Asymptotically safe gravity-matter systems: functional and lattice perspectives

Quantum Spacetime and the Renormalization Group 2023

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Motivation: towards combining functional and lattice methods

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  - Outlook: comparing EDT and FRG studies

The "weak gravity bound"

- Specifically in asymptotically safe gravity:
  - There exist indications that metric fluctuations must not be too strong.
  - Interacting nature of gravity induces novel interactions in the matter sector. [Eichhorn and Gies, 2011], [Eichhorn, 2012], [Meibohm and Pawlowski, 2016], [Eichhorn, Held and Pawlowski, 2016], [Christiansen and Eichhorn, 2017], [Eichhorn and Held, 2017], [Eichhorn, Lippoldt and Skinjar, 2017] [Eichhorn, Lippoldt and MS, 2018]
  - Beyond the weak-gravity regime, metric fluctuations can induce novel divergences in these interactions.

Effect of gravity on matter: Induced interactions

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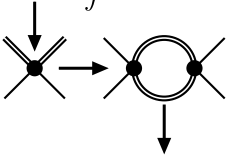

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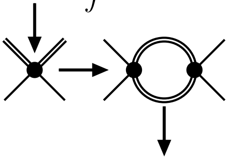

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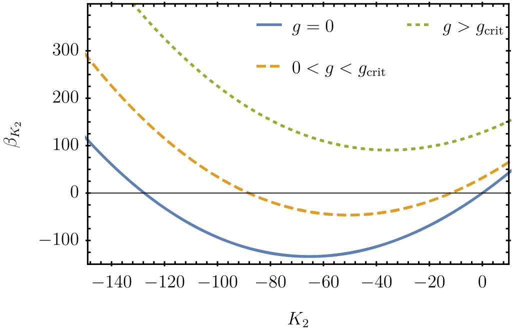
$$\beta_{K_2} = C_0(g) + C_1(g)\,K_2 + C_2\,K_2^2$$



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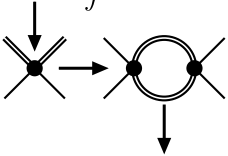

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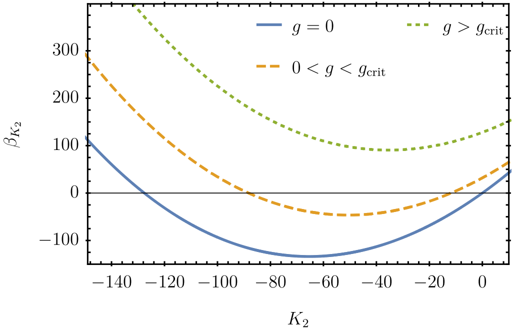

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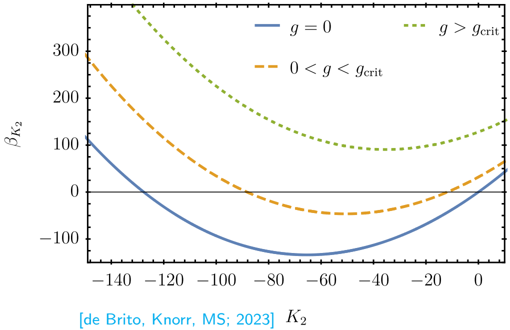
- WGB from fixed-point collision

  [Eichhorn; 2012], [Eichhorn, Held; 2017], ...

The WGB



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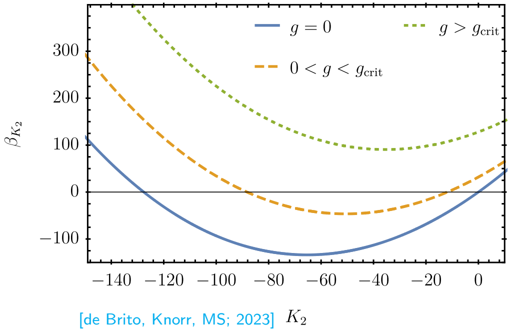

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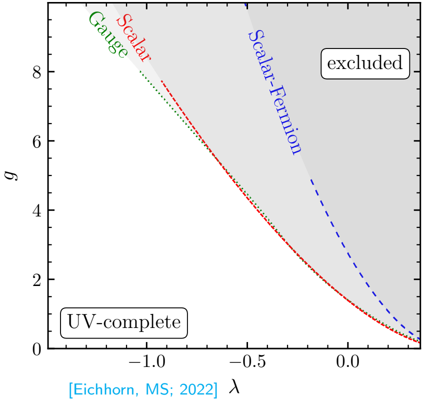

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  - nice feature; constrains gravitational dynamics
  - together with gravitational fixed point: might rule out certain scenarios
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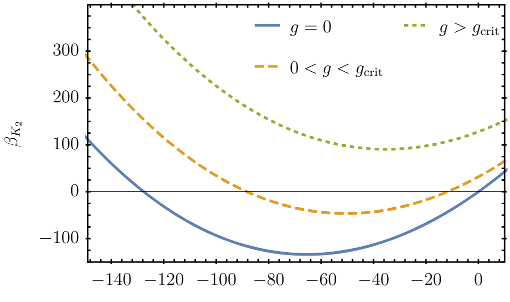

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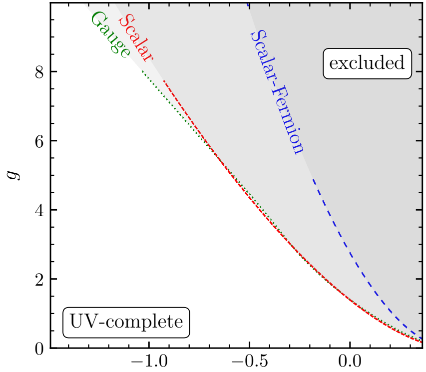

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  - "non-perturbative effect"; large deviation from canonical mass dimension;
  - might require larger truncations to properly understand

Induced shift-symmetric scalar interactions

$$\Gamma_k^{\text{scal.}} = \int \mathrm{d}^4x \, \sqrt{g} \, k^4 \, K(X) \,, \quad \text{and} \quad K(X) \approx X + \sum_{n=2}^{N_{\max}} K_n X^n \, ; g = 0$$

[Laporte, Locht, Pereira, Saueressig; 2022], [de Brito, Knorr, MS; 2023]

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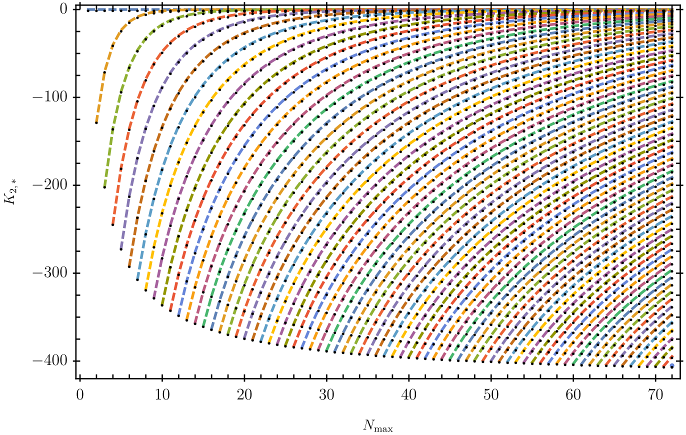

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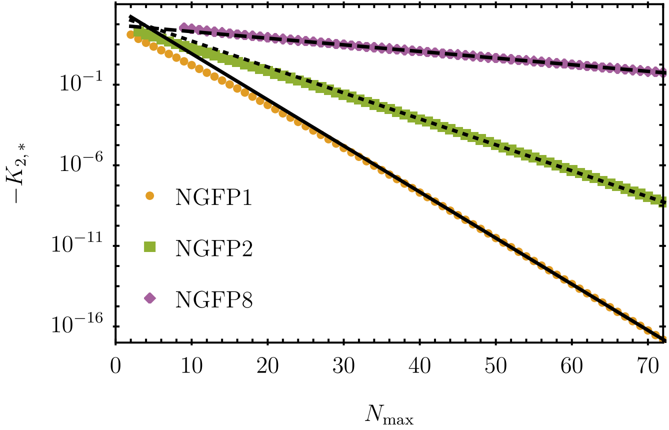

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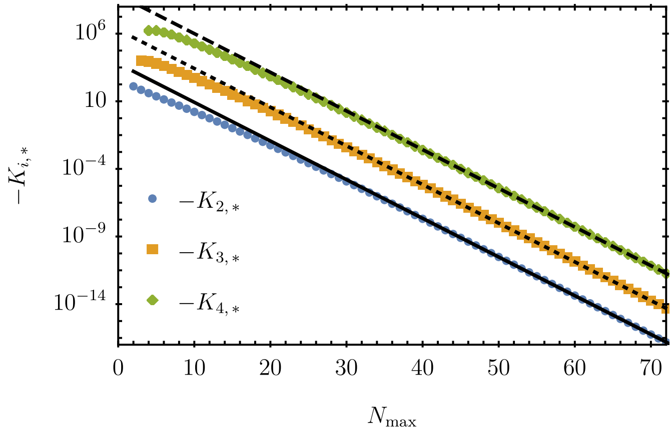

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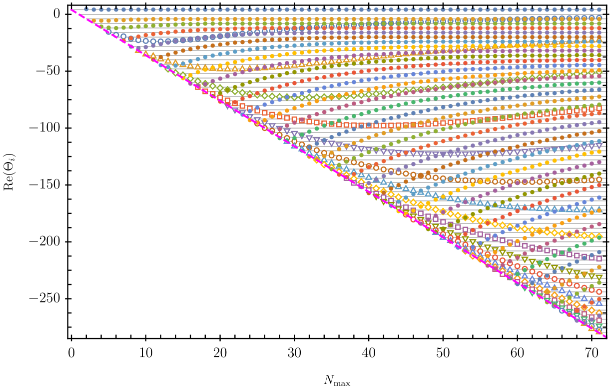

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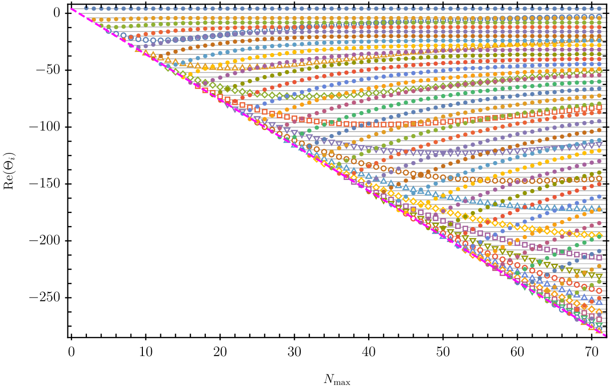

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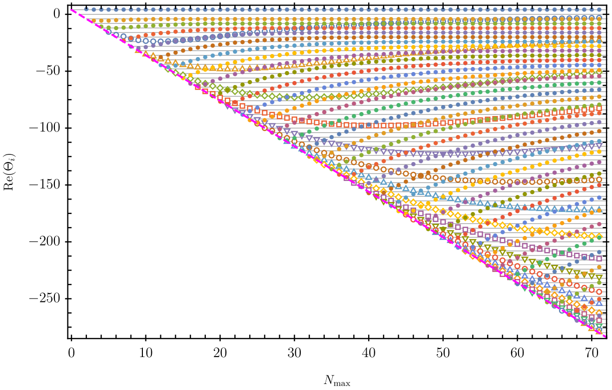

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Key Conclusion

All interacting pure scalar fixed points seem spurious and artefacts of finite-order truncations

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  $\Gamma_k = \Gamma_k^{\mathrm{EH}} + \Gamma_k^{\mathrm{scal.}}\,, \quad \Lambda = 0\,,$ and $\quad g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$
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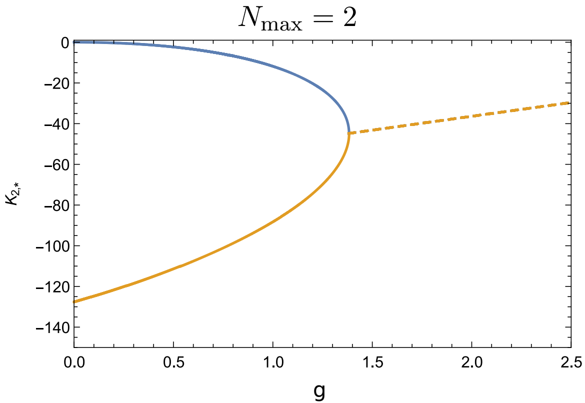

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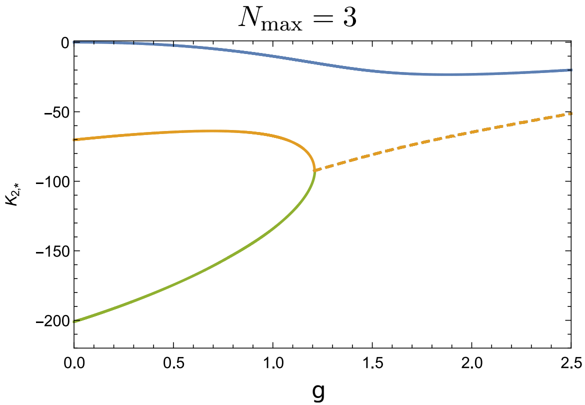

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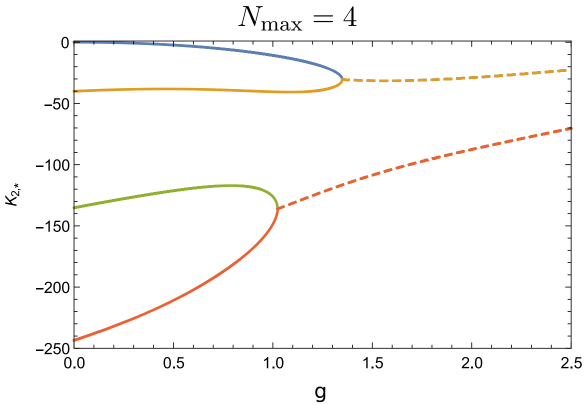

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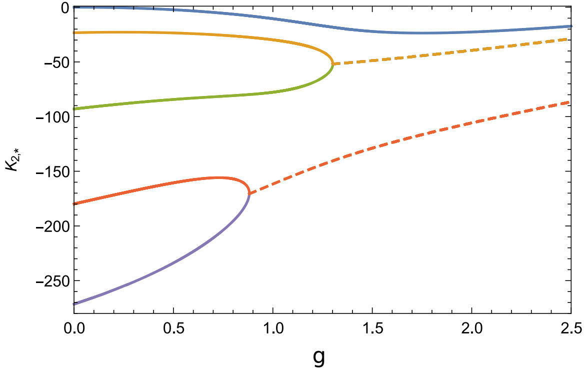
$N_{\text{max}} = 5$



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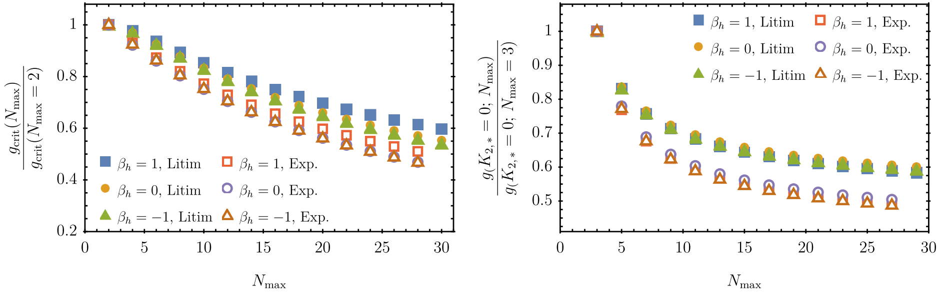

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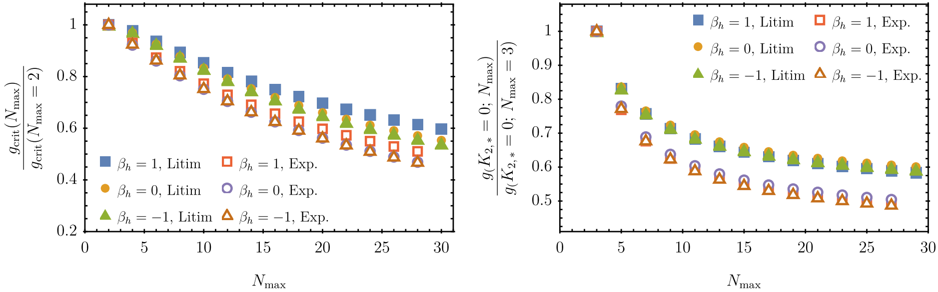

- No apparent convergence of g_{crit}
- For odd N_{max}: convergent and stable SGFP up to (at least) $g \approx 2$.
- WGB as result of FP collision: likely spurious
- New notion of WGB based on number of relevant directions

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- Extend study to $U(1)$ gauge fields [de Brito, Knorr, MS; WIP]
- Couple charged matter, investigate induced interactions at $e_* \neq 0$ [MS; WIP]

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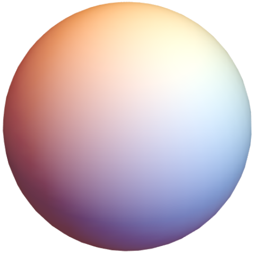
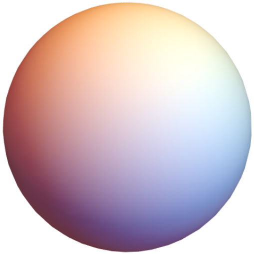


- Discretization of spacetime in terms of triangulations

Evidence for asymptotic safety from Euclidean Dynamical Triangulations



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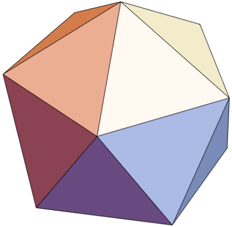

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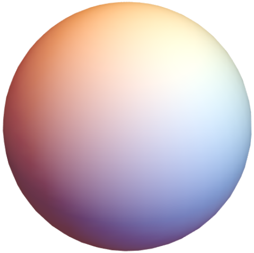

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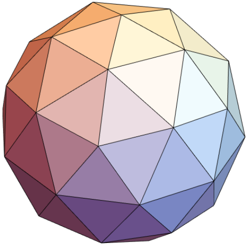

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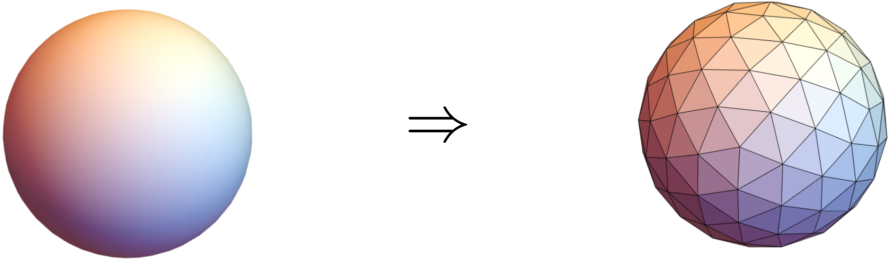

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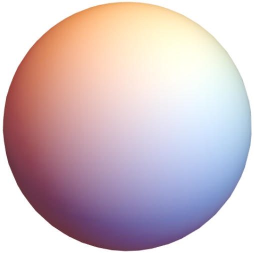

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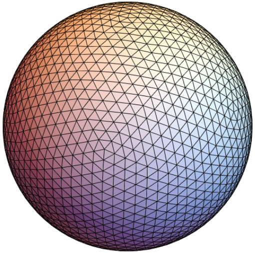

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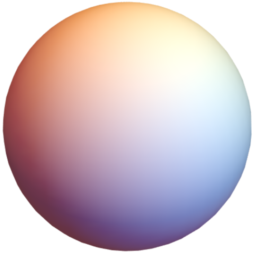

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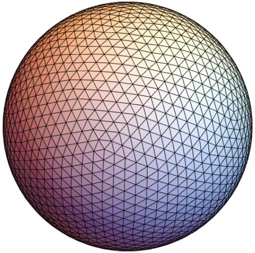

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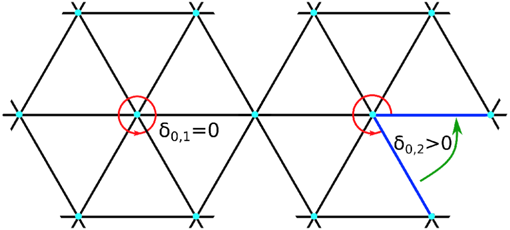
  [Ambjørn and Jurkiewicz, 1992], [Agishtein and Migdal, 1992], . . .

$$\int \mathcal{D}g \, e^{-S[g]} \to \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}} \, e^{-S_{\mathrm{ER}}}$$

with Euclidean Einstein-Regge action $S_{\mathrm{ER}} = -\kappa_2 N_2 + \kappa_4 N_4$ [Regge, 1961]

The Einstein-Regge action



- starting point: Einstein-Hilbert action

$$S_{\mathrm{EH}} = -\frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}^4x \sqrt{g}\, (R - 2\Lambda)$$

- Einstein-Regge action: [Regge, 1961]

$$S_{\mathrm{ER}} = -\frac{1}{8\pi G_{\mathrm{N}}} \left(\sum_{s_2} V_{s_2} \delta_{s_2} - \Lambda \sum_{s_4} V_{s_4} \right) = \kappa_4\, N_4 - \kappa_2\, N_2$$

Lattice quantum gravity in $d = 4$

- Discretization of spacetime in terms of triangulations

$$\int \mathcal{D}g \, e^{-S[g]} \rightarrow \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}} \qquad\qquad e^{-S_{\mathrm{ER}}}$$

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- in $d = 4$: no physical phase, no indications for higher-order transition in κ_2-κ_4 - space

[Ambjørn, Jain, Jurkiewicz, Kristjansen, 1993], [Bakker, Smit, 1994]
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$$\int \mathcal{D}g\, e^{-S[g]} \rightarrow \sum_{\mathcal{T}} \frac{1}{C_{\mathcal{T}}} \qquad\qquad e^{-S_{\mathrm{ER}}}$$

- in $d = 4$: no physical phase, no indications for higher-order transition in κ_2-κ_4 - space
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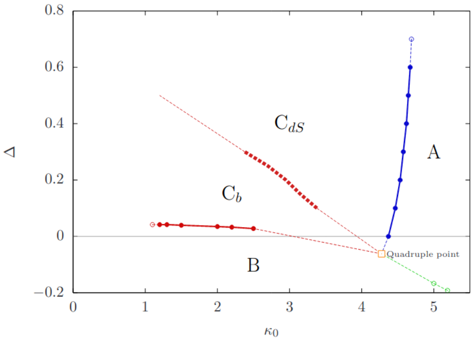

Taken from [Loll, 2020]

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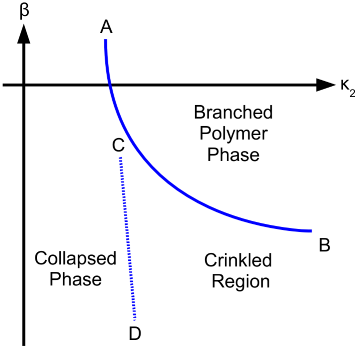

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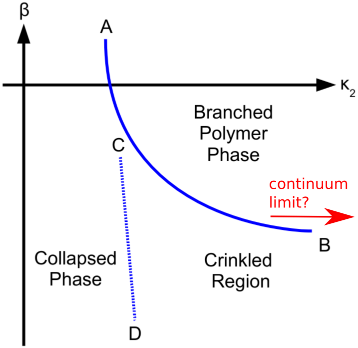

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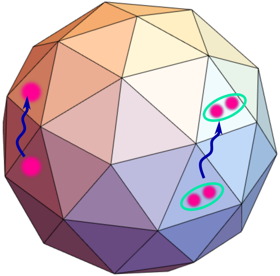
Newtonian binding energy from EDT



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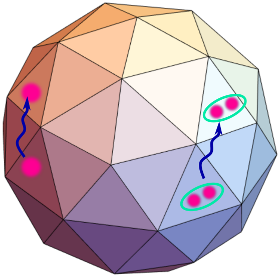

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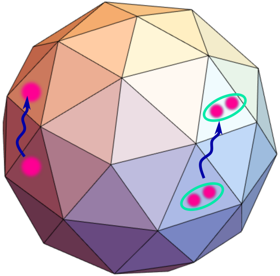

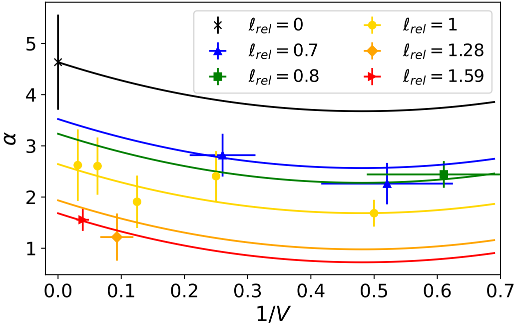

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- Continuum, non-relativistic limit: $E_{\mathrm{b}} = \frac{G\, m^5}{4}$ (in $d = 4$)
- EDT fit: $\alpha = 4.6 \pm 0.9$ $\Rightarrow\ d = 3.9 \pm 0.2$

EDT might feature well-behaved non-relativistic limit! [Laiho, MS, Unmuth-Yockey; 2021]

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- Generalize algorithms used in studies of dynamical systems (e.g., growth of crystals)

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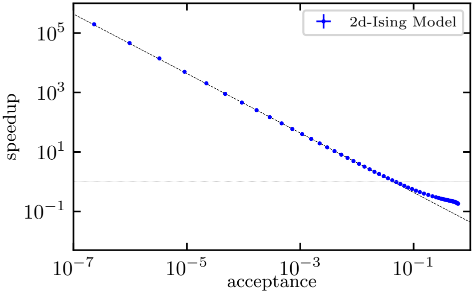

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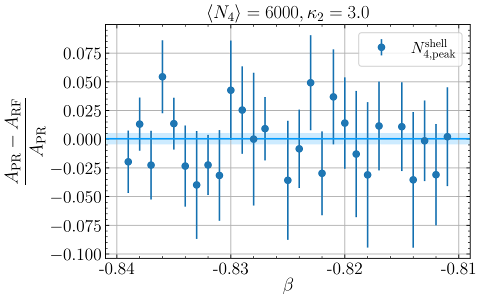

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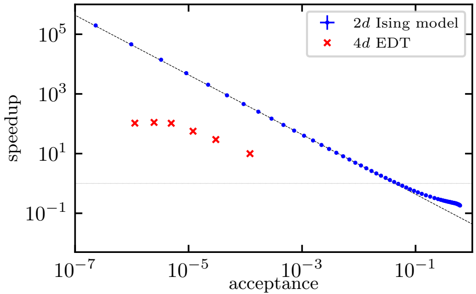

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Significant speedup: allows to simulate efficiently at larger κ_2 (finer lattices)

Towards a de Sitter universe in EDT?

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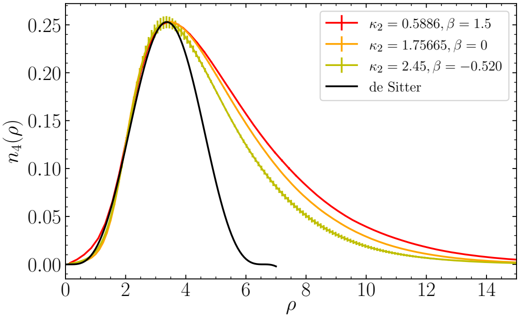

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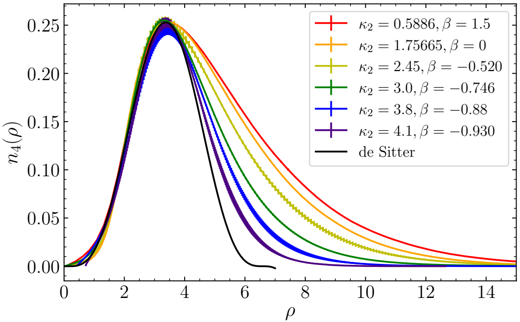

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Lattice volume profiles: approximate de Sitter profile better for larger κ_2

Summary and Outlook

- Evidence that EDT might be suitable tool to discover asymptotic safety
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  and compare with FRG results
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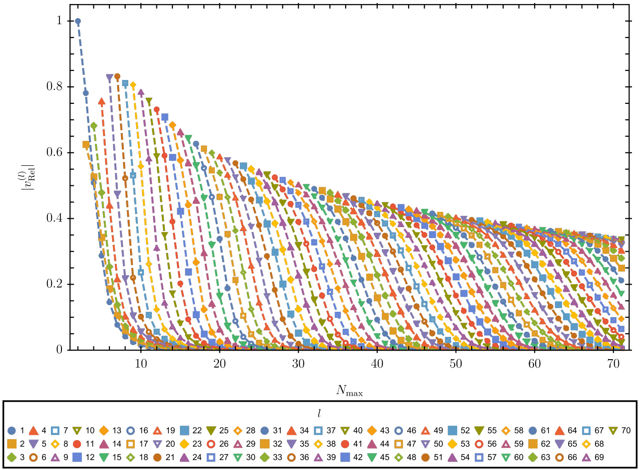
Stay tuned! **Thank you for your attention!**

Pure scalar system: Eigenvectors

- Focus on relevant direction v_{Rel} of NGFP1
- $v^{(l)}_{\mathrm{Rel}}$ points in direction of K_{l+1}



- Procedure: [Kluth, Litim; 2020]
  - Rescale couplings s.t. rows and columns of matrix of EV's are normalized
    $\Rightarrow$ all K_n contribute equally to system of EV's
- Key properties:
  - Overlap with $K_{l<N_{\mathrm{max}}}$: decreases rapidly
  - Most dominant overlap with v_{Rel}: $K_{N_{\mathrm{max}}}$
  - If existent: FP is highly non-perturbative

Pure scalar system: Eigenperturbations

- Since $K_{2,*} \to 0$:
  understand Eigenperturbations of NGFP1 by perturbing around GFP;

$$K(X) = X + \epsilon\, e^{-\Theta\, t}\, \delta K\,, \qquad \eta_\Phi = 0 + \epsilon\, e^{-\Theta\, t}\, \delta\eta_\Phi\,.$$

  At order ϵ: Flow equation is inhomogeneous differential equation for δK
- Absorb $\delta\eta_\Phi$ by shift $\delta\tilde{K} = \delta K - \delta\eta_\Phi\,(a + b\,X)$ (for $\Theta \neq 0$ and $\Theta \neq 4$).
- Bring in Sturm-Liouville form (with $y \sim X$):

$$\partial_y \left[p(y)\, \delta\tilde{K}'(y)\right] = -\lambda\, w(y)\, \delta\tilde{K}(y)\,,$$

$$\text{with} \quad p(y) = y^2 e^{-y} \geq 0\,, \quad w(y) = y\, e^{-y} \geq 0\,, \quad \lambda = 1 - \frac{\Theta}{4}\,.$$

- Expect discrete Eigenspectrum for λ, which is bounded from below;
  Corresponding Eigenfunctions: square integrable with respect to measure $w(y)$.

Pure scalar system: Eigenperturbations II

- Solutions (for $\Theta \neq 0$ and $\Theta \neq 4$):

$$\delta\tilde{K}(y) = c_1\,{}_1F_1\left(\frac{\Theta}{4} - 1; 2\middle| y\right) + c_2\, G^{2,0}_{1,2}\left(y\middle|\begin{array}{c} 2-\frac{\Theta}{4} \\ -1, 0 \end{array}\right),$$

- Regularity at $y = 0 \Rightarrow c_2 = 0$
- Normalisability w.r.t $w(y)$:

  investigate asymptotic behavior of $\delta\tilde{K}(y)$:

$$\delta\tilde{K}(y) \sim \frac{c_1}{\Gamma\left(\frac{\Theta}{4} - 1\right)} y^{\frac{\Theta}{4}-3}\, e^y, \qquad y \to \infty.$$

  Not normalisable w.r.t. $w(y)$, except if $\Theta = 4 - 4n, \quad n \in \mathbb{N}, n > 1$.

  - Similarly,

$$\left.\begin{array}{llll} \Theta = 0: & \delta\tilde{K}(y) \sim & \frac{4c_1}{(5-2\gamma)y^3} e^y, & y \to \infty, \\ \Theta = 4: & \delta\tilde{K}(y) \sim & \frac{c_1}{y^2} e^y, & y \to \infty, \end{array}\right\} \text{Not normalisable w.r.t. } w(y).$$

- $\Theta = 4$ can arise as eigenperturbation of GFP,

  but is not normalisable, hence should be discarded.

Gravity-scalar system: Expansion in g

- Alternative expansion ($\tilde{X} = \frac{3}{2}(16\pi)^2 X$):

$$K(X) \approx X + \left(\frac{1}{16\pi}\right)^2 \sum_{n=1}^{N_{\max}} \left(\frac{g}{16\pi}\right)^n L_n(\tilde{X})\,, \qquad \text{and} \qquad \eta_\phi = \sum_{n=1}^{N_{\max}} \left(\frac{g}{16\pi}\right)^n \eta_n\,.$$

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$$L_{2,*}(\tilde{X}) = -\frac{32}{9}\tilde{X}^2 \,, \qquad \eta_{2,*} = -128 \,, \qquad \text{generally} \qquad L_{n,*}(\tilde{X}) = \sum_{i=2}^{n} \ell_{n,i}\tilde{X}^i \,.$$

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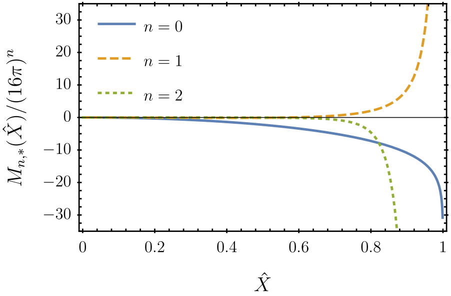
- Unfortunately, no global X-information
- But: Regularity + non-exponential growth fixes **all** constants of integration

Gravity-scalar system: Combined expansion

- Use combined expansion to capture global X dependence:

$$K(X) \approx X + \sum_{n=0}^{N_{\max}} \left(\frac{g}{16\pi}\right)^n M_n(\hat{X}), \quad \text{with } \hat{X} = 16\pi\, g\, X\,.$$

- Rather complicated expression;
  Only achieved up to M_2.



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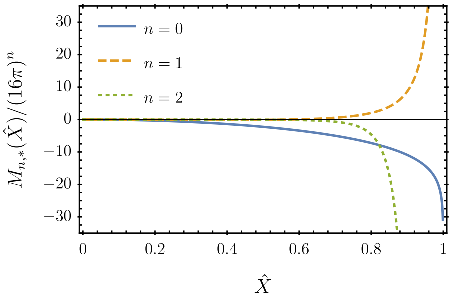
- Rather complicated expression;
  Only achieved up to M_2.
- Common feature:
  divergence at $\hat{X} = 1$

$$M_{n,*}^{\text{pole}}(\hat{X}) \sim \frac{\sqrt{2}\,(-6)^{n+1}\mathcal{B}_n}{(1-\hat{X})^{3(n-1)+3/2}}\,.$$

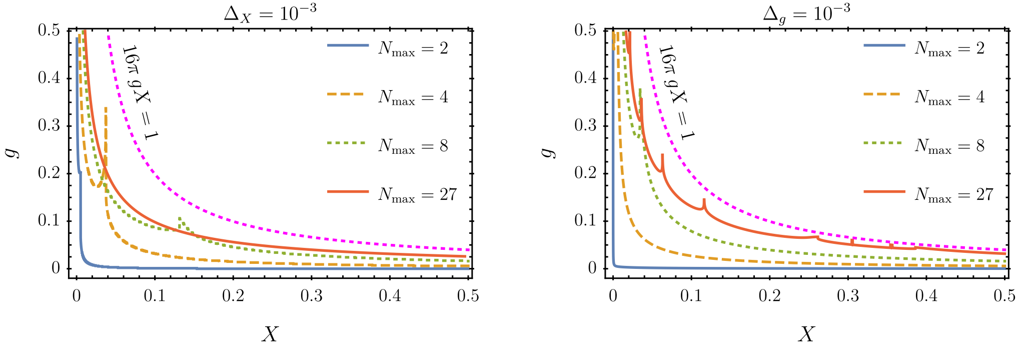


- Nature of pole: likely off-shell

Gravity-scalar system: Analysis of truncation error

- Upon expansion of $K(X)$ to finite order:

$$\Delta(X,g) = \left|\frac{\text{LHS} - \text{RHS}}{\text{LHS}}\right|_* \neq 0$$



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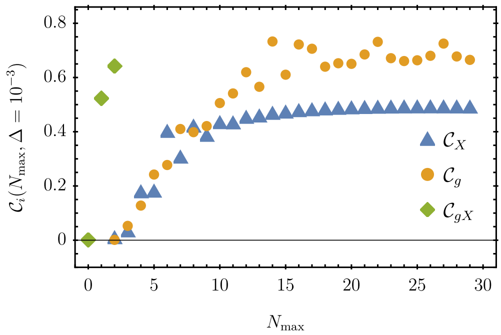
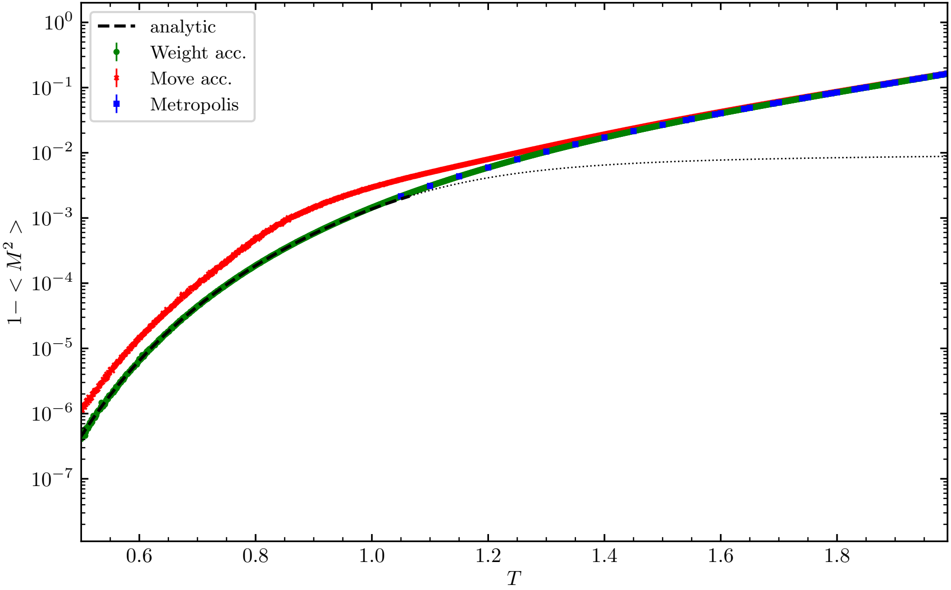


Ising validation I



Ising validation II