

Particle physics & quantum black holes from asymptotically safe correlation functions

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STRUCTURES
CLUSTER OF
EXCELLENCE



Outline

- Asymptotically safe correlation functions
- Applications I: asymptotically safe Standard Model
- Applications II: asymptotically black holes
- Summary

Asymptotically safe correlation functions

**JMP, Reichert, Front.in Phys. 8 (2021) 527
2309.10785**

Background (in)dependence in gravity

aka
background and fluctuations fields, modified STIs and their importance

free energy

$$k\partial_k\Gamma_k[\bar{g}, \phi] = \frac{1}{2} \text{gauge fields} - \text{fermions} + \frac{1}{2} \text{bosons} + \frac{1}{2} \text{gravity}$$

Linear split
 $g = \bar{g} + h$

Effective action

$$\Gamma_k[\bar{g}, \bar{h}] = \Gamma_k[\bar{g}] + \Gamma_k^{(0,1)}[\bar{g}] * \bar{h} + \frac{1}{2} \Gamma_k^{(0,2)}[\bar{g}] * \bar{h}^2 + \frac{1}{6} \Gamma_k^{(0,3)}[\bar{g}] * \bar{h}^3 + \dots$$

$\bar{h} = \langle h \rangle$

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$\bar{h} = \langle h \rangle$

$$\left\{ \Gamma_k[\bar{g}], \Gamma_k^{(0,1)}[\bar{g}], \Gamma_k^{(0,2)}[\bar{g}], \Gamma_k^{(0,3)}[\bar{g}], \dots \right\}$$

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Se vogliamo che tutto rimanga come è,
bisogna che tutto cambi.

From vertex dressings/distribution functions to physics

aka
form factors

Effective action

$$\Gamma[\bar{g}, h, c_\mu, \bar{c}_\mu] = \int_x \left[\frac{2\Lambda - R}{16\pi G_N} + R f_R(\Delta) R + C f_C(\Delta) C + \dots \right]_{\text{BRST-inv}} + S_{\text{gf}} + S_{\text{gh}}$$

Background effective action

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g} \left\{ \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu} + \dots \right\}$$

JMP, Tränkle, 2309.17043

Enforced by IR-UV consistence

$$R f_{R^2}(\Delta, R) R = \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R$$

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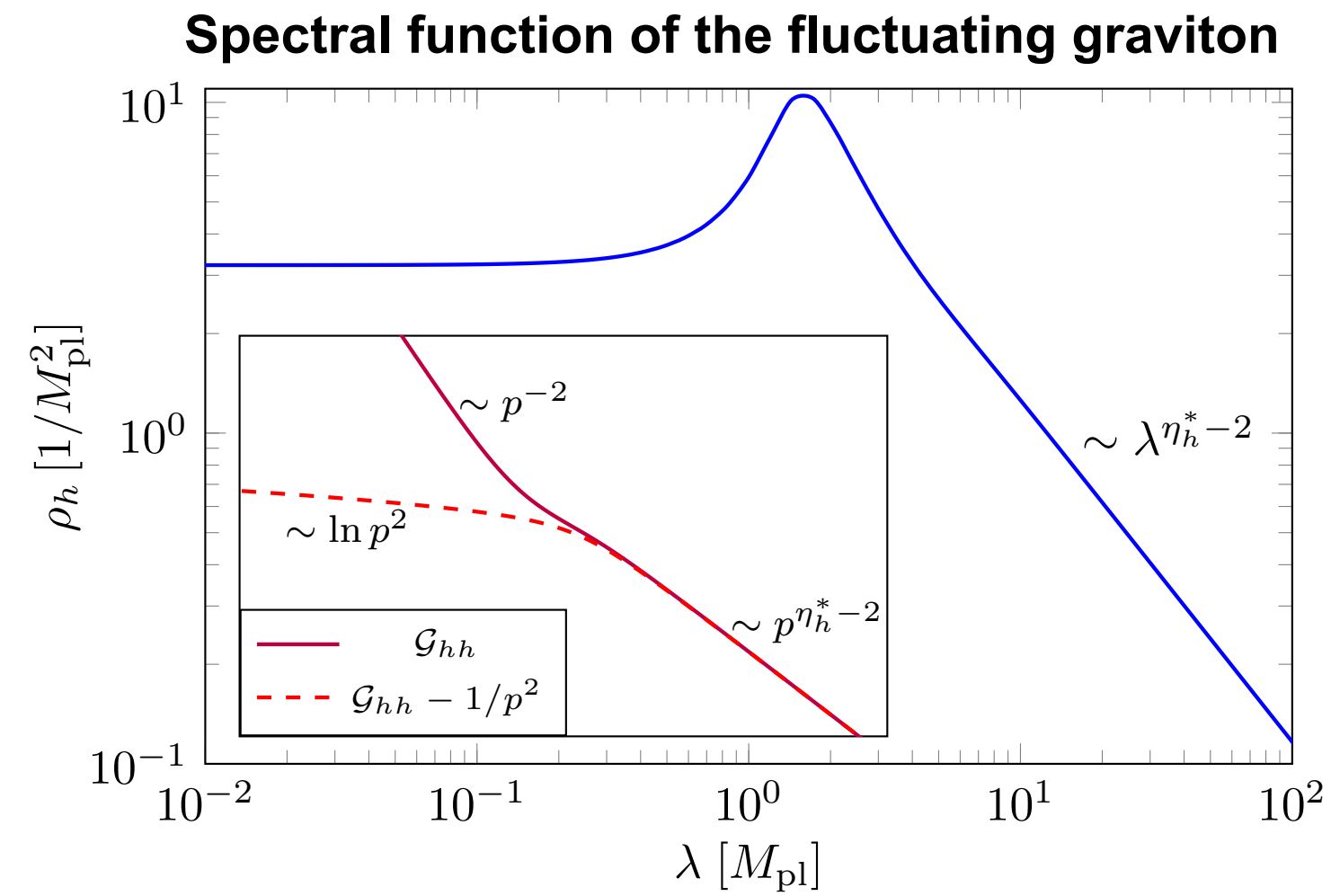
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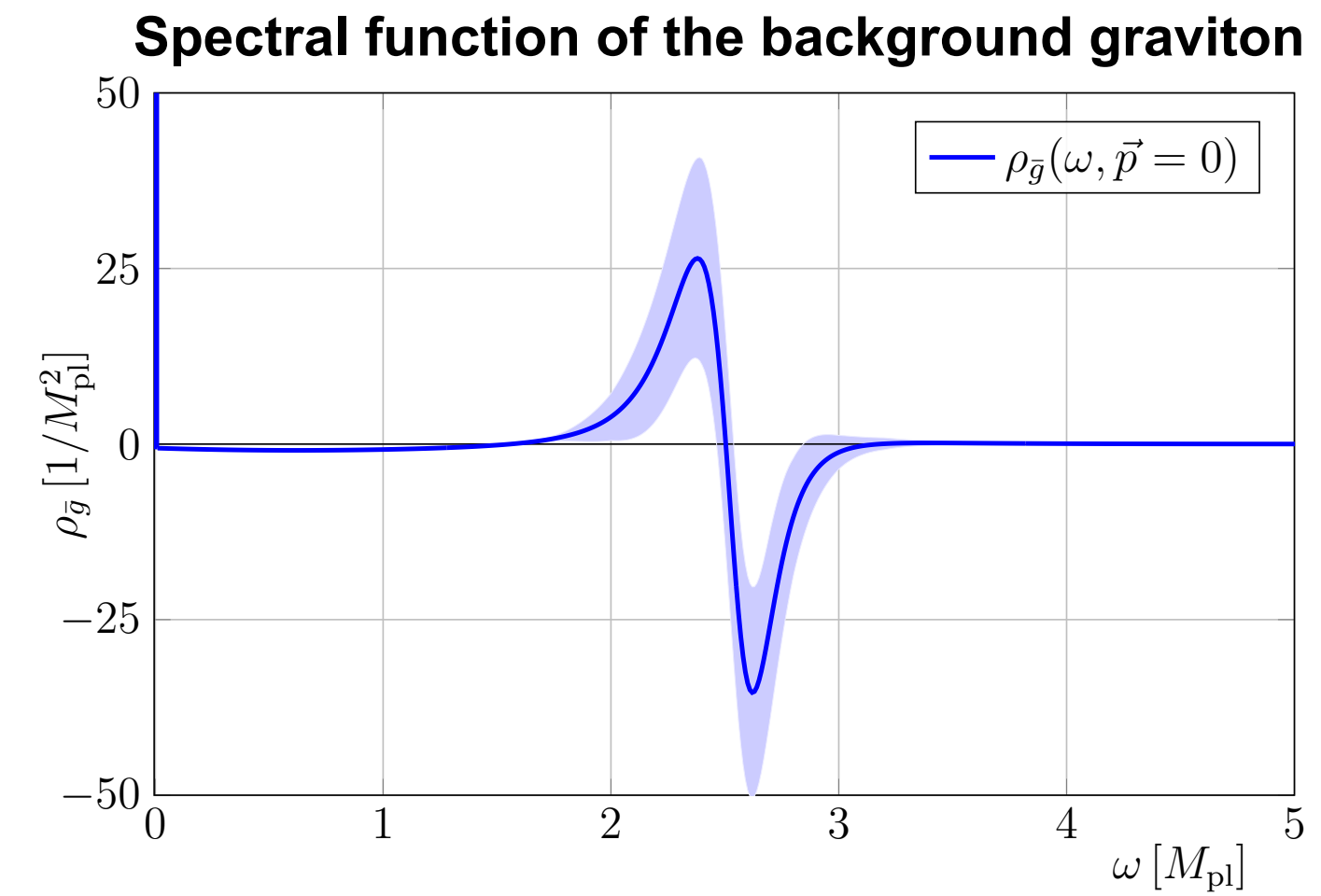
How much do they differ?

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A lesson from graviton spectral functions



Fehre, Litim, JMP, Reichert, PRL 130 (2023) 081501



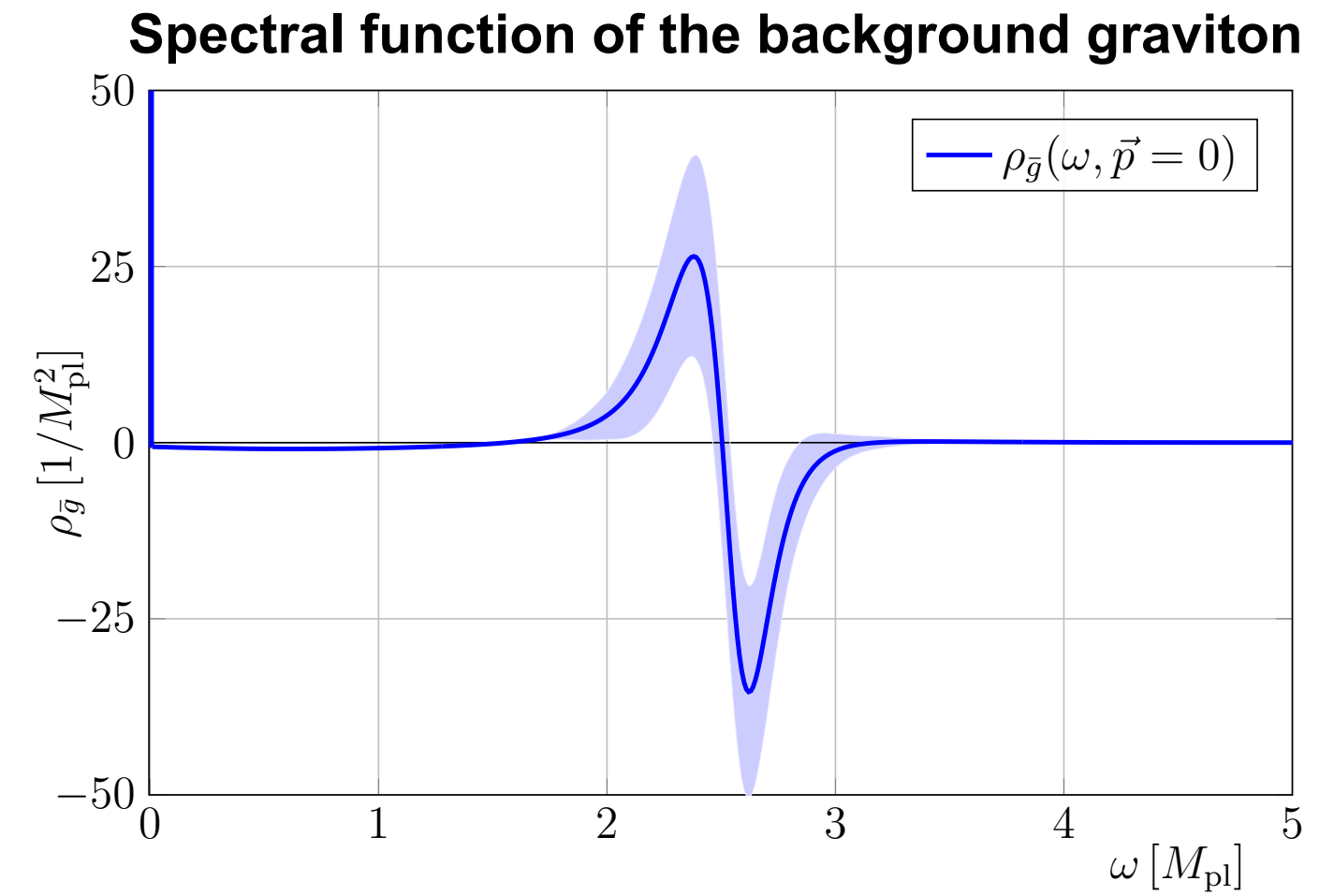
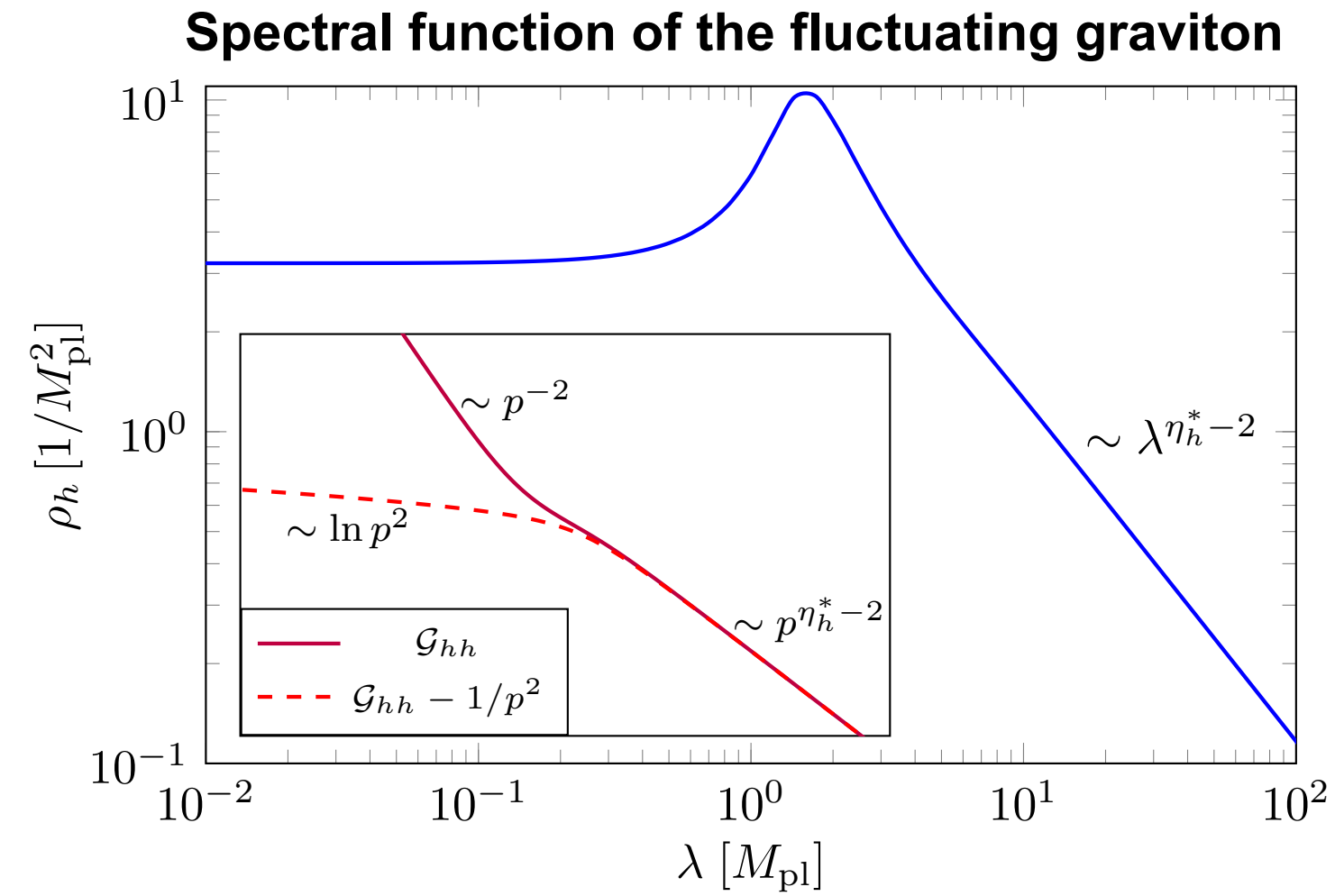
Bonanno, Denz, JMP, Reichert, SciPost Phys. 12 (2022) 1, 001

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Il Gattopardo

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Spectral properties 'resemble' that of an asymptotic

$$\rho_h(\lambda) \in \mathbb{R}^+ \quad \int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \rho_h(\lambda) = \infty$$

Spectral properties of an unphysical mode

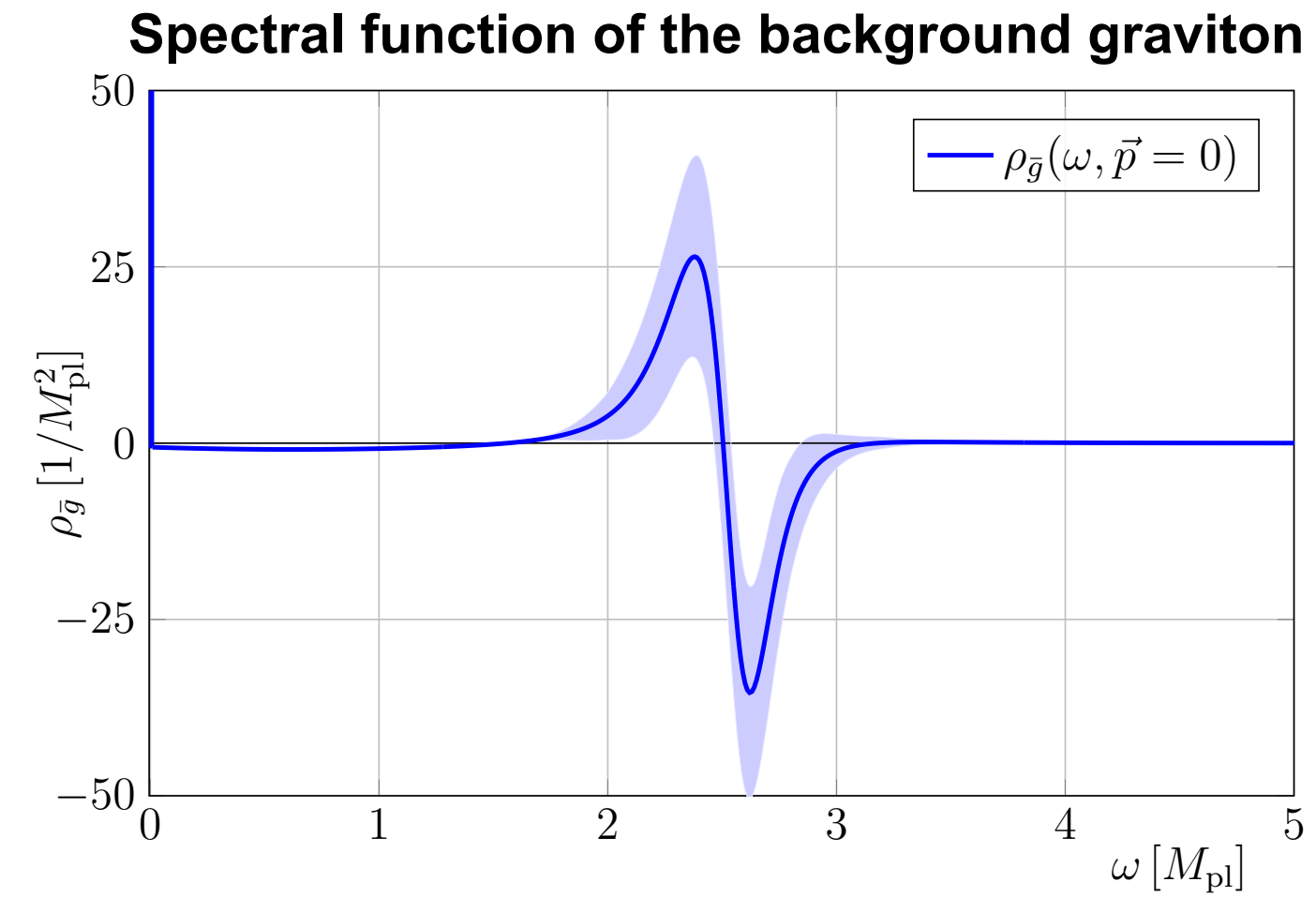
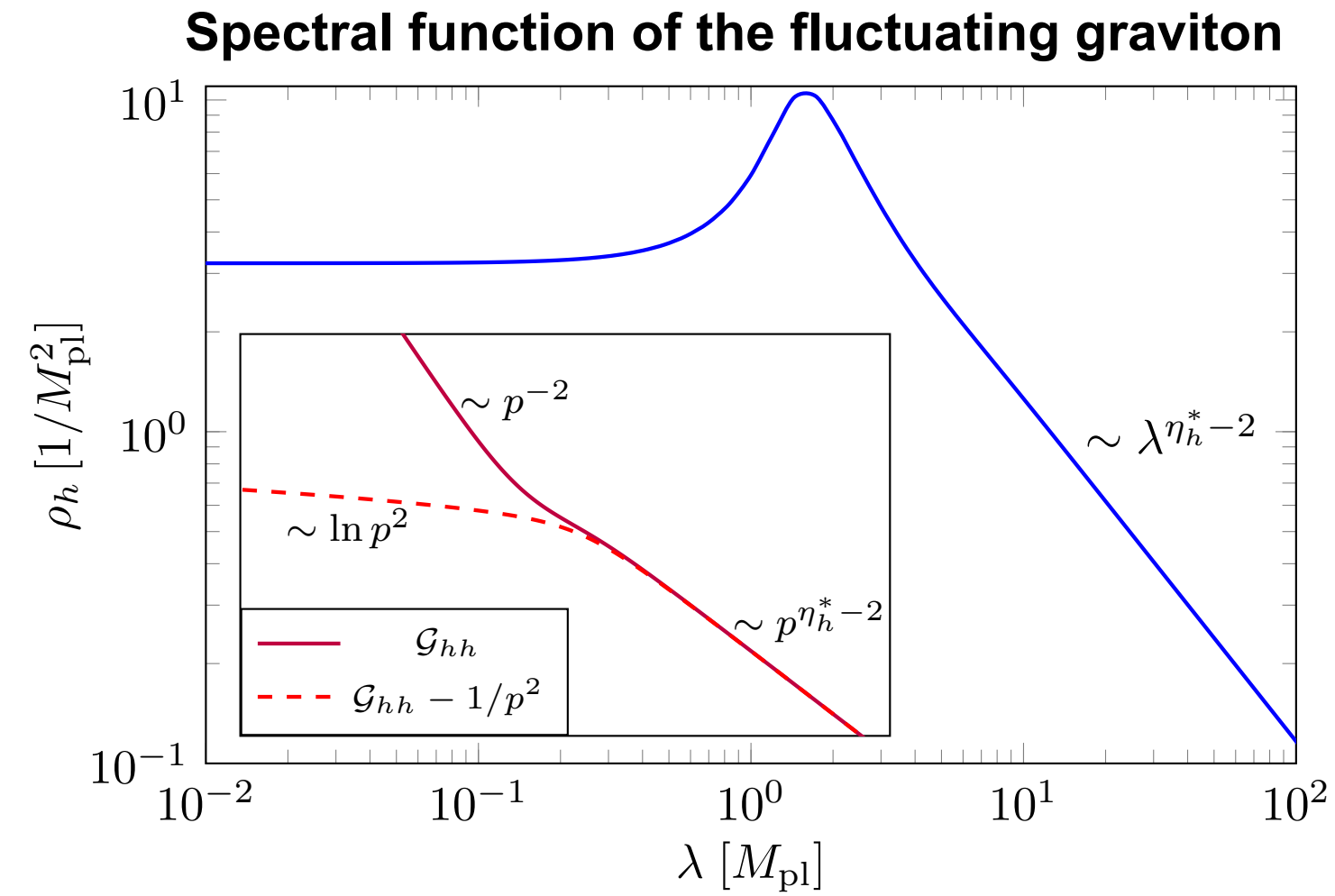
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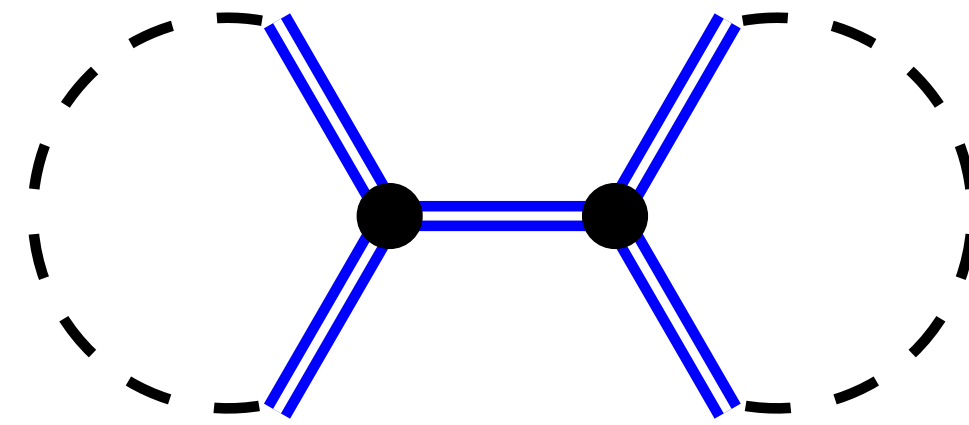
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Il Gattopardo

much !

Background spectral function and scattering amplitudes

Graviton-graviton scattering



Bonanno, Denz, JMP, Reichert, *SciPost Phys.* 12 (2022) 1, 001

Background spectral function and scattering amplitudes

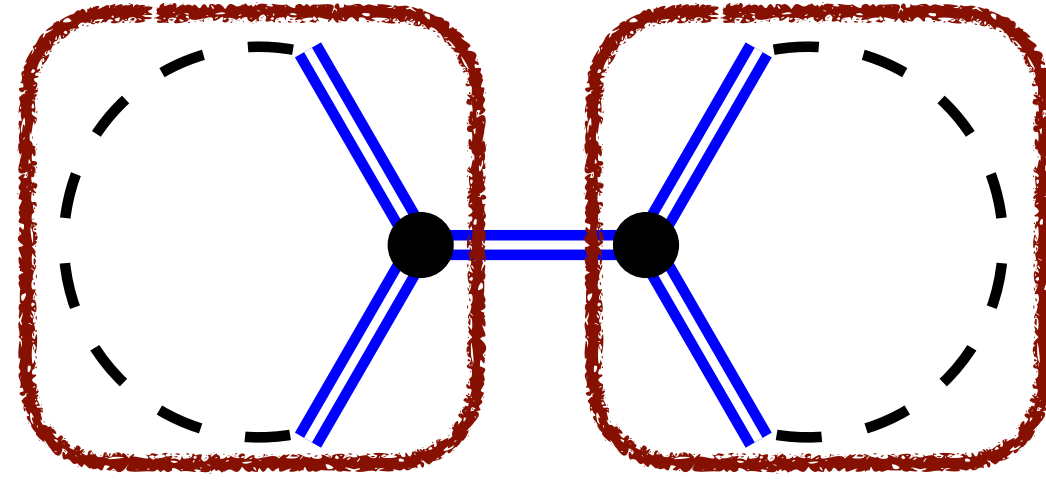
RG-invariant vertex

$$\frac{\Gamma_{hhh}^{(3)}(p_1, p_2, p_3)}{Z_h^{\frac{1}{2}}(p_1)Z_h^{\frac{1}{2}}(p_2)Z_h^{\frac{1}{2}}(p_3)}$$

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**RG-invariant coupling
/form factor**

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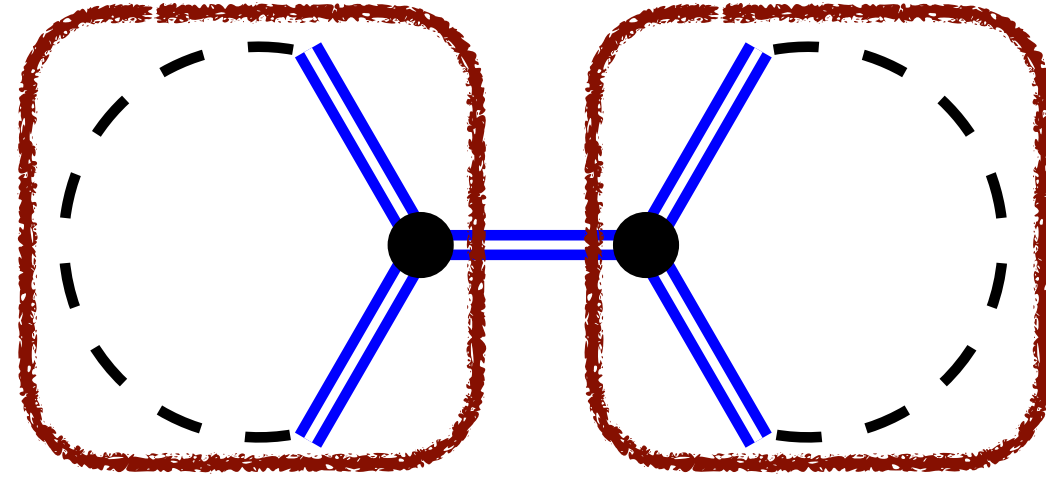
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Fluctuation approach: 2012 ...

Form factor approach: 2018 ...

Knorr, Ripken, Saueressig, ...

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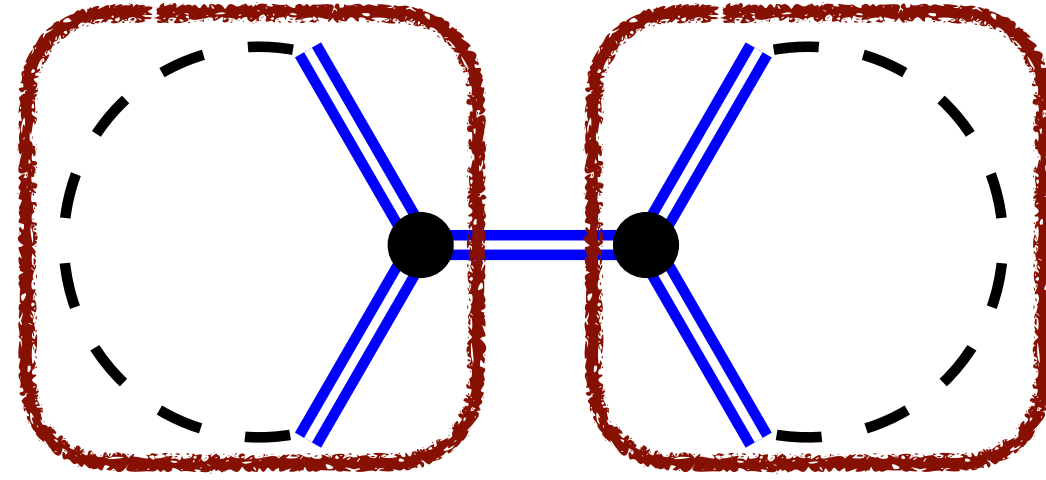
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Suggestive educated guess

$$\bar{\Gamma}_{\bar{g}^n}^{(n)}(p_1, \dots, p_n) \approx \frac{\Gamma_{h^n}^{(n)}(p_1, \dots, p_n)}{Z_h^{\frac{1}{2}}(p_1) \cdots Z_h^{\frac{1}{2}}(p_n)}$$

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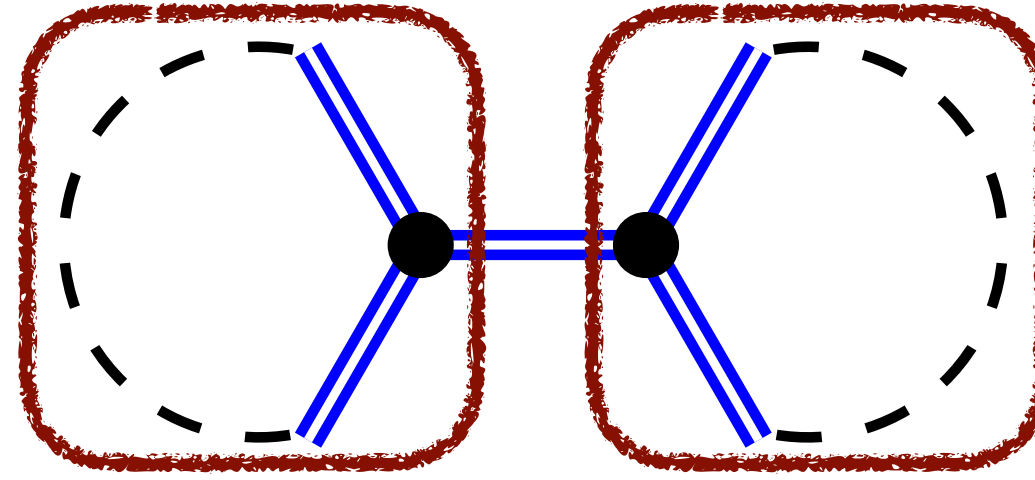
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Final comment

'Bar' propagator

$$\bar{\Gamma}^{(2)}(p_1, p_2) \quad \rho_g(\lambda) \in \mathbb{R}^+$$

$$\int_{\mathbb{R}} \frac{d\lambda}{2\pi} \lambda \bar{\rho}(\lambda) = 1$$

A bit of gauge invariance and all that

Some thoughts on vertex expansion schemes/fluctuation approach and covariant expansion schemes/background approximation

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Clear cut case: Einstein-Hilbert truncation & minimally coupled gravity matter systems

Exact map with additional truncation

Fluctuation approach with flat vertex expansion



Background approximation with heat kernel methods

Additional truncation: background approximation

Background approximation flows can be derived also in the flat vertex expansion

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claim also tested in QCD

where there are enough benchmark results from the lattice

Fluctuation approach rules

Relation to background approach via pinch technique

aka: if there are qualitative differences between results in the fluc. approach and the background approx., the former is conceptually more trustworthy

- (a) better account of dynamics
- (b) better account of **physical** diffeomorphism invariance
- (c) better -tested- systematics

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Beware!

there are qualitative differences between flows in the fluc. approach & the background approximation in pure gravity and minimally coupled systems

The physics of thresholds

Bonanno et al., Critical reflections on asymptotically safe gravity, Front.in Phys. 8 (2020) 269

QCD & SM thresholds in the RG since (many) decades

QCD with the fRG since a decade

The physics of thresholds

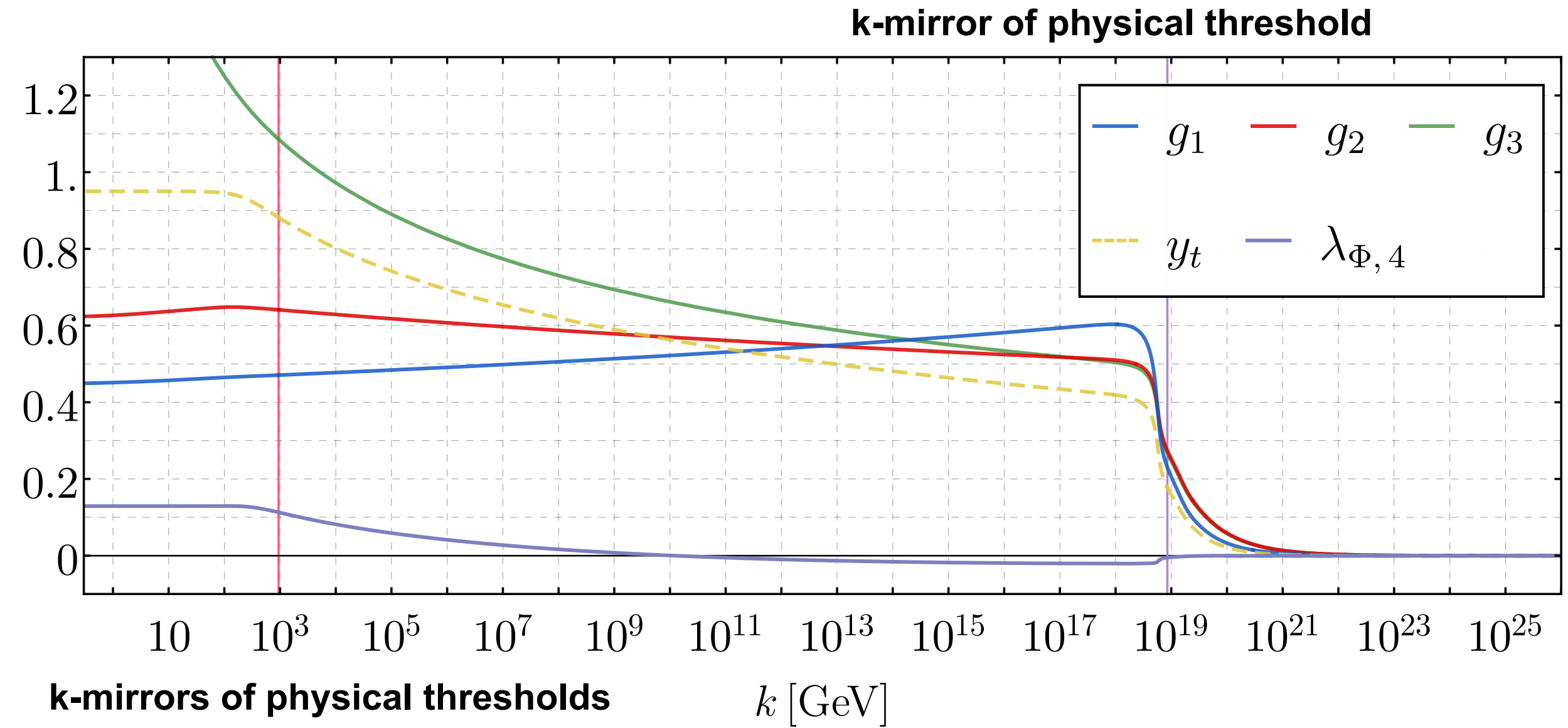
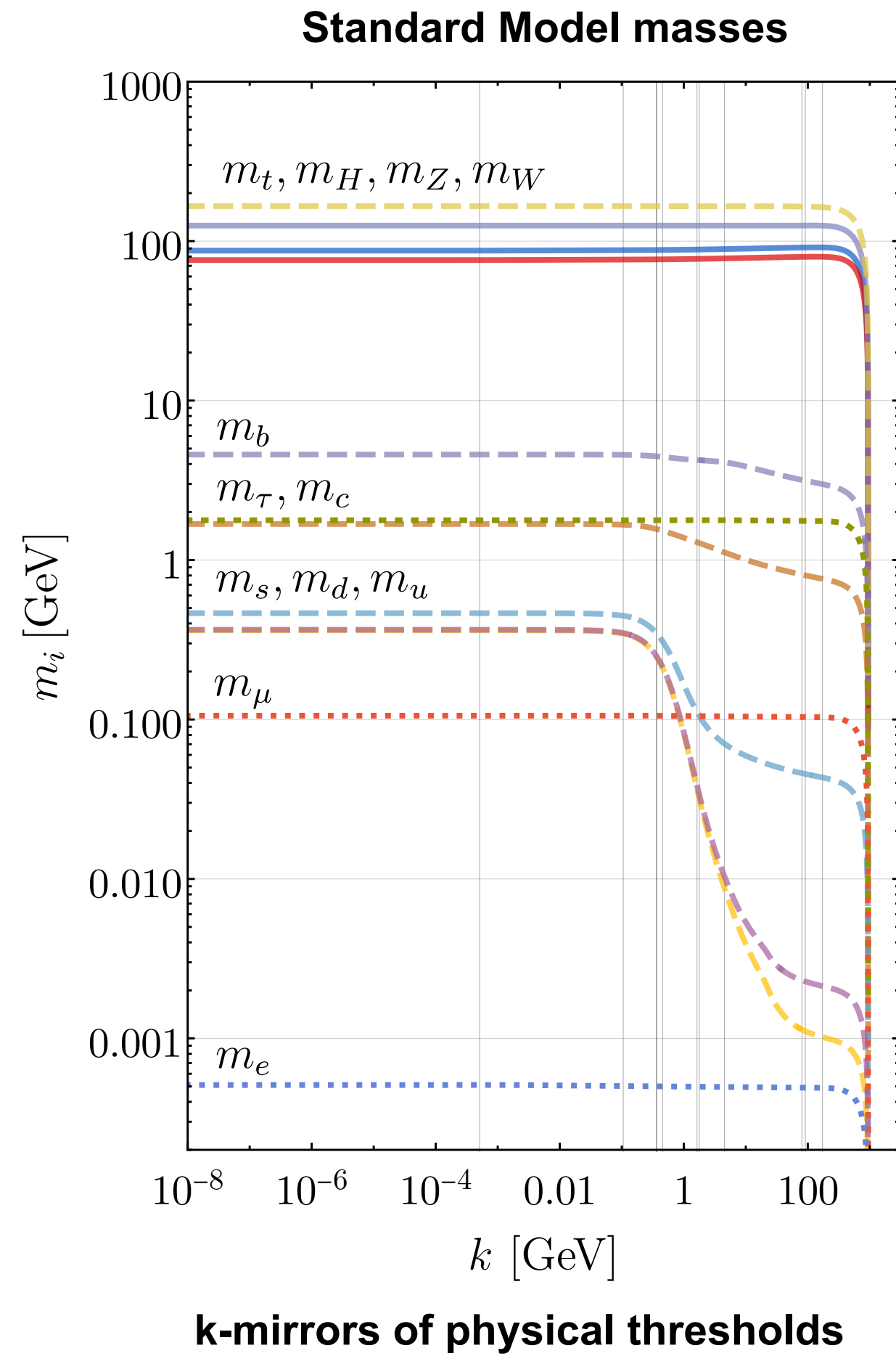
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Example: asymptotically safe Standard Model

Pastor-Gutiérrez, JMP, Reichert, SciPost Phys. 15 (2023) 105



The physics of thresholds

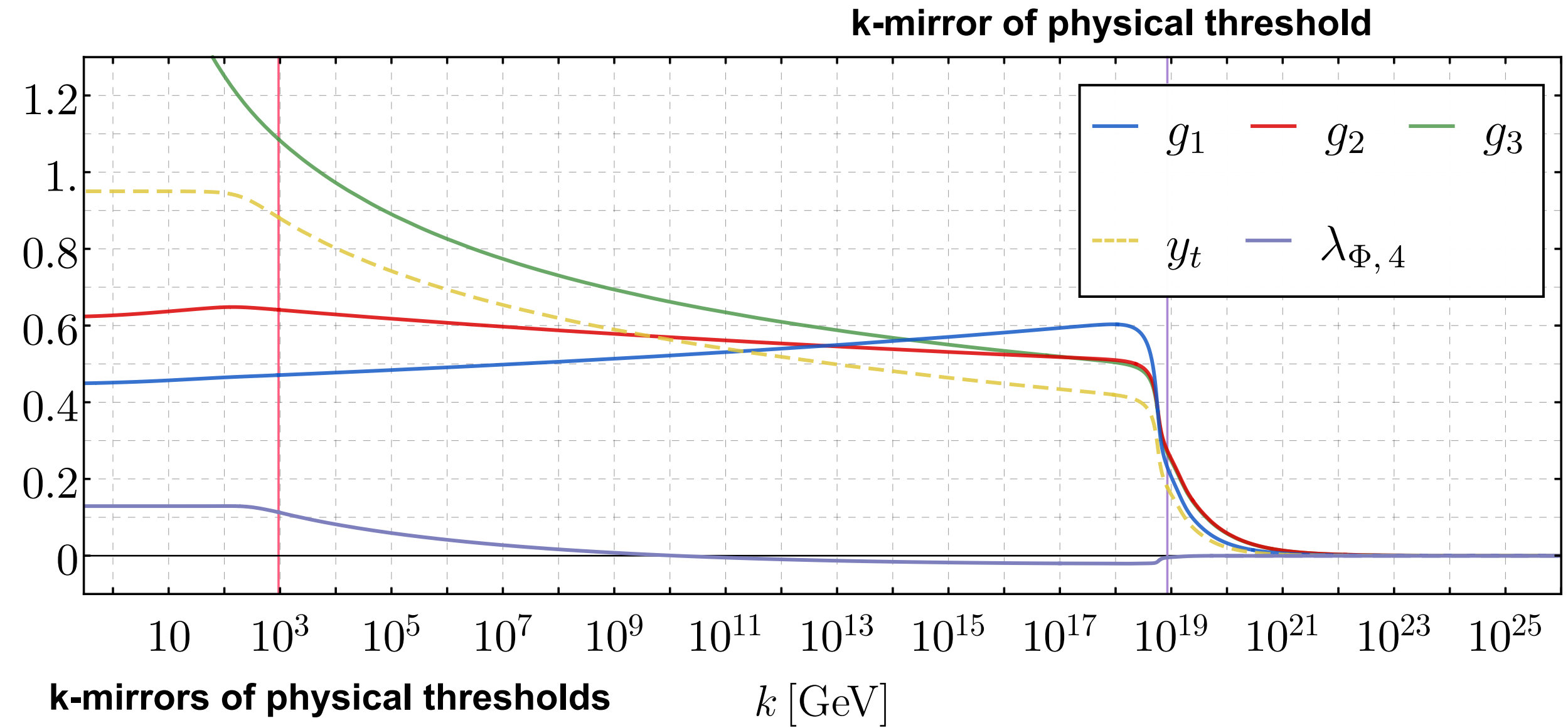
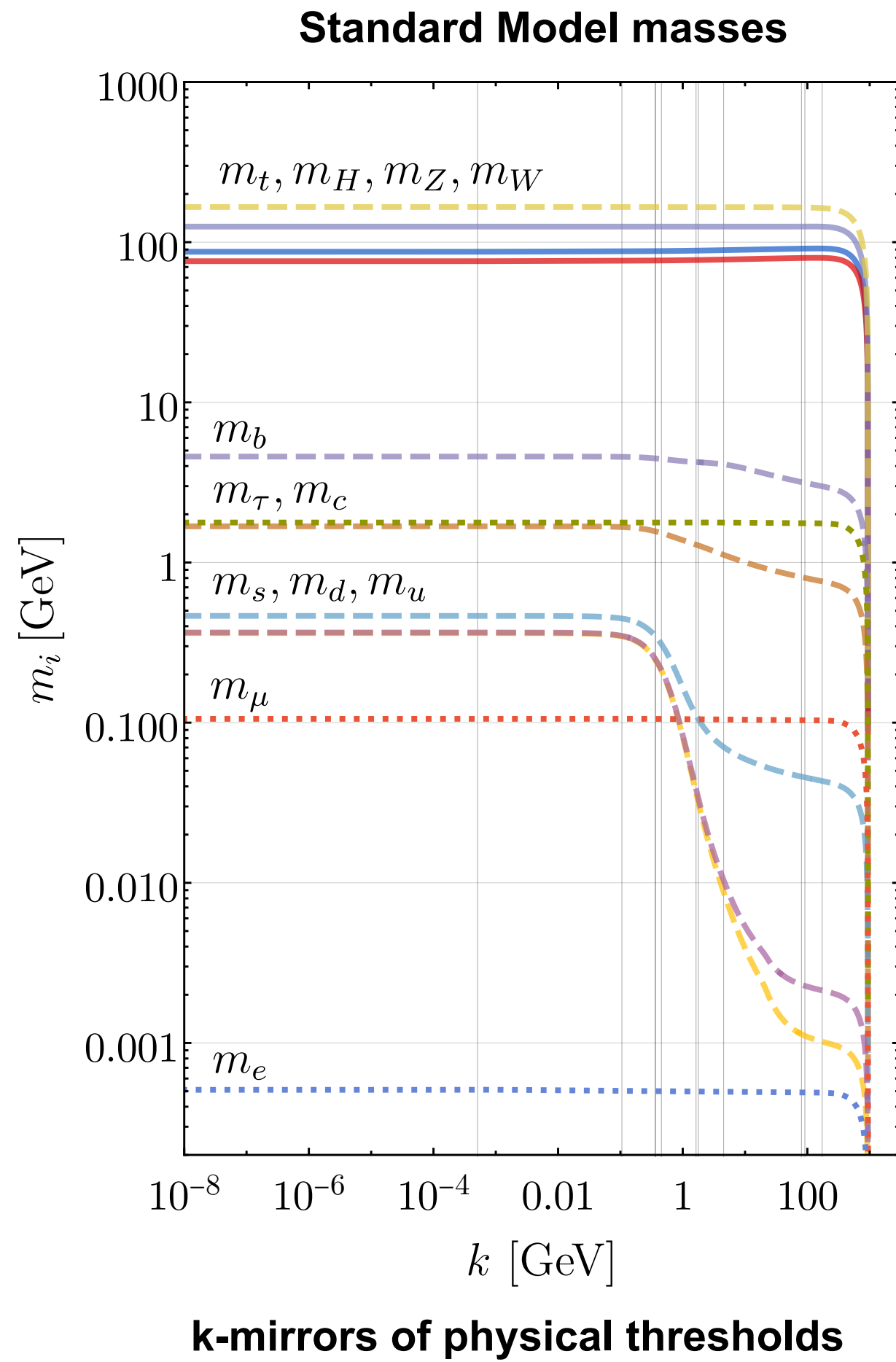
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k-mirrors of physical thresholds: feature, not bug!

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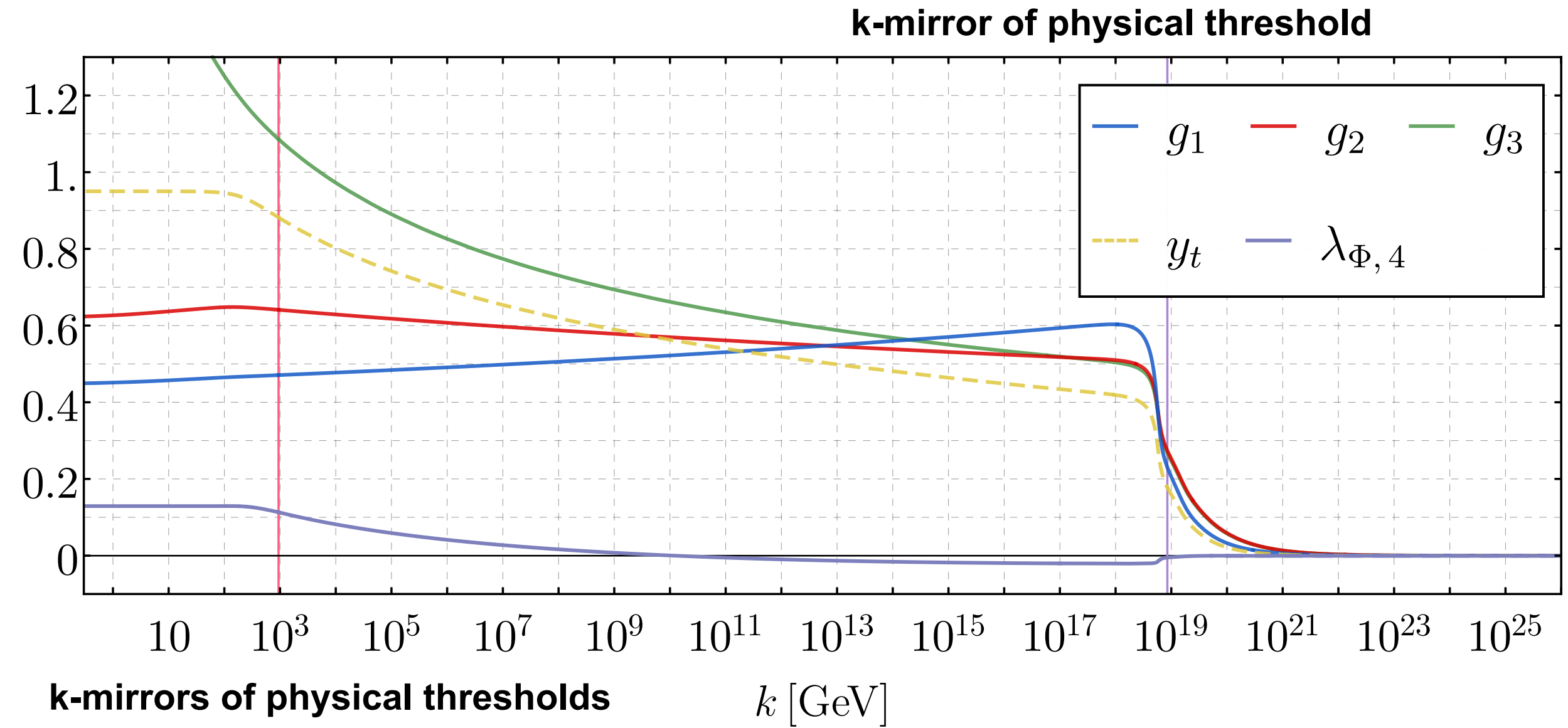
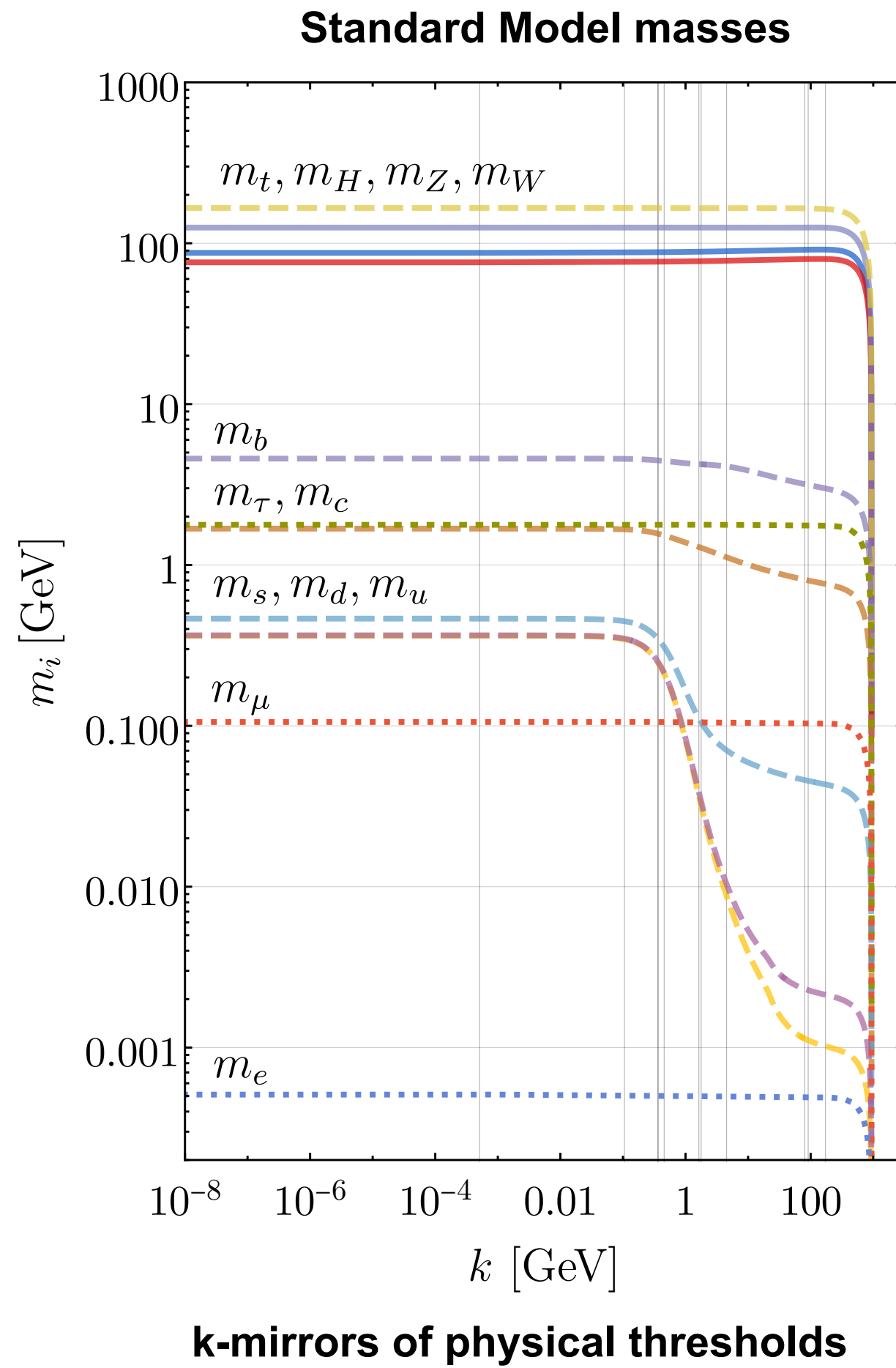
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Full physics: momentum-dependent correlation functions & S-matrix elements at $k=0$

Towards apparent convergence in quantum gravity

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

$$\partial_t \Gamma_k = \frac{1}{2} \text{Diagram 1} - \text{Diagram 2}$$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{Diagram 3} + \text{Diagram 4}$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{Diagram 5} + \text{Diagram 6} - 2 \text{Diagram 7}$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{Diagram 8} + \text{Diagram 9}$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{Diagram 10} + 3 \text{Diagram 11} - 3 \text{Diagram 12} + 6 \text{Diagram 13}$$

$$\partial_t \Gamma_k^{(4h)} = -\frac{1}{2} \text{Diagram 14} + 3 \text{Diagram 15} + 4 \text{Diagram 16} - 6 \text{Diagram 17}$$

$$- 12 \text{Diagram 18} + 12 \text{Diagram 19} - 24 \text{Diagram 20}$$

The diagrams represent Feynman-like graphs in quantum gravity. Diagrams 1, 3, 5, 10, 14, 18, and 20 are blue circles with a vertex marked by a circle containing an 'X'. Diagrams 2, 4, 7, 9, 11, 12, 13, 15, 16, 17, 19, and 20 are red dashed circles with vertices marked by red arrows. Diagrams 1, 3, 5, 10, 14, 18, and 20 have external lines (double blue lines) attached to their vertices. Diagrams 2, 4, 7, 9, 11, 12, 13, 15, 16, 17, 19, and 20 have external lines (double blue lines) attached to their vertices. The coefficients in the equations are: 1/2, -1/2, -1/2, 1, -1/2, 3, -3, 6, -1/2, 3, 4, -6, -12, 12, -24.

Aiming at apparent convergence

Towards apparent convergence in quantum gravity

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

Reuter, PRD 57 (1998) 971,

$$\partial_t \Gamma_k = \frac{1}{2} \left(\text{blue circle with } \otimes \right) - \left(\text{red dashed circle with } \otimes \right)$$

background approximation: $\Gamma^{(m,n)} \approx \Gamma^{(m+n,0)}$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \left(\text{blue circle with } \otimes \text{ and } \bullet \right) + \left(\text{red dashed circle with } \otimes \text{ and } \bullet \right)$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \left(\text{blue circle with } \otimes \text{ and } \bullet \text{ and } \bullet \right) + \left(\text{blue circle with } \otimes \text{ and } \bullet \text{ and } \bullet \right) - 2 \left(\text{red dashed circle with } \otimes \text{ and } \bullet \text{ and } \bullet \right)$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \left(\text{red dashed circle with } \otimes \text{ and } \bullet \text{ and } \bullet \right) + \left(\text{blue circle with } \otimes \text{ and } \bullet \text{ and } \bullet \right)$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \left(\text{blue circle with } \otimes \text{ and } \bullet \text{ and } \bullet \text{ and } \bullet \right) + 3 \left(\text{blue circle with } \otimes \text{ and } \bullet \text{ and } \bullet \text{ and } \bullet \right) - 3 \left(\text{blue circle with } \otimes \text{ and } \bullet \text{ and } \bullet \text{ and } \bullet \right) + 6 \left(\text{red dashed circle with } \otimes \text{ and } \bullet \text{ and } \bullet \text{ and } \bullet \right)$$

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$$\Gamma^{(m,n \geq 2)}$$

bi-metric approach: Manrique, Reuter, Saueressig, *Annals Phys.* 326 (2011) 463

$$\partial_t \Gamma_k = \frac{1}{2} \text{[Diagram 1]} - \text{[Diagram 2]}$$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{[Diagram 3]} + \text{[Diagram 4]}$$

level 1: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-1,1)}$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{[Diagram 5]} + \text{[Diagram 6]} - 2 \text{[Diagram 7]}$$

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geometrical approach: Donkin, JMP, arXiv:1203.4207

flat expansion: Christiansen, Litim, JMP, Rodigast, PLB 728 (2014) 114

$$\partial_t \Gamma_k = \frac{1}{2} \text{[Diagram: Circle with two vertices and a cross]} - \text{[Diagram: Circle with two vertices, a cross, and a red dashed loop]$$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{[Diagram: Circle with two vertices, a cross, and a blue line]} + \text{[Diagram: Circle with two vertices, a cross, and a red dashed loop]$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{[Diagram: Circle with two vertices, a cross, and two blue lines]} + \text{[Diagram: Circle with two vertices, a cross, and two blue lines]} - 2 \text{[Diagram: Circle with two vertices, a cross, and a red dashed loop]$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{[Diagram: Circle with two vertices, a cross, and a red dashed loop]} + \text{[Diagram: Circle with two vertices, a cross, and a red dashed loop]}$$

level 2: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-2,2)}$

$$Z_h(p), Z_c(p), \mu = -2\lambda_2$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{[Diagram: Circle with two vertices, a cross, and three blue lines]} + 3 \text{[Diagram: Circle with two vertices, a cross, and three blue lines]} - 3 \text{[Diagram: Circle with two vertices, a cross, and three blue lines]} + 6 \text{[Diagram: Circle with two vertices, a cross, and a red dashed loop]}$$

$$\partial_t \Gamma_k^{(4h)} = -\frac{1}{2} \text{[Diagram: Circle with two vertices, a cross, and four blue lines]} + 3 \text{[Diagram: Circle with two vertices, a cross, and four blue lines]} + 4 \text{[Diagram: Circle with two vertices, a cross, and four blue lines]} - 6 \text{[Diagram: Circle with two vertices, a cross, and four blue lines]}$$

$$- 12 \text{[Diagram: Circle with two vertices, a cross, and four blue lines]} + 12 \text{[Diagram: Circle with two vertices, a cross, and four blue lines]} - 24 \text{[Diagram: Circle with two vertices, a cross, and a red dashed loop]}$$

Aiming at apparent convergence

Towards apparent convergence in quantum gravity

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

Christiansen, Knorr, Maibohm, JMP, Reichert, PRD 92 (2015) 12, 121501

$$\partial_t \Gamma_k = \frac{1}{2} \text{[Diagram 1]} - \text{[Diagram 2]}$$

level 3: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-3,3)}$

$$\partial_t \Gamma_k^{(h)} = -\frac{1}{2} \text{[Diagram 3]} + \text{[Diagram 4]}$$

$$\partial_t \Gamma_k^{(2h)} = -\frac{1}{2} \text{[Diagram 5]} + \text{[Diagram 6]} - 2 \text{[Diagram 7]}$$

$$Z_h(p), Z_c(p), \mu = -2\lambda_2$$

$$\partial_t \Gamma_k^{(c\bar{c})} = \text{[Diagram 8]} + \text{[Diagram 9]}$$

$$\partial_t \Gamma_k^{(3h)} = -\frac{1}{2} \text{[Diagram 10]} + 3 \text{[Diagram 11]} - 3 \text{[Diagram 12]} + 6 \text{[Diagram 13]}$$

$g_3(p), \lambda_3$

$$\partial_t \Gamma_k^{(4h)} = -\frac{1}{2} \text{[Diagram 14]} + 3 \text{[Diagram 15]} + 4 \text{[Diagram 16]} - 6 \text{[Diagram 17]}$$

$$- 12 \text{[Diagram 18]} + 12 \text{[Diagram 19]} - 24 \text{[Diagram 20]}$$

Aiming at apparent convergence

Towards apparent convergence in quantum gravity

vertex expansion

$$\Gamma^{(m,n \geq 2)}$$

Denz, JMP, Reichert, EPJ C78 (2018) 4, 336

level 4: $\Gamma^{(m,n)} \approx \Gamma^{(m+n-4,4)}$

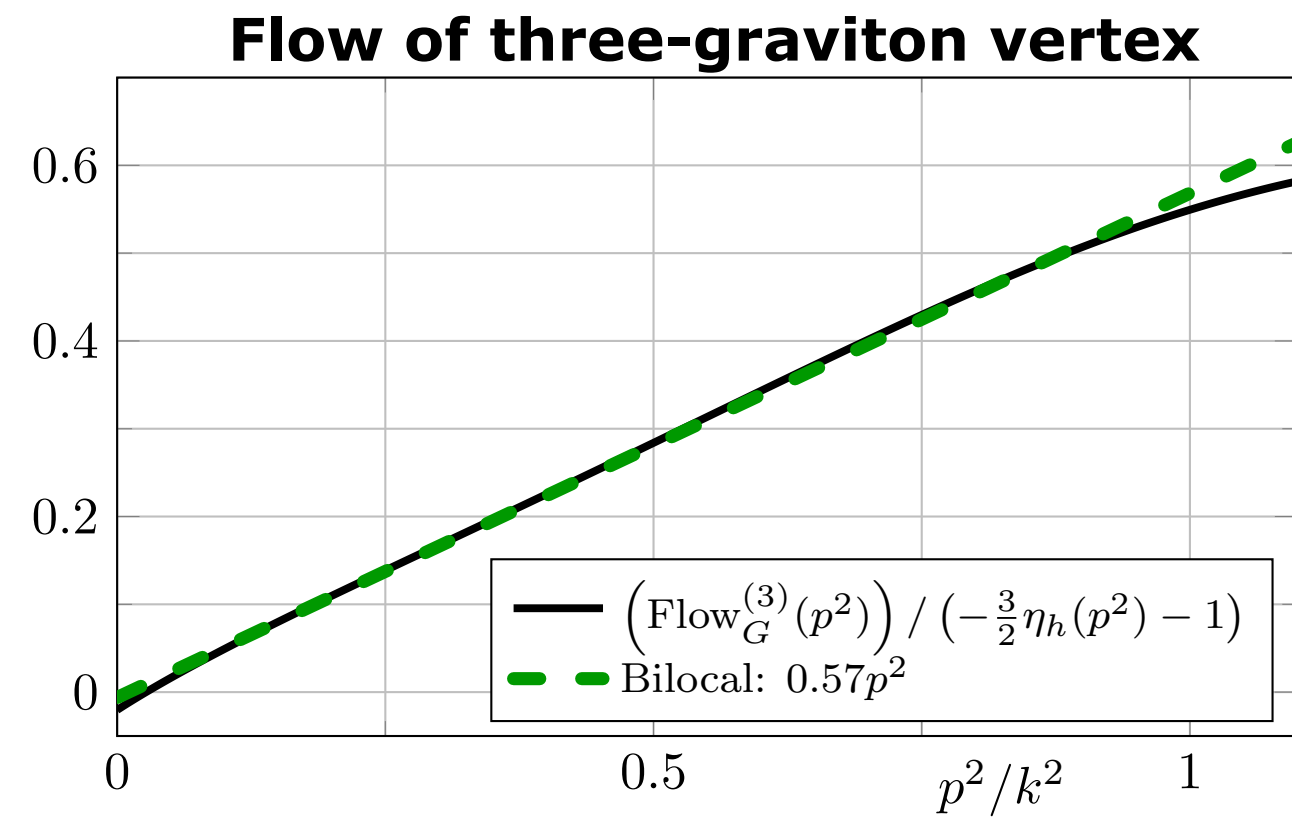
$$\begin{aligned} \partial_t \Gamma_k &= \frac{1}{2} \text{[Diagram 1]} - \text{[Diagram 2]} \\ \partial_t \Gamma_k^{(h)} &= -\frac{1}{2} \text{[Diagram 3]} + \text{[Diagram 4]} \\ \partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} \text{[Diagram 5]} + \text{[Diagram 6]} - 2 \text{[Diagram 7]} \\ \partial_t \Gamma_k^{(c\bar{c})} &= \text{[Diagram 8]} + \text{[Diagram 9]} \\ \partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} \text{[Diagram 10]} + 3 \text{[Diagram 11]} - 3 \text{[Diagram 12]} + 6 \text{[Diagram 13]} \\ \partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} \text{[Diagram 14]} + 3 \text{[Diagram 15]} + 4 \text{[Diagram 16]} - 6 \text{[Diagram 17]} \\ &\quad - 12 \text{[Diagram 18]} + 12 \text{[Diagram 19]} - 24 \text{[Diagram 20]} \end{aligned}$$

$Z_h(p), Z_c(p), \mu = -2\lambda_2$

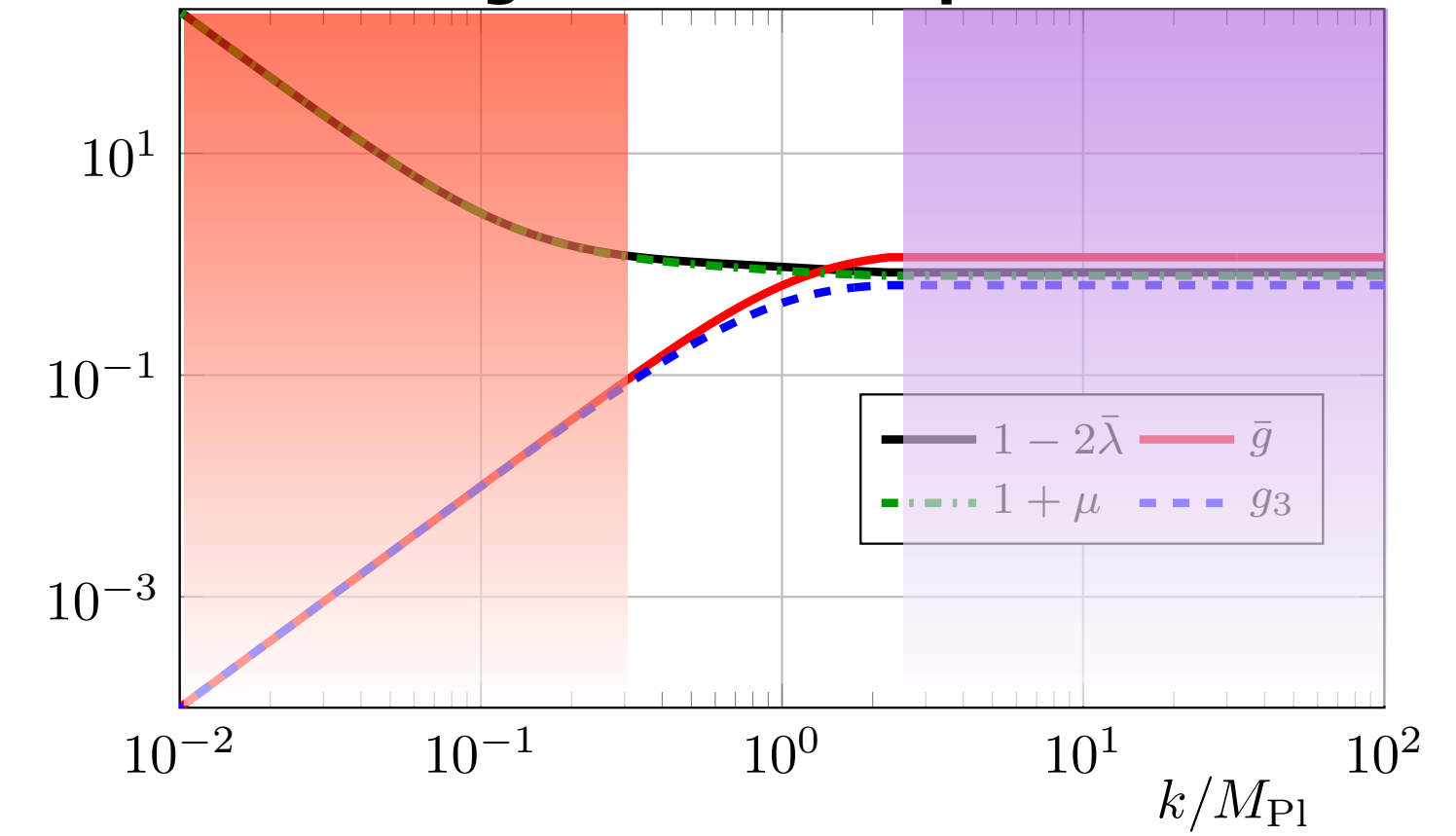
$g_3(p), \lambda_3$

$g_4(p), \lambda_4$

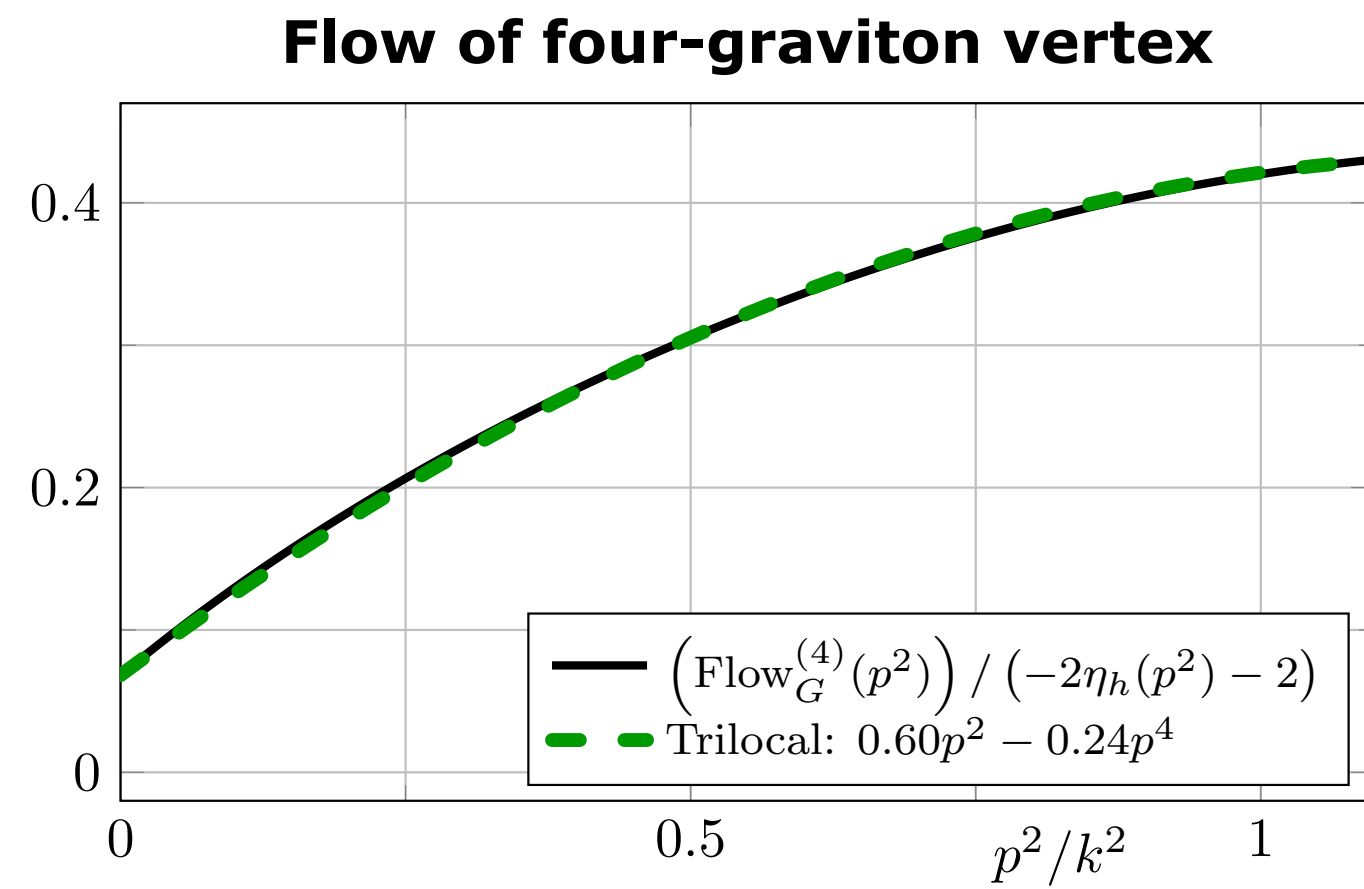
Momentum-dependent vertices



IR fine-tuning of diffeomorphism invariance



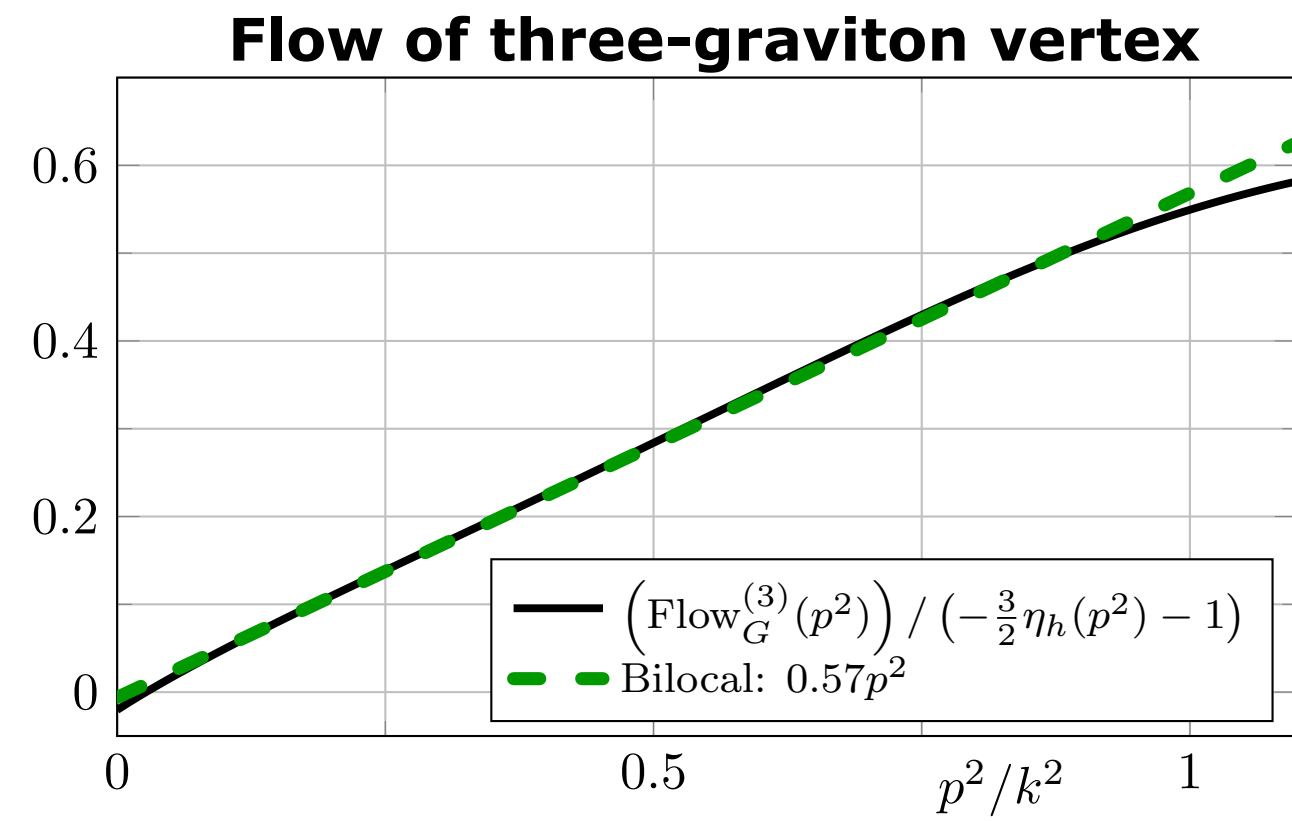
Denz, JMP, Reichert, EPJ C78 (2018)



classical general relativity

asymptotically safe fixed point scaling

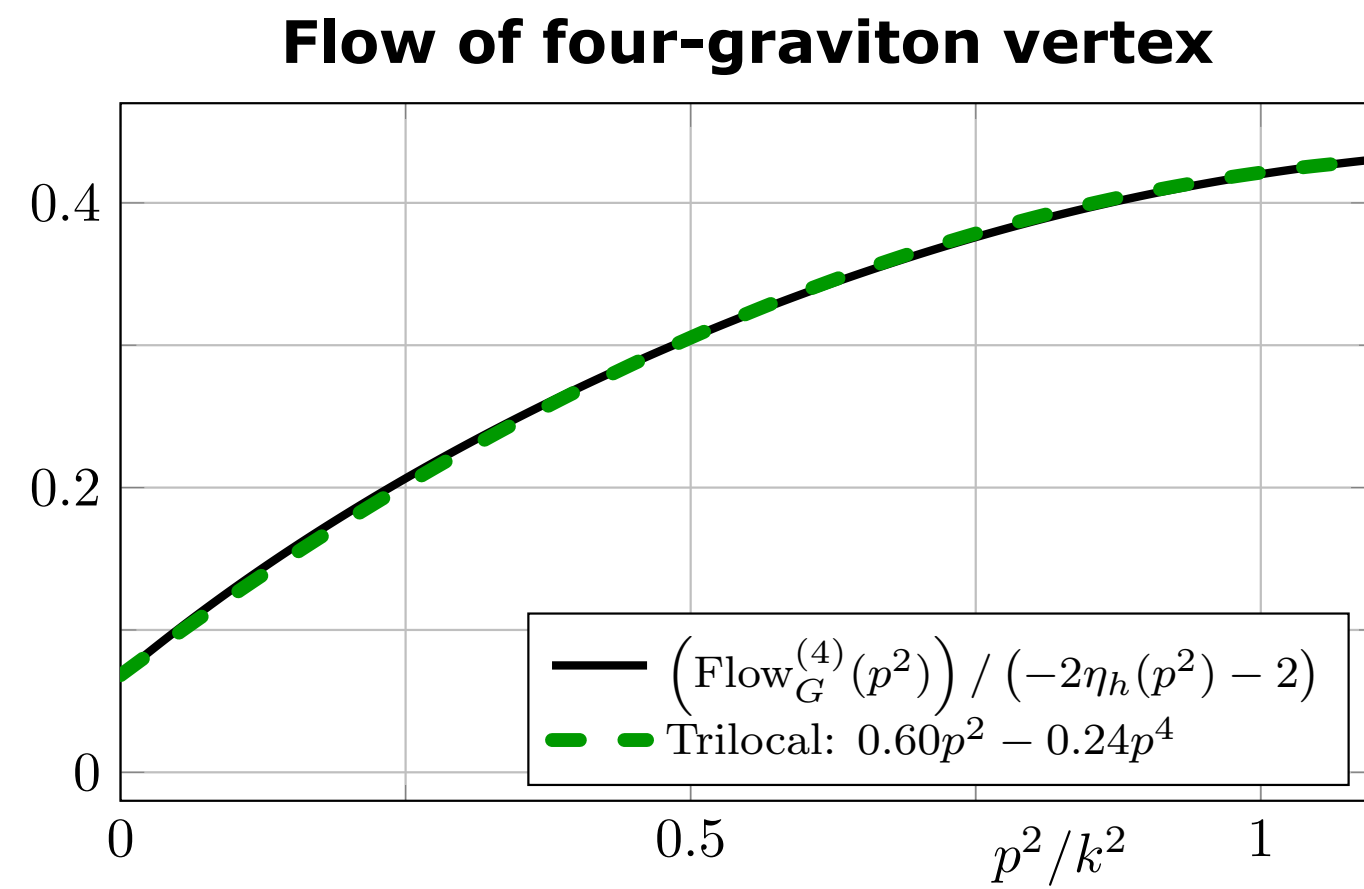
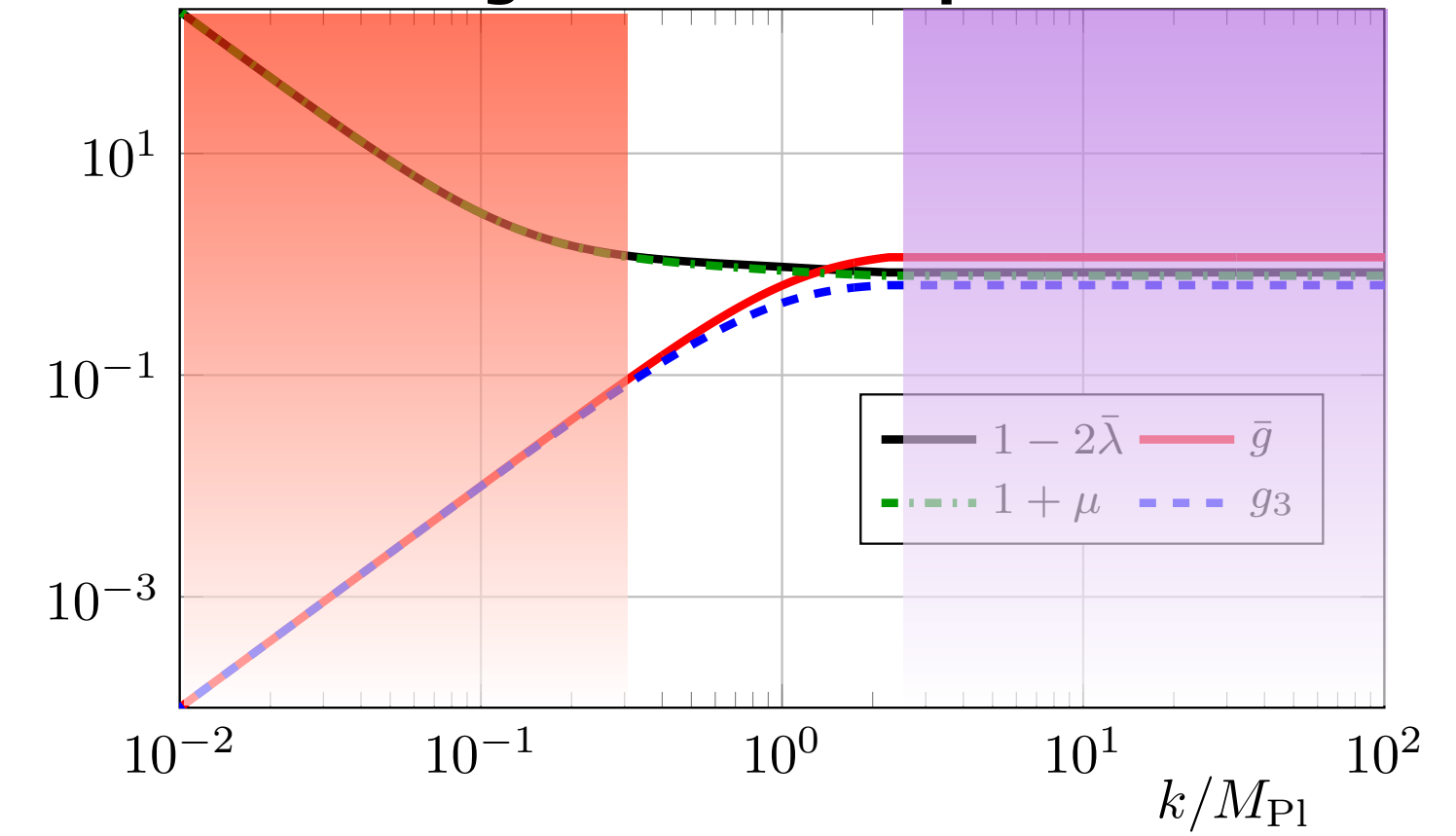
Momentum-dependent vertices



no $R_{\mu\nu}^2$ - tensor structure generated!

Denz, JMP, Reichert, EPJ C78 (2018)

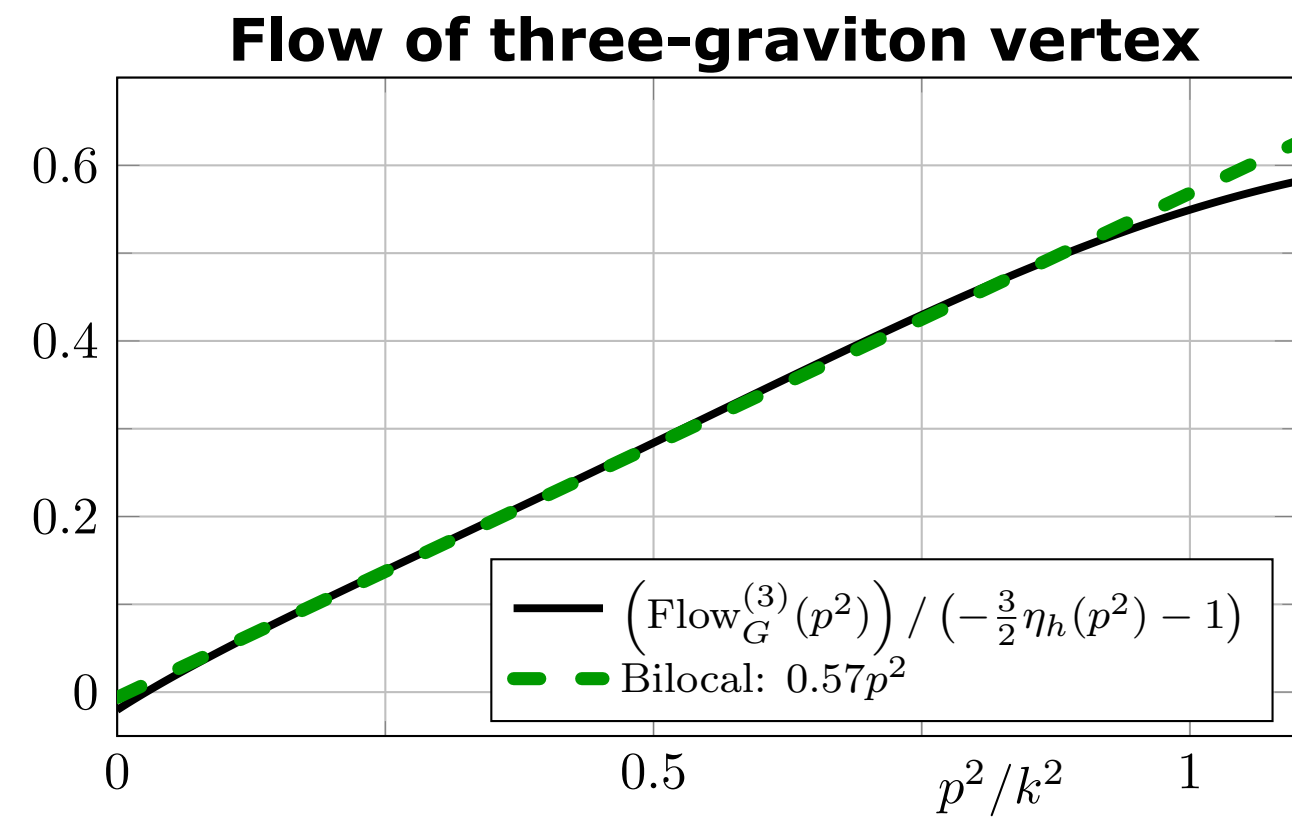
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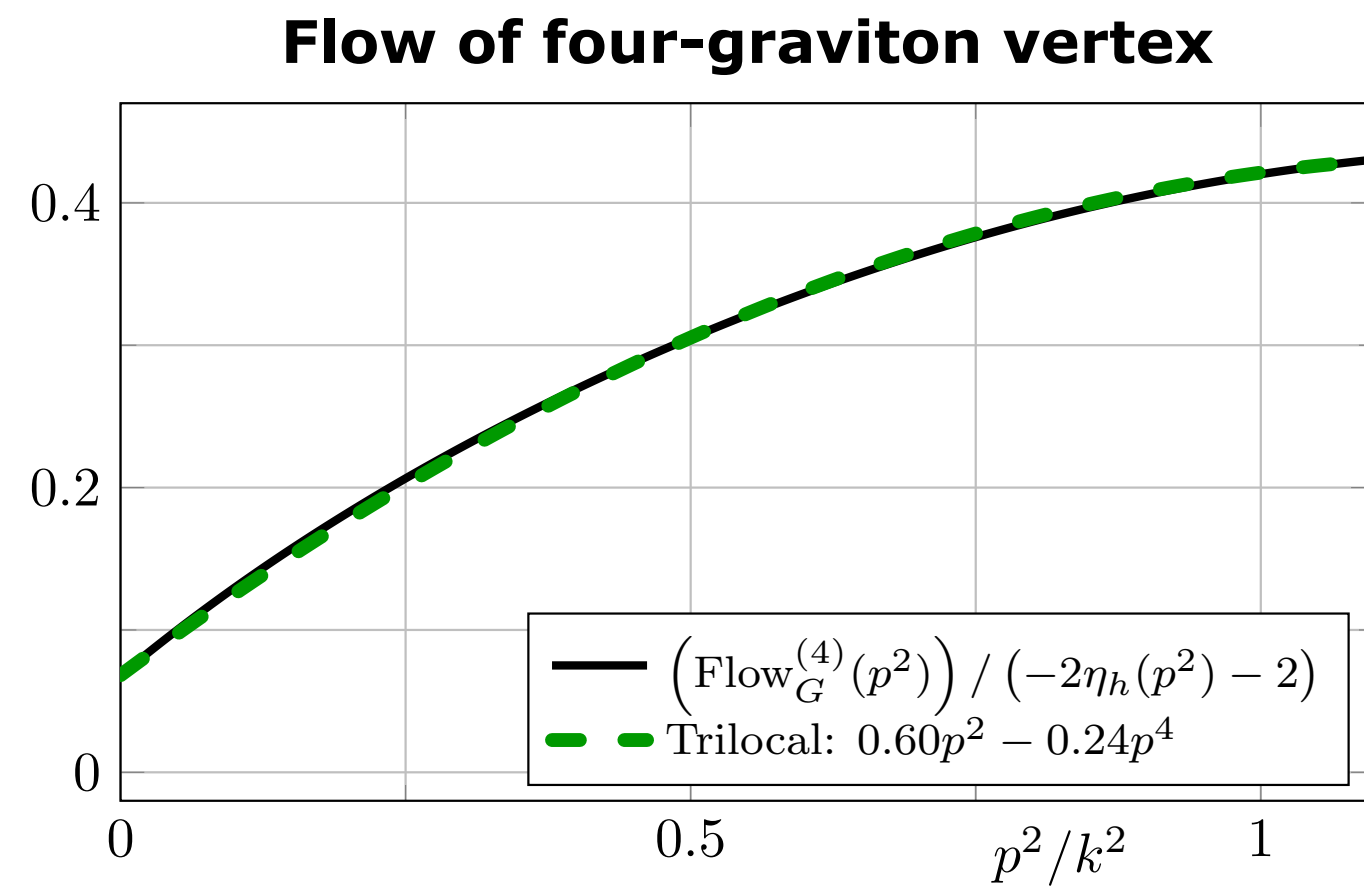
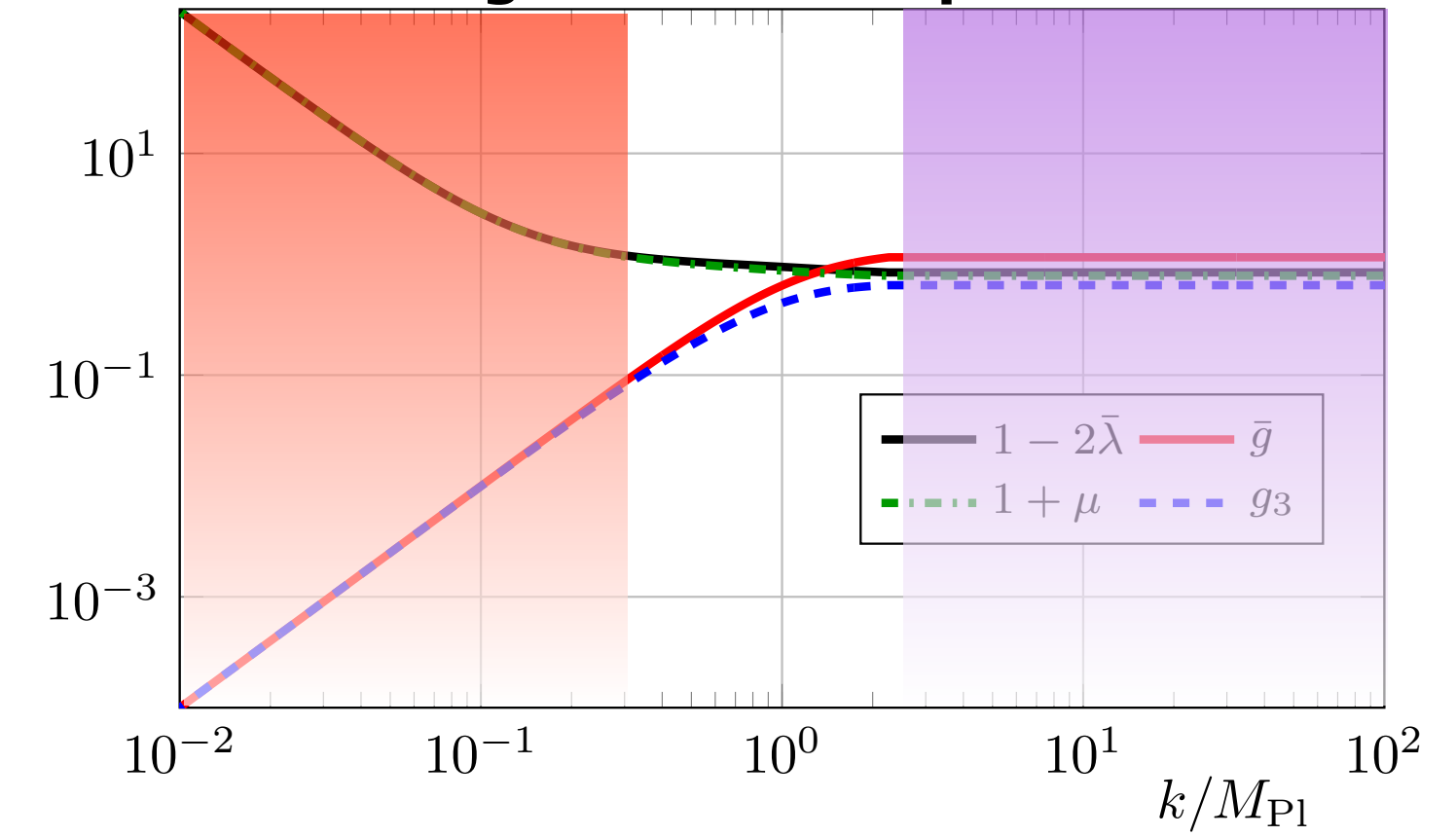
Momentum-dependent vertices



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Denz, JMP, Reichert, EPJ C78 (2018)

IR fine-tuning of diffeomorphism invariance



R^2 - tensor structure generated

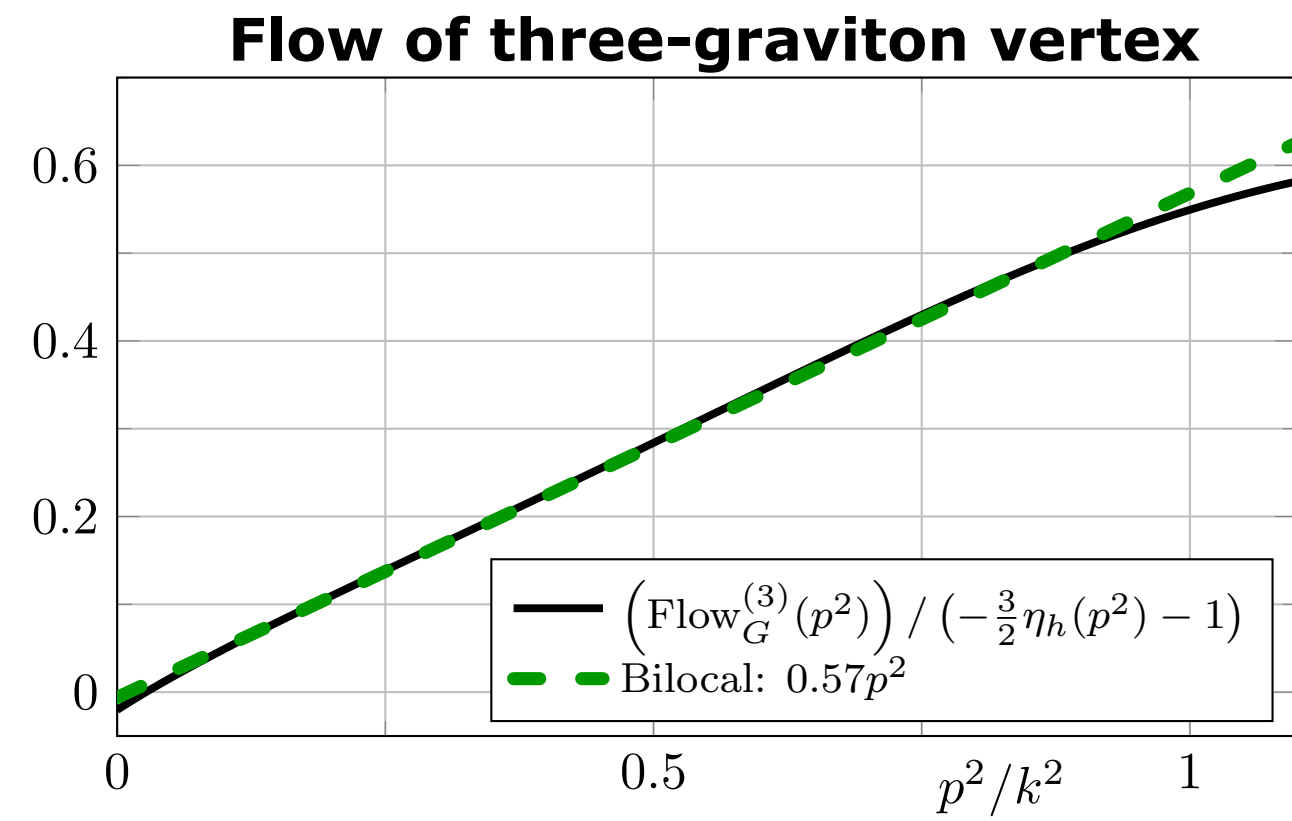
R - tensor structure sustained



classical general relativity

asymptotically safe fixed point scaling

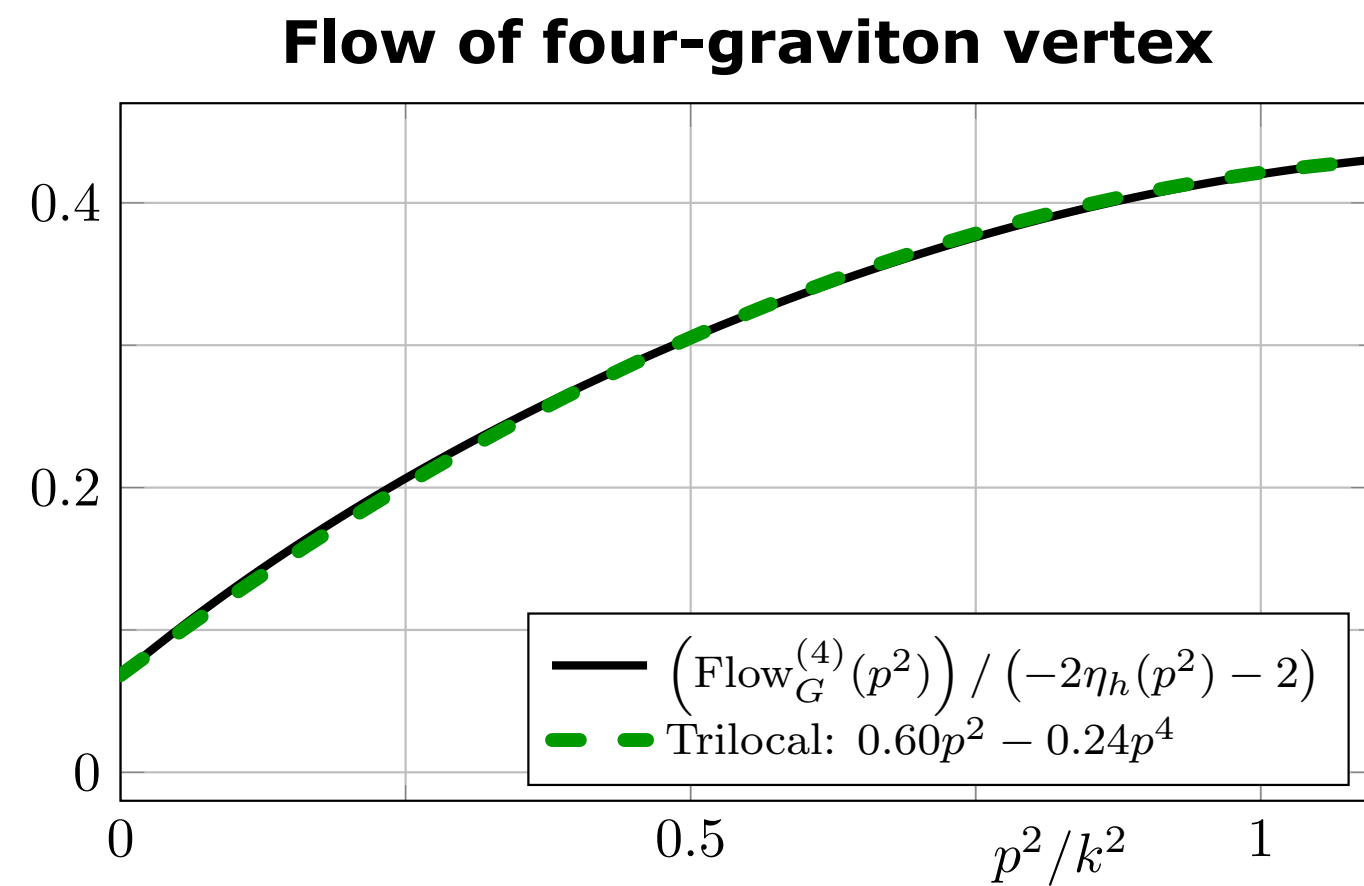
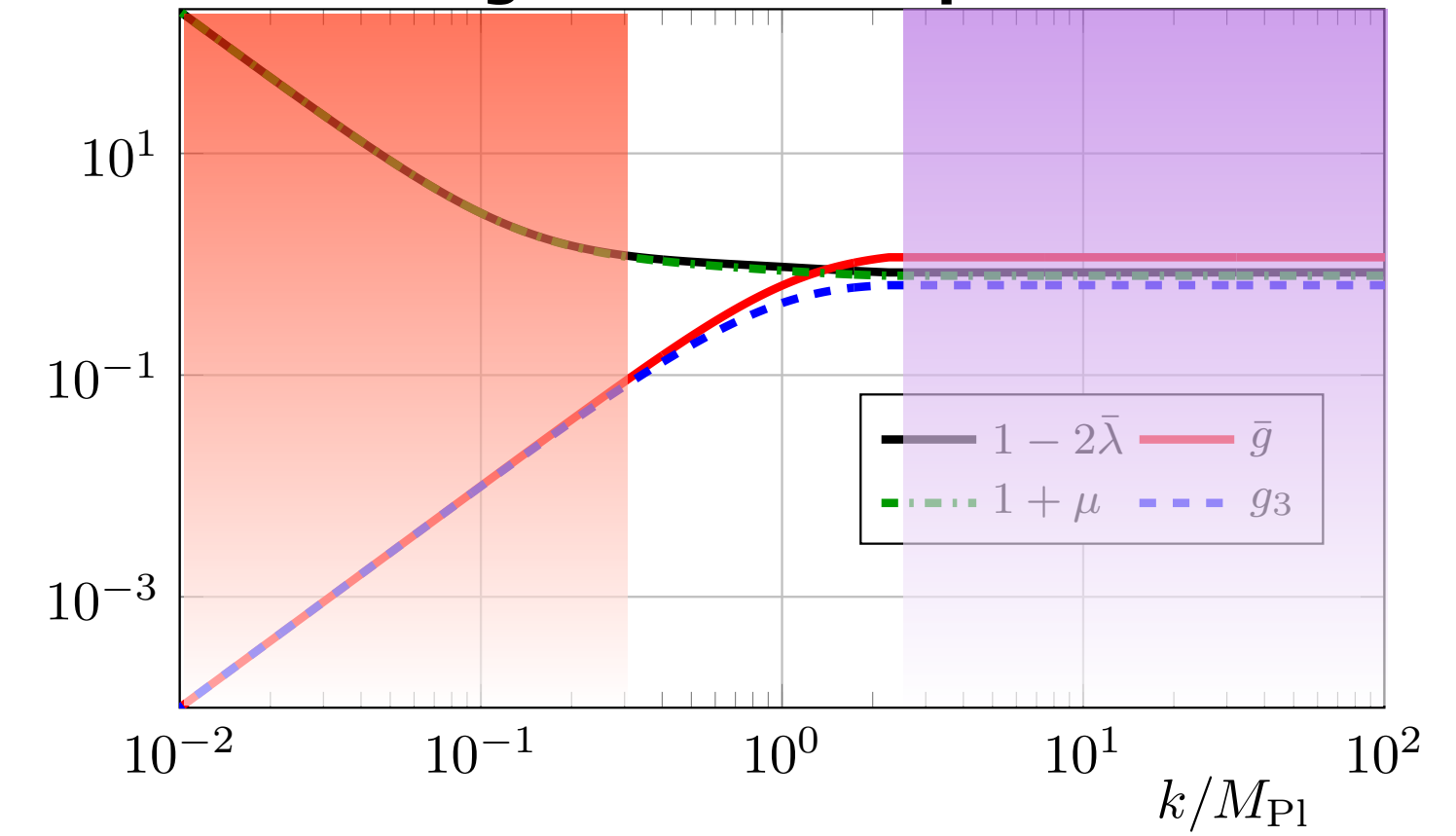
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Denz, JMP, Reichert, EPJ C78 (2018)

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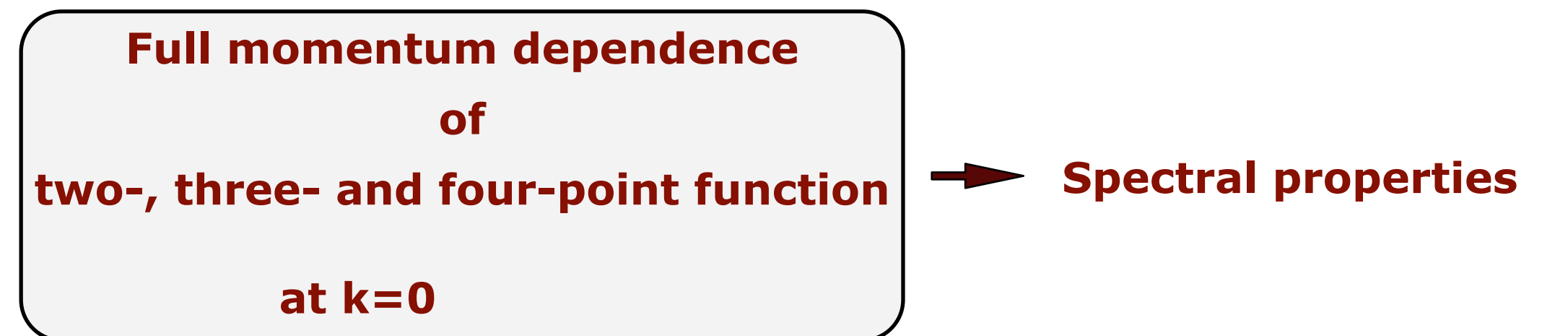
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classical general relativity

asymptotically safe fixed point scaling



Towards apparent convergence in quantum gravity

Why does/could it work?

Typically diagrams with higher order vertices are strongly suppressed

(a) couplings stay finite

(b) combinatorial suppression of diagrams with higher vertices

(c) phase space (angular) suppression of diagrams with higher vertices

turns out to be very efficient!

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Why does/could it fail?

Resonant interaction channels and their interactions circumvent (b) and make (a) irrelevant

(a) couplings diverge

(b) hadrons, diquarks, glueballs, ... in QCD



Emergent composites, BSE

Gies, Wetterich, PRD 65 (2002) 0650016

JMP, AP 322 (2007) 2831

Flörchinger, Wetterich, PLB 680 (2009) 371

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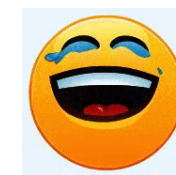
Emergent composites, BSE

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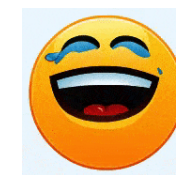
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Flörchinger, Wetterich, PLB 680 (2009) 371

(c) graviballs in gravity



QG as perturbative as possible & apparently converging

... slight oversimplification for the sake of this talk ...

JMP, Reichert, Front.in Phys. 8 (2021) 527

2309.10785

Applications I: asymptotically safe Standard Model

Pastor-Gutiérrez, JMP, Reichert, SciPost Phys. 15 (2023) 105

Dona, Eichhorn, Percacci, PRD 89 (2014) 084035

Meibohm, JMP, Reichert, EPJC 76 (2016) 285

Christiansen, Litim, JMP, Reichert PRD 97 (2018) 4, 046007



Shaposhnikov, Wetterich, PLB 683 (2010) 196

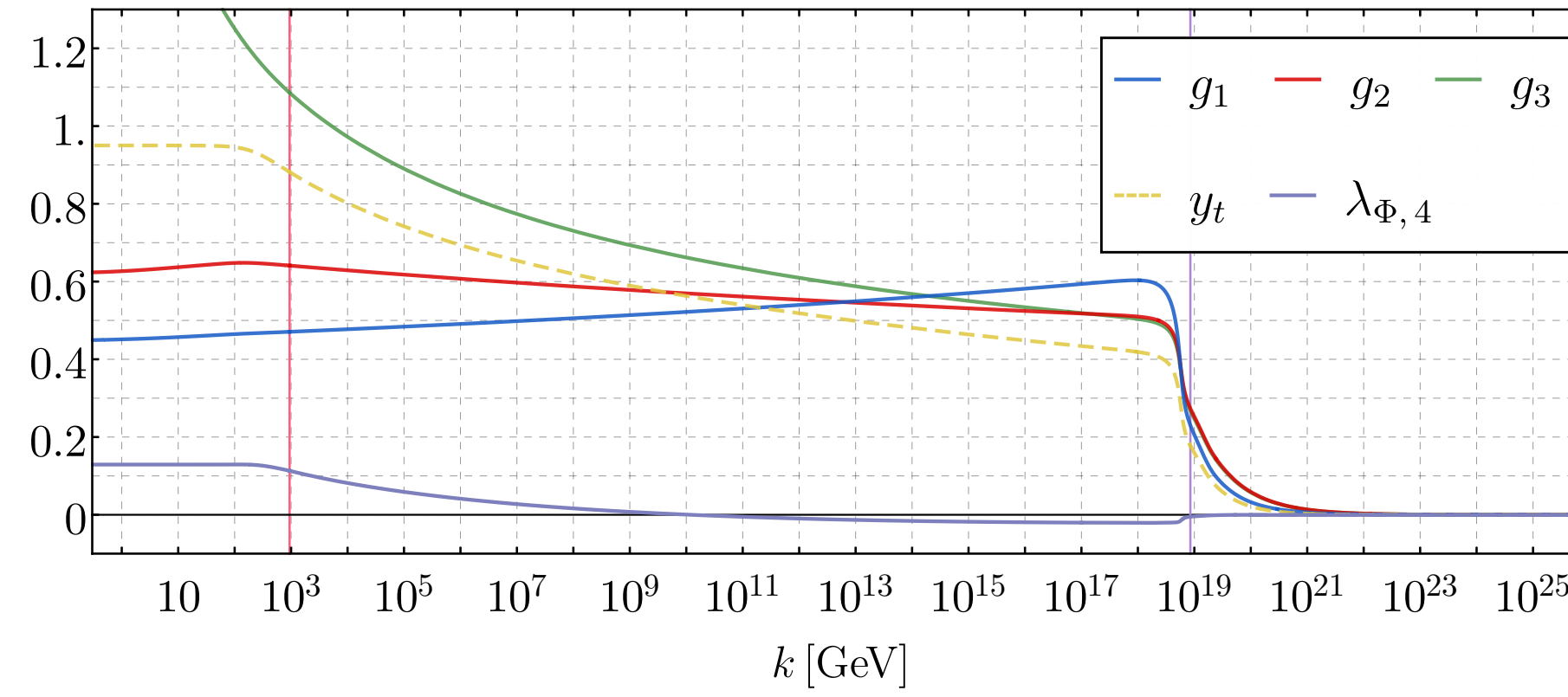
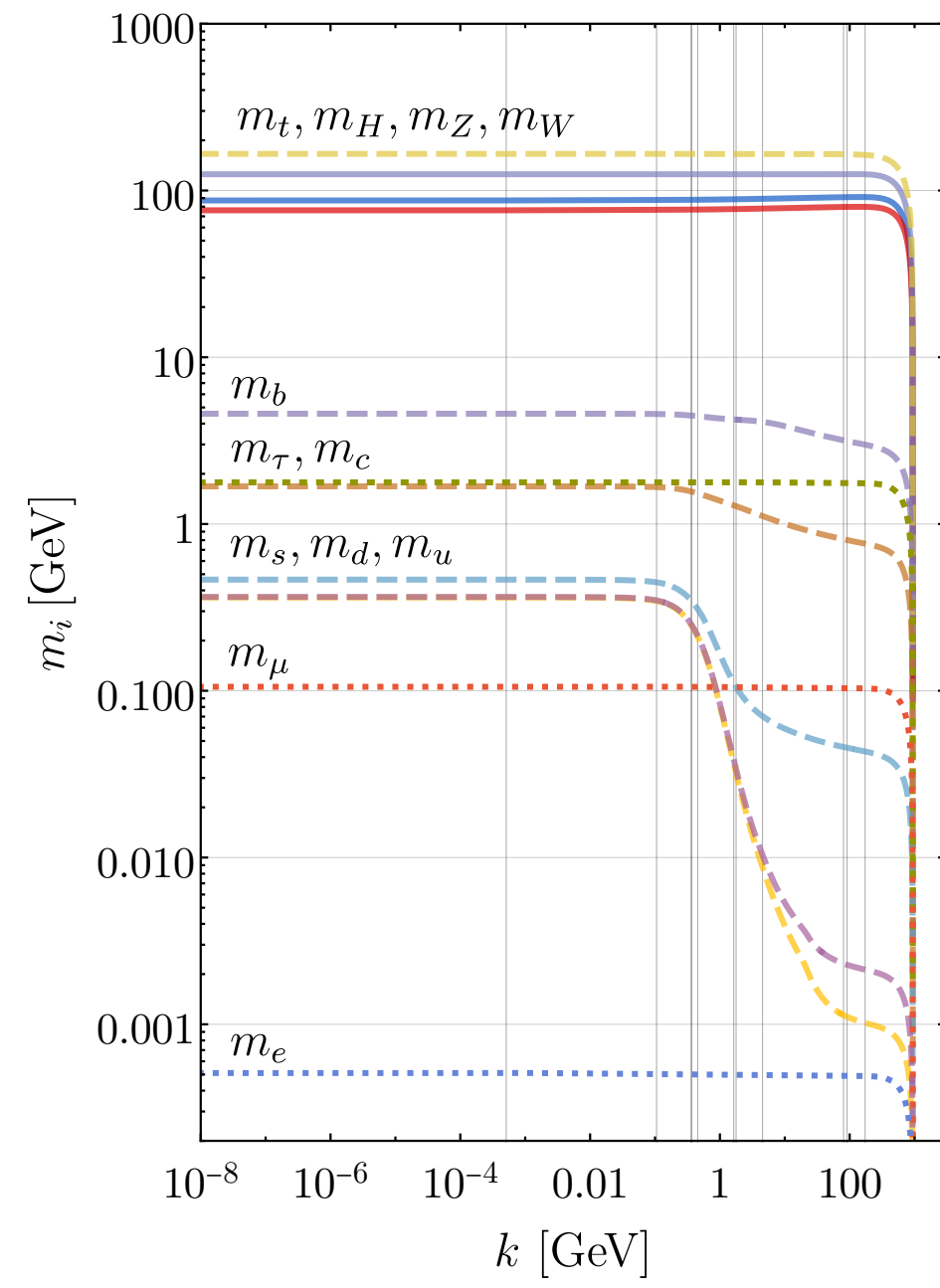
Eichhorn, Versteegen, JHEP 1801 (2018) 030

Eichhorn, Held, PRL 121 (2018) 151302



Latest 'status report': Eichhorn, Schiffer, 2212.07456

Asymptotically safe Standard Model

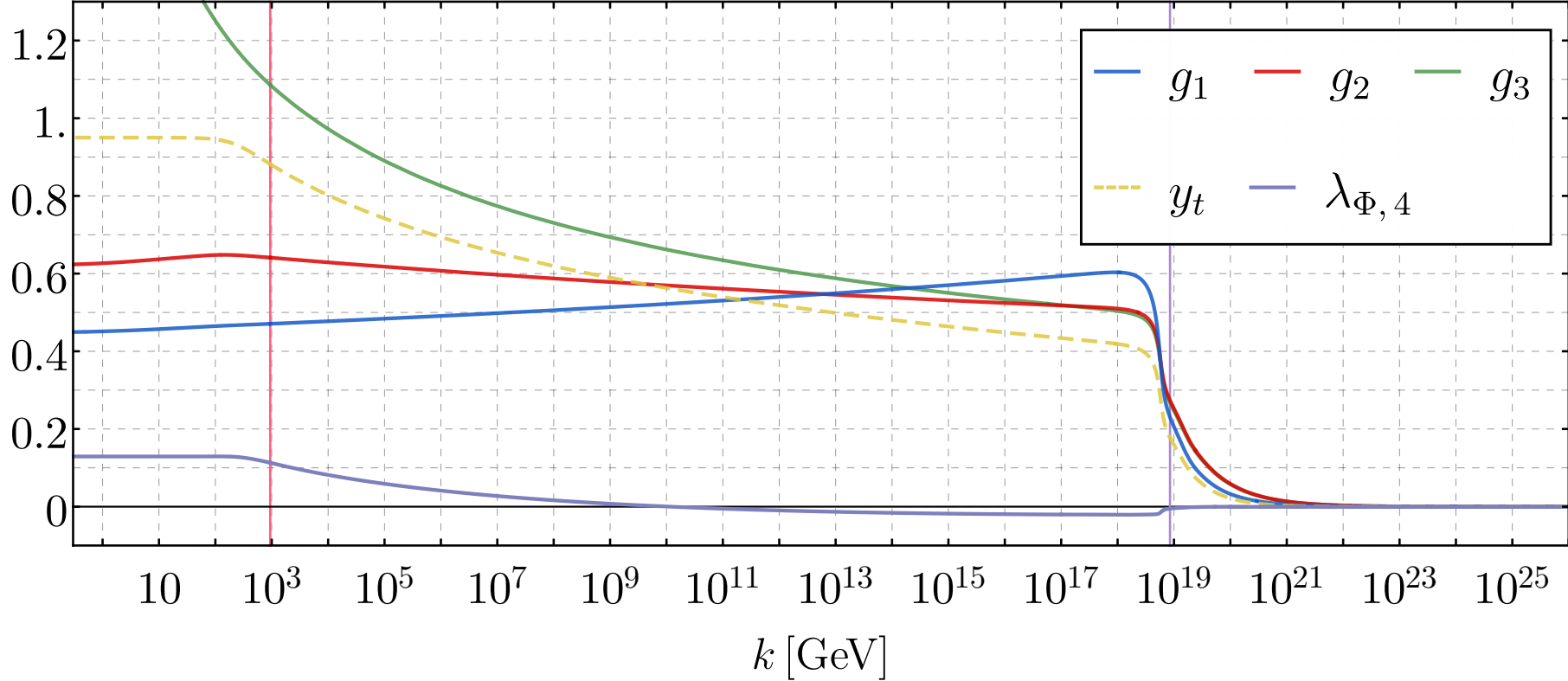
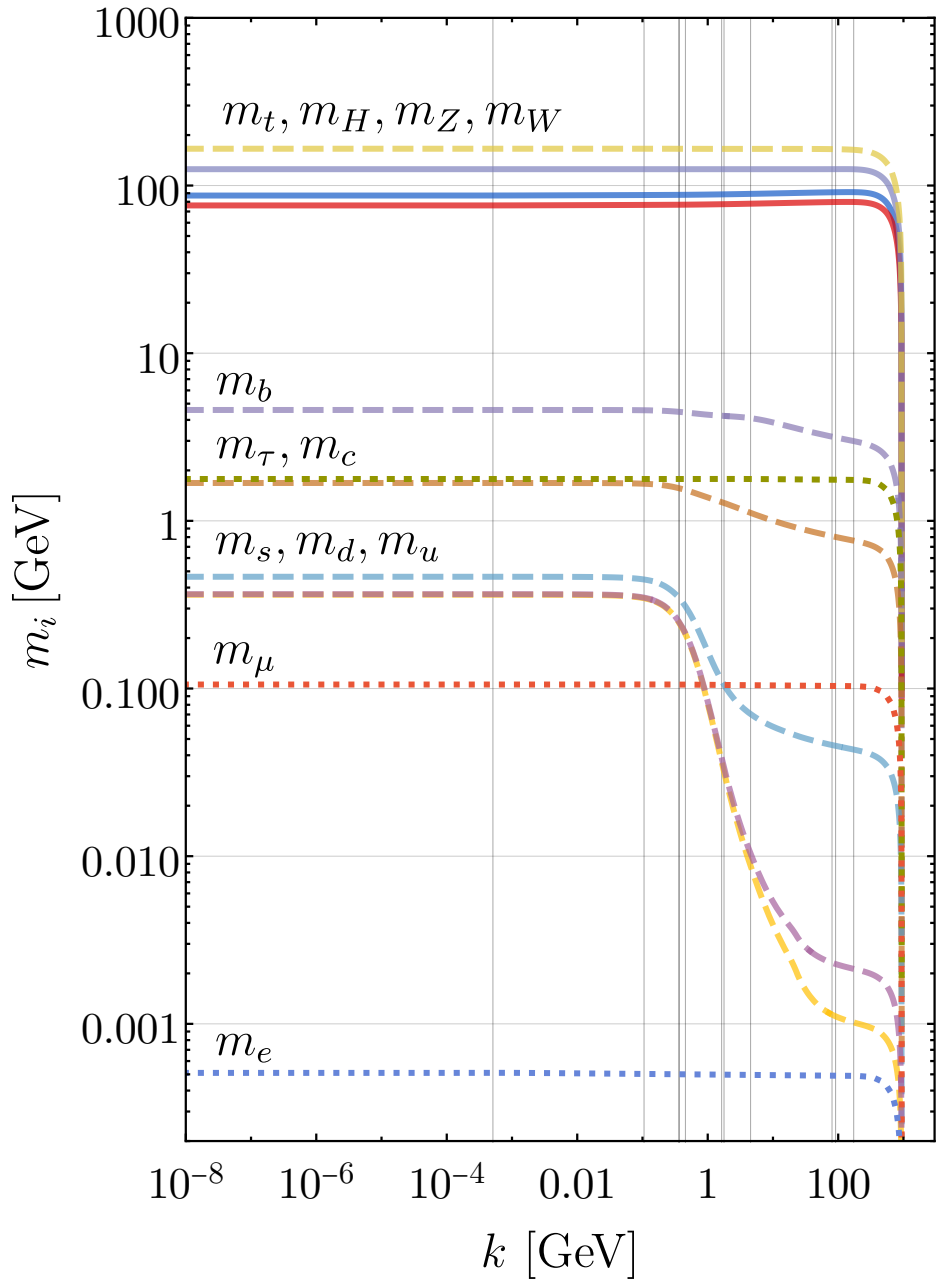


top pole mass (getting real)

$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7 \text{ GeV}$$

Experimental value (PDG)

Asymptotically safe Standard Model

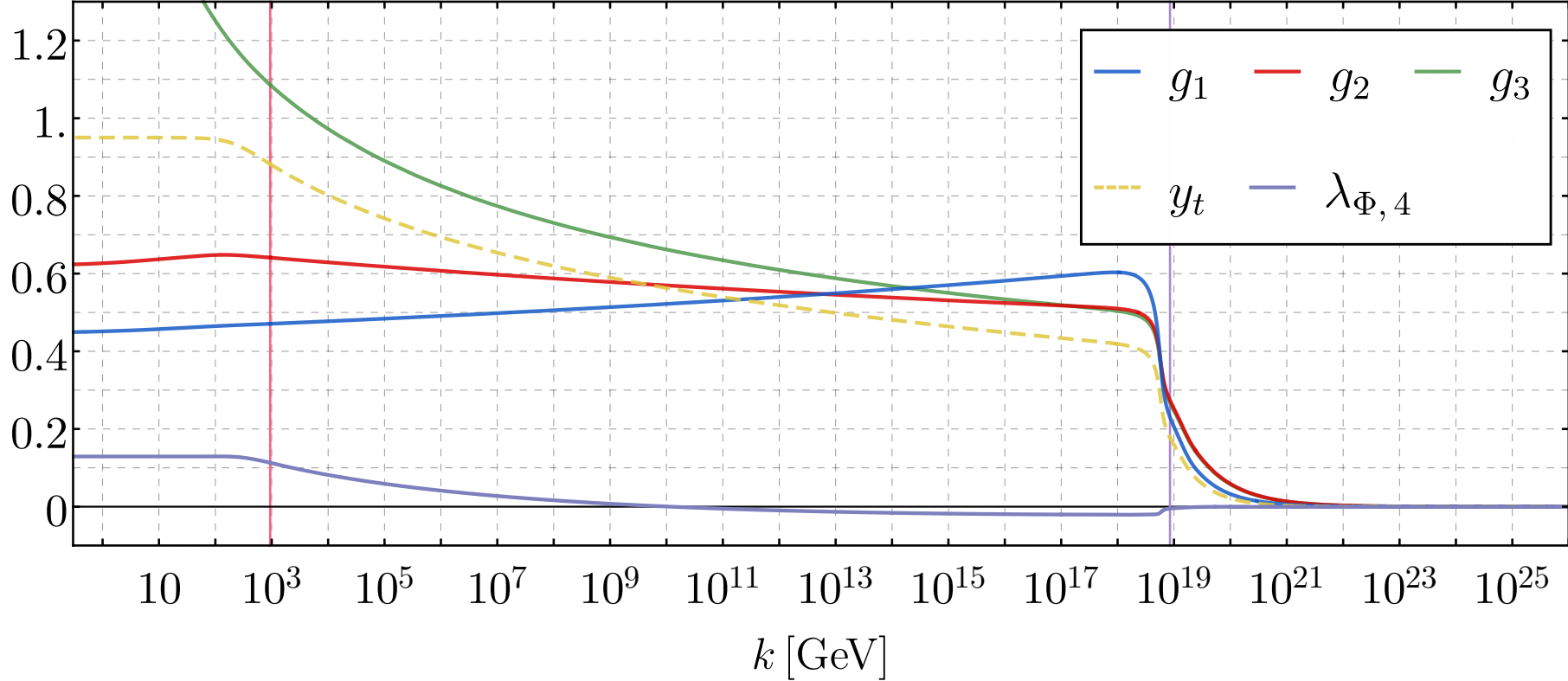
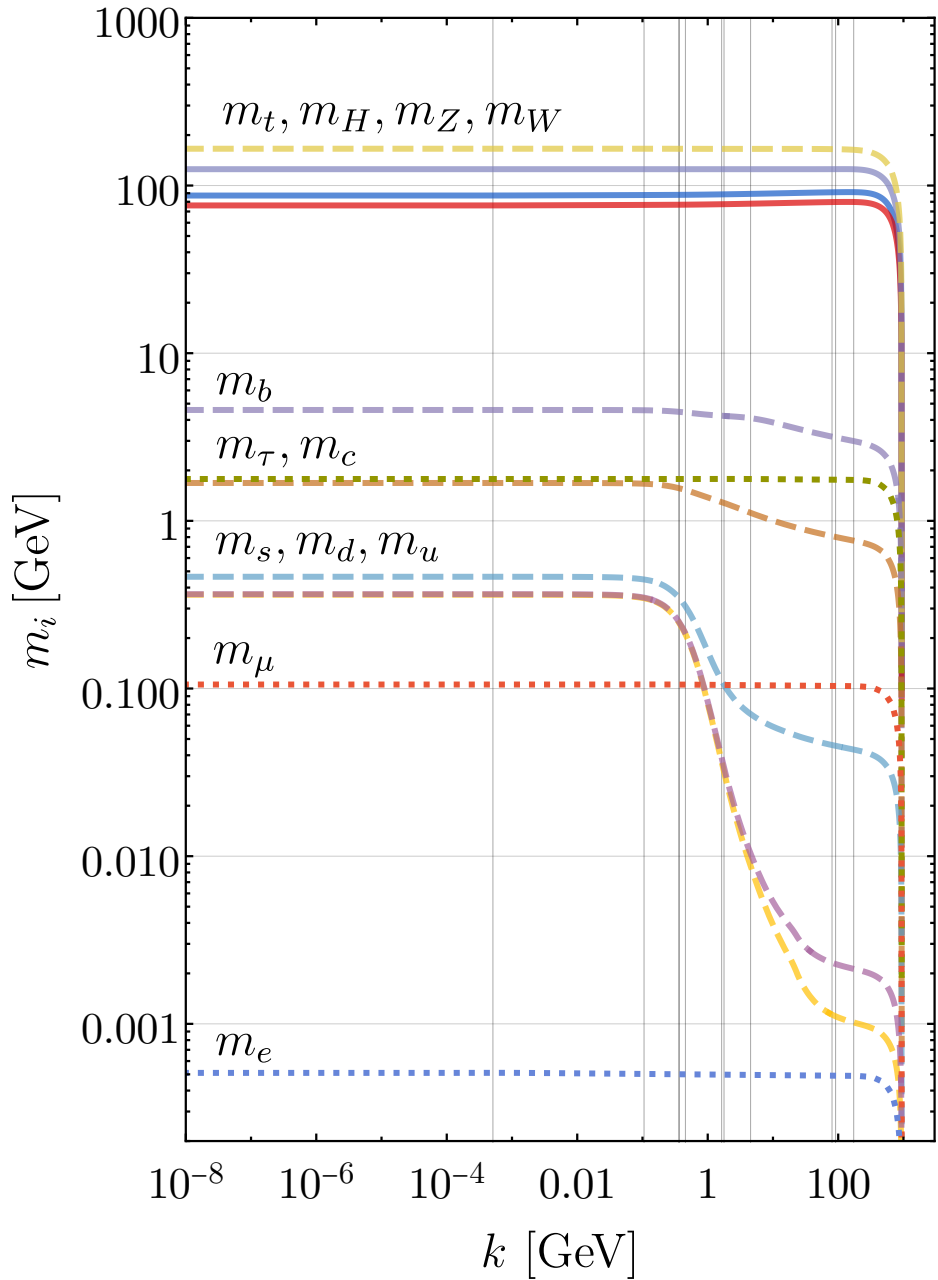


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$$M_{t,\text{pole}}^{(\text{exp})} = 172.5 \pm 0.7 \text{ GeV} \quad \leftarrow \quad m_t = 165.4_{-0.2}^{+0.9} \text{ GeV}$$

Experimental value (PDG)
Euclidean curvature mass

Asymptotically safe Standard Model



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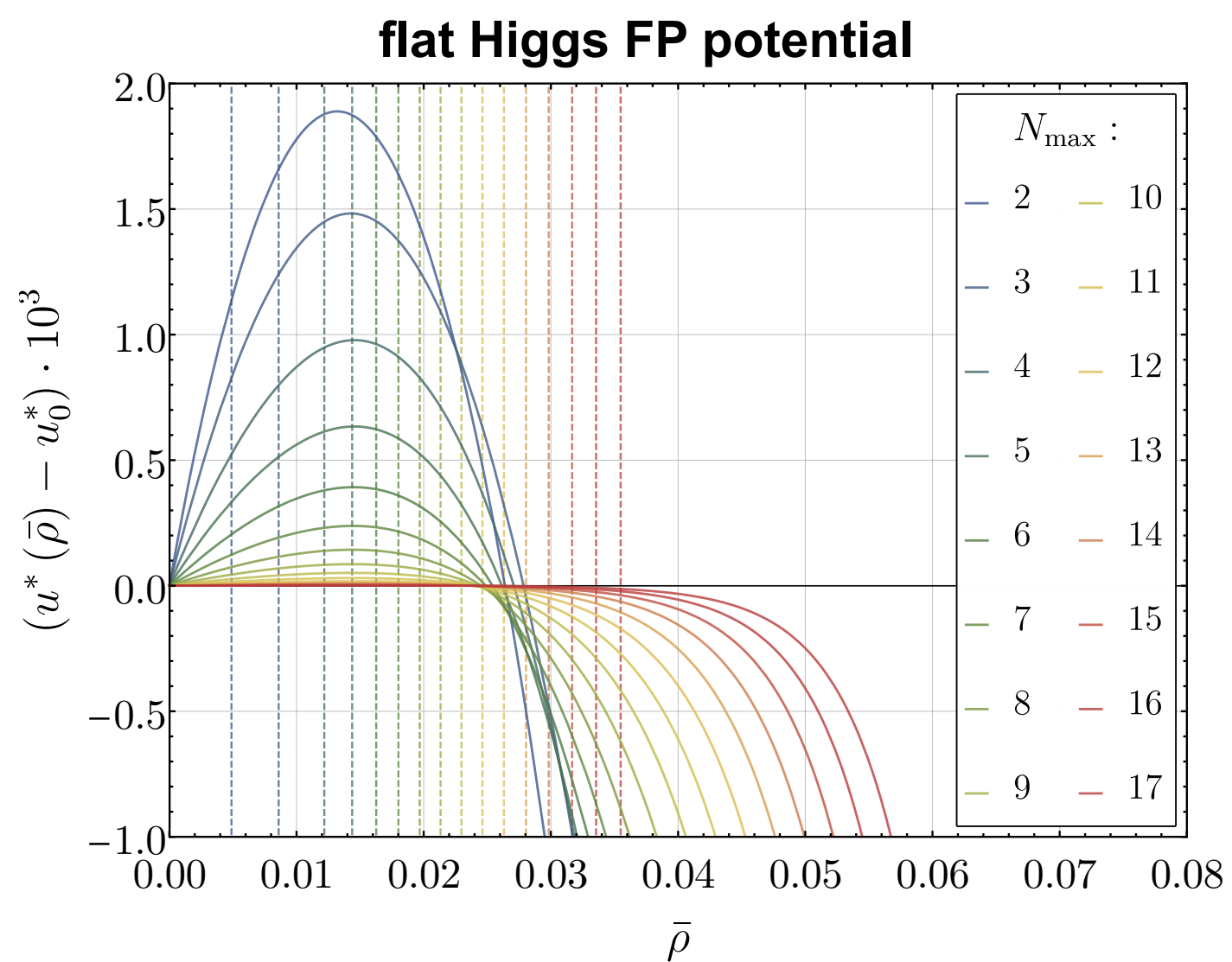
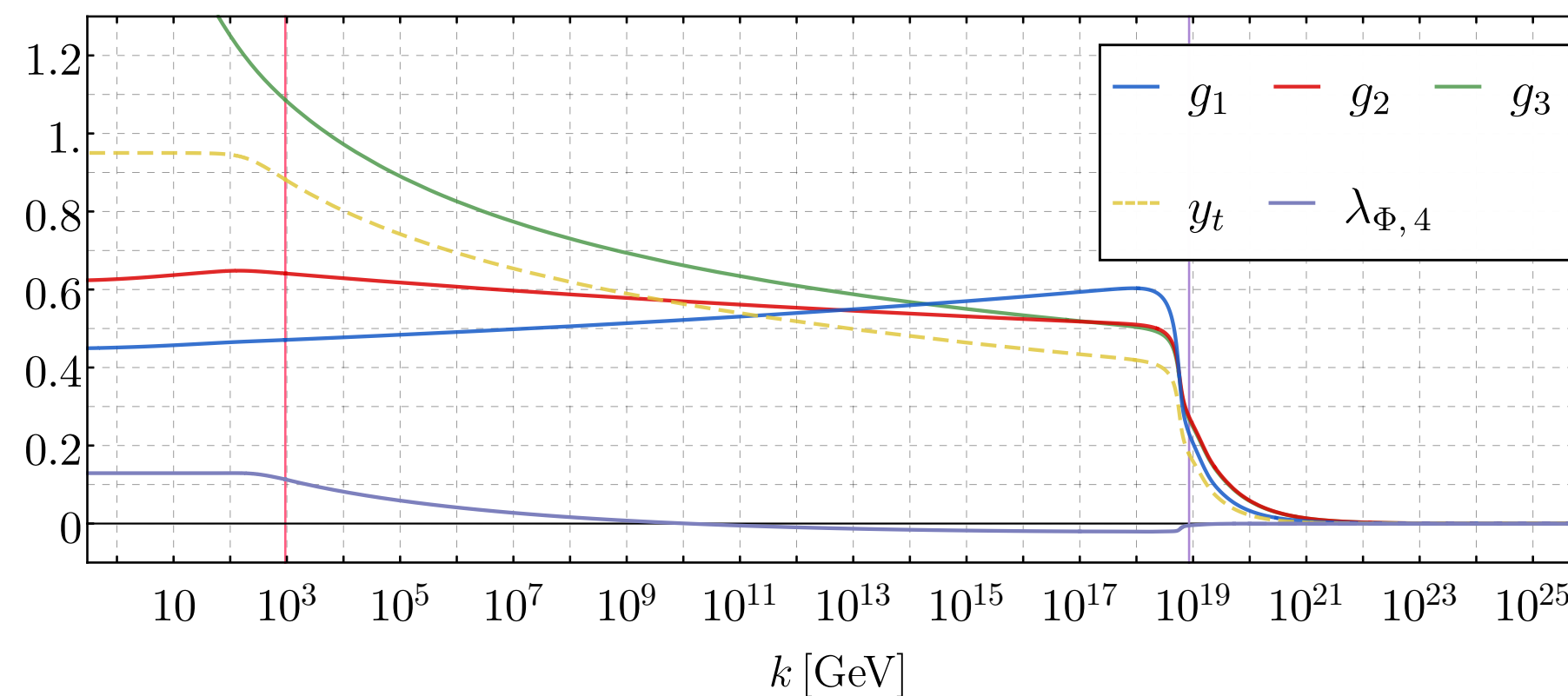
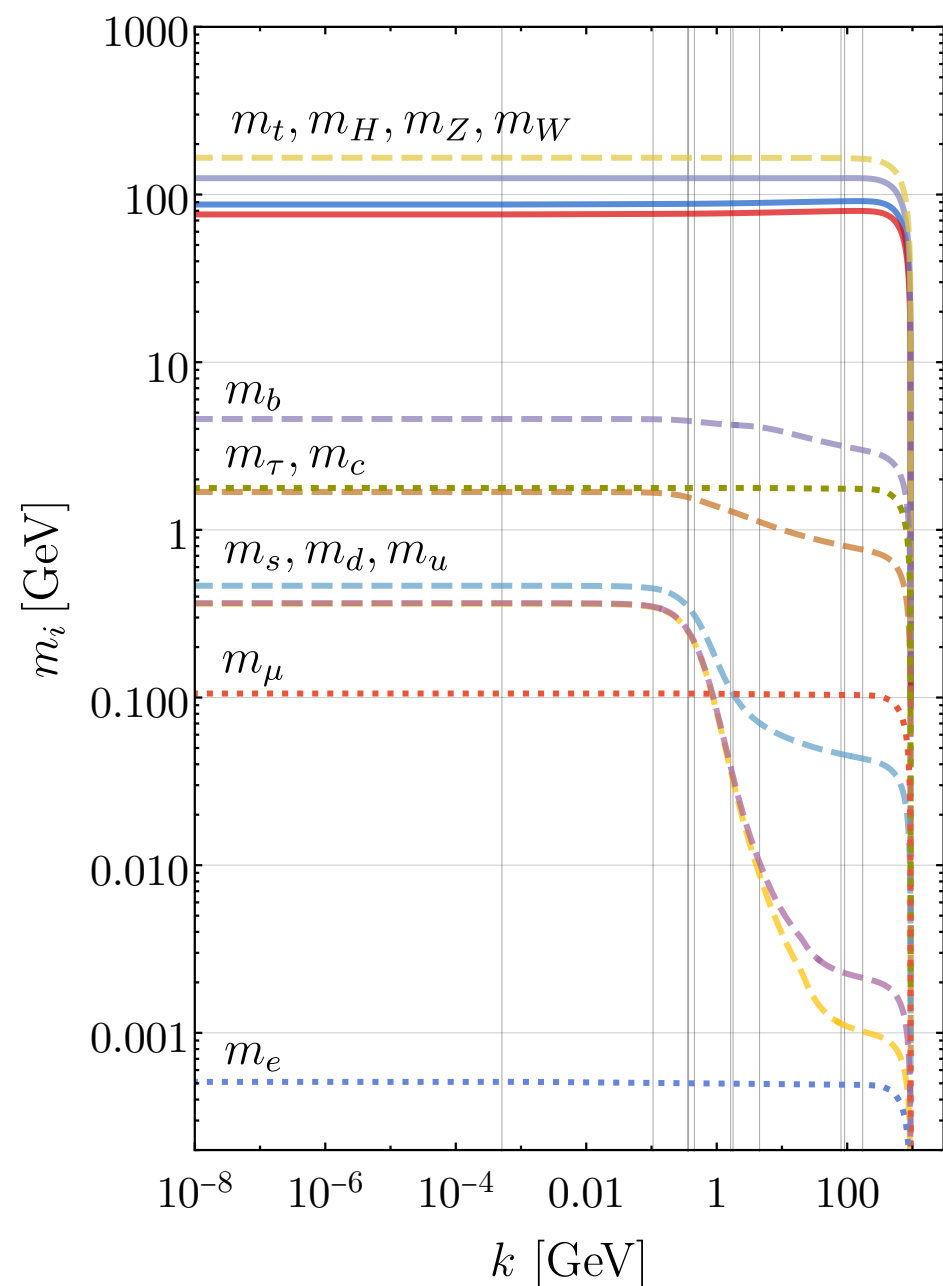
Experimental value (PDG)
Euclidean curvature mass

Prediction of decay width

$$\Gamma_{t,\text{pole}}^{(\text{theo})} = 1.72_{-0.41}^{+0.09} \text{ GeV} \quad \Gamma_{t,\text{pole}}^{(\text{exp})} = 1.42_{-0.15}^{+0.19} \text{ GeV}$$

Experimental value (PDG)

Asymptotically safe Standard Model



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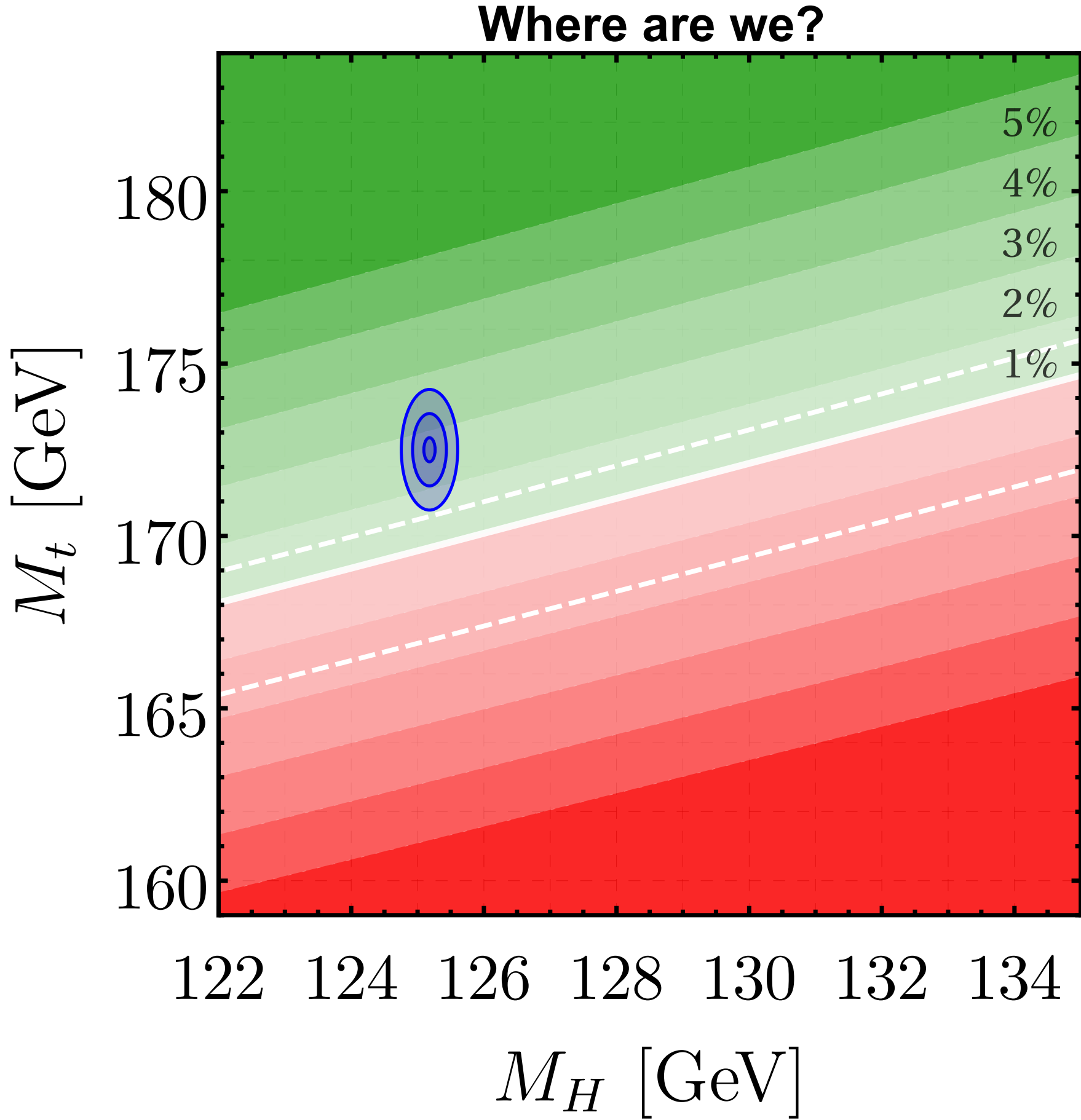
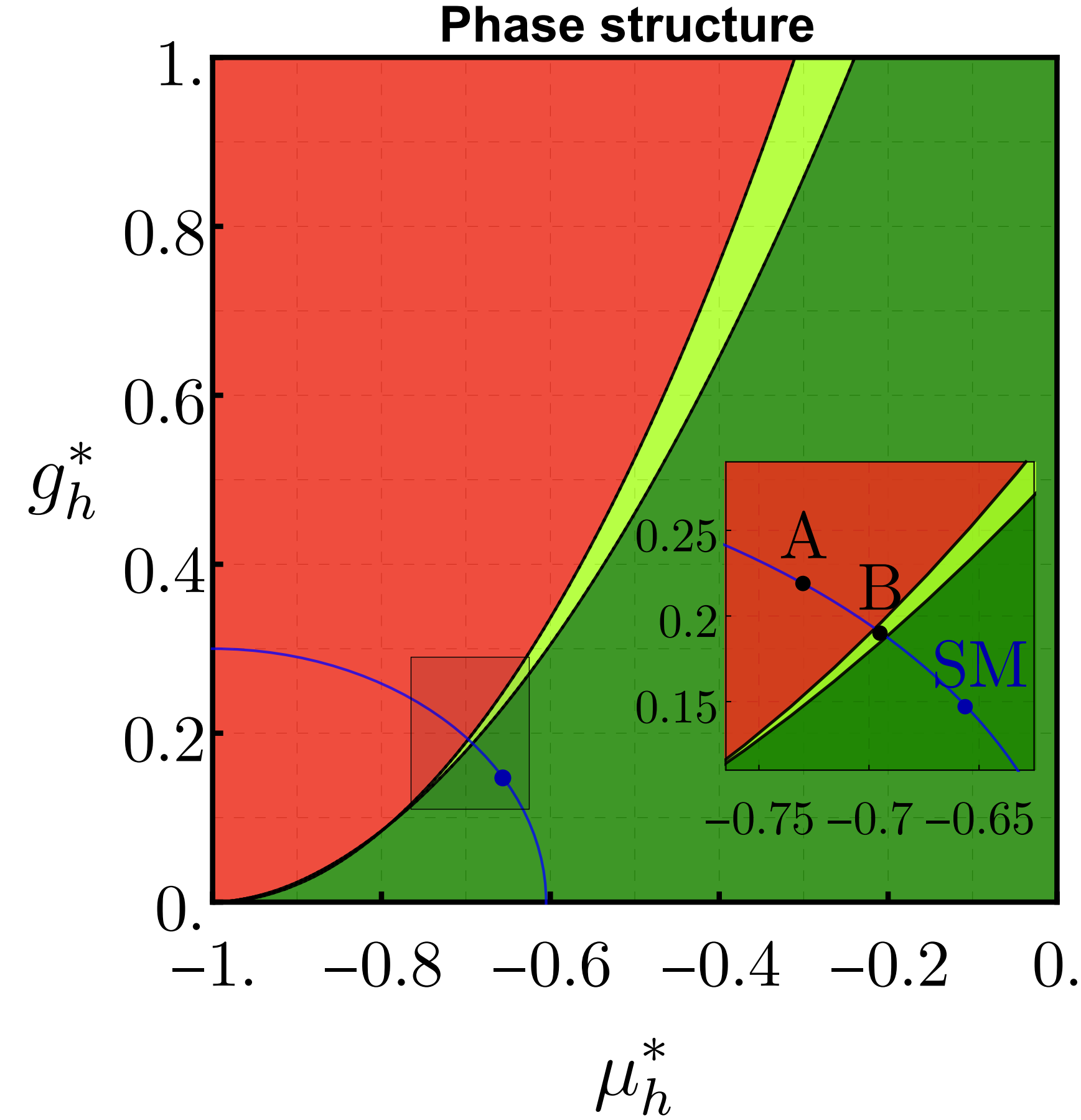
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Experimental value (PDG)

Asymptotically safe Standard Model



Applications II: asymptotically black holes

JMP, Tränkle, 2309.17043

Black Holes in Asymptotically Safe Gravity:

Platania, 2309.17043

and beyond: Held, Eichhorn, 2212.09495

Asymptotically black holes

Unfolding the background effective action

$$\Gamma[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{g} \left\{ \mathcal{R}(\Delta, R) + R f_{R^2}(\Delta) R + R_{\mu\nu} f_{R_{\mu\nu}^2}(\Delta) R^{\mu\nu} + \dots \right\}$$

gauge dependent

RG-invariant

$$\bar{\Gamma}_{hh}^{(2)}(p)$$

$$\bar{\Gamma}_{\bar{g}^3}^{(3)}(p)$$

$$\bar{\Gamma}_{h^3}^{(3)}(p)$$



$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p)$$

$$\bar{\Gamma}_{h^4}^{(4)}(p)$$

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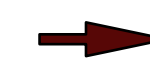
gauge independent

$$\bar{\Gamma}_{hh}^{(2)}(p)$$

$$\bar{\Gamma}_{\bar{g}^3}^{(3)}(p)$$

$$\mathcal{R}(\Delta, R)$$

$$\bar{\Gamma}_{h^3}^{(3)}(p) \quad \rightarrow$$



$$R f_{R^2}(\Delta) R$$

$$\bar{\Gamma}_{h^4}^{(4)}(p)$$

$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p) \quad \rightarrow$$

$$R_{\mu\nu} f_{R_{\mu\nu}^2}(\Delta) R^{\mu\nu}$$

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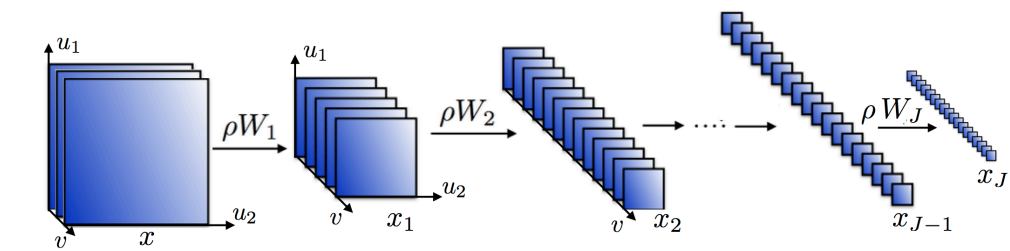
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gauge independent

$$\bar{\Gamma}_{hh}^{(2)}(p)$$

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$\mathcal{R}(\Delta, R)$
Unfolding



$$\bar{\Gamma}_{h^3}^{(3)}(p) \quad \rightarrow$$

$R f_{R^2}(\Delta) R$

$$\bar{\Gamma}_{h^4}^{(4)}(p)$$

$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p) \quad \rightarrow$$

Maps

$R_{\mu\nu} f_{R^2_{\mu\nu}}(\Delta) R^{\mu\nu}$

Asymptotically black holes

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gauge dependent

RG-invariant

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$$\bar{\Gamma}_{hh}^{(2)}(p)$$

$$\bar{\Gamma}_{\bar{g}^3}^{(3)}(p)$$

$$\mathcal{R}(\Delta, R)$$

Unfolding

Educated guess

$$\bar{\Gamma}_{h^3}^{(3)}(p)$$



$$R f_{R^2}(\Delta) R$$



$$\bar{\Gamma}_{h^4}^{(4)}(p)$$

$$\bar{\Gamma}_{\bar{g}^4}^{(4)}(p)$$

Maps

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Unfolding

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$$\bar{\Gamma}_{h^3}^{(3)}(p)$$



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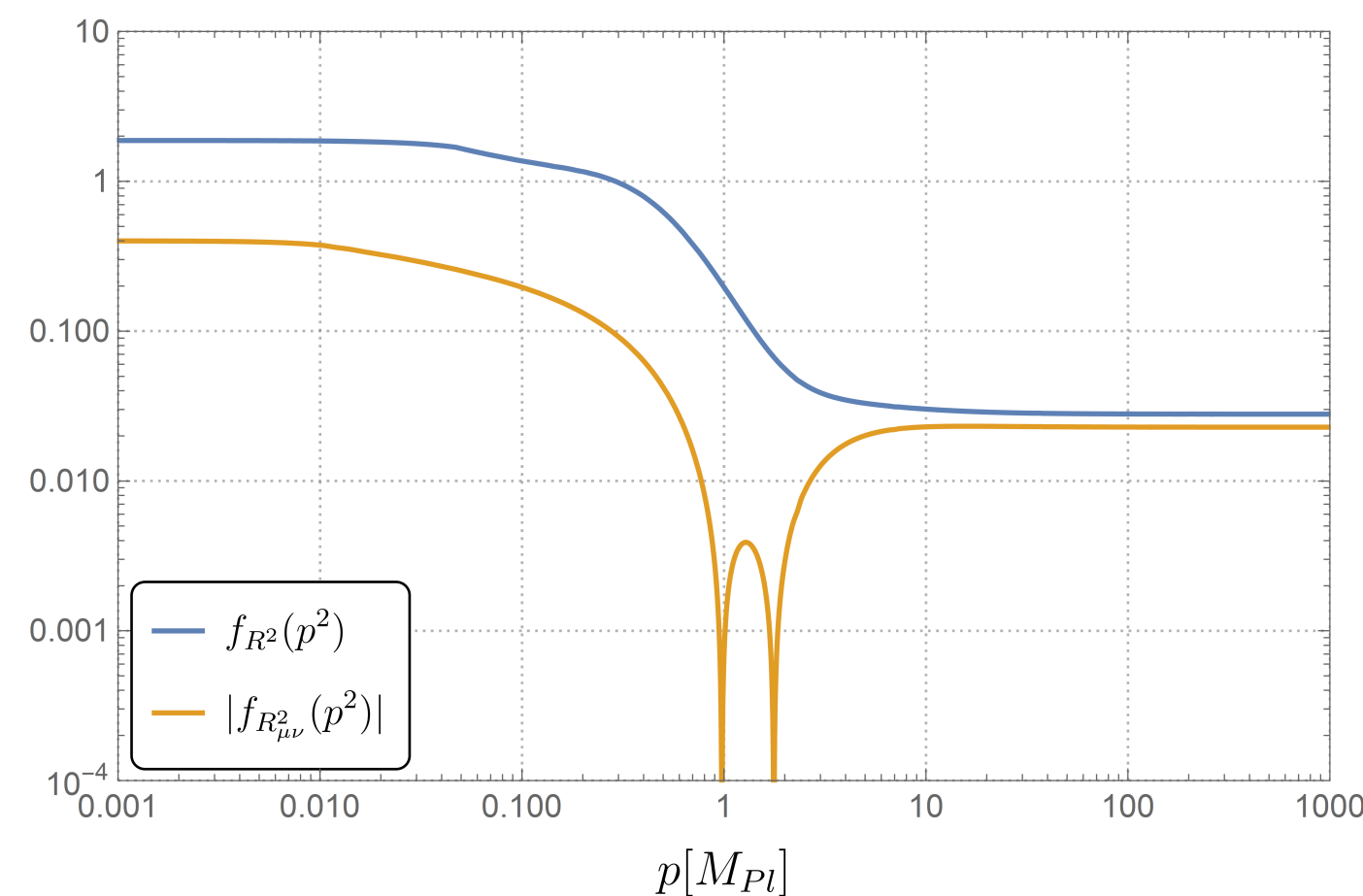
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Maps

$$R_{\mu\nu} f_{R_{\mu\nu}^2}(\Delta) R^{\mu\nu}$$

Results for form factors



$$\mathcal{R}(\Delta, R) = R \frac{\gamma_g^{(3)}(\Delta) - \bar{\gamma}_3 \Delta}{\Delta + R} R$$

Asymptotically black holes

Infrared asymptotic effective action

$$\Gamma_{\text{IR}}[g_{\mu\nu}] = \frac{1}{16\pi} \int_x \sqrt{-g} (G_N^{-1} R + g_{R^2_{\mu\nu}} R_{\mu\nu} R^{\mu\nu} + g_{R^2} R^2 + c_1 R_{\mu\nu} \square R^{\mu\nu} + c_2 R \square R)$$

Spherical symmetric solution

$$ds^2 = -f(r) dt^2 + \frac{1}{g(r)} dr^2 + r^2 d\Omega^2$$

Weak field solutions

$$f(r) = 1 - \frac{2M}{r} + S_0 \frac{e^{-m_0 r}}{r} + S_2 \frac{e^{-m_2 r}}{r}$$

$$g(r) = 1 - \frac{2M}{r} - S_0 \frac{e^{-m_0 r}}{r} (1 + m_0 r) + \frac{1}{2} S_2 \frac{e^{-m_2 r}}{r} (1 + m_2 r)$$

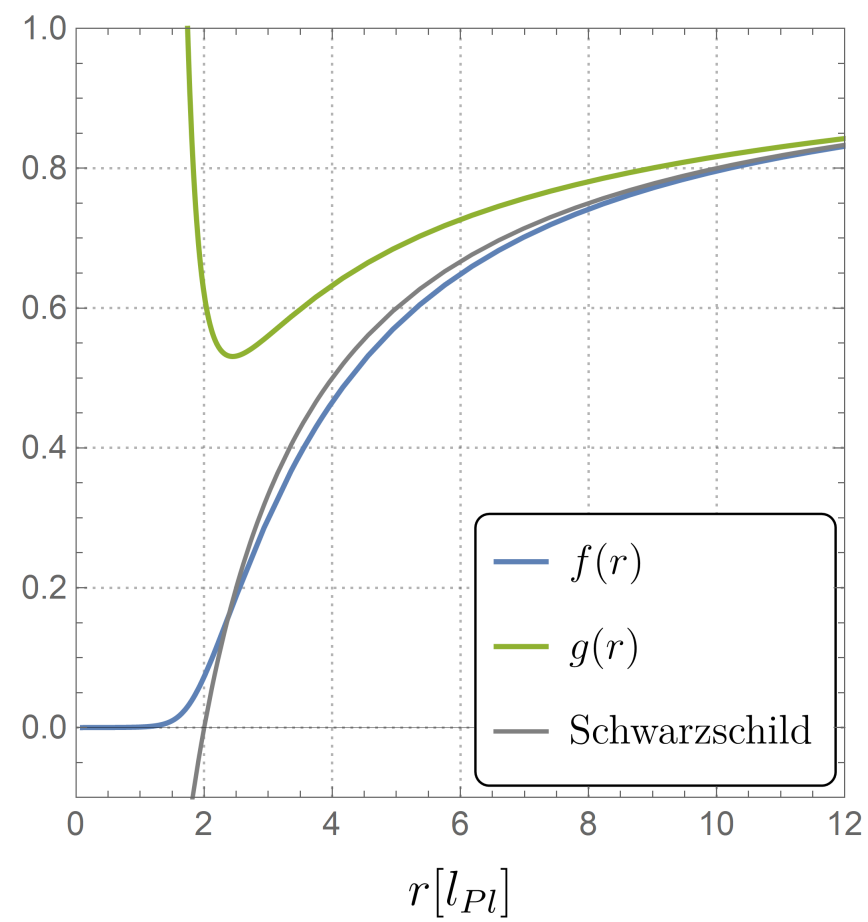
Asymptotically black holes

Weak field solutions

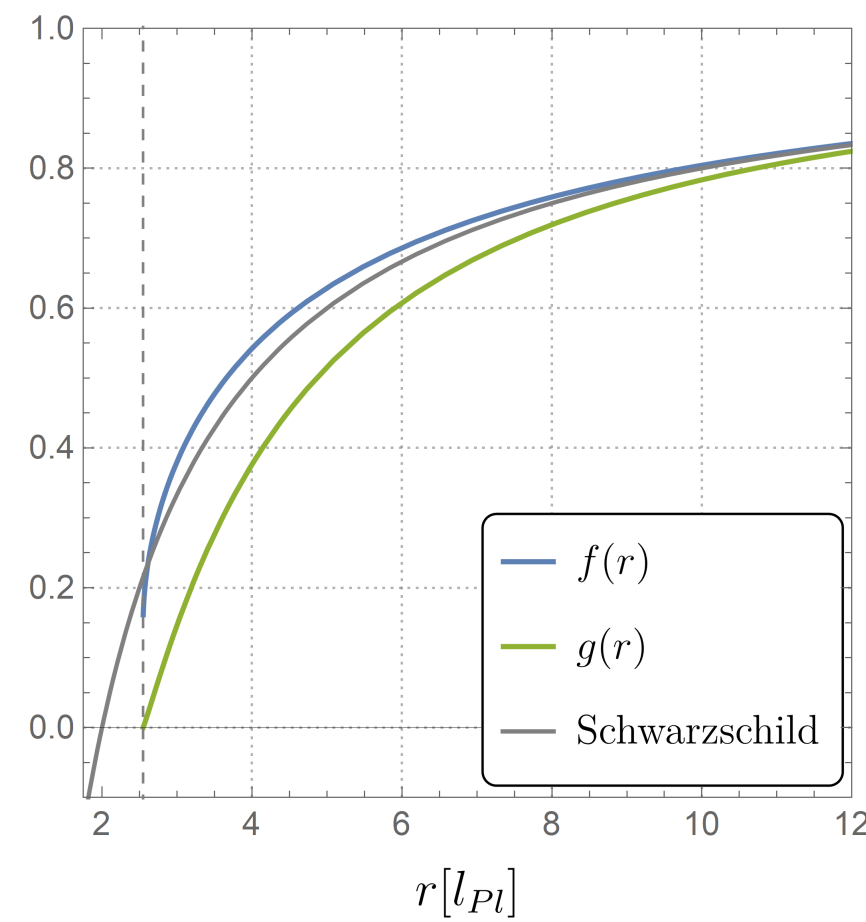
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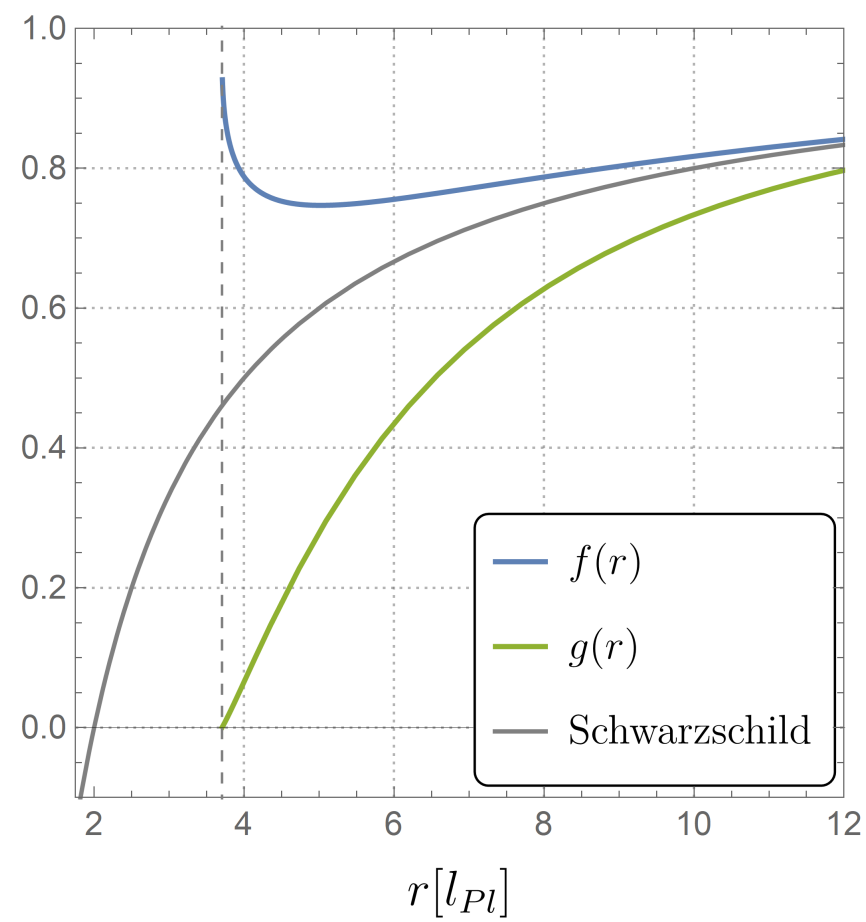
Numerical Solutions for $S_2 = 1 = M$



$$S_0 = -1$$



$$S_0 = 1$$



$$S_0 = 4$$

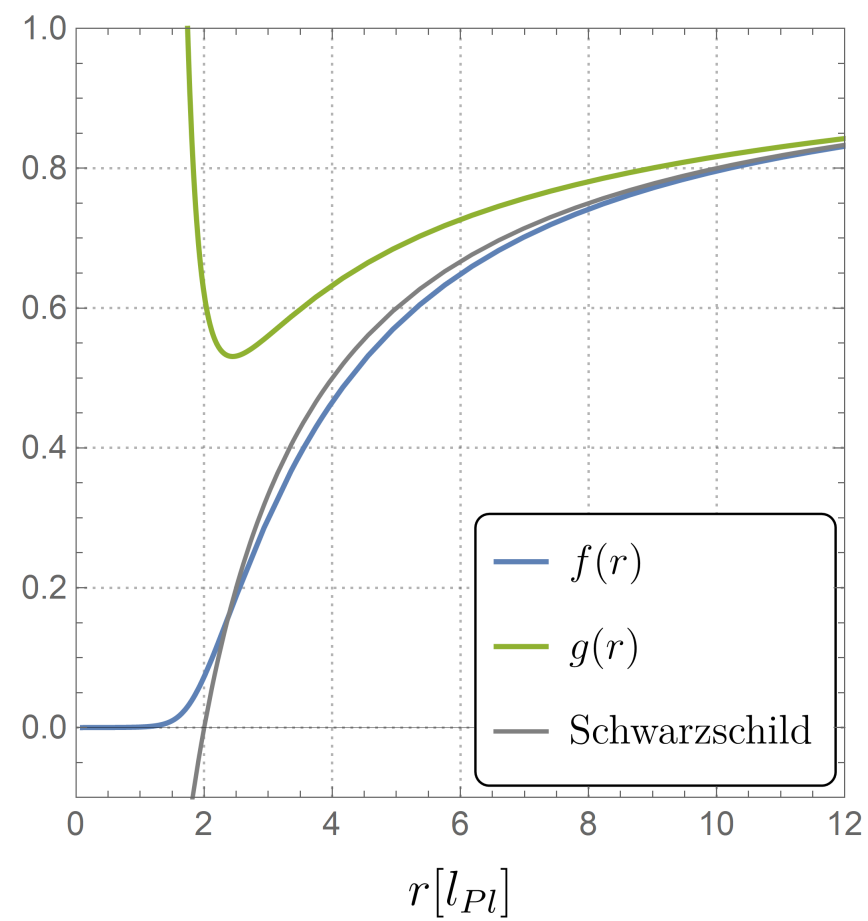
Asymptotically black holes

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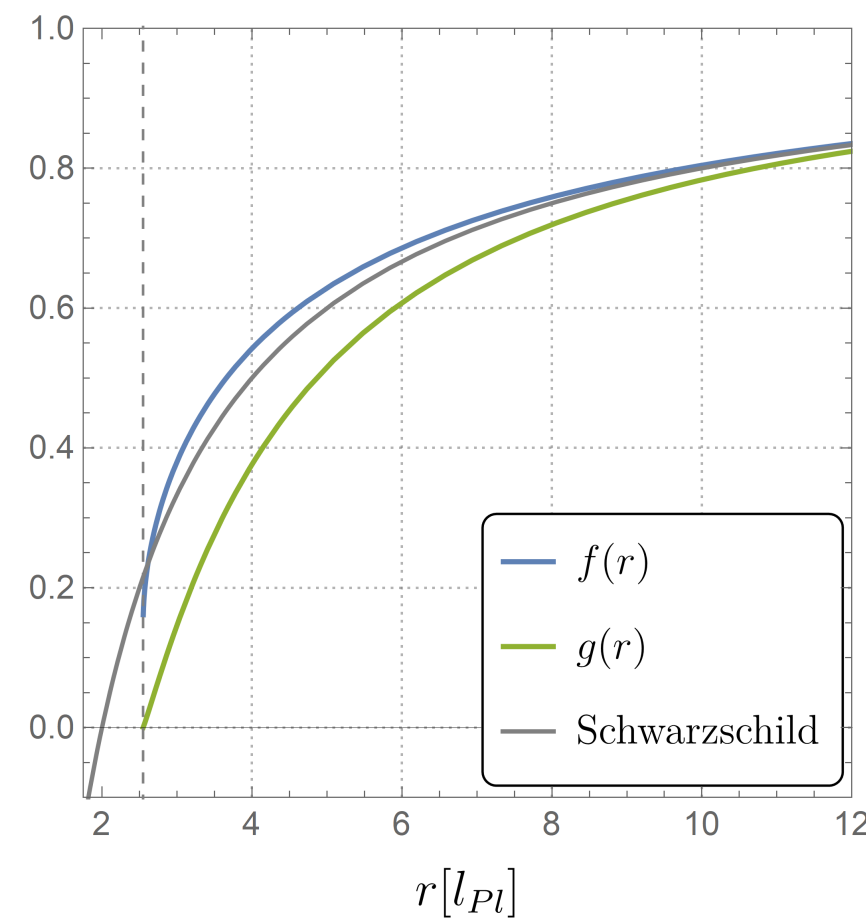
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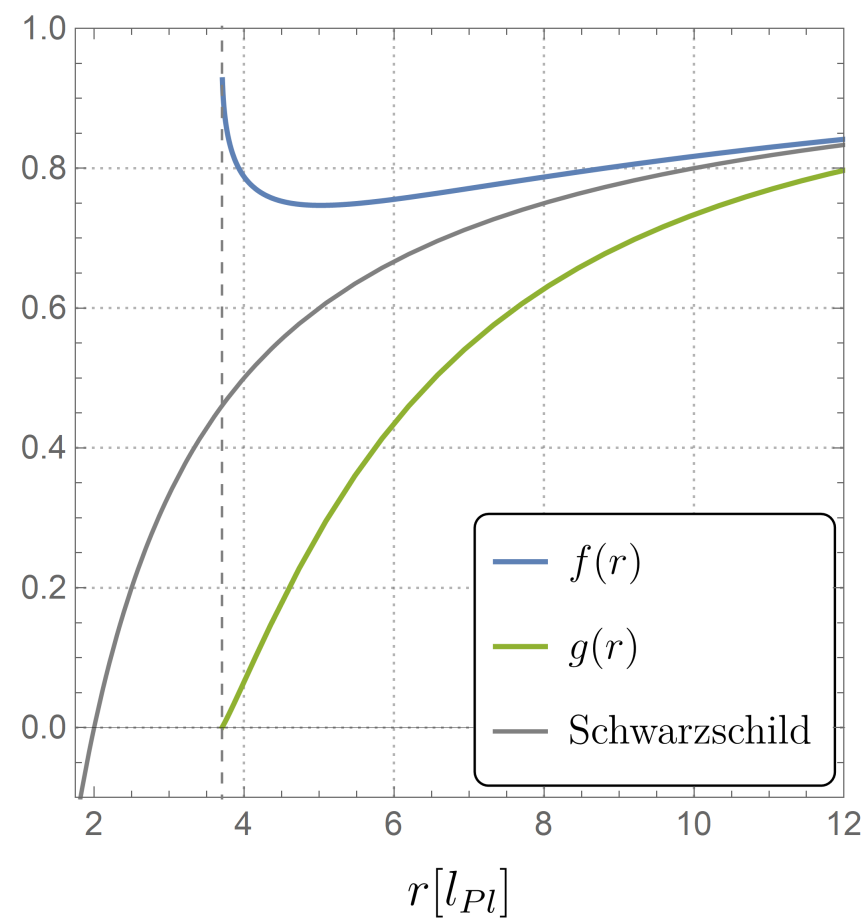
Numerical Solutions for $S_2 = 1 = M$



$S_0 = -1$

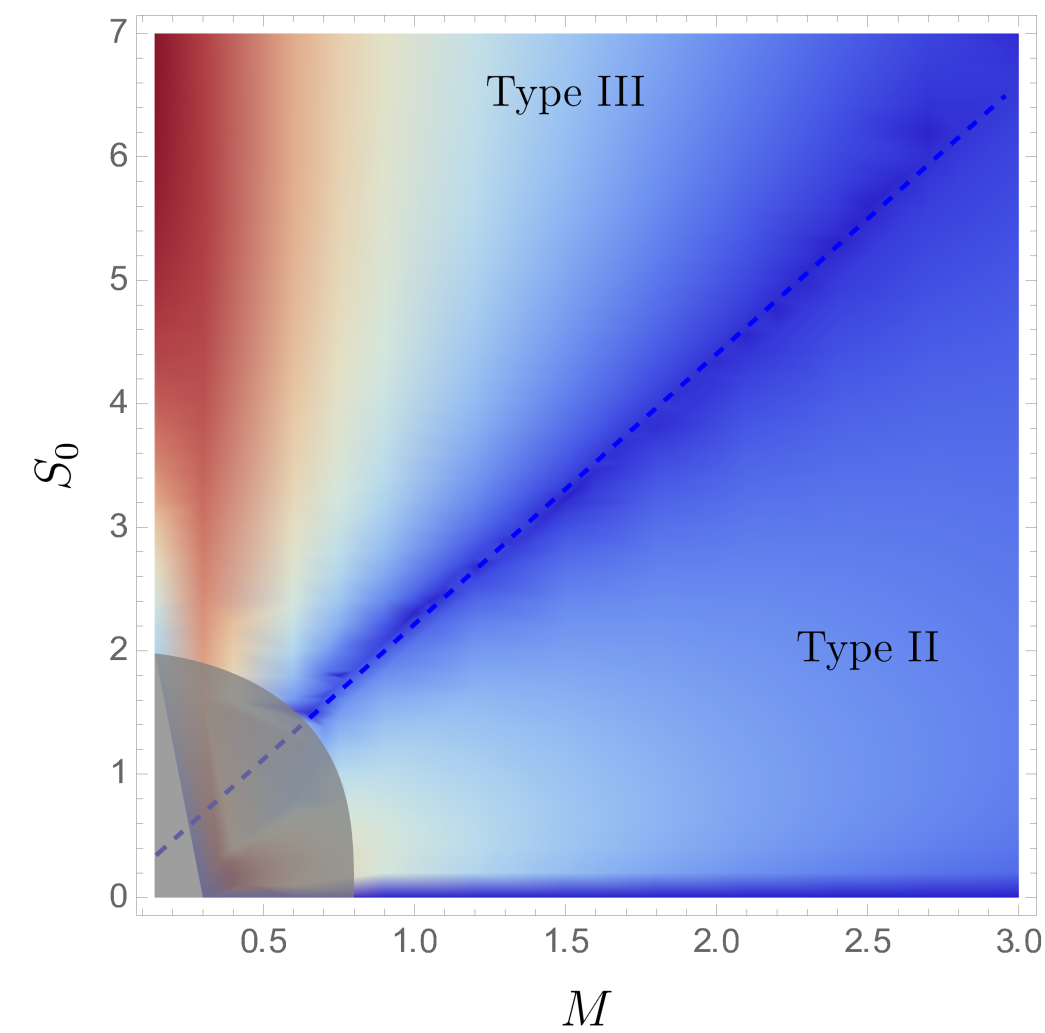


$S_0 = 1$



$S_0 = 4$

Phase structure



$$T = \frac{1}{4\pi} \sqrt{|f'(r_h)g'(r_h)|}$$

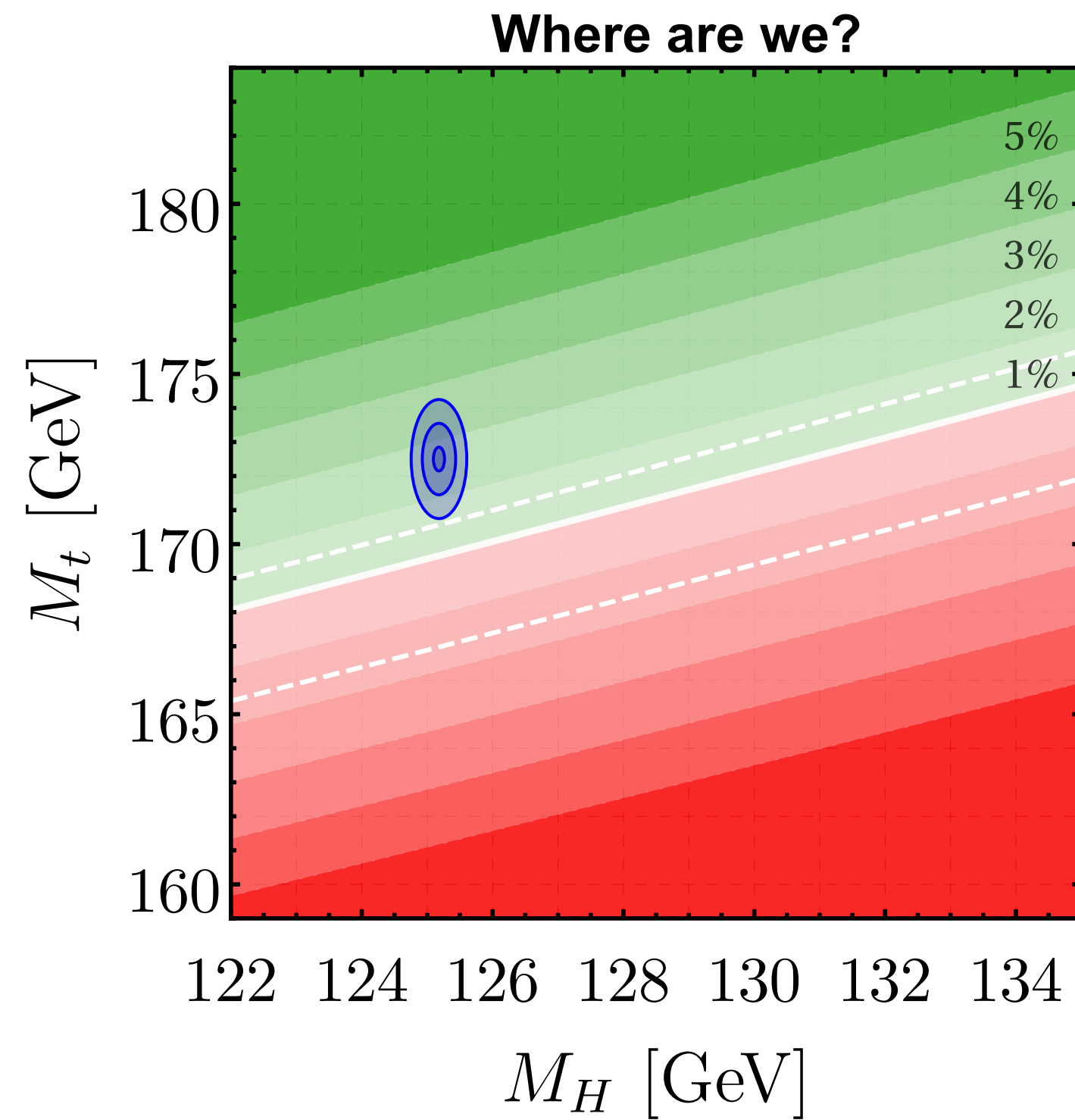
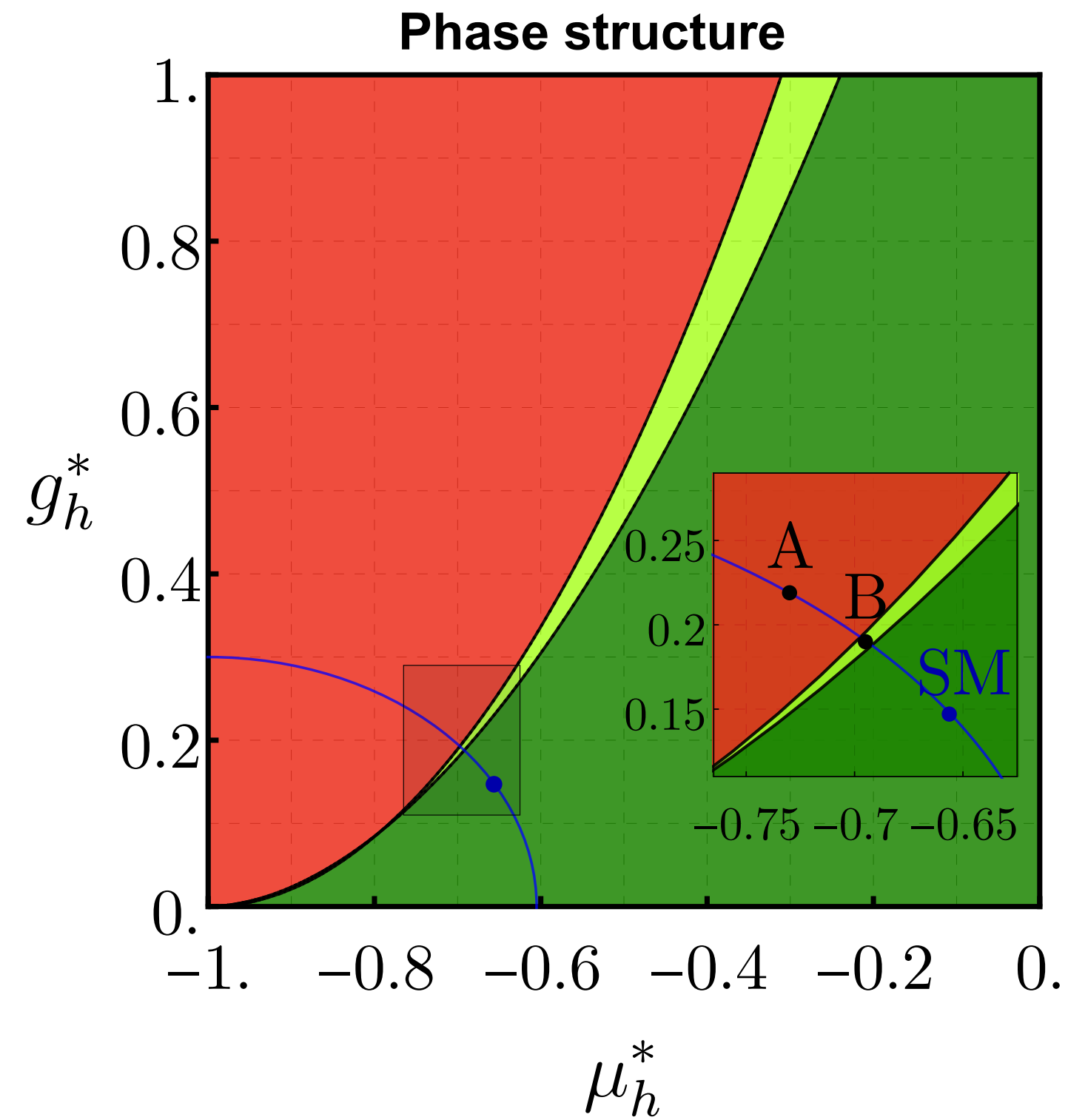
‘Hawking temperature’

see also [Borissova, Held, Afshordi, CQuant.Grav. 40 \(2023\) 7, 075011](#)

Summary

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{[Orange loop]} - \text{[Dashed loop]} - \text{[Black loop]} + \frac{1}{2} \text{[Blue loop with arrow]} + \frac{1}{2} \text{[Double blue loop]} - \text{[Red loop]}$$

Asymptotically safe SM



Quantum black holes

