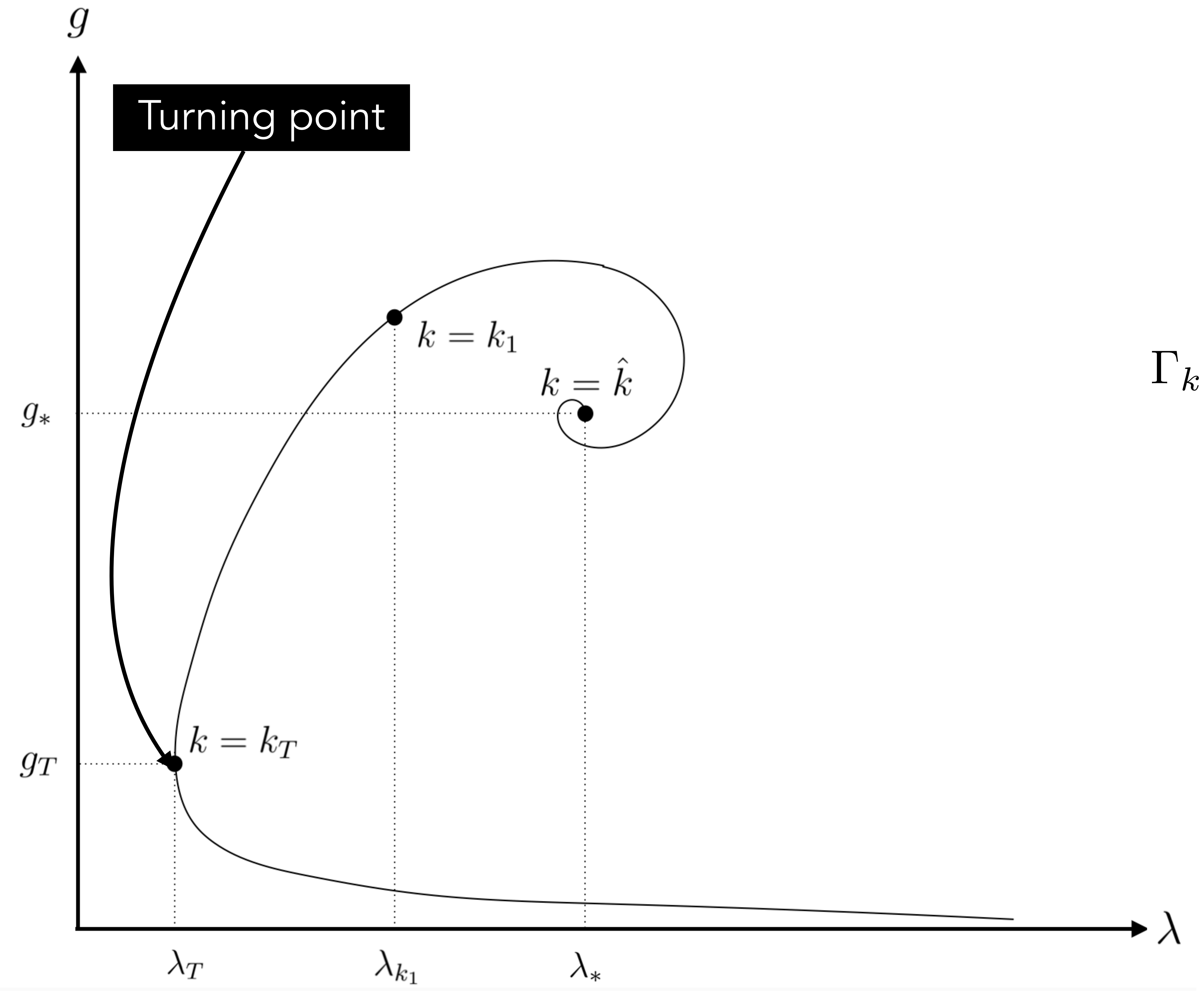
Remarks

The cutoff modes are located precisely at the threshold between "**already integrated out at RG scale**", and "**not yet integrated out**" if the fluctuations propagate on a background which is self-consistent at that given k.

COMs are a valuable **link between the bare off-shell world** under the path integral **and the effective level** of the on-shell expectation values.

1 Quantum waves: spectral flow

RG trajectory: **Trajectory of the Type IIIa**



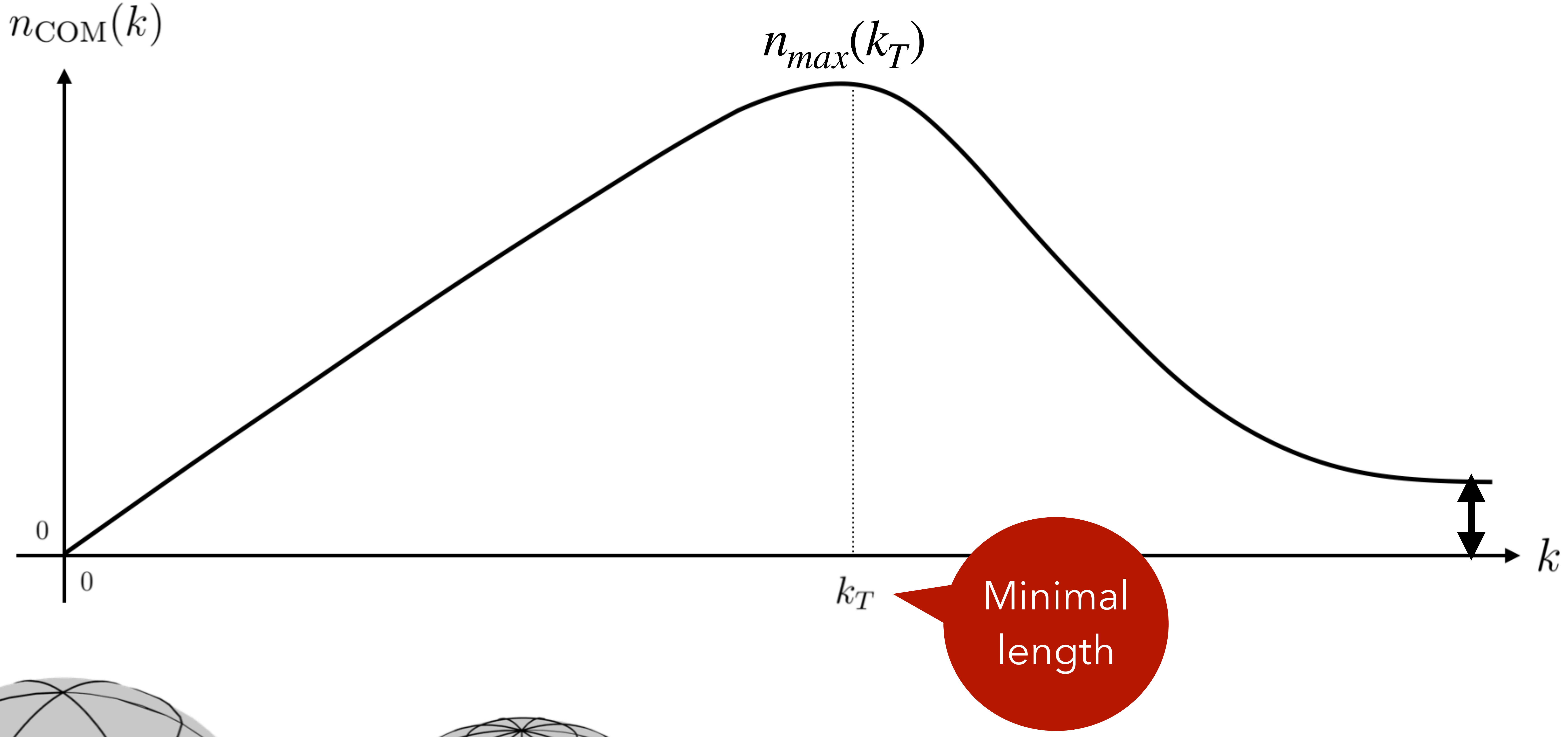
We restrict the analysis to pure quantum gravity, or matter-coupled gravity in a vacuum dominated regime.

$$\Gamma_k[h;\bar{g}] = \frac{1}{16\pi\, G(k)} \int d^4x\, \sqrt{-g}\, \Big(R(g) - 2\Lambda(k)\Big)\Big|_{g=\bar{g}+h} + \cdots$$

Caricature trajectory

$$\lambda(k) = \begin{cases} \frac{1}{2}\lambda_T \left[\left(\frac{k_T}{k}\right)^2 + \left(\frac{k}{k_T}\right)^2 \right] & \text{for} \quad 0 \le k \le \hat{k} \quad \text{(semiclassical)} \\ \lambda_* & \text{for} \quad \hat{k} < k < \infty \quad \text{(fixed point)} . \end{cases}$$

Euclidean



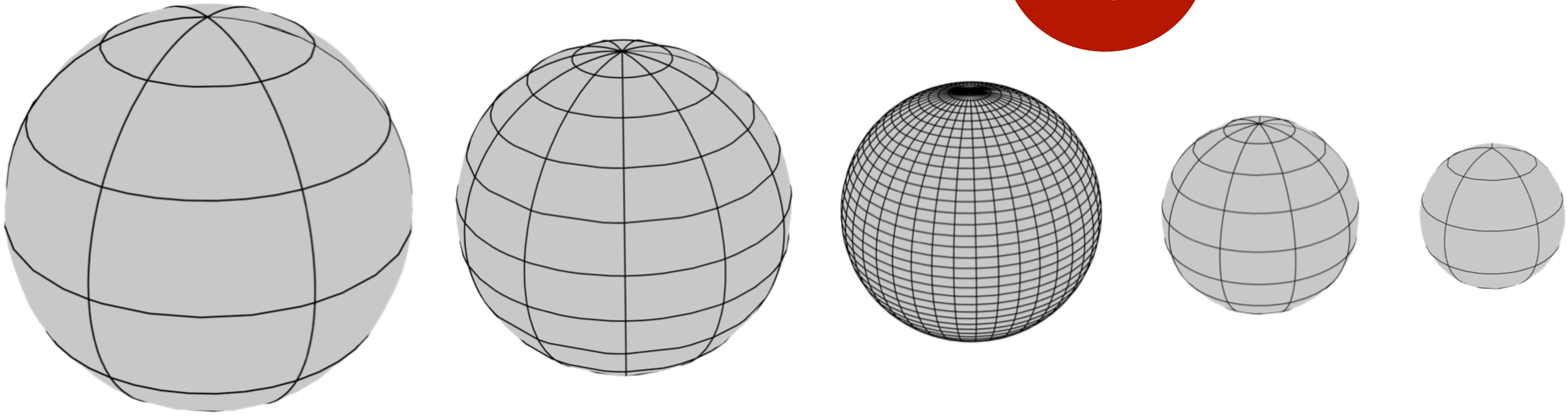
$S^4(L^{sc}(k))$

self-consistent spheres

$\mathcal{F}_n \sim n^2/L^2$

$n_{\mathrm{COM}}(k) \sim k\ L^{sc}(k)$

Less DOFs in the UV!



Limitations on the distinguishability of spacetime points

Resolution $2\pi/n_{\mathrm{COM}}(k)$

Timelike vs. spacelike fluctuation modes

How to integrate out negative/positive momentum squared modes in the flow equation?

There is ambiguity, no unique right choice.

However: experimental settings might suggest a specific choice.

Observer-dependent RG.

Path integral approach

Piecemeal integrating out of modes that underlies Γ_k as a procedure of performing the basic path integral in a stepwise fashion, rather than solving a flow equation.

Trajectories used

Einstein-Hilbert truncation

Rigid de Sitter Space (off-shell)

Conformal coordinates: $$\mathrm{d}s^2 = b(\eta)^2 \left[-\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2\right] = \frac{-\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2}{H^2\,\eta^2}$$

Eigenvalue equation: $$-\Box_{dS_4}\ \chi_{\nu,\mathbf{p}}(\eta,\mathbf{x}) = \mathcal{F}_\nu\ \chi_{\nu,\mathbf{p}}(\eta,\mathbf{x})$$

$$v''_{\nu,p}(\eta) + \left[p^2 - \frac{\nu^2 - 1/4}{\eta^2}\right] v_{\nu,p}(\eta) = 0$$

Eigenfunctions: $$\chi_{\nu,\mathbf{p}}(\eta,\mathbf{x}) = -\eta\ v_{\nu,p}(\eta)\ e^{i\mathbf{p}\cdot\mathbf{x}}\,,$$

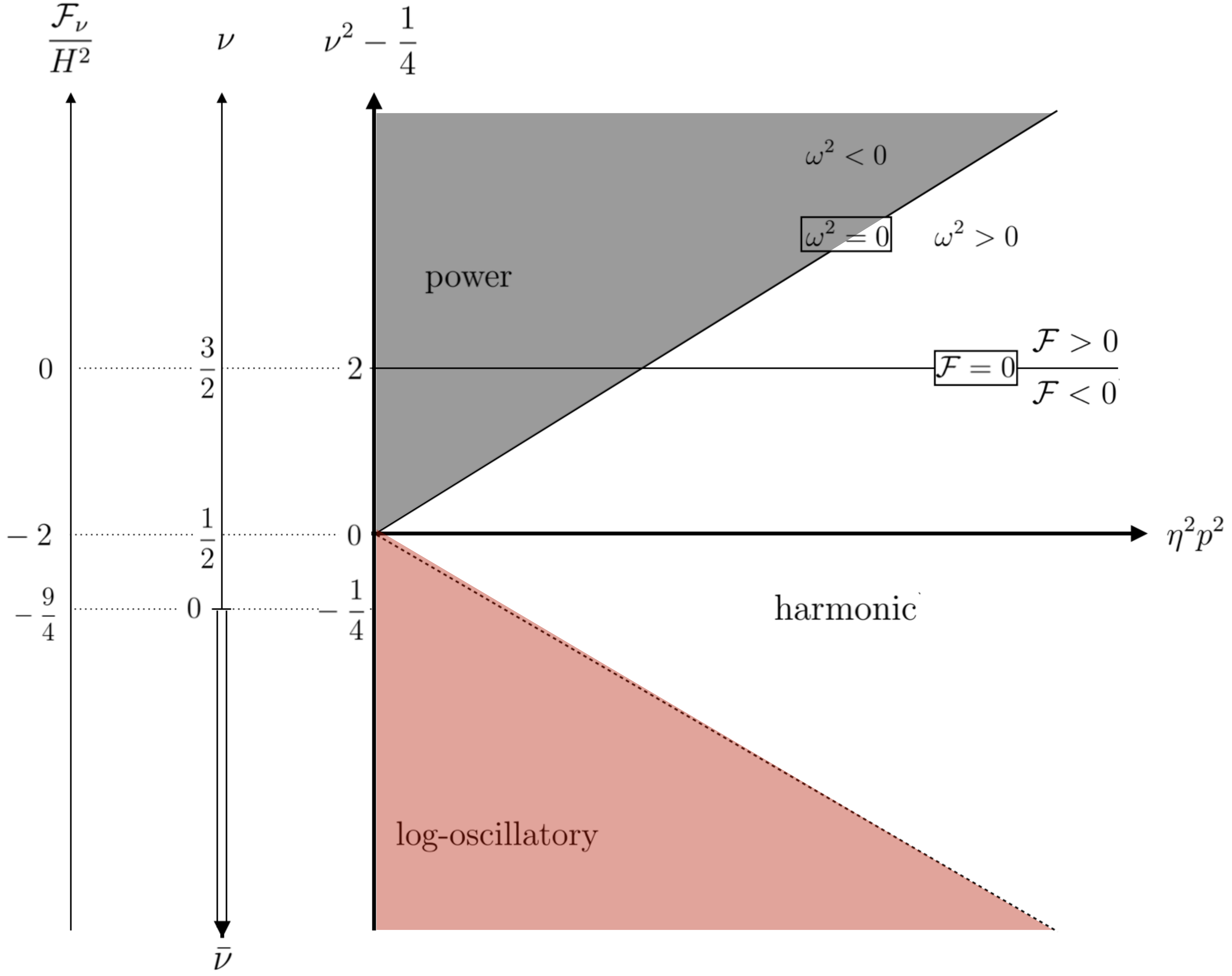
$$v_{\nu,p}(\eta) = (p\,|\eta|)^{1/2}\left[A_p\ J_\nu\left(p\,|\eta|\right) + B_p\ Y_\nu\left(p\,|\eta|\right)\right]$$

Bessel functions

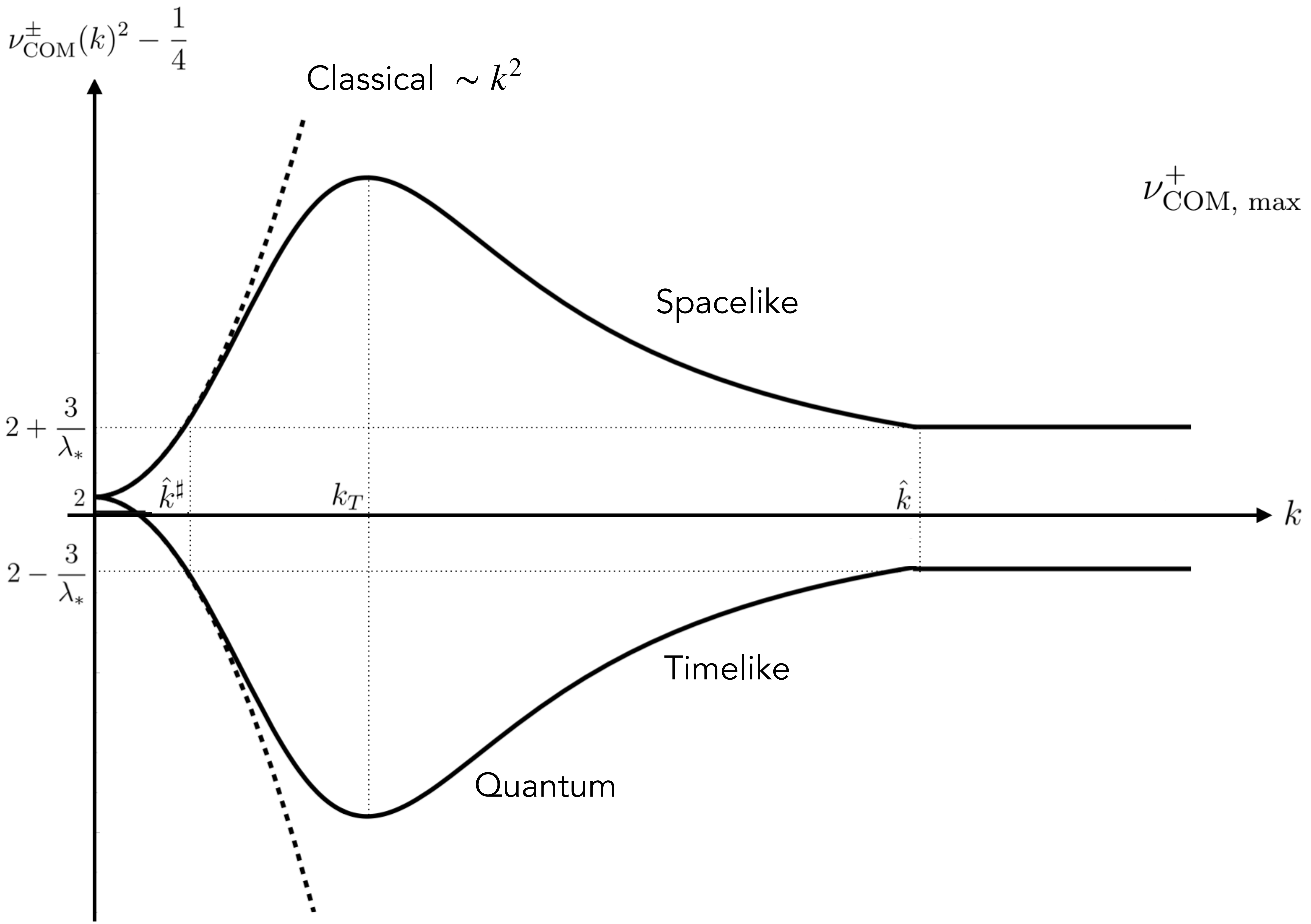
Eigenvalues: $$\mathcal{F}_\nu = \left(\nu^2 - \frac{9}{4}\right) H^2$$

Type	**Eigenvalue**	$\boldsymbol{\mathcal{F}_\nu/\mathrm{H}^2 + 2}$	**Index**
spacelike: $\mathcal{F} > 0$	$\mathcal{F}_\nu \in (0,\infty)\ H^2$	$\nu^2 - \frac{1}{4} \in (2,\infty)$	$\nu \in \left(\frac{3}{2},\infty\right)$
null: $\mathcal{F} = 0$	$\mathcal{F}_\nu = 0$	$\nu^2 - \frac{1}{4} = 2$	$\nu = \frac{3}{2}$
timelike: $\mathcal{F} < 0$	$\mathcal{F}_\nu \in \left(-\frac{9}{4},0\right)\ H^2$	$\nu^2 - \frac{1}{4} \in \left(-\frac{1}{4},2\right)$	$\nu \in \left(0,\frac{3}{2}\right)$
	$\mathcal{F}_\nu \in \left(-\infty,-\frac{9}{4}\right)\ H^2$	$\nu^2 - \frac{1}{4} \in \left(-\infty,-\frac{1}{4}\right)$	$i\,\nu \equiv \bar{\nu} \in (0,\infty)$

The ν-p plane



Evolving COM quantum numbers



$$\nu^{+}_{\rm COM,\ max} = \nu^{+}_{\rm COM}(k_T) \approx \left(\frac{3}{\lambda_T}\right)^{1/2}$$

What can we conclude about the resolution?

EFT and cutoff modes

Effective quantum geometry at scale k

Which are the geometrical features that are displayed by the "on-shell'" mean field configurations?

Are there structures which have a size that is comparable to the length scale at which Γ_k defines a "**good effective field theory**"?

Resolving structures on a time slice: effective spatial geometry

For every fixed time η and scale k the modes possess **unlimited resolving power** for spatial structures on the respective 3D time slice of the dS manifold.

We have **no way of controlling the η-dependence of the modes** if we use up all our freedom by optimizing the spatial resolution.

Impose conditions on the space of detectable modes inspired by experimental setting

The characteristic COM proper lengths

Proper wavelength

$$L_p(\eta, k) \equiv b_k(\eta)\, \Delta x_p = \frac{2\pi}{|\eta|\; p\; H(k)}$$

Transition wavelength

It is the largest possible **proper** wavelength a cutoff mode can posses in the harmonic regime.

$$L^+_{COM}(k) = \frac{2\pi}{k} \sqrt{\frac{3}{3 + 2\,\lambda(k)}}$$

depends on k, it is independent of η

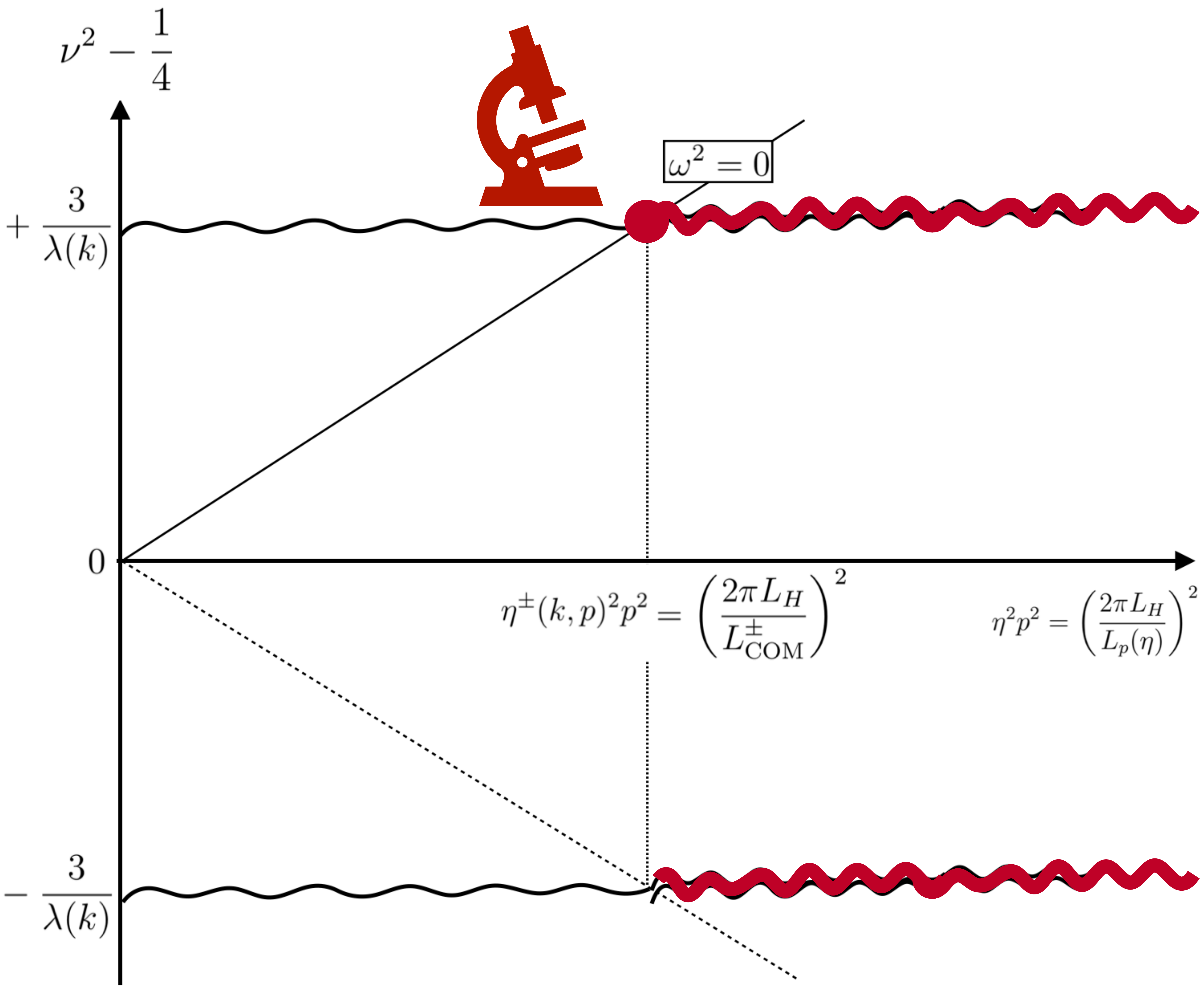
Consider the ratio:

$$\frac{L^+_{COM}(k)}{L_H(k)} = 2\pi \sqrt{\frac{\lambda(k)}{3 + 2\,\lambda(k)}}$$

Near the turning point

$$\left(\frac{L^+_{COM}(k)}{L_H(k)}\right)_{max} \approx \frac{2\pi}{\nu^+_{COM}(k_T)} \approx 2\pi \left[\frac{4}{9}\;\; G_0\;\Lambda_0\right]^{1/4} \quad \longrightarrow \quad L^+_{COM}(k) \ll L_H(k)$$

Two models



MODEL A

For every fixed k, only η-independent cutoff modes and combinations thereof are registered. All observed structures of field configurations are strictly **time independent**.

This model comes close to the ideal of reducing the wealth of physical patterns to the eternal geometric properties one would ascribe to **3D space**.

MODEL B

For every fixed k, only cutoff modes in the **harmonic regime**, and combinations thereof are registered.

The modes selected in this model are a generalization of the familiar **sub-horizon modes** on classical de Sitter spacetime. They are solutions to the Klein-Gordon equation, and yet are almost unaffected by curvature effects.

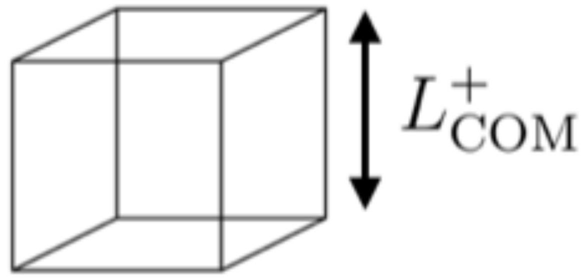
$$L_p \leq L^{+}_{COM}(k)$$

Coherence length and fragmentation of space

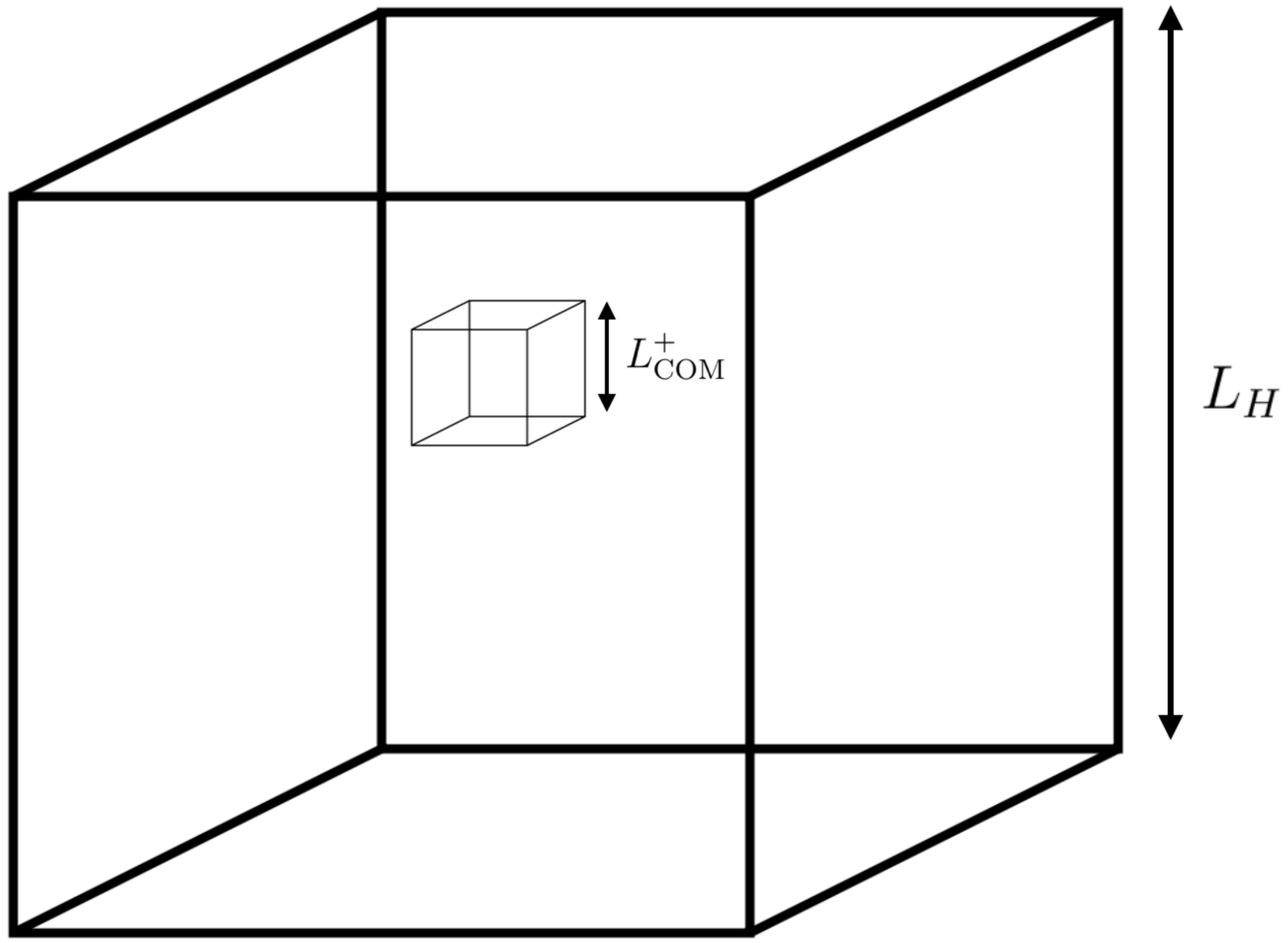
Patterns observed in the Universe should display a maximum size which is significantly smaller than the Hubble radius (**CAUSALITY**).

$$L_p \leq L^{+}_{COM}(k) \ll L_H(k)$$

Coherence length



Physics and geometry is well described within a patch by one of the effective field theories $\{\Gamma_k\}_{k\geq 0}$.



How many of those "COM boxes" would fit into one Hubble volume?

$$N_b(k) = \left(\frac{L_H(k)}{L^{+}_{COM}(k)}\right)^3 = \frac{1}{(2\pi)^3}\left[2 + \frac{3}{\lambda(k)}\right]^{3/2}$$

$$N_b^{max} = N_b(k_T) \approx \frac{1}{(2\pi)^3}\left[\frac{4}{9}\,\varpi\, G_0\, \Lambda_0\right]^{-3/4}$$

$$N_b^{max} \approx 10^{90}$$

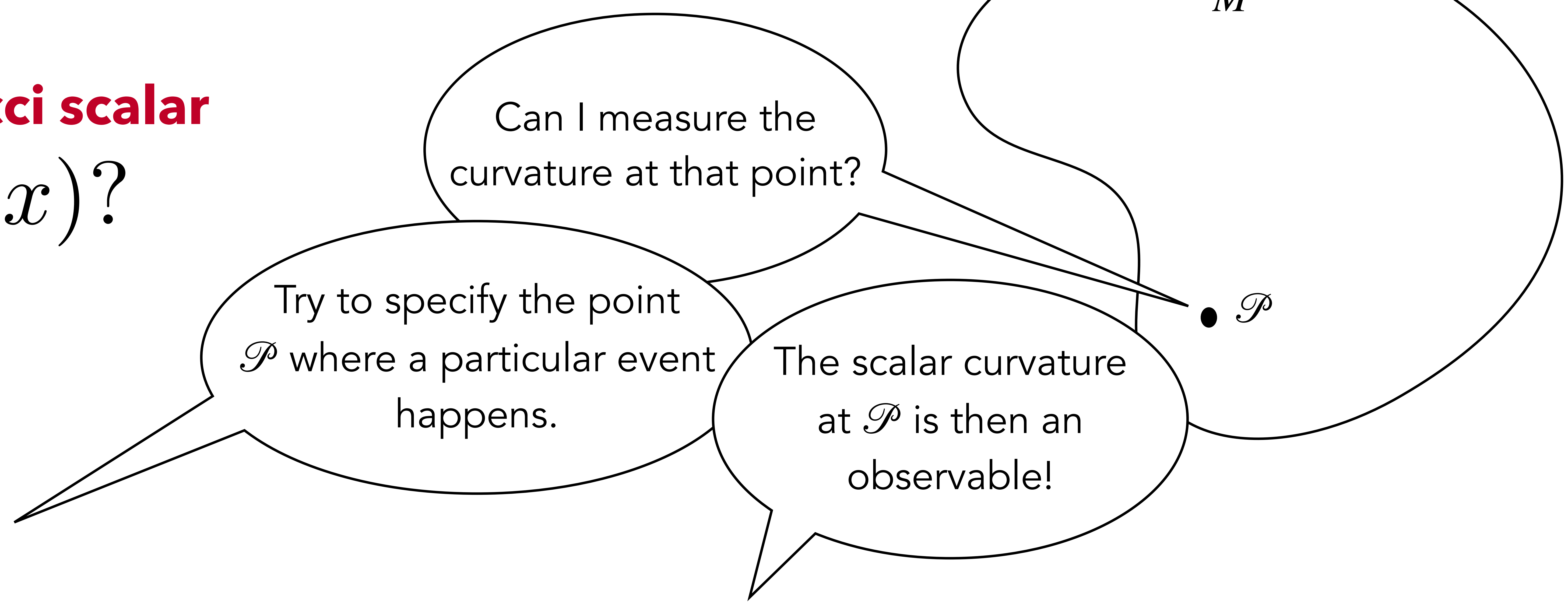
inter-domain entropy

Running relational observables

Diffeomorphism-invariant observables

Example: The Ricci scalar

$$R(x)?$$



In GR there are no local diffeomorphism-invariant observables: $R(x) \mapsto \varphi * R(x)$

However, if X denotes the (spacetime) position of a particle, a diffeomorphism will map

$$X \mapsto \varphi^{-1}(X)$$

Thus $R(X)$ at the position of the particle, is diffeomorphism invariant, and hence observable.

$$R(X) \mapsto \varphi * R(\varphi^{-1}(X)) = R(X)$$

2 Running relational observables

Physical reference system

Construct a physical coordinate frame, s.t. composed transformation leaves the tensor invariant.

add matter fields

Composite operators

Observables require information about operators which are not taken into account in a truncated EAA.

One can couple them to an external source so that it can be inserted into correlation functions.

$$\Gamma_k[\phi,\varepsilon] = \Gamma_k[\phi] + \int \mathrm{d}^d x\, \varepsilon(x)\, \mathcal{O}_k(x) + O(\varepsilon^2)$$

$$\int \mathrm{d}^d x\, \varepsilon\, k\partial_k \mathcal{O}_k = -\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\left(\int \mathrm{d}^d x\, \varepsilon \mathcal{O}_k^{(2)}\right)\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

Concrete method to compute expectation values of observables.

$$\lim_{k\to\infty} \mathcal{O}_k = \hat{\mathcal{O}}|_{\hat{\phi}\to\phi}$$

$$\mathcal{O}_{k=0} = \langle \hat{\mathcal{O}} \rangle$$

The critical exponents do not depend on the cutoff scheme, they are **universal**.

$$u^i(k) = u^i_* + \sum_I C_I V^i_I \left(\frac{k_0}{k}\right)^{\theta_I}$$

Flow of the relational observables

$$e^{\hat{\mu}}_{\mu}(x) = \partial_\mu \hat{X}^{\hat{\mu}}(x)$$

$$\tilde{e} = \det e^{\hat{\mu}}_{\mu}$$

Relational EAA

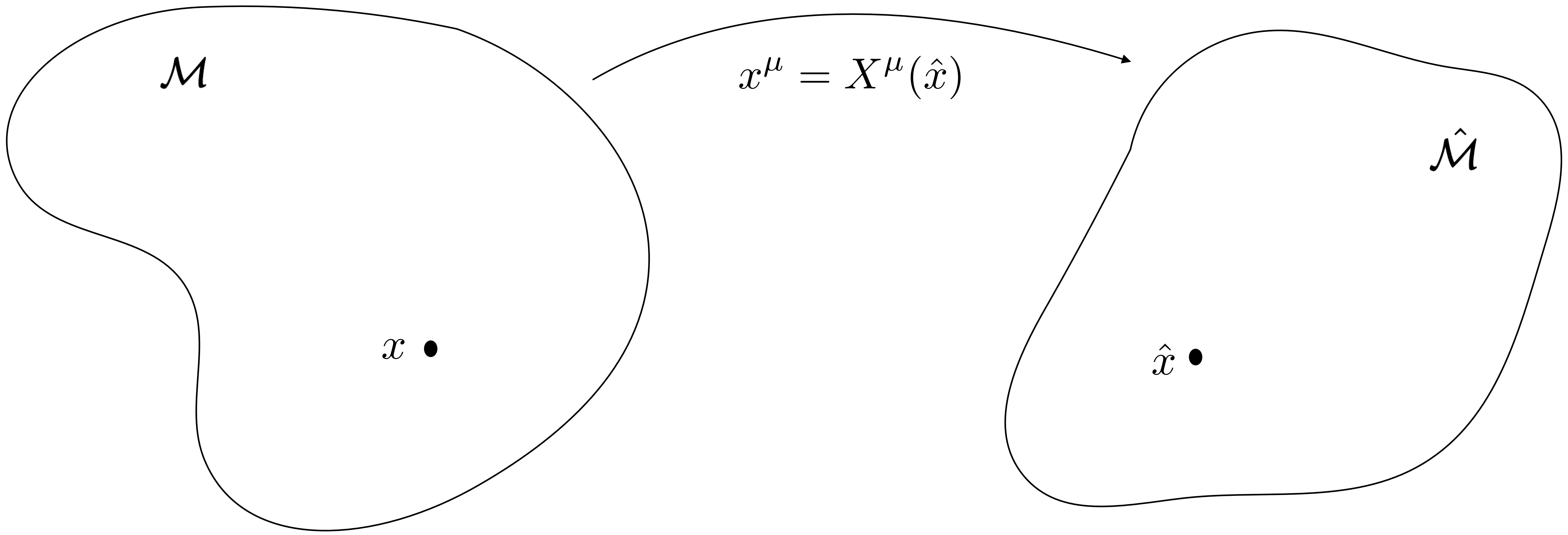
$$\Gamma_k^{\text{rel.}} \equiv \int \mathrm{d}^4x\, \tilde{e}(x) \mathcal{L}_k^{\text{rel.}}(x) := \int \mathrm{d}^4x\, \tilde{e}(x) \sum_i a_{\hat{I}_i}(k) O^{\hat{I}_i}(x)$$

Flow equation of the relational EAA

$$k\partial_k \Gamma_k^{\text{rel.}} = -\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\left(\Gamma_k^{\text{rel.}(2)}\right)\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

$$\hat{X}^{\hat{\mu}}(x) = \hat{x}^{\hat{\mu}}$$

$$x^\mu = X^\mu(\hat{x})$$



Find the fixed points

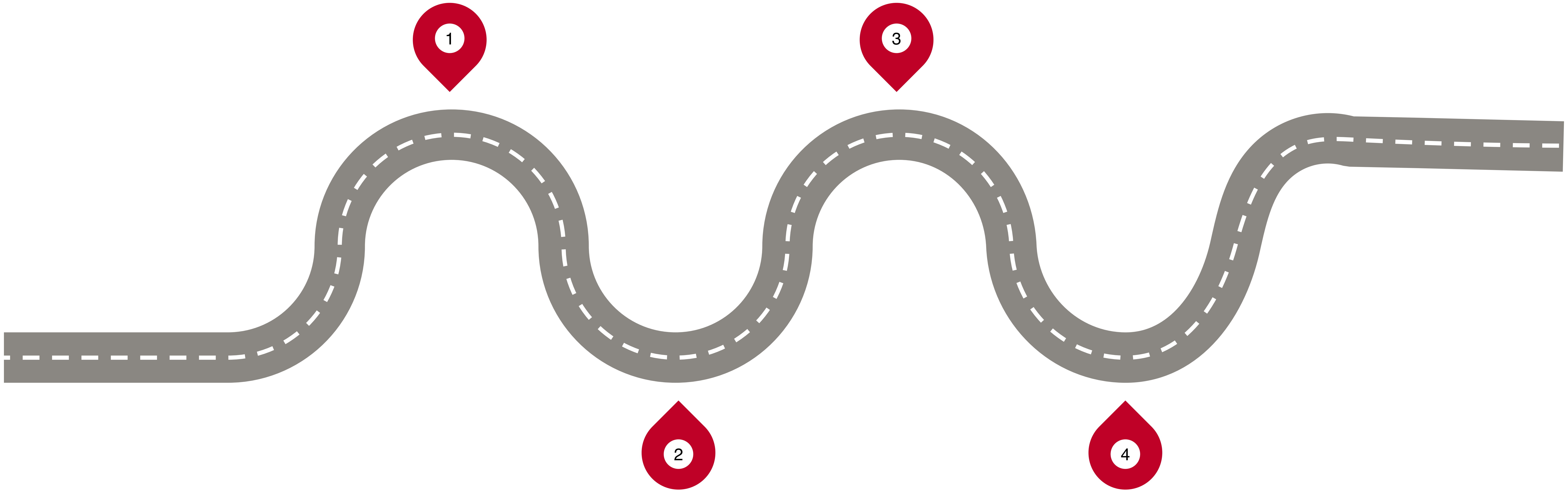
EAA

Identify the relational observables

Compute the flow of the observables

Relational EAA

Scaling dimension at the FP



Application

Find the fixed points

1

$$\Gamma_k = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{16\pi G_N(k)}(R-2\Lambda(k)) + \frac{1}{2}\delta_{\mathrm{AB}}g^{\mu\nu}\partial_\mu\varphi^{\mathrm{A}}\partial_\nu\varphi^{\mathrm{B}}\right) + \text{gauge fixing and ghosts}\ldots$$

Identify the relational observables

2

$$\varphi^{\hat\mu} = \hat X^{\hat\mu}$$

Compute the flow of the observables

3

Relational curvature

$$\Gamma_k^{\text{rel.}} = \int \mathrm{d}^4x\,\tilde e\left(\alpha_0(k) + \alpha_R(k)R + \alpha_1(k)\delta_{\hat\mu\hat\nu}g^{\mu\nu}(\partial_\mu\hat X^{\hat\mu})(\partial_\nu\hat X^{\hat\nu})\right)$$

Volume term

Relational metric

Scaling dimension at the FP

4

Matter content	θ_0	θ_R	θ_1
SM (type II)	-4	-5.97643	-7.92358
SM (type I)	-4	-5.97467	-7.8177
SM + SF (type II)	-4	-5.97505	-7.80603
SM + 3 ν (type II)	-4	-5.98015	-7.78084

Small quantum corrections

Let us suppose that we have an asymptotically safe theory of quantum gravity.

How can we characterize the quantum geometry of spacetime?

We need to take into account the **backreaction of a scaling spacetime**

We need to **adapt how we integrate out to the experimental setting**

How can we make contact with observables or make predictions?

We need some **"reference"**: matter, fixed volume, boundary,…

Scaling depends on the physical reference (also via gauge fixing)