functional flows for higher curvature quantum gravity

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Workshop
Quantum Space-Time and the Renormalisation Group
Calgiari / Sant Elmo
03 Oct 2023

critical points in 4d

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UV critical points

fundamental definition of QFT Wilson '71

asymptotic freedom Gross, Wilzcek '73 , Politzer '73

asymptotic near freedom Bailin, Love '74

asymptotic safety Weinberg '79 Reuter '98

IR critical points

Banks-Zaks conformal window Caswell '74 Banks, Zaks, '82

weak-strong dualities Seiberg '95

today:

canonical mass dimension as ordering principle

weakly-coupled 4d QFTs

strongly-coupled fermions

higher curvature quantum gravity

perturbative 4d QFTs

fields **vectors** A_μ^a , **fermions** ψ_I , **scalars** ϕ^A

path integral

$$Z[J] = \exp -i \int d^4x (L + L_{\rm gf} + L_{\rm gh} + J^i \, \Phi_i)$$

action

$$L = \frac{1}{4g_a^2} \mathrm{Tr}\, F^a_{\mu\nu} F_a^{\mu\nu} + i \psi_I \not{D} \psi_I + \frac{1}{2} (D_\mu \phi^A)^2$$

$$+ \frac{1}{2} Y^A{}_{IJ} \phi^A \, \psi_I \, \xi \, \psi_J + \frac{1}{4!} \lambda_{ABCD} \phi^A \phi^B \phi^C \phi^D$$

gauge → $\frac{1}{4g_a^2}$

Yukawa → $Y^A{}_{IJ}$

quartics → λ_{ABCD}

Perturbativity guarantees that

mass terms remain relevant

higher dimension interactions remain irrelevant

classically marginal couplings are key

strongly-coupled fermions

Gross-Neveu

U(N) symmetric fermions

4-fermion interactions $G(\bar{\psi}\psi)^2$

Gross, Neveu '74

chiral symmetry $\psi \to \gamma^5\psi \quad \bar{\psi} \to -\bar{\psi}\gamma^5$

3d: perturbatively non-renormalisable… $[G] = 2 - d$

…yet non-perturbatively renormalisable

interacting UV fixed point

Gawedzki, Kupiainen '85
Rosenstein, War, Park '89
de Calan, Faria da Veiga, Magnen, de Seneor '91

Gross-Neveu (-Yukawa)

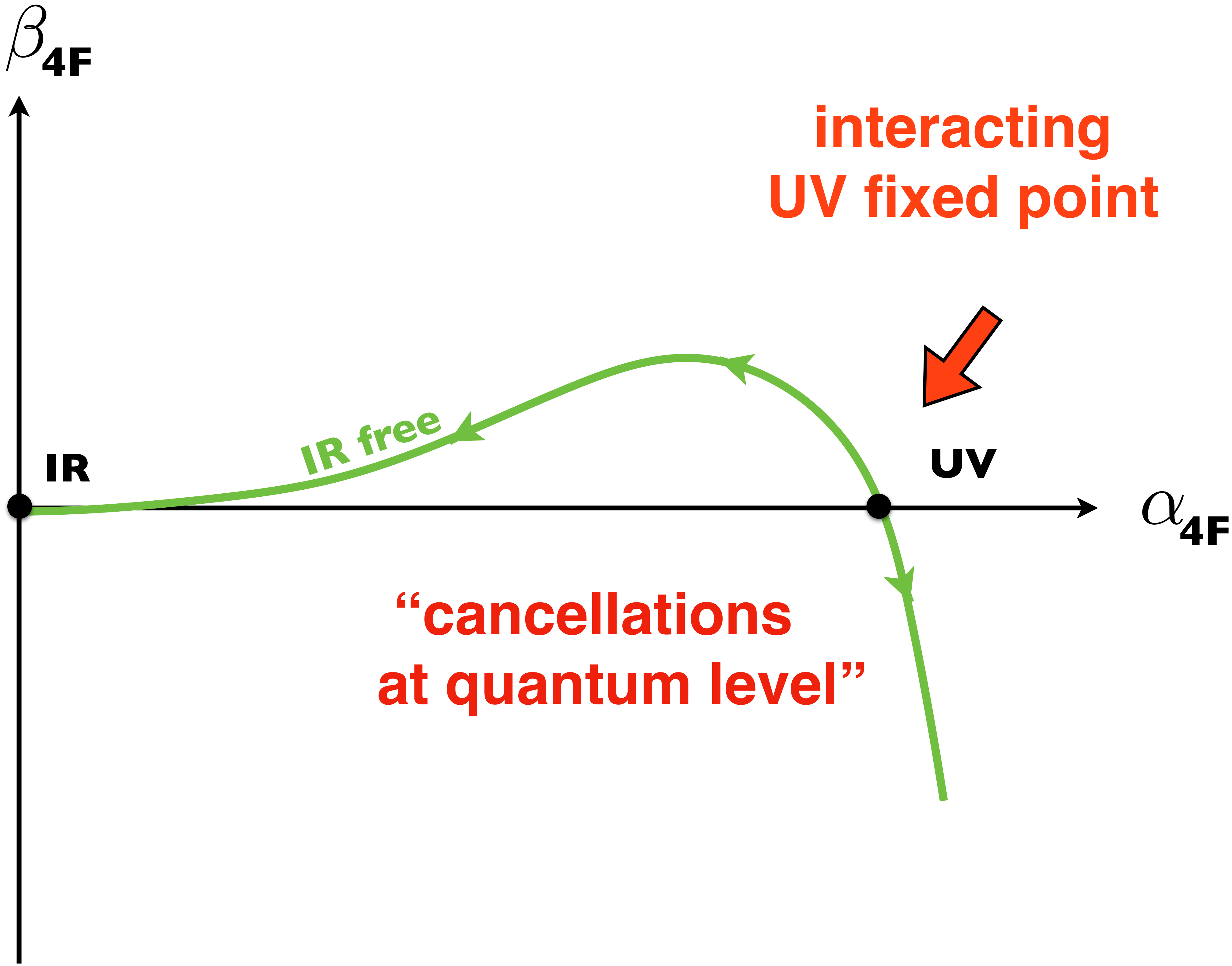
PT, large Nf

functional RG

Gracey '90
Moshe Moshe, Zinn-Justin '03

Gies, MM Scherer '10
Braun, Gies, DD Scherer '12
Jakovac, Patkos '13, '14

renormalisation group



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Gross-Neveu+

relax chiral symmetry

$$S = \int \mathrm{d}^d x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G (\bar{\psi}_a \psi_a)^2 + \frac{1}{3!} H (\bar{\psi}_a \psi_a)^3 \right\}.$$

mass term permitted $m\bar{\psi}\psi$

6-fermion interactions permitted $[H] = 3 - 2d$

functional RG

$$\partial_t \Gamma_k = \tfrac{1}{2}\, \mathrm{tr} \left\{ \left[\Gamma_k^{(2)} + R_k \right]^{-1} \cdot \partial_t R_k \right\}$$

exactly solvable at infinite Nf

Charlie Cresswell-Hogg, DFL, PRL '23 and PRD '23

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"theory space"

in 3d	classical
$\lambda_1 \, \bar\psi\psi$	relevant
$\lambda_2 (\bar\psi\psi)^2$	irrelevant
$\lambda_3 (\bar\psi\psi)^3$	.
$\lambda_n (\bar\psi\psi)^n$	.
.	.
.	
.	

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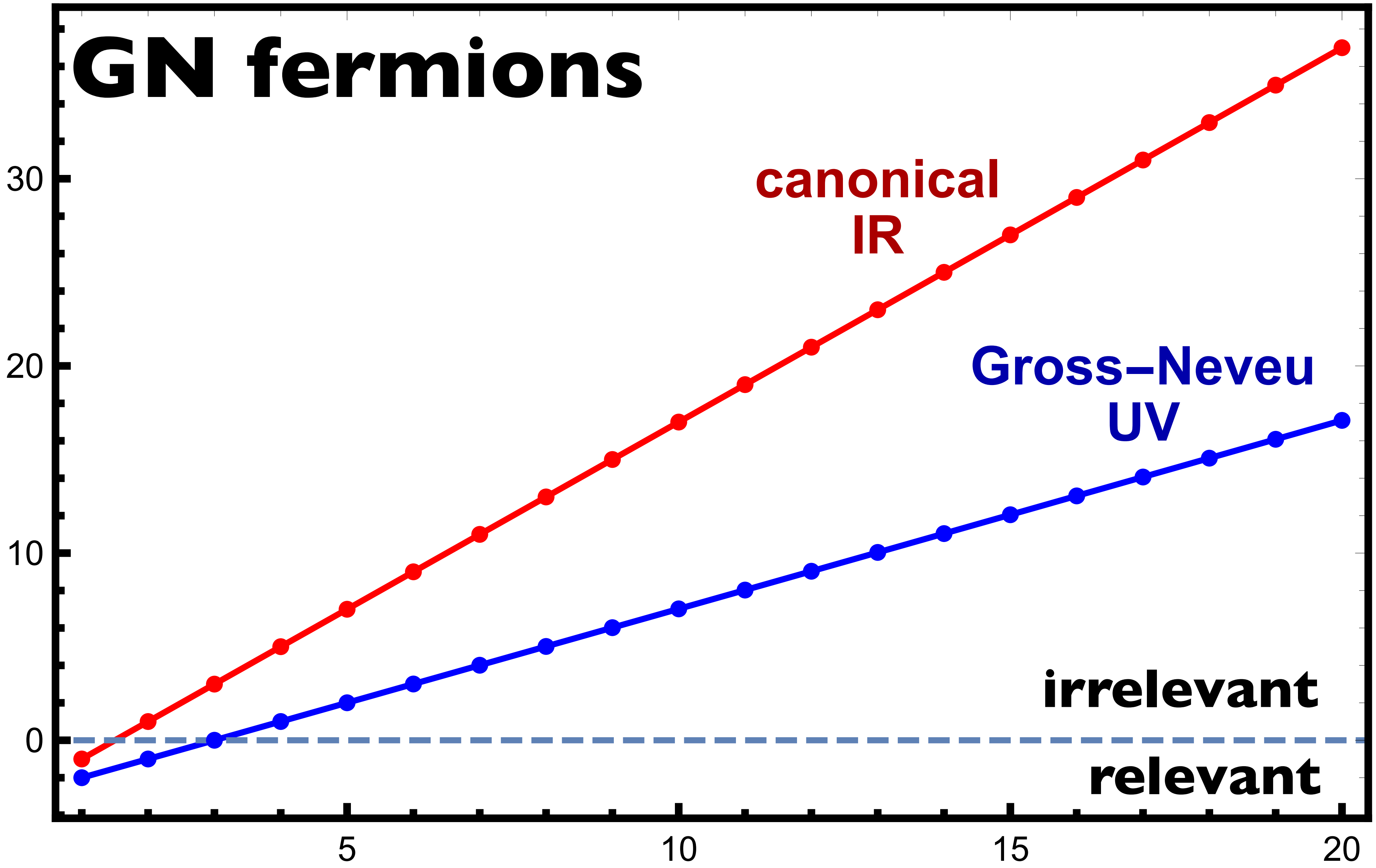
"theory space"

in 3d	classical	quantum
$\lambda_1\,\bar\psi\psi$	relevant	relevant
$\lambda_2(\bar\psi\psi)^2$	irrelevant	relevant
$\lambda_3(\bar\psi\psi)^3$	⋮	marginal
$\lambda_n(\bar\psi\psi)^n$		irrelevant
⋮		⋮

Charlie Cresswell-Hogg, DFL, PRL '23 and PRD '23

universal scaling dimensions

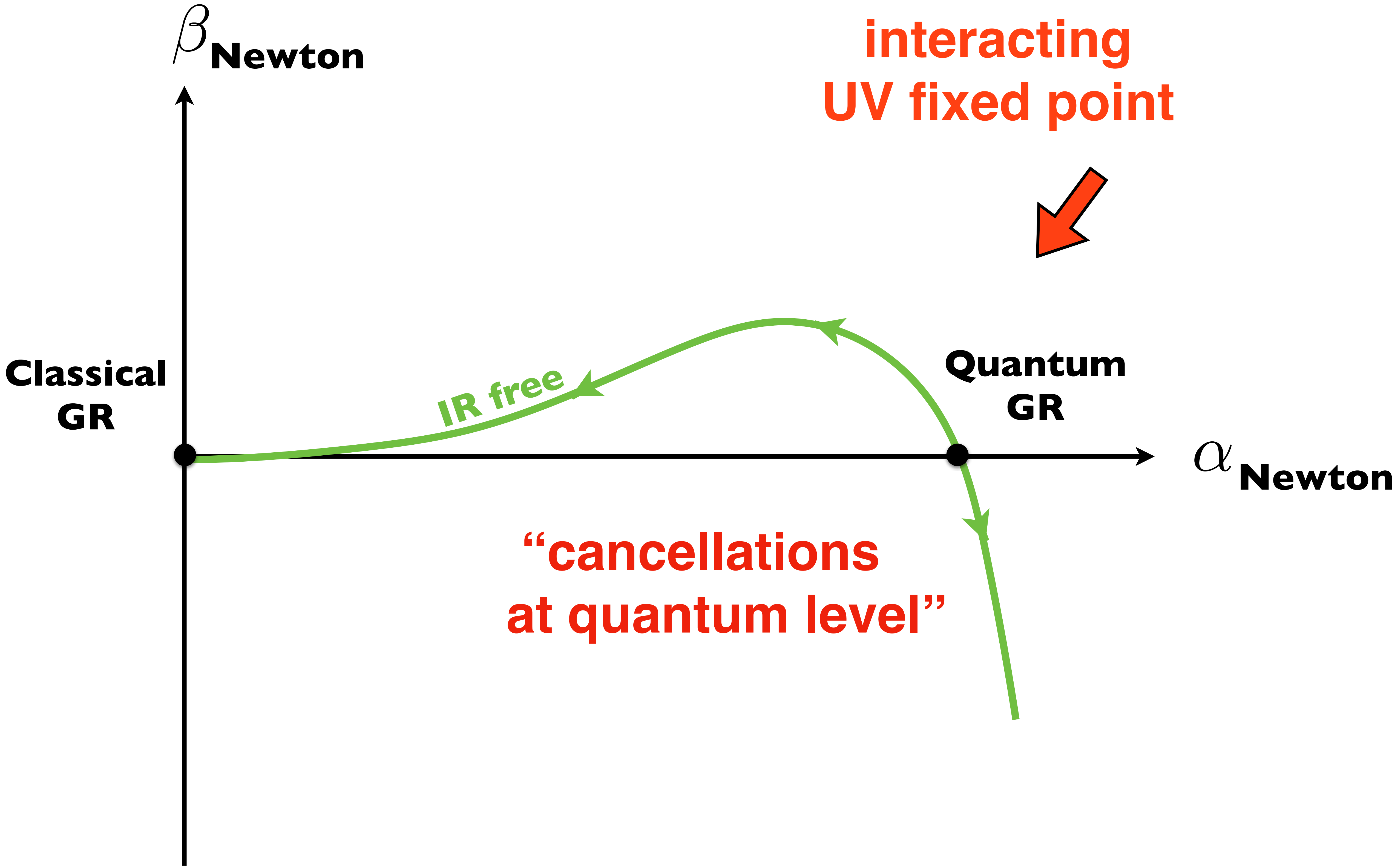


large quantum effects

$$\frac{\vartheta_n^{(\mathrm{IR})} - \vartheta_n^{(\mathrm{UV})}}{\vartheta_n^{(\mathrm{IR})}} = \frac{n+1}{2n-1}$$

quantum gravity

renormalisation group



what's the trouble with gravity?

degrees of freedom: **spin 2**

Newton's coupling is **dimensionful** $[G_N] = 2 - D < 0$

perturbatively non-renormalisable

interacting fixed point requires **large** anomalous dimensions

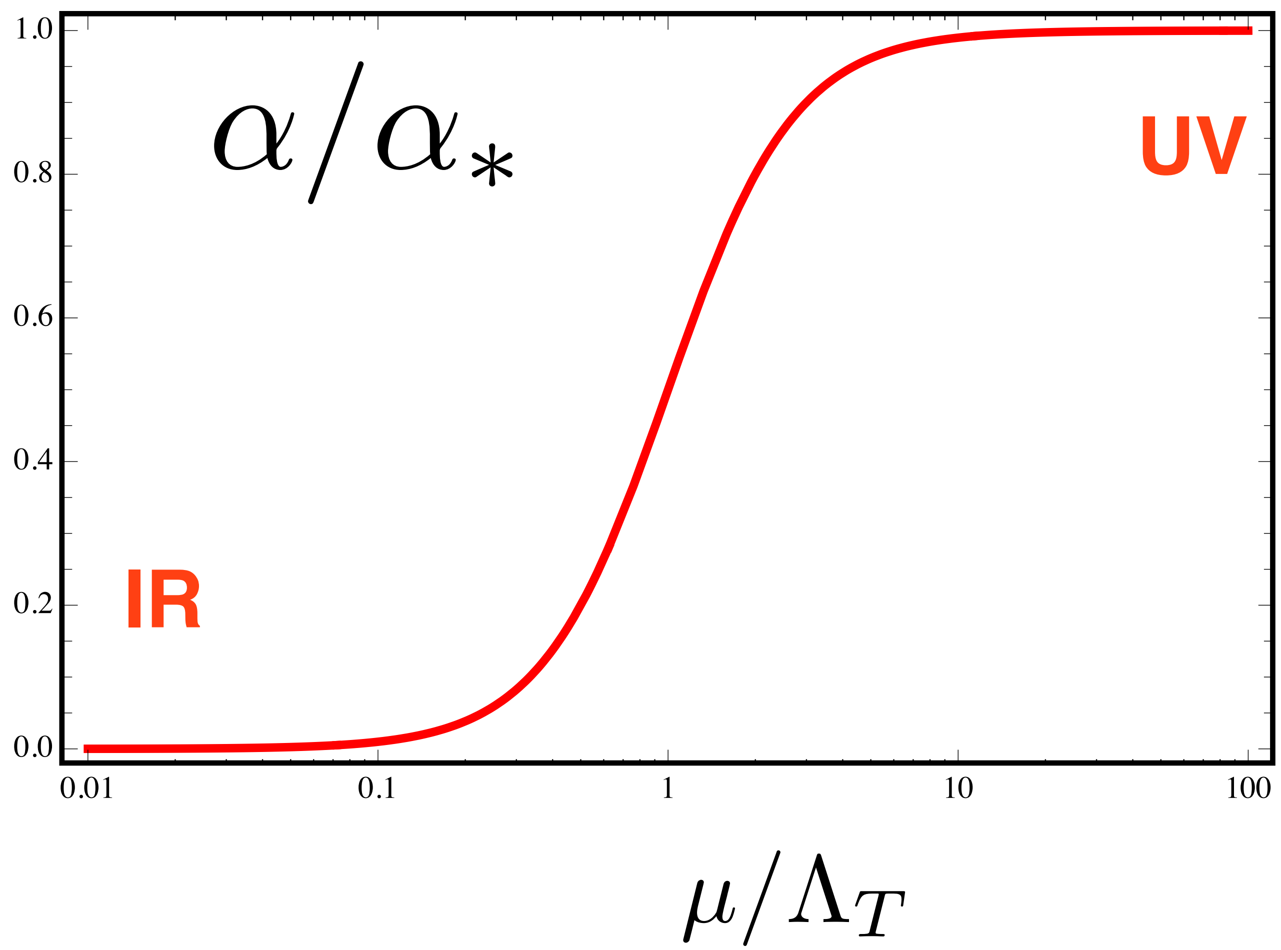
higher curvature interactions non-negligible

Einstein-Hilbert

	dimension	coupling	
gravitons	$D = 2 + \epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Christensen, Duff '78 Weinberg '79 Kawai et al '90

$$G(\mu) \approx G_N$$

classical GR



$$G(\mu) \approx \frac{\alpha_*}{\mu^{D-2}}$$

quantum GR

UV fixed point implies weakened gravity

bootstrap

Falls, Litim, Nikolakopoulos, Rahmede '13

Ricci scalars $\Gamma_k \propto f(R)$

$$\Gamma_k = \int d^4x \sqrt{\det g_{\mu\nu}} \frac{1}{16\pi\, G} \left[-R + 2\Lambda\right] + \sum_{n=2}^{N-1} \lambda_n \, R^n$$

effective action with invariants up to mass dimension $D = 2(N-1)$

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bootstrap

Ricci scalars $\Gamma_k \propto f(R)$

$$\Gamma_k = \int d^4x \sqrt{\det g_{\mu\nu}} \frac{1}{16\pi G} [-R + 2\Lambda] + \sum_{n=2}^{N-1} \lambda_n R^n$$

effective action with invariants up to mass dimension $D = 2(N-1)$

up to order $N = 2$ Souma, '99, Reuter, Lauscher '01, Litim '03

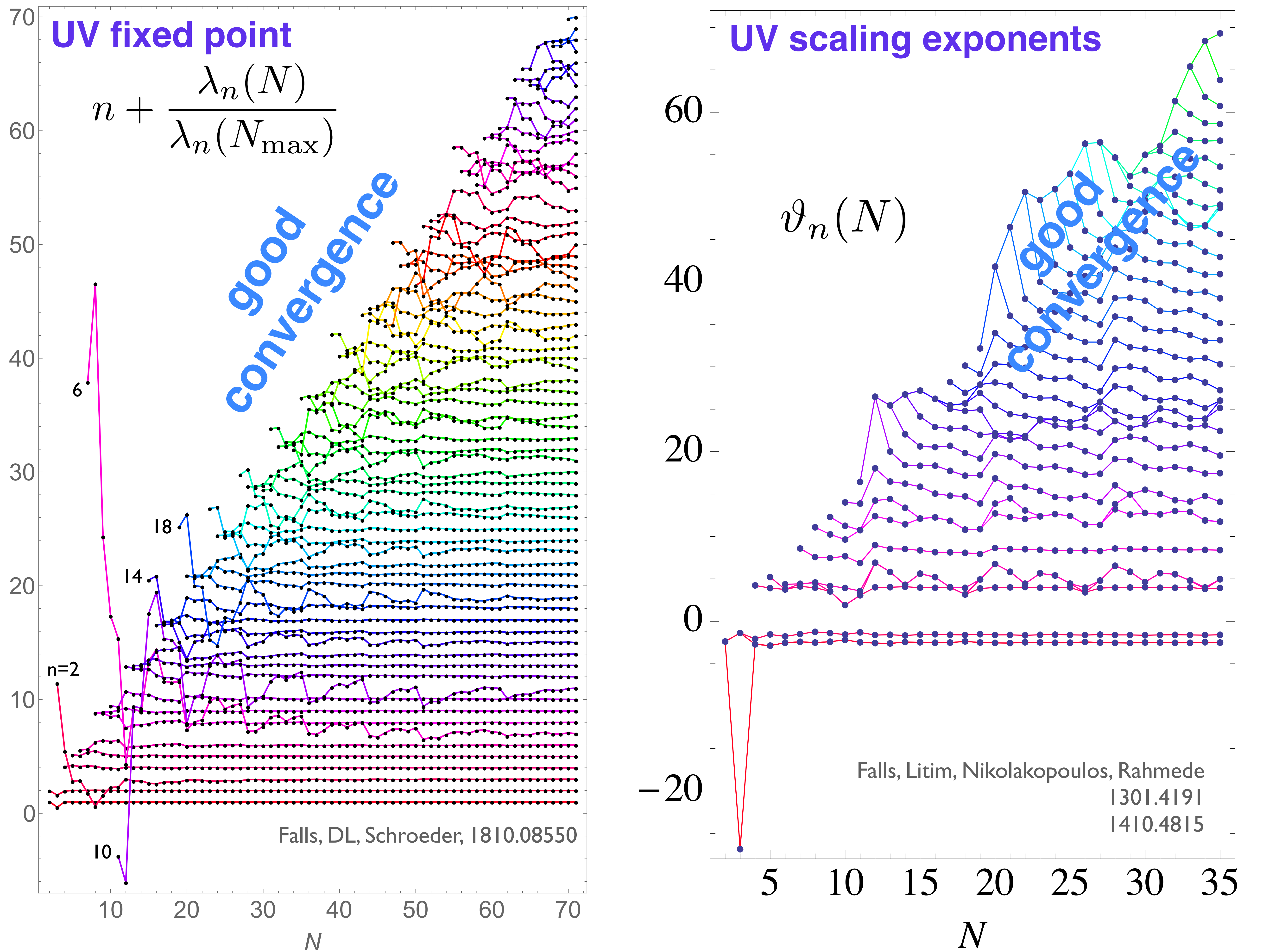
$N = 3$ Reuter, Lauscher '01

$N = 7$ Codello, Percacci, Rahmede '07

$N = 11$ Bonanno, Contillo,, Percacci '10

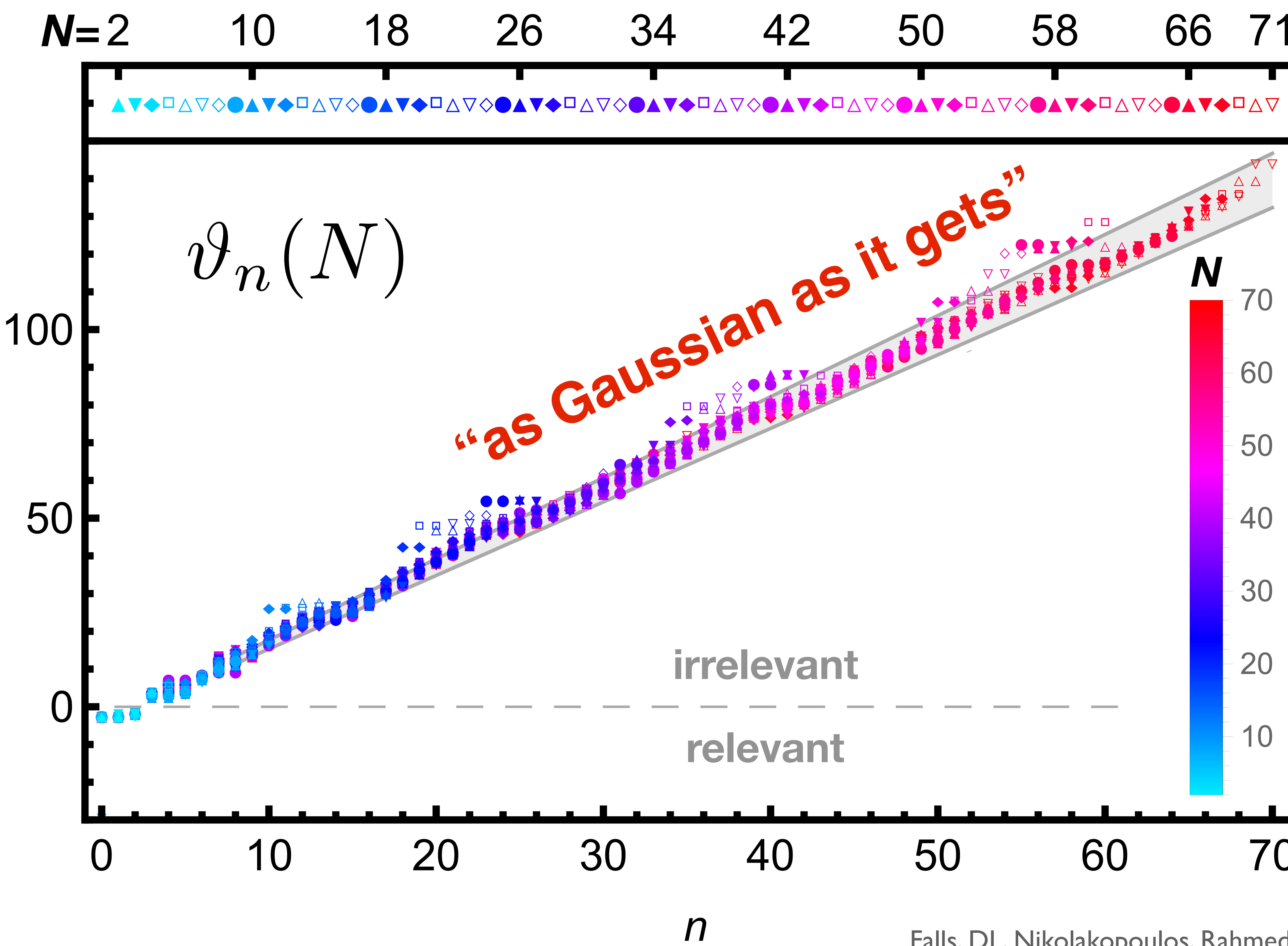
$N = 35$ Falls, Litim, Nikolakopoulos, Rahmede '13, '14, '16

$N = 71$ Falls, Litim, Schroeder '18



UV fixed point
$n + \frac{\lambda_n(N)}{\lambda_n(N_{\max})}$
good convergence
n=2
6
10
14
18
Falls, DL, Schroeder, 1810.08550
N
UV scaling exponents
$\vartheta_n(N)$
good convergence
Falls, Litim, Nikolakopoulos, Rahmede
1301.4191
1410.4815
N

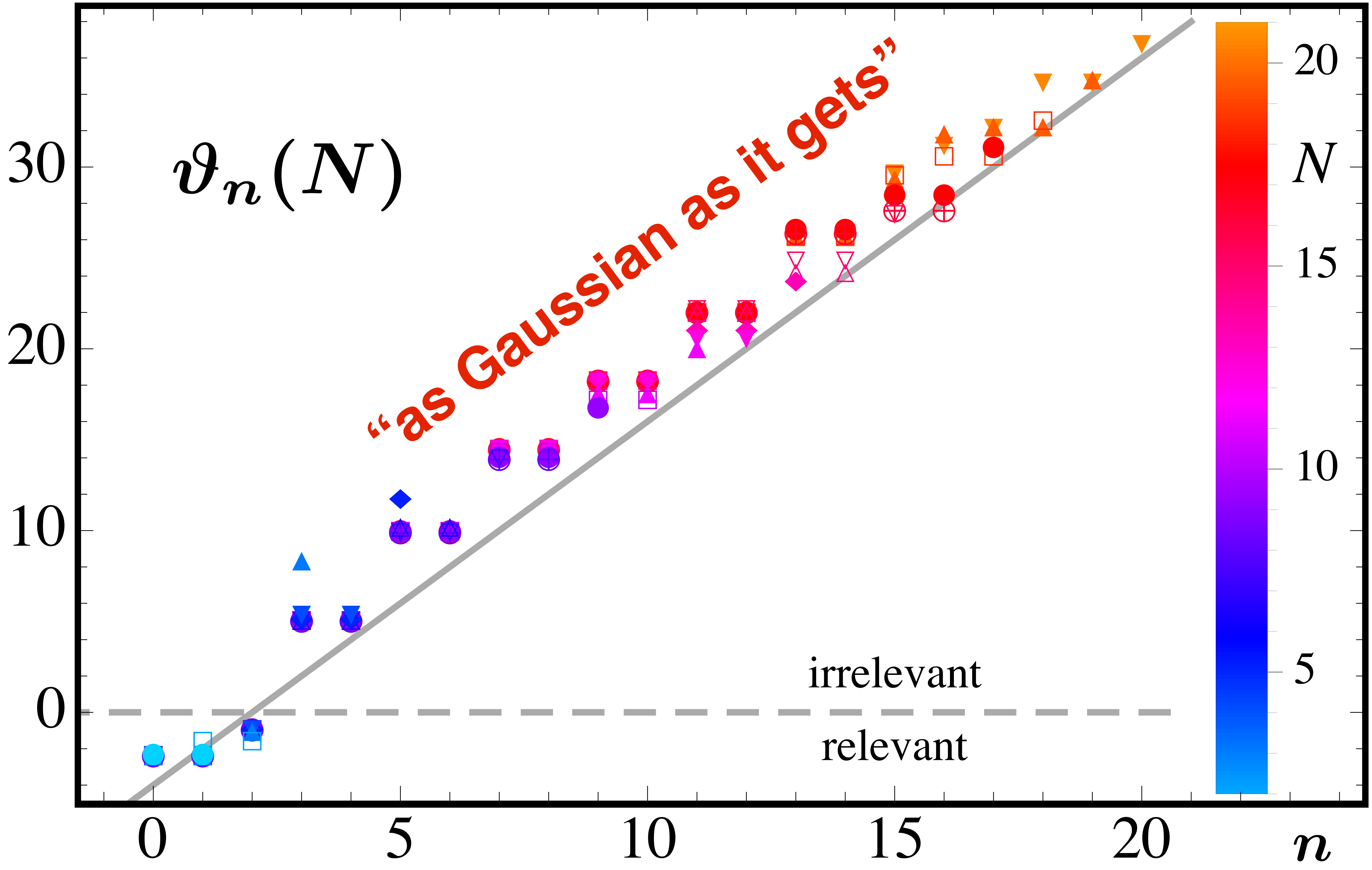
Ricci scalars



Falls, DL, Nikolakopoulos, Rahmede '13, '14, '16
Falls, DL, Schroeder, '18

$$\Gamma_k = \int d^dx\sqrt{g}[F_k(\mathrm{Ric}^2) + R\cdot Z_k(\mathrm{Ric}^2)]$$

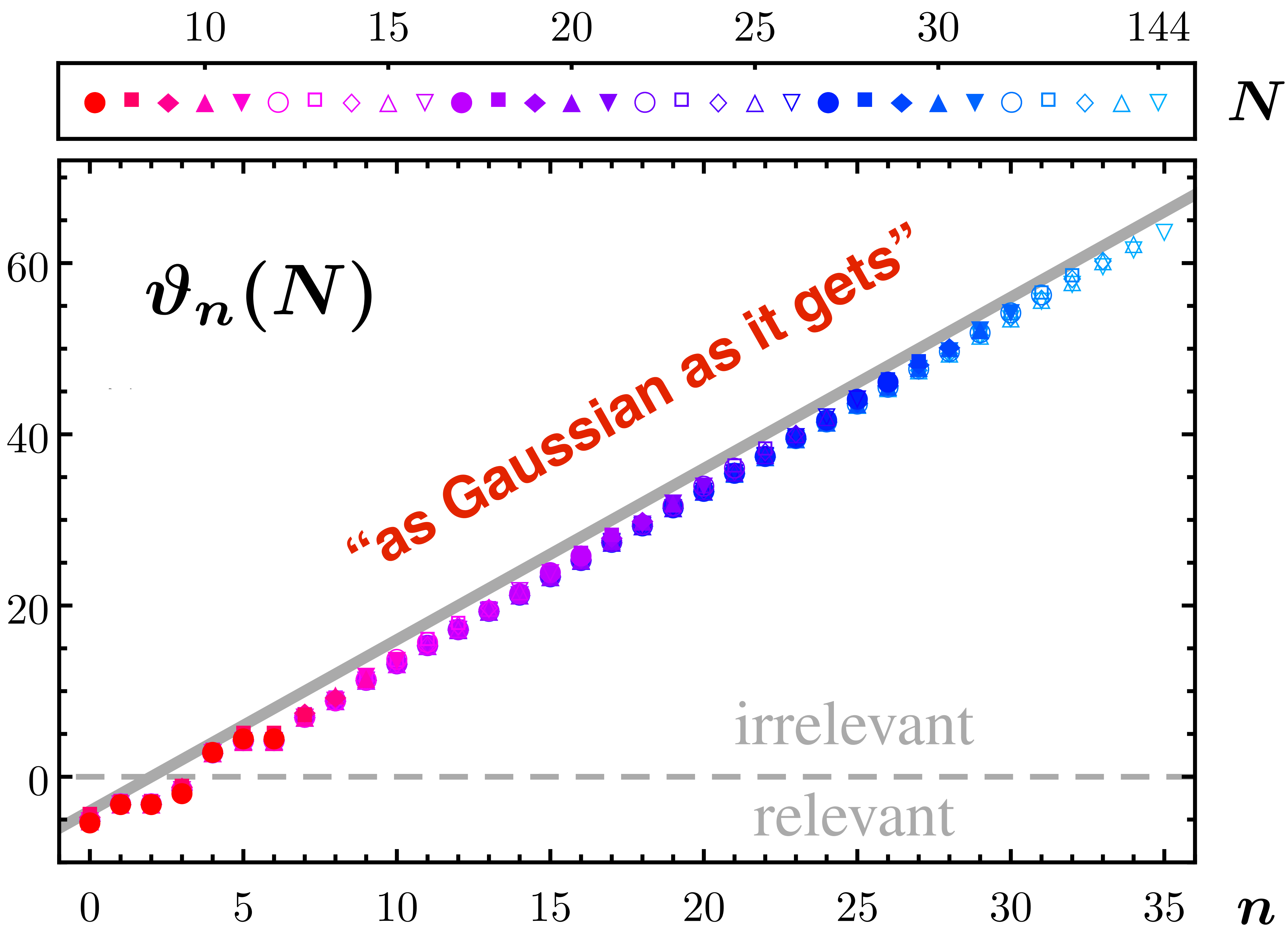
Ricci tensors



Falls, King, DL, Nikolakopoulos, Rahmede, '18

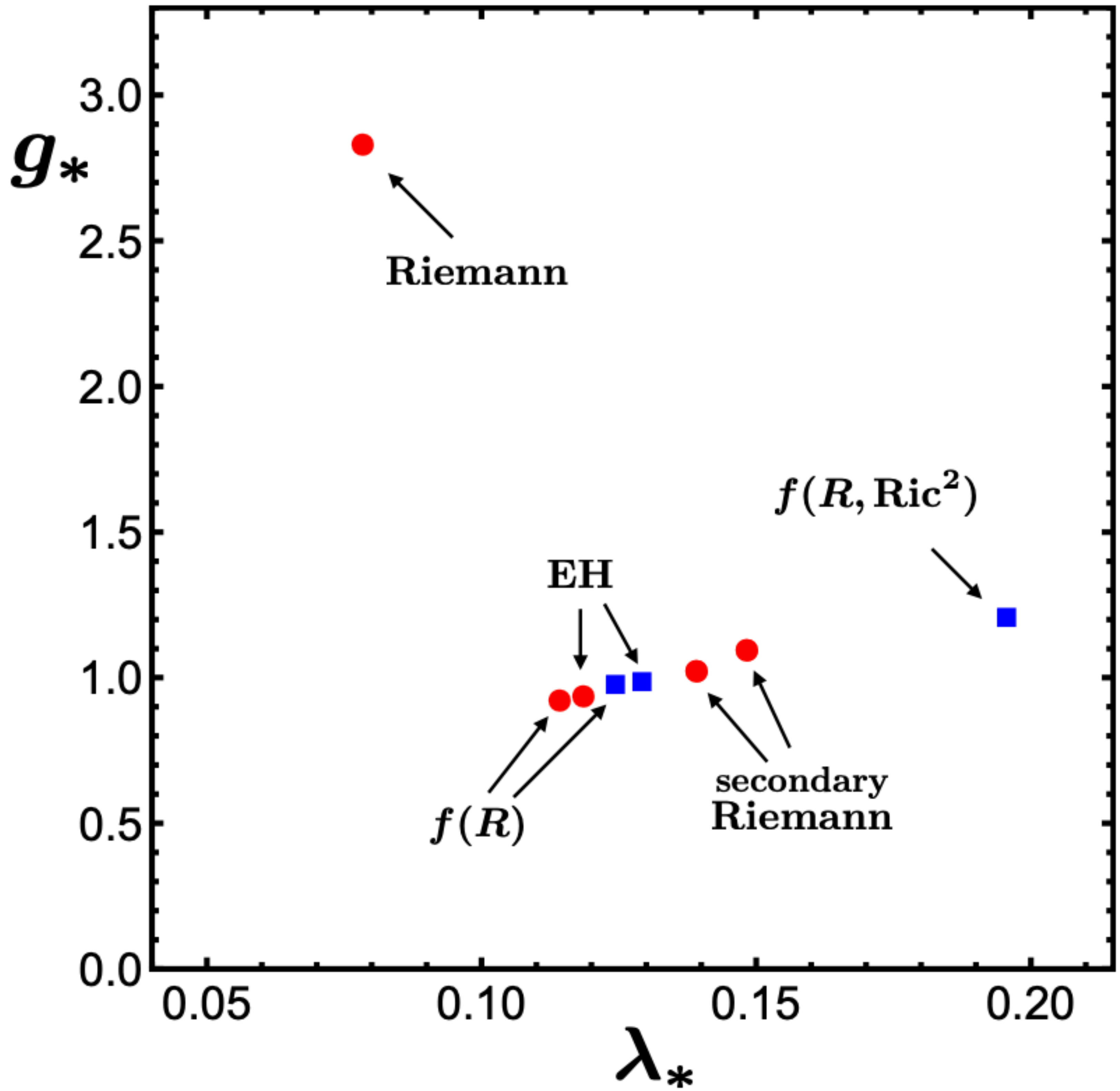
$$\Gamma_k = \int d^dx\sqrt{g}[F_k(\mathrm{Riem}^2) + R \cdot Z_k(\mathrm{Riem}^2)]$$

Riemann



Yannick Kluth, DF Litim, **Fixed points of quantum gravity and the dimensionality of the UV critical surface,** 2008.09181

comparison



Yannick Kluth, DF Litim, **Fixed points of quantum gravity and the dimensionality of the UV critical surface,** 2008.09181

status

+ consistency of findings
+ signatures of weak coupling
+ FPs in various extensions of EH

- pragmatic choices in theory space
- prospect of global FPs unclear
- systematic search in theory space missing

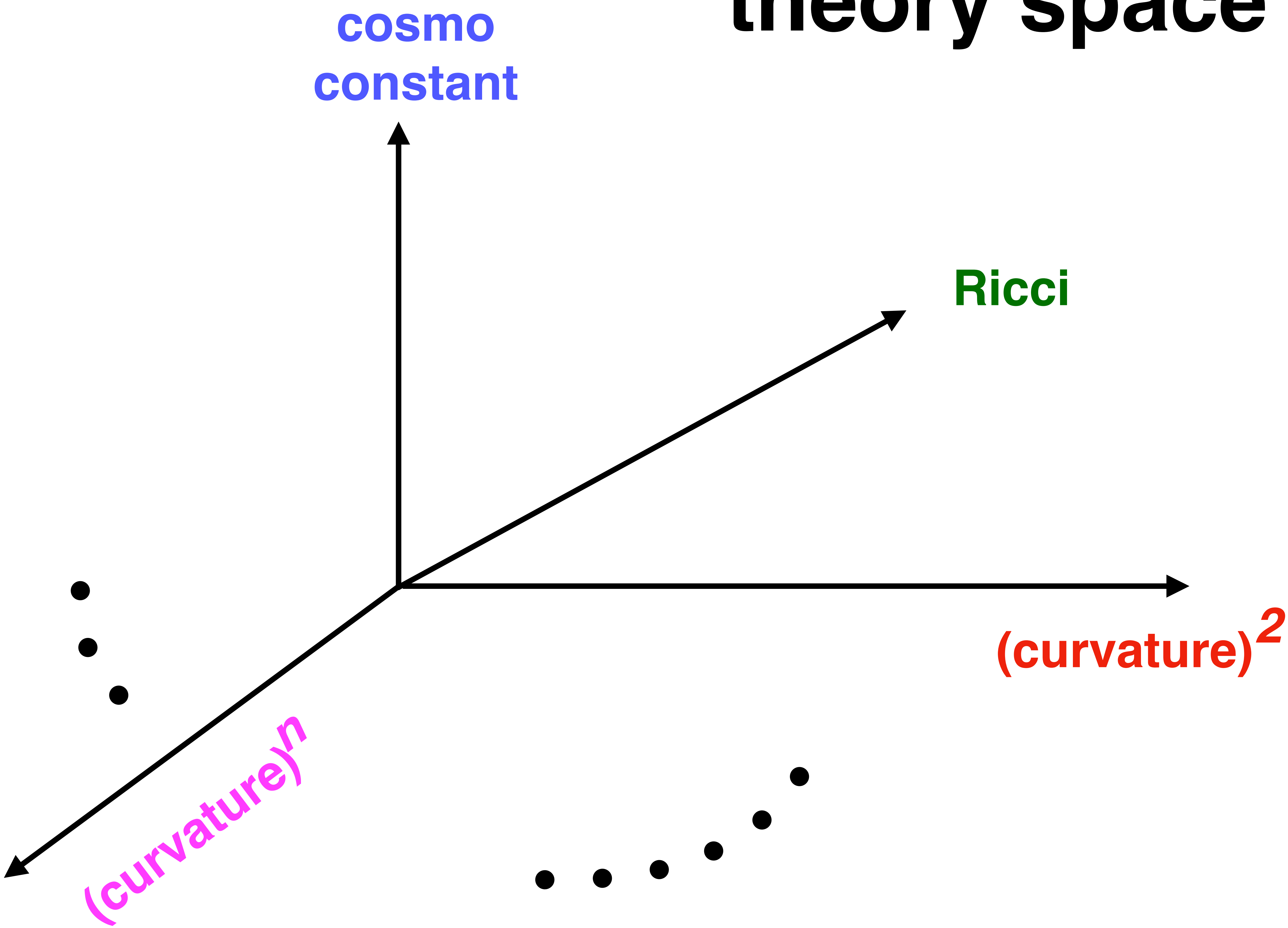
fRG for f(Riemann)

consider fRG for actions

$$\overline{\Gamma}_k[g_{\mu\nu}] = \int \mathrm{d}^d x \sqrt{g}\ \mathcal{L}(R_{\rho\sigma\mu\nu}, g^{\alpha\beta})$$

Yannick Kluth, DF Litim, **Functional Renormalisation of f(Riemann) Quantum Gravity,** (PRD) 2202.10436

"theory space"



fRG for f(Riemann)

	Curvature Invariants
dim-0	1
dim-2	R
dim-4	$R^2, R_{\mu\nu}R^{\mu\nu}, R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, \Box R$
dim-6	$R^3, RR_{\mu\nu}R^{\mu\nu}, RR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, R_{\mu\nu}R^{\nu\rho}R^{\mu}_{\rho}, R_{\mu\rho}R_{\nu\sigma}R^{\mu\nu\rho\sigma}, R^{\mu}_{\nu}R_{\nu\alpha\beta\gamma}R^{\mu\alpha\beta\gamma},$ $R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}R^{\nu\alpha\sigma\beta}, \nabla_{\mu}R\nabla^{\mu}R, \nabla_{\rho}R_{\mu\nu}\nabla^{\rho}R^{\mu\nu}$

Table 1. List of curvature invariants up to canonical mass dimension six.

Yannick Kluth, DF Litim, **Fixed points of QG and the dimensionality of the UV critical surface** (PRD) 2008.09181
Yannick Kluth, DF Litim, **Functional Renormalisation of f(Riemann) Quantum Gravity** (PRD) 2202.10436

fRG for f(Riemann)

	Curvature Invariants
dim-0	1
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dim-4	$R^2, R_{\mu\nu}R^{\mu\nu}, R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, \Box R$
dim-6	$R^3, RR_{\mu\nu}R^{\mu\nu}, RR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, R_{\mu\nu}R^{\nu\rho}R^{\mu}_{\rho}, R_{\mu\rho}R_{\nu\sigma}R^{\mu\nu\rho\sigma}, R^{\mu}_{\nu}R_{\nu\alpha\beta\gamma}R^{\mu\alpha\beta\gamma},$ $R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}R^{\nu\alpha\sigma\beta}, \nabla_{\mu}R\nabla^{\mu}R, \nabla_{\rho}R_{\mu\nu}\nabla^{\rho}R^{\mu\nu}$

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fRG for f(Riemann)

1st variation:

$$\frac{\partial \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}}$$ = **it's complicated**

2nd variation:

$$\frac{\partial^2 \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}\partial R_{\alpha\beta\gamma\delta}}$$ = **even more so…**

fRG for f(Riemann)

1st variation:

$$\left.\frac{\partial \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}}\right|_{\text{msb}}$$

max symmetric backgrounds

2nd variation:

$$\left.\frac{\partial^2 \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}\partial R_{\alpha\beta\gamma\delta}}\right|_{\text{msb}}$$

max symmetric backgrounds

fRG for f(Riemann)

1st variation:

$$\left.\frac{\partial \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}}\right|_{\rm msb} \equiv \mathcal{W}^{\rho\sigma\mu\nu}\big|_{\rm msb} = E\,\mathcal{P}^{\rho\sigma\mu\nu} \qquad\qquad \mathcal{P}^{\rho\sigma\mu\nu} = g^{\rho[\mu}g^{\nu]\sigma}$$

2nd variation:

$$\left.\frac{\partial^2 \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}\partial R_{\alpha\beta\gamma\delta}}\right|_{\rm msb} = A(R)\,\mathcal{A}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta} + B(R)\,\mathcal{B}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta} + C(R)\,\mathcal{C}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta}$$

$$\begin{aligned}
\mathcal{A}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta} &= \mathcal{P}^{\rho\sigma\mu\nu}\,\mathcal{P}^{\alpha\beta\gamma\delta}\,,\\
\mathcal{B}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta} &= \frac{1}{4}\Big[g^{\beta][\rho}g^{\sigma][\mu}g^{\nu]\gamma}g^{\delta[\alpha} + g^{\sigma][\alpha}g^{\beta][\gamma}g^{\delta]\mu}g^{\nu[\rho} + g^{\beta][\mu}g^{\nu][\rho}g^{\sigma]\gamma}g^{\delta[\alpha} + g^{\nu][\alpha}g^{\beta][\gamma}g^{\delta]\rho}g^{\sigma[\mu}\Big]\\
\mathcal{C}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta} &= \frac{1}{6}\Big[2g^{\alpha[\rho}g^{\sigma]\beta}g^{\gamma[\mu}g^{\nu]\delta} + 2g^{\alpha[\mu}g^{\nu]\beta}g^{\gamma[\rho}g^{\sigma]\delta} - g^{\alpha[\rho}g^{\mu]\beta}g^{\gamma[\nu}g^{\sigma]\delta} - g^{\alpha[\nu}g^{\sigma]\beta}g^{\gamma[\rho}g^{\mu]\delta}\\
&\qquad - g^{\alpha[\rho}g^{\nu]\beta}g^{\gamma[\sigma}g^{\nu]\delta} - g^{\alpha[\sigma}g^{\mu]\beta}g^{\gamma[\rho}g^{\nu]\delta}\Big]\,.
\end{aligned}$$

fRG for f(Riemann)

1st variation:

$$\left.\frac{\partial \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}}\right|_{\rm msb} \equiv \mathcal{W}^{\rho\sigma\mu\nu}|_{\rm msb} = E\,\mathcal{P}^{\rho\sigma\mu\nu} \qquad \mathcal{P}^{\rho\sigma\mu\nu} = g^{\rho[\mu}g^{\nu]\sigma}$$

2nd variation:

$$\left.\frac{\partial^2 \mathcal{L}}{\partial R_{\rho\sigma\mu\nu}\partial R_{\alpha\beta\gamma\delta}}\right|_{\rm msb} = A(R)\,\mathcal{A}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta} + B(R)\,\mathcal{B}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta} + C(R)\,\mathcal{C}^{\rho\sigma\mu\nu\alpha\beta\gamma\delta}$$

2nd variation only depends on three functions

parameter count

5 functions of curvature **L, E, A, B, C**

- 2 conditions $\frac{\partial L}{\partial R} = E$ $\frac{\partial^2 L}{\partial R^2} = A + \frac{2}{d(d-1)} B + \frac{1}{d} C$

= 3 independent functions **L, B, C**

are sufficient to characterise the fRG flow

$$\partial_t L = I[L, B, C]$$

mapping invariants onto parameters

$$\Lambda = \frac{R}{d(d-1)}$$

Curvature Invariants	L	E	A	B	C
R	$(d-1)d\Lambda$	1	0	0	0
R^2	$(d-1)^2d^2\Lambda^2$	$2(d-1)d\Lambda$	2	0	0
$R^{\mu\nu}R_{\mu\nu}$	$(d-1)^2d\Lambda^2$	$2(d-1)\Lambda$	0	2	0
$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$	$2(d-1)d\Lambda^2$	4Λ	0	0	2
R^3	$(d-1)^3d^3\Lambda^3$	$3(d-1)^2d^2\Lambda^2$	$6(d-1)d\Lambda$	0	0
$RR^{\mu\nu}R_{\mu\nu}$	$(d-1)^3d^2\Lambda^3$	$3(d-1)^2d\Lambda^2$	$4(d-1)\Lambda$	$2(d-1)d\Lambda$	0
$R^{\nu\rho}R_{\mu\nu}R^{\mu}{}_{\rho}$	$(d-1)^3d\Lambda^3$	$3(d-1)^2\Lambda^2$	0	$6(d-1)\Lambda$	0
$RR^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$	$2(d-1)^2d^2\Lambda^3$	$6(d-1)d\Lambda^2$	8Λ	0	$2(d-1)d\Lambda$
$R_{\rho\sigma}R_{\mu\nu}R^{\mu\rho\nu\sigma}$	$(d-1)^3d\Lambda^3$	$3(d-1)^2\Lambda^2$	2Λ	$2(2d-3)\Lambda$	0
$R^{\nu\alpha\sigma\beta}R_{\mu\nu\rho\sigma}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}$	$(d-2)(d-1)d\Lambda^3$	$3(d-2)\Lambda^2$	0	6Λ	-3Λ
$R^{\rho\sigma\alpha\beta}R_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}$	$4(d-1)d\Lambda^3$	$12\Lambda^2$	0	0	12Λ
$R^{\nu\rho\sigma\alpha}R_{\mu\nu}R^{\mu}{}_{\rho\sigma\alpha}$	$2(d-1)^2d\Lambda^3$	$6(d-1)\Lambda^2$	0	8Λ	$2(d-1)\Lambda$
R^4	$(d-1)^4d^4\Lambda^4$	$4(d-1)^3d^3\Lambda^3$	$12(d-1)^2d^2\Lambda^2$	0	0
$R^2R^{\mu\nu}R_{\mu\nu}$	$(d-1)^4d^3\Lambda^4$	$4(d-1)^3d^2\Lambda^3$	$10(d-1)^2d\Lambda^2$	$2(d-1)^2d^2\Lambda^2$	0
$R^{\rho\sigma}R_{\rho\sigma}R^{\mu\nu}R_{\mu\nu}$	$(d-1)^4d^2\Lambda^4$	$4(d-1)^3d\Lambda^3$	$8(d-1)^2\Lambda^2$	$4(d-1)^2d\Lambda^2$	0
$RR^{\nu\rho}R_{\mu\nu}R^{\mu}{}_{\rho}$	$(d-1)^4d^2\Lambda^4$	$4(d-1)^3d\Lambda^3$	$6(d-1)^2\Lambda^2$	$6(d-1)^2d\Lambda^2$	0
$R^{\nu}{}_{\sigma}R^{\rho\sigma}R_{\mu\nu}R^{\mu}{}_{\rho}$	$(d-1)^4d\Lambda^4$	$4(d-1)^3\Lambda^3$	0	$12(d-1)^2\Lambda^2$	0
$R^2R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$	$2(d-1)^3d^3\Lambda^4$	$8(d-1)^2d^2\Lambda^3$	$20(d-1)d\Lambda^2$	0	$2(d-1)^2d^2\Lambda^2$
$R^{\rho\sigma\alpha\beta}R_{\rho\sigma\alpha\beta}R^{\mu\nu}R_{\mu\nu}$	$2(d-1)^3d^2\Lambda^4$	$8(d-1)^2d\Lambda^3$	$16(d-1)\Lambda^2$	$4(d-1)d\Lambda^2$	$2(d-1)^2d\Lambda^2$
$R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$	$4(d-1)^2d^2\Lambda^4$	$16(d-1)d\Lambda^3$	$32\Lambda^2$	0	$8(d-1)d\Lambda^2$
$RR_{\rho\sigma}R_{\mu\nu}R^{\mu\rho\nu\sigma}$	$(d-1)^4d^2\Lambda^4$	$4(d-1)^3d\Lambda^3$	$2(d-1)(4d-3)\Lambda^2$	$2(d-1)d(2d-3)\Lambda^2$	0
$R_{\rho\sigma}R^{\sigma}{}_{\alpha}R_{\mu\nu}R^{\mu\rho\nu\alpha}$	$(d-1)^4d\Lambda^4$	$4(d-1)^3\Lambda^3$	$4(d-1)\Lambda^2$	$4(d-1)(2d-3)\Lambda^2$	0
$RR^{\nu\alpha\sigma\beta}R_{\mu\nu\rho\sigma}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}$	$(d-2)(d-1)^2d^2\Lambda^4$	$4(d-2)(d-1)d\Lambda^3$	$6(d-2)\Lambda^2$	$6(d-1)d\Lambda^2$	$-3(d-1)d\Lambda^2$
$RR^{\rho\sigma\alpha\beta}R_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}$	$4(d-1)^2d^2\Lambda^4$	$16(d-1)d\Lambda^3$	$24\Lambda^2$	0	$12(d-1)d\Lambda^2$
$RR^{\nu\rho\sigma\alpha}R_{\mu\nu}R^{\mu}{}_{\rho\sigma\alpha}$	$2(d-1)^3d^2\Lambda^4$	$8(d-1)^2d\Lambda^3$	$12(d-1)\Lambda^2$	$8(d-1)d\Lambda^2$	$2(d-1)^2d\Lambda^2$
$R^{\nu\alpha\sigma\beta}R_{\rho\sigma}R_{\mu\nu}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}$	$2(d-1)^3d\Lambda^4$	$8(d-1)^2\Lambda^3$	$2\Lambda^2$	$2(9d-10)\Lambda^2$	$2(d-1)^2\Lambda^2$
$R^{\nu\sigma\alpha\beta}R_{\rho\sigma}R_{\mu\nu}R^{\mu\rho}{}_{\alpha\beta}$	$2(d-1)^3d\Lambda^4$	$8(d-1)^2\Lambda^3$	$4\Lambda^2$	$4(4d-5)\Lambda^2$	$2(d-1)^2\Lambda^2$
$R^{\rho\alpha}{}_{\gamma\delta}R^{\sigma\beta\gamma\delta}R_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\alpha\beta}$	$4(d-1)d\Lambda^4$	$16\Lambda^3$	0	0	$24\Lambda^2$
$R_{\rho\sigma}R^{\rho\alpha\sigma\beta}R_{\mu\nu}R^{\mu}{}_{\alpha}{}^{\nu}{}_{\beta}$	$(d-1)^4d\Lambda^4$	$4(d-1)^3\Lambda^3$	$2(3d-4)\Lambda^2$	$2(d(3d-8)+6)\Lambda^2$	0
[illegible]	$(d-1)d(2d-5)\Lambda^4$	$4(2d-5)\Lambda^3$	0	[illegible]	[illegible]

Examples:

a)
$$L = f(R)\,,$$
$$E = f'(R)\,,$$
$$A = f''(R)\,,$$
$$B = 0\,,$$
$$C = 0\,,$$

b)
$$f(R, \mathrm{Ric}^2) = F(\mathrm{Ric}^2) + R \cdot Z(\mathrm{Ric}^2)\,.$$
$$L = F(x) + R\,Z(x)\,,$$
$$E = \tfrac{1}{2}\left[F'(x) + R\,Z'(x)\right] R + Z(x)\,,$$
$$A = \tfrac{1}{4}\left[F''(x) + R\,Z''(x)\right] R^2 + R\,Z'(x)\,,$$
$$B = 2\,F'(x) + 2\,R\,Z'(x)\,,$$
$$C = 0\,,$$

c)
$$f(R, \mathrm{Riem}^2) = F(\mathrm{Riem}^2) + R \cdot Z(\mathrm{Riem}^2)\,.$$
$$L = F(x) + R\,Z(x)\,,$$
$$E = \tfrac{1}{3}\left[F'(x) + R\,Z'(x)\right] R + Z(x)\,,$$
$$A = \tfrac{1}{9}\left[F''(x) + \tfrac{2}{3} R\,Z''(x)\right] R^2 + R\,Z'(x)\,,$$
$$B = 0\,,$$
$$C = 2\,F'(x) + 2\,R\,Z'(x)\,,$$

Yannick Kluth, DF Litim, **Functional Renormalisation of f(Riemann) Quantum Gravity,** (PRD) 2202.10436

2nd variation

$$
\begin{aligned}
\delta^2\overline{\Gamma}_k = \int \mathrm{d}^d x\sqrt{\overline{g}}\Bigg\{ h^T_{\mu\nu}\Bigg[&\frac{R^2}{(d-1)d^2}\left(\frac{B}{d-1}+2C\right)-\frac{R}{d(d-1)}L'+(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right)\\
&+\left(\frac{(d+1)R}{d(1-d)}\left(\frac{B}{d+1}+C\right)+\frac{L'}{2}\right)\overline{\nabla}^2+\left(\frac{B}{4}+C\right)\overline{\nabla}^4\Bigg] h^T_{\mu\nu}\\
&-2\xi_\mu(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right)\left[\frac{R}{d}+\overline{\nabla}^2\right]\xi^\mu\\
&+\sigma\left[(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right)\left(\frac{R}{d}-\frac{(1-d)}{d}\overline{\nabla}^2\right)+\Xi\,\overline{\nabla}^2\right]\overline{\nabla}^2\sigma\\
&-2h\,\Xi\,\overline{\nabla}^2\sigma\\
&+h\left[\Xi+\frac{(\tau-1+\frac{d}{2})}{d}\left(\frac{L}{2}-\frac{R}{d}L'\right)\right]h\Bigg\},
\end{aligned}
$$

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2nd variation

$$
\begin{aligned}
\delta^2\overline{\Gamma}_k = \int \mathrm{d}^d x\sqrt{\bar{g}}\Bigg\{ & h^T_{\mu\nu}\Bigg[\frac{R^2}{(d-1)d^2}\left(\frac{B}{d-1}+2C\right)-\frac{R}{d(d-1)}L'+(\tau-1)\boxed{\left(\frac{L}{2}-\frac{R}{d}L'\right)} \\
& +\left(\frac{(d+1)R}{d(1-d)}\left(\frac{B}{d+1}+C\right)+\frac{L'}{2}\right)\overline{\nabla}^2+\left(\frac{B}{4}+C\right)\overline{\nabla}^4\Bigg]h^T_{\mu\nu} \\
& -2\xi_\mu(\tau-1)\boxed{\left(\frac{L}{2}-\frac{R}{d}L'\right)}\left[\frac{R}{d}+\overline{\nabla}^2\right]\xi^\mu \\
& +\sigma\left[(\tau-1)\boxed{\left(\frac{L}{2}-\frac{R}{d}L'\right)}\left(\frac{R}{d}-\frac{(1-d)}{d}\overline{\nabla}^2\right)+\Xi\,\overline{\nabla}^2\right]\overline{\nabla}^2\sigma \\
& -2h\,\Xi\,\overline{\nabla}^2\sigma \\
& +h\left[\Xi+\frac{(\tau-1+\frac{d}{2})}{d}\boxed{\left(\frac{L}{2}-\frac{R}{d}L'\right)}\right]h\Bigg\},
\end{aligned}
$$

= e.o.m.

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2nd variation

$$
\begin{aligned}
\delta^2\overline{\Gamma}_k = \int \mathrm{d}^d x\sqrt{\bar g}\Bigg\{ & h^T_{\mu\nu}\Bigg[\frac{R^2}{(d-1)d^2}\left(\frac{B}{d-1}+2C\right)-\frac{R}{d(d-1)}L'+(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right) \\
& +\left(\frac{(d+1)R}{d(1-d)}\left(\frac{B}{d+1}+C\right)+\frac{L'}{2}\right)\overline{\nabla}^2+\left(\frac{B}{4}+C\right)\overline{\nabla}^4\Bigg]h^T_{\mu\nu} \\
& -2\xi_\mu(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right)\left[\frac{R}{d}+\overline{\nabla}^2\right]\xi^\mu \\
& +\sigma\left[(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right)\left(\frac{R}{d}-\frac{(1-d)}{d}\overline{\nabla}^2\right)+\Xi\,\overline{\nabla}^2\right]\overline{\nabla}^2\sigma \\
& -2h\,\Xi\,\overline{\nabla}^2\sigma \\
& +h\left[\Xi+\frac{(\tau-1+\frac{d}{2})}{d}\left(\frac{L}{2}-\frac{R}{d}L'\right)\right]h\Bigg\},
\end{aligned}
$$

= e.o.m.

= exp. split

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fRG for f(Riemann)

cosmological constant for general backgrounds

$$\delta^2 \left(\int \mathrm{d}^d \sqrt{g} \lambda_0 \right) = \int \mathrm{d}^d x \sqrt{g} \left[\frac{1}{4} h h - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} + \frac{\tau}{2} h_{\mu\nu} h^{\mu\nu} \right] \lambda_0 \,.$$

standard metric split generates contributions to all modes, irrespective of background geometry

exp split only generates contributions to trace mode, irrespective of the background

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d.o.f.

$$
\begin{aligned}
\delta^2\overline{\Gamma}_k = \int \mathrm{d}^d x\sqrt{\bar{g}}\Bigg\{ & h^T_{\mu\nu}\Bigg[\frac{R^2}{(d-1)d^2}\left(\frac{B}{d-1}+2C\right)-\frac{R}{d(d-1)}L'+(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right) \\
& +\left(\frac{(d+1)R}{d(1-d)}\left(\frac{B}{d+1}+C\right)+\frac{L'}{2}\right)\overline{\nabla}^2+\left(\frac{B}{4}+C\right)\overline{\nabla}^4\Bigg]h^T_{\mu\nu} \\
& -2\xi_\mu(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right)\left[\frac{R}{d}+\overline{\nabla}^2\right]\xi^\mu \\
& +\sigma\left[(\tau-1)\left(\frac{L}{2}-\frac{R}{d}L'\right)\left(\frac{R}{d}-\frac{(1-d)}{d}\overline{\nabla}^2\right)+\Xi\,\overline{\nabla}^2\right]\overline{\nabla}^2\sigma \\
& -2h\,\Xi\,\overline{\nabla}^2\sigma \\
& +h\left[\Xi+\frac{(\tau-1+\frac{d}{2})}{d}\left(\frac{L}{2}-\frac{R}{d}L'\right)\right]h\Bigg\},
\end{aligned}
$$

=F

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auxiliary function

$$\Xi = \frac{R^2}{d^2}\left(\frac{d-3}{(d-1)d}(B-C)+L''\right) - \frac{(d-2)R}{2d^2}L'$$
$$+\left[\frac{R}{d^2}\left(\frac{d^2+4d-20}{4d}B - \frac{d-4}{d}C + 2(d-1)L''\right) - \frac{(d-2)(d-1)}{2d^2}L'\right]\overline{\nabla}^2$$
$$+\frac{d-1}{d^2}\boxed{\left(\frac{d^2-8}{4d}B + \frac{1}{d}C + (d-1)L''\right)}\overline{\nabla}^4 .$$

=S

e.g. Einsteinian gravities: S=0 F=0

→ see poster by Gabriel Assant

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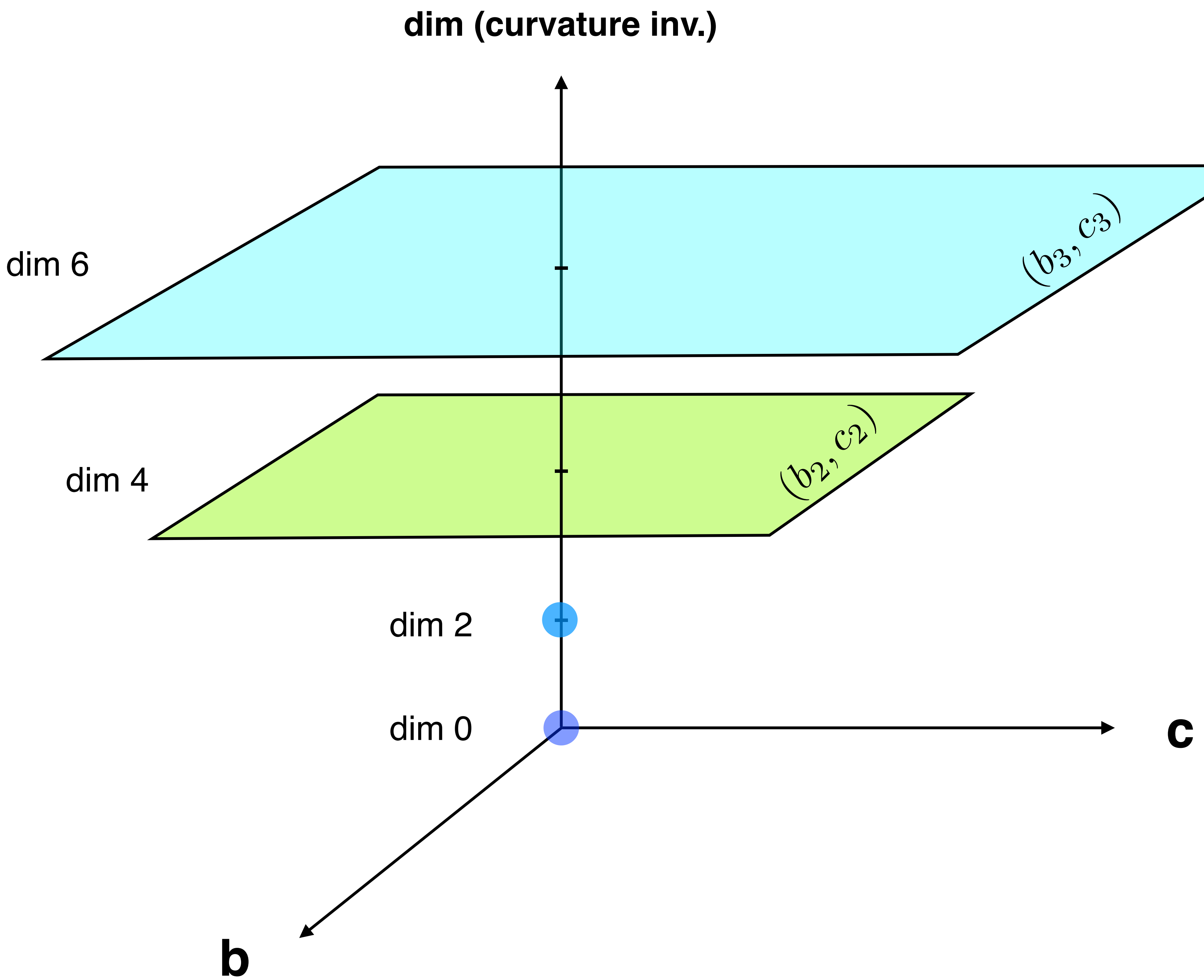
enables “horizontal" and “vertical" searches



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University of Sussex

conclusions

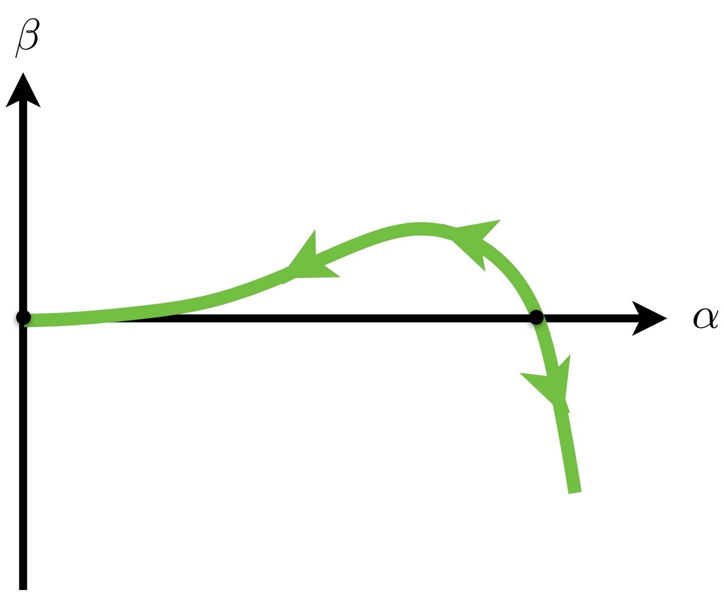
new fRG flows

flow for general f(Riemann) actions

one curvature invariant at a time, LPA

vertical and horizontal searches

global fixed points



challenges

same mass dim curvature invariants

backgrounds beyond msb

include (DRiem)^n interactions