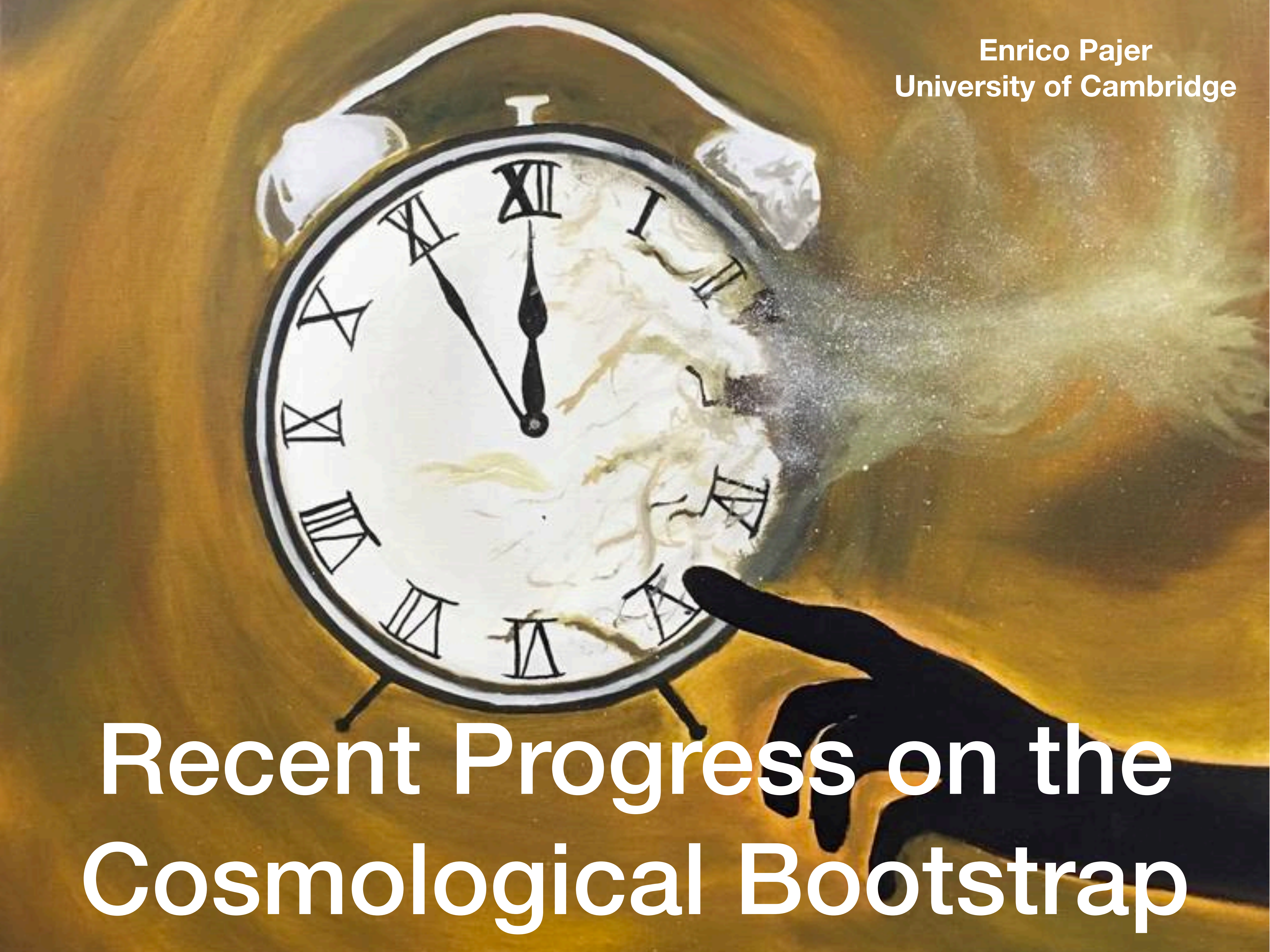
Enrico Pajer
University of Cambridge



Recent Progress on the Cosmological Bootstrap

Table of contents

Primordial Cosmo = QFT in curved spacetime / Quantum Gravity

- On large scales (>> Mpc) cosmological surveys measure QFT correlators of metric fluctuation

$$\langle \prod^{n} \delta(k_a) \rangle \sim \int_k \left[\prod^{n} \Delta^{(Y)}(k_a) \right] \langle \prod^{n} \zeta(k_a) \rangle$$

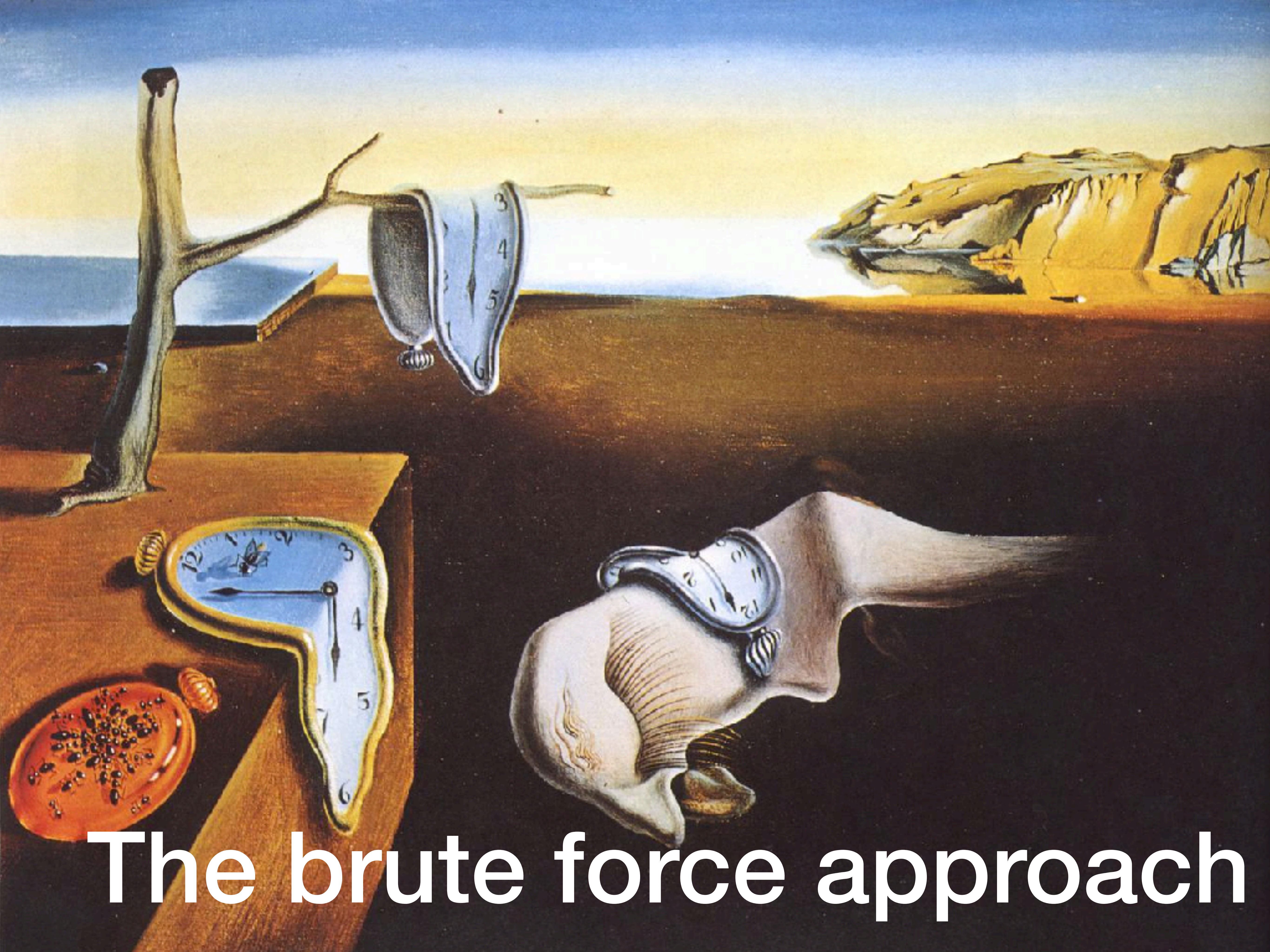
CMB temperature
density of galaxies
dark matter, …

QFT / QG
in de Sitter

- Gravitational floor of non-Gaussianity in single-clock inflation: $f_{NL}^{eq} \gtrsim 10^{-2}$
- The goal of primordial cosmology is to understand QFT and QG in (approximately, asymptotically) de Sitter

Aspirations

- We want to learn about fundamental physics from cosmo:
  - *New degrees of freedom and their interactions*: Inflation requires at least one degree of freedom and three energy scales beyond the standard model.
  - The laws of gravity at short distances/high energies: probe GR and beyond at high energies
  - QFT in FLRW/de Sitter: which theories are consistent?
  - Quantum gravity in dS? Holography, string theory?



The brute force approach

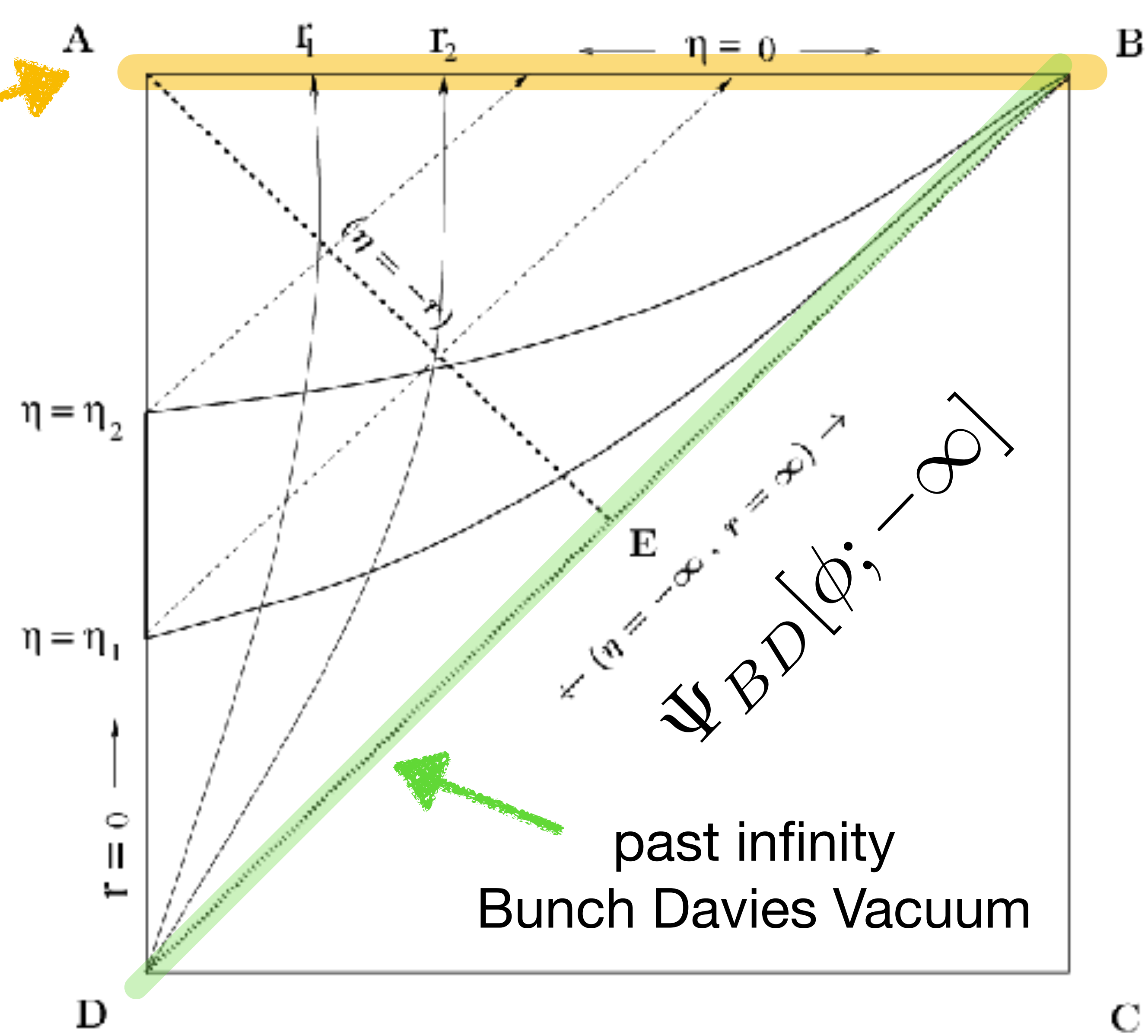
Penrose diagram

- We work in the Poincare' patch (half of dS)

$$ds^2 = -dt^2 + a^2 dx^2 = a^2(-d\eta^2 + dx^2)$$

$$\Psi[\phi; \eta \to 0]$$

The future (conformal) boundary can be thought as the reheating surface after inflation and determines the statistics of *LSS and CMB observations*



Correlators

- The observables of cosmology are correlators of the product of equal-time local operators $\mathcal{O}$ at $\eta \to 0$

$$\lim_{\eta \to 0} \langle \Omega | \prod_{a=1}^{n} \mathcal{O}(\mathbf{k}_a, \eta) | \Omega \rangle \equiv \langle \prod_{a=1}^{n} \mathcal{O}(\mathbf{k}_a) \rangle \equiv \langle \mathcal{O}^n \rangle \, .$$

- they are usually computed in the interaction picture

$$\langle \mathcal{O}(\eta) \rangle = \langle 0 | \left[\bar{T} e^{\left(i \int_{-\infty(1+i\epsilon)}^{\eta} d\eta' H_{\rm int}(\eta') \right)} \right] \mathcal{O}_I(\eta) \left[T e^{\left(-i \int_{-\infty(1-i\epsilon)}^{\eta} d\eta' H_{\rm int}(\eta') \right)} \right] | 0 \rangle$$

Bunch-Davies

time evolution operator

- We know the Feynman rules to compute correlators in perturbation theory

The wavefunction

- The field theoretic *wavefunction* is the projection of the quantum state $|\Psi\rangle$ of the system onto eigenstates $|\phi\rangle$ of the field operators, $\hat{\phi}(x,\eta)|\phi\rangle = \phi(x,\eta)|\phi\rangle$, namely: $\Psi[\phi,\eta] \equiv \langle\phi|\Psi,\eta\rangle$

- It is a functional of the all fields in the theory (including the metric) at some time. It can be parameterised in terms of wavefunction coefficients ψ_n

$$\Psi[\phi,\eta] = \exp\left[\sum_{n=2}^{\infty}\int_{\mathbf{k}} \psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n;\eta)\prod_a^n \phi(\mathbf{k}_a)\right]$$

- This Ψ is a large-volume solution of the wavefunction of the universe, which solves the Wheeler de Witt equation

From Ψ to correlators

- All probabilities can be computed from Ψ as in QM

$$\langle \mathcal{O} \rangle = \int d\phi \Psi^* \mathcal{O} \Psi$$

- The Ψ_n are closely related to *cosmological correlators*, which determine the statistics of the Cosmic Microwave Background and Large Scale Structures. For example

$$\langle \phi_{\mathbf{p}}(\eta_0) \phi_{-\mathbf{p}}(\eta_0) \rangle' = \frac{1}{2 \operatorname{Re} \psi_2(p)},$$

$$\left\langle \prod_{a=1}^{3} \phi_{\mathbf{p}_a}(\eta_0) \right\rangle' = -2 \prod_{a=1}^{3} \frac{1}{2 \operatorname{Re} \psi_2'(p_a)} \left[\psi_3(\mathbf{p}) + \psi_3(-\mathbf{p}) \right],$$

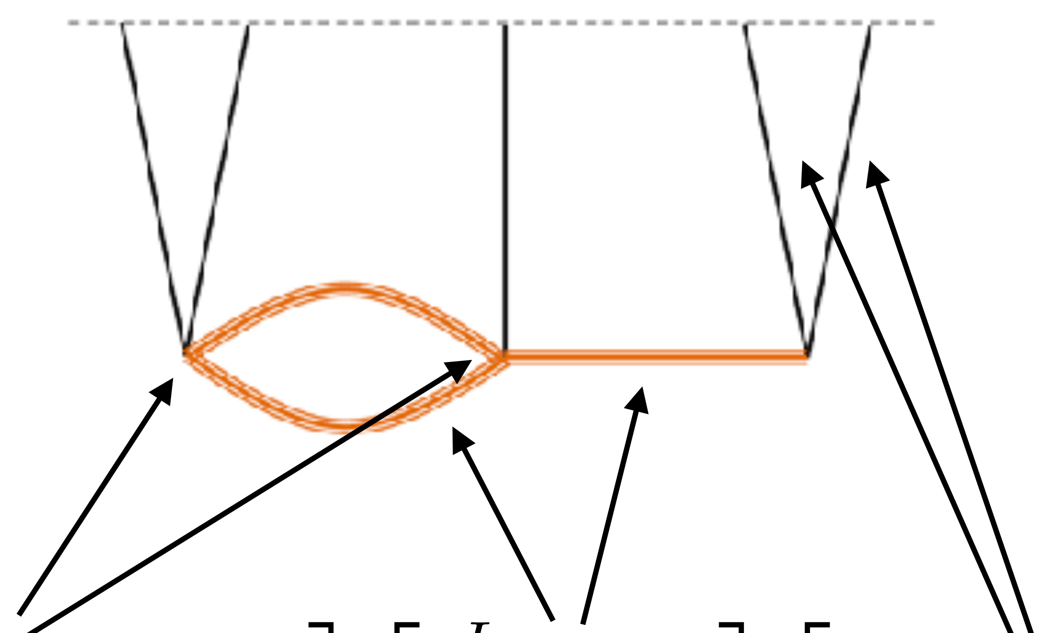
$$\left\langle \prod_{a=1}^{4} \phi_{\mathbf{p}_a}(\eta_0) \right\rangle' = -2 \prod_{a=1}^{4} \frac{1}{2 \operatorname{Re} \psi_2(p_a)} \left[\left[\psi_4(\mathbf{p}) + \psi_4(-\mathbf{p}) \right] \right.$$
$$\left. - \frac{\left[\psi_3(\mathbf{p}_1, \mathbf{p}_2, -\mathbf{s}) + \psi_3(-\mathbf{p}_1, -\mathbf{p}_2, -\mathbf{s}) \right] \left[\psi_3(\mathbf{p}_3, \mathbf{p}_4, \mathbf{s}) + \psi_3(-\mathbf{p}_3, -\mathbf{p}_4, -\mathbf{s}) \right]}{\operatorname{Re} \psi_2'(s)} - t - u \right].$$

Feynman Diagrams

- Given a model, the wavefunction can be computed perturbatively from a path integral

$$\Psi[\phi(\mathbf{x});\eta_0] = \int_{\text{vacuum}}^{\phi(\mathbf{x},\eta_0)} [\mathcal{D}\phi]\, e^{iS[\phi]}$$

with the following Feynman rules (equivalent to in-in):



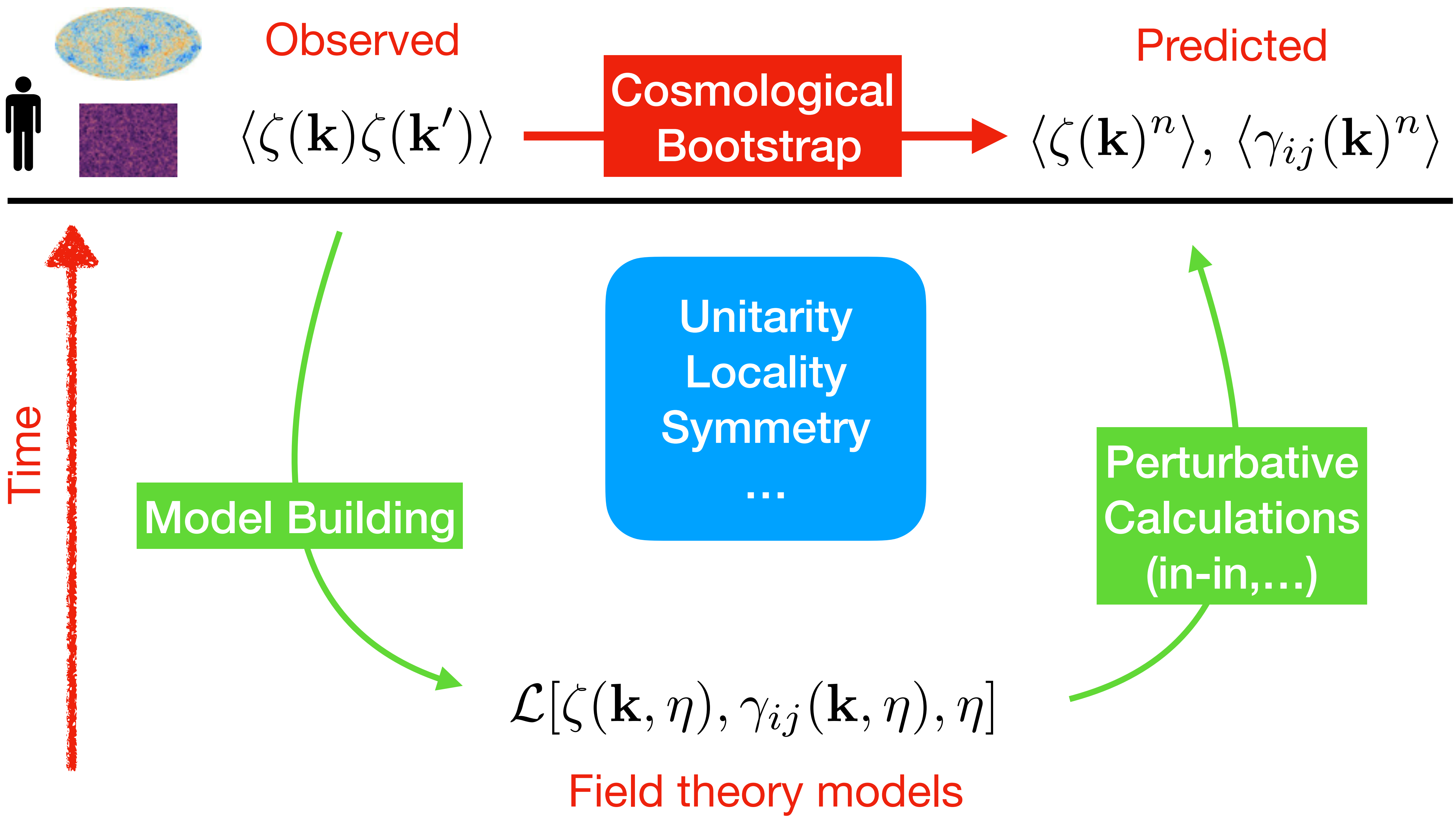
$$\psi_n = \left[\prod_A^V \int d\eta_A\, F_A\right]\left[\prod_m^I G(p)\right]\left[\prod_a^n K(k_a)\right]$$

Problems

Here's some obstacles:

- Calculations are hard: V-nested time integrals at tree-level!
- Gravity is harder: $\langle \gamma_{ij}^4 \rangle$ only computed last year…
- *Too many models* for too little data on primordial universe. Models describe time evolution, which is not observable.
- Little *theoretical guidance*: what EFT's admit a UV-completion?
- Are there any non-perturbative/quantum gravity signals to be searched for?

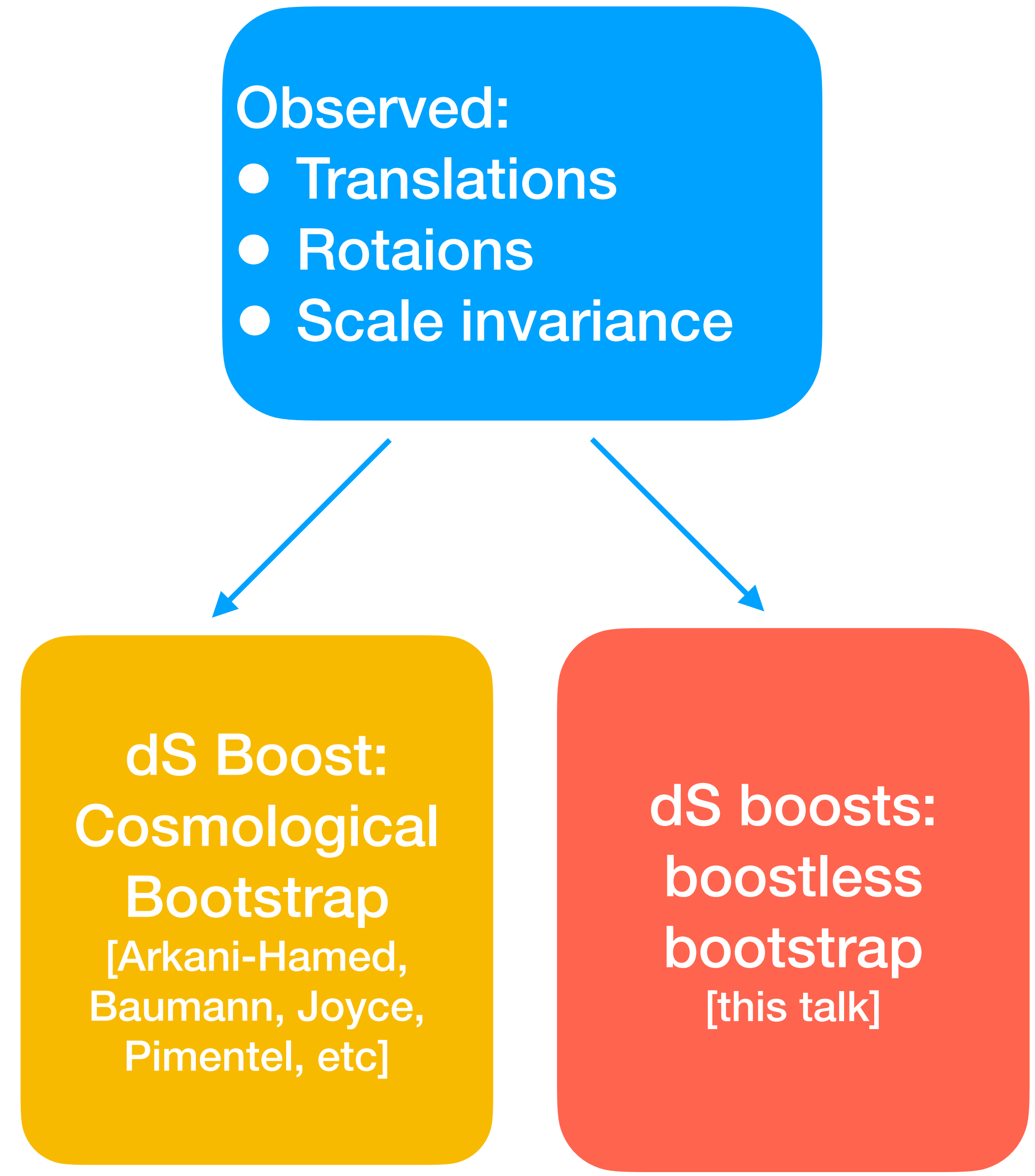
The Cosmological Bootstrap



The Cosmo Bootstrap

Observed symmetries

- Cosmological perturbations are observed to be statistically *homogeneous* and *isotropic*
- *Primordial* perturbations are also observed to be approximately scale invariant
- Anything else?
    - With *de Sitter boost* we can derive general results and connect with Conformal Field Theory and holography. See beautiful progress by Maldacena, Pimentel, Arkani-Hamed, Baumann, Joyce, etc…
    - If we are instead more interested in phenomenology, we cannot assume Boost invariance.



Broken boost [Green & EP '20]

Assuming only homogeneity, isotropy and scale invariance we have

Theorem: de Sitter symmetries are the largest possible set of symmetries for any single scalar field

Theorem: In single-clock inflation, the only theory of *curvature perturbations* ζ with full de Sitter symmetries is the free theory

Hence we allow dS boosts to be broken by the inflaton background

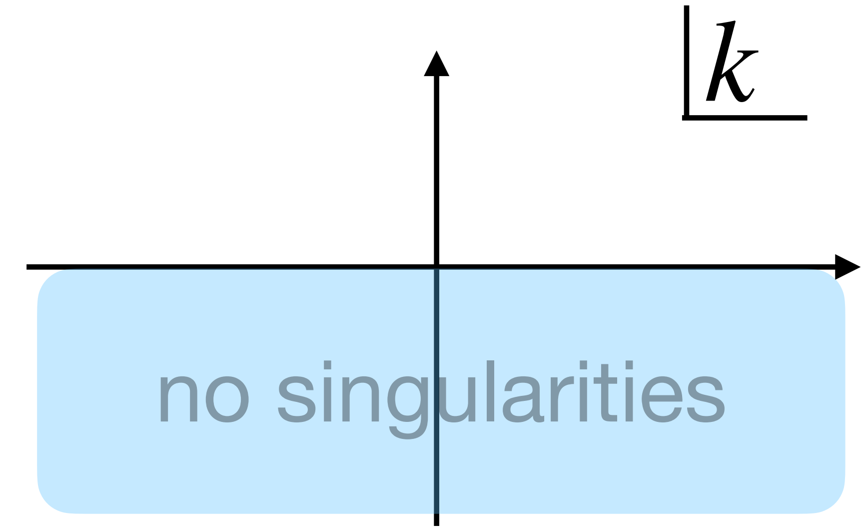
$$\sum_{a=1}^{n} \vec{k}_a \langle \phi(k_1) \dots \phi(k_n) \rangle = 0 \quad \text{translations}$$

$$\sum_{a=1}^{n} k_a^{[i} \partial_{k_a^{j]}} \langle \phi(k_1) \dots \phi(k_n) \rangle = 0 \quad \text{rotations}$$

$$\sum_{a=1}^{n} \left(3 - \Delta + k_a \partial_{k_a}\right) \langle \phi(k_1) \dots \phi(k_n) \rangle = 0 \quad \text{dilations}$$

n

Analyticity



- The ψn are analytic in $k_a = |\mathbf{k}_a|$ (defined off-shell) in the lower-half complex plane because the integral is even more convergent.

- This is a consequence of causality and it is true non-perturbatively [Aguei-Salcedo, Lee, Melville & EP 23]

$$\psi_n(k_1, \ldots) \sim \int_{-\infty}^{0} d\eta\, e^{ik_1\eta} F(k)$$

- Singularities are only on the negative real axis

Singularities at tree-level

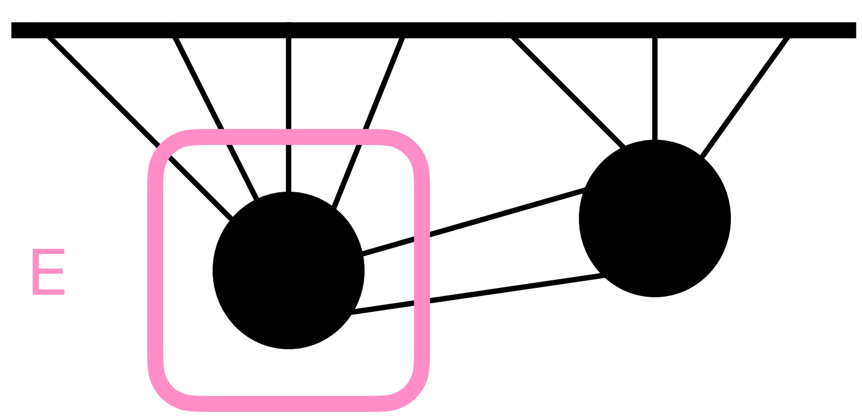
- Assuming locality, and a Bunch Davies initial state, the tree-level wavefunction has singularities when the “total energy” vanishes. The leading residue is the UV-limit of the flat-space amplitude [Maldacena & Pimentel ’11; Raju ’12; Arkani-Hamed et al ’17-‘18; Benincasa ’18]

$$\lim_{k_T \to 0} \psi_n \sim \frac{A_n}{k_T^p} \qquad k_T = \sum_a^n |\mathbf{k}_a|$$

- Only other singularities are at *vanishing partial energy*

$$\lim_{E \to 0} \psi_n \sim \frac{C_n}{E^p} \qquad E = \sum_a^{\text{int}} |\mathbf{k}_a| + \sum_m^{\text{ext}} |\mathbf{p}_m|$$

- All residues of partial energy singularities are fixed by unitarity! [Jazayeri, EP & Stefanyszyn ’21]



Amplitude limit

- The residue of the total-energy pole (kT=k1+…+kn=0) of (tree-level) correlators is fixed by the (UV-limit of the) amplitude [Maldacena & Pimentel ’11; Raju ’12; Arkani-Hamed et al ’17-‘18; Benincasa ’18]

- The precise relation is [Goodhew, Jazayeri & EP ’20]

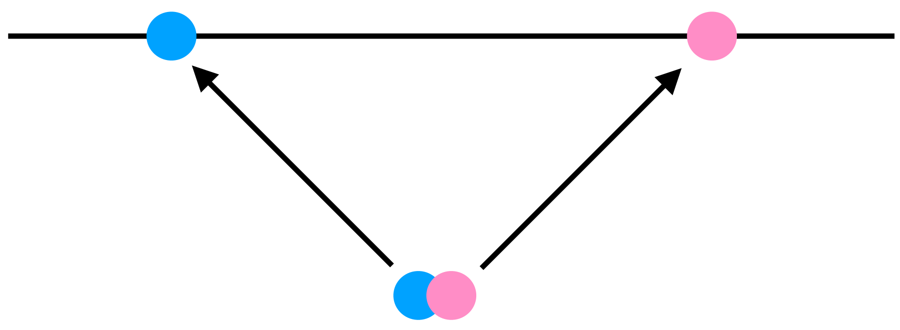
$$\lim_{k_T \to 0} B_n = \frac{(-1)^n H^{p+n-1}(p-1)!}{2^{n-1}} \times \frac{\mathrm{Re}\left(i^{1+n+p} A_n\right)}{\left(\prod_{a=1}^n k_a\right)^2 k_T^p} \,,$$

- p is fixed by dimensional analysis and scale invariance [EP ’20]

$$p = 1 + \sum_\alpha (D_\alpha - 4)$$

Locality

- Locality: what happens here cannot affect what happens far away. Operators commute for space-like separation and correlators factorise at large distances (*cluster decomposition*).



- There is no cluster decomposition in dS

- A common sufficient condition is *Manifest Locality*: Lagrangian interactions are products of operators *at the same spacetime point, e.g.*

$$\mathcal{L} \supset \dot{\phi}^3 + (\partial\phi)^2\dot{\phi} + \dot{\phi}^4 + \ldots$$

- The wavefunction of light *scalars and spin-2 fields* ($m^2 < 2H^2$) satisfies *non-perturbatively* the *Manifestly Local Test (MLT)* [Jazayeri, EP & Stefanyszyn '21]

$$\frac{\partial}{\partial k_c}\psi_n(k_1, ..., k_n; \{p\}; \{\mathbf{k}\})\Big|_{k_c=0} = 0\,, \qquad \forall c = 1, \ldots, n\,,$$

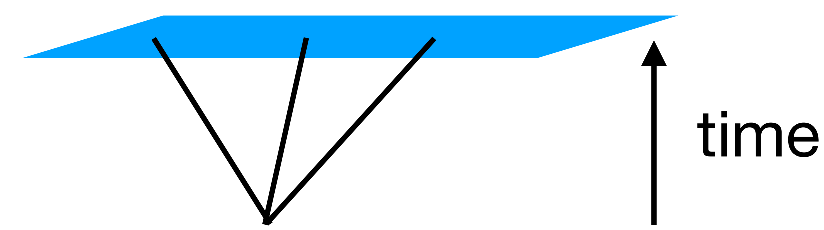
The Cosmological Optical Theorem (COT)

[Goodhew, Jazayeri, EP '20]

- Unitary time evolution, $UU^\dagger = 1$, from a Bunch-Davies state implies infinitely many relations. Let's see some examples

- The simplest example is a contact n-point functions

$$\psi_n(\{k\}, \{\mathbf{k}\}) + \psi_n^*(-\{k\}, -\{\mathbf{k}\}) = 0$$

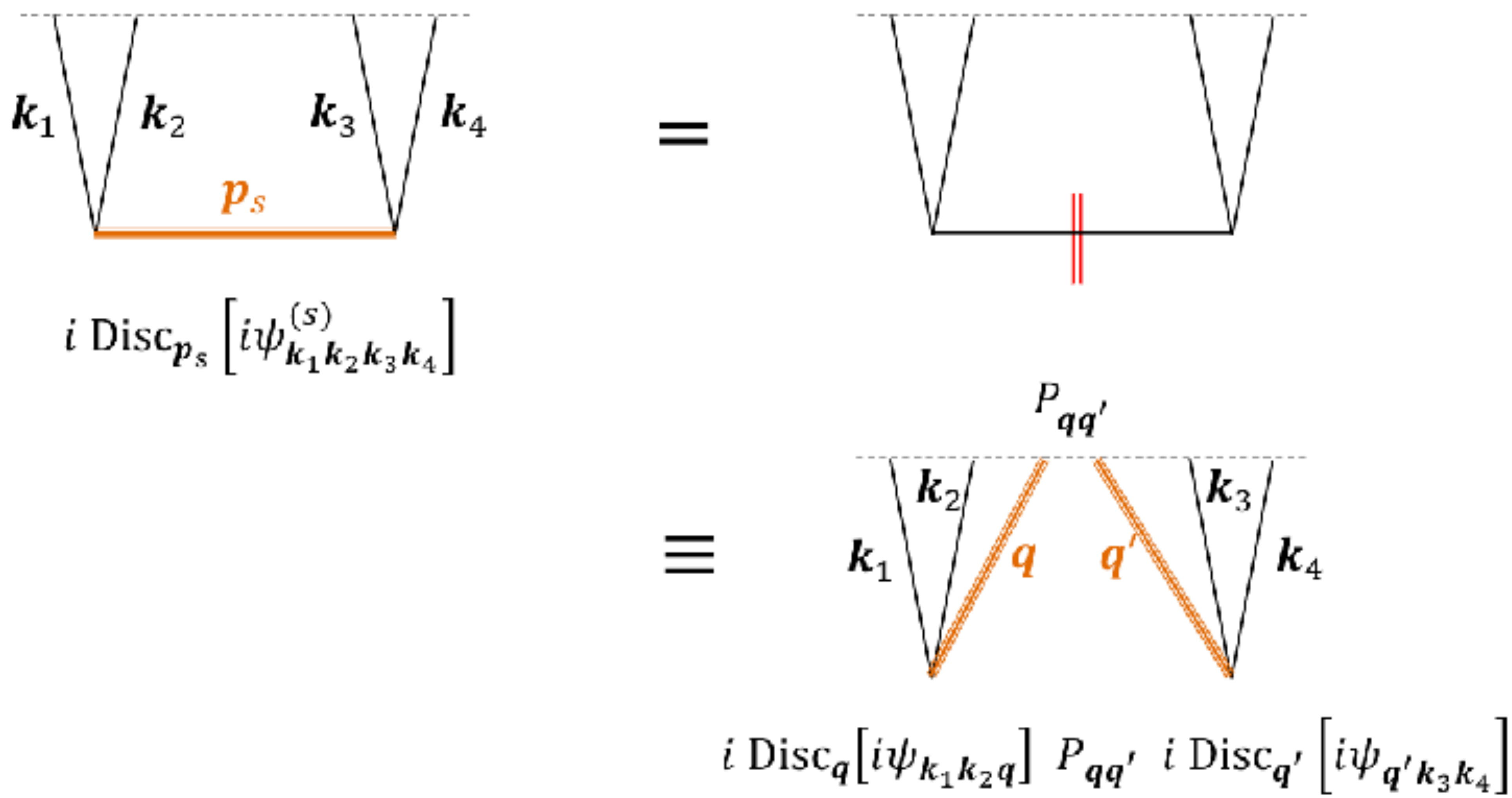


- This is a Cosmological Optical Theorem (COT) and can be interpreted as fixing a "discontinuity"

$$\text{Disc } \psi_n \equiv \psi_n(\{k\}, \{\mathbf{k}\}) + \psi_n^*(-\{k\}, -\{\mathbf{k}\}) = 0$$

Exchange diagrams

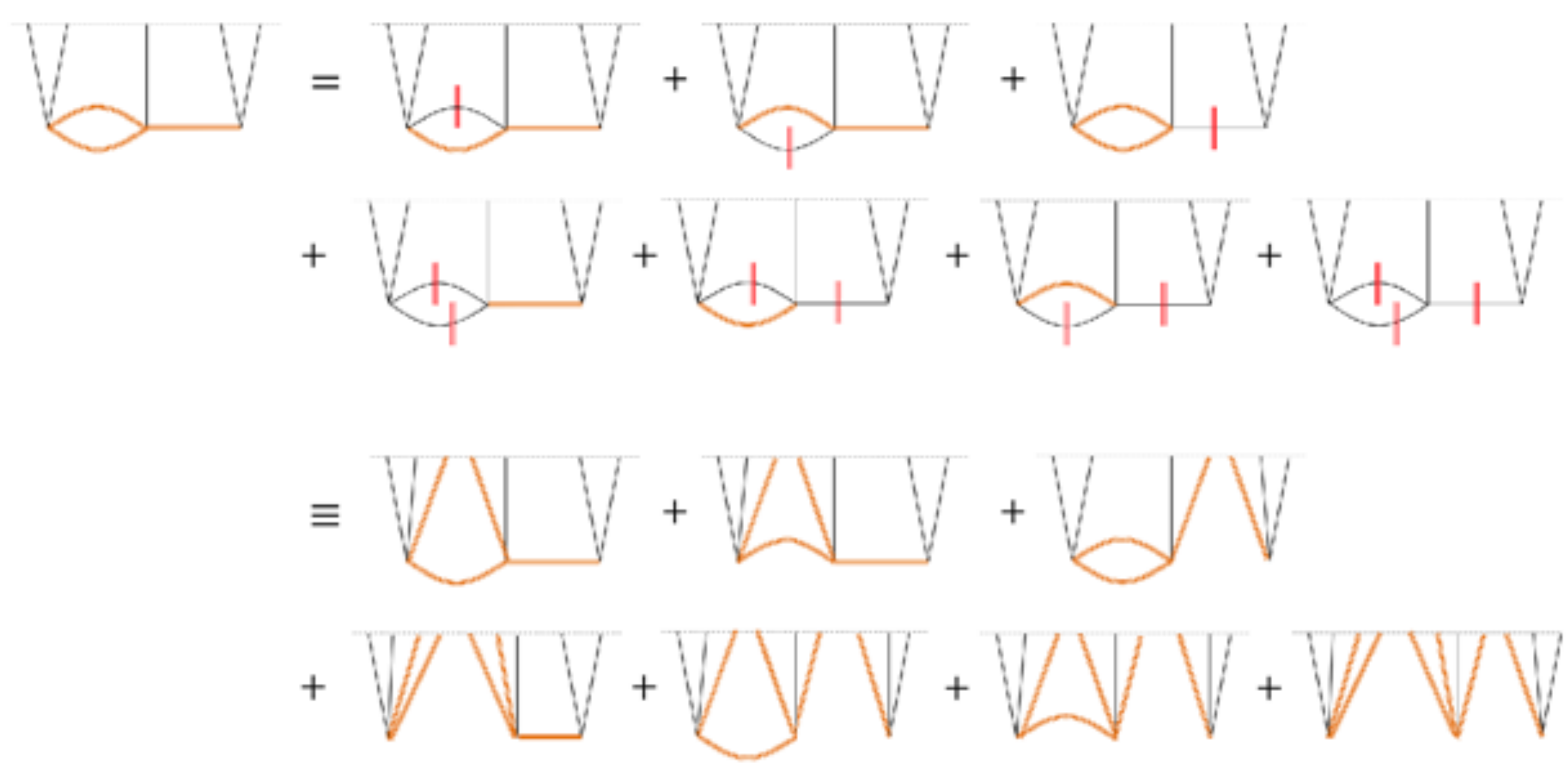
- The next simplest case is a 4-particle exchange diagram (trispectrum). The Cosmo Optical Theorem (COT) is



General diagrams

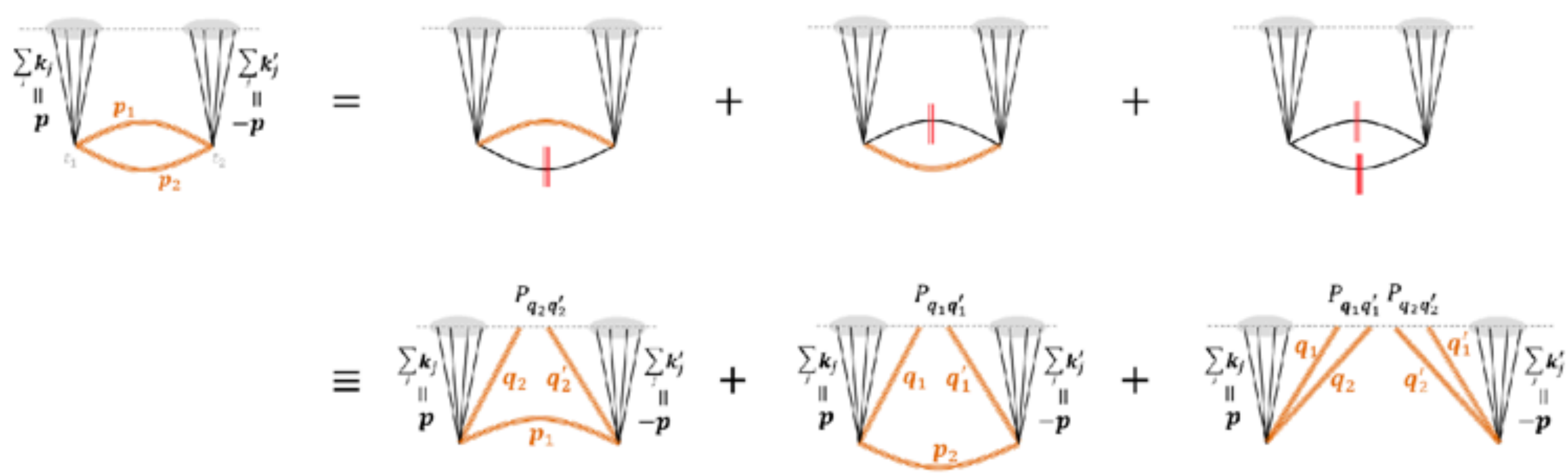
- These relations are valid to *all order in perturbation theory to any number of loops for fields of any mass and spin and arbitrary interactions (around any FLRW admitting a Bunch Davies initial condition)* [Goodhew, Jazayeri & EP '21; Melville & EP '21]

$$i \underset{\substack{\text{internal}\\ \text{lines}}}{\text{disc}} \left[i\,\psi^{(D)} \right] = \sum_{\text{cuts}} \left[\prod_{\substack{\text{cut}\\ \text{momenta}}} \int P \right] \prod_{\text{subdiagrams}} (-i) \underset{\substack{\text{internal }\&\\ \text{cut lines}}}{\text{disc}} \left[i\,\psi^{(\text{subdiagram})} \right] ,$$



Loop corrections

- Unitarity gives us also (the disc of) *loop corrections*!
- For example we can relate the disc of the 1-loop corrections to the power spectrum (or any other 2n-correlator) to a simpler integral over tree-level 3- and 4-point correlators



Poster child N°1:
All tree-level
scalar bispectra

Scalar bispectra

All scale-invariant tree-level bispectra of a massless scalar such as ζ from interaction with up to p derivatives are [Pajer 20; Jazayeri, EP & Stefanyszyn 21]

Scale invariance

Bose symmetry

$$k_T \equiv k_1 + k_2 + k_3$$
$$e_2 \equiv k_1 k_2 + k_2 k_3 + k_1 k_3$$
$$e_3 \equiv k_1 k_2 k_3$$

$$B_3 = \frac{\mathrm{Poly}_{p+3}(k_T, e_2, e_3)}{(k_1 k_2 k_3)^3 k_T^p}$$

tree level in dS

Bunch Davies vacuum

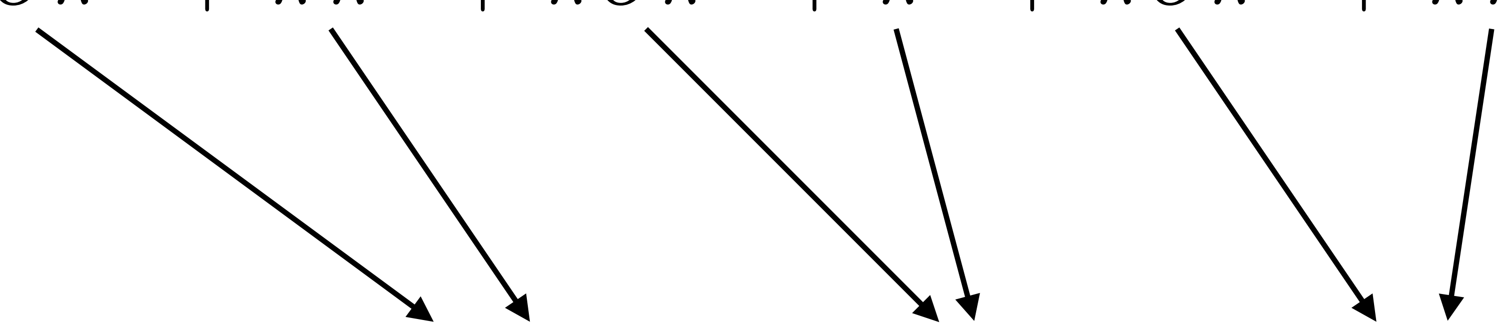
such that $$\partial_{k_1} \left[\frac{\mathrm{Poly}_{p+3}}{k_T^p} \right]_{k_1=0} = 0$$

Manifestly Local Test

Shapes of non-Gaussianity

- Here p equals the number of derivatives in the Lagrangian interaction
- The solution $B_3^{p=0}$ is local non-Gaussianity. In single-clock inflation it must be slow-roll suppressed by Maldacena's consistency relation
- The solution for $p \geq 1$ are in one-to-many relation to the operators in the EFT of Inflation (decoupling limit).

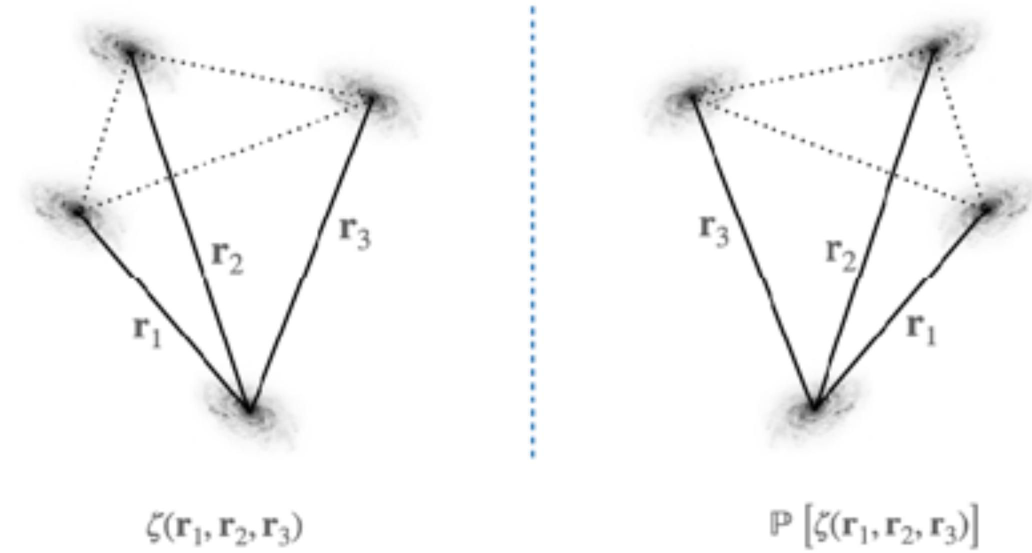
$$\mathcal{L}_{EFT} \sim \pi\partial\pi^2 + \pi\dot{\pi}^2 + \dot{\pi}\partial\pi^2 + \dot{\pi}^3 + \ddot{\pi}\partial\pi^2 + \pi\ddot{\pi}^2 + \dots$$



$$B_3 \sim \{p=1\} + \{p=2\} + \{p=3\} + \{p=4\} + \dots$$

Poster child N˚2: No-go theorems for the parity-odd trispectrum

Parity violation



- The scalar power spectrum and bispectrum are always parity even ($\{\mathbf{k}\} \to -\{\mathbf{k}\}$), non-perturbatively.

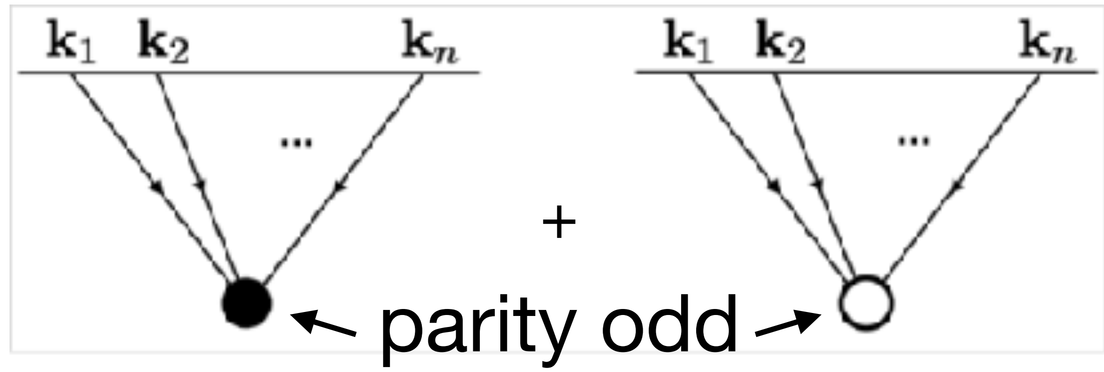
- Parity violation shows up first in the scalar trispectrum B_4

- There are infinitely many parity violating interactions

$$\mathcal{L} \sim \sum_{n_1, n_2, n_3, n_4} \epsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \prod_{a=1}^{4} \partial_{\mu_a} \partial^{n_a} \phi$$

however…

Contact interactions



- Contact interactions contribute to correlators as

$$\langle \varphi(\mathbf{k}_1) \ldots \varphi(\mathbf{k}_n) \rangle = \frac{\int \mathcal{D}\varphi \ \Psi \Psi^* \ \varphi(\mathbf{k}_1) \ldots \varphi(\mathbf{k}_n)}{\int \mathcal{D}\varphi \ \Psi \Psi^*} ,$$

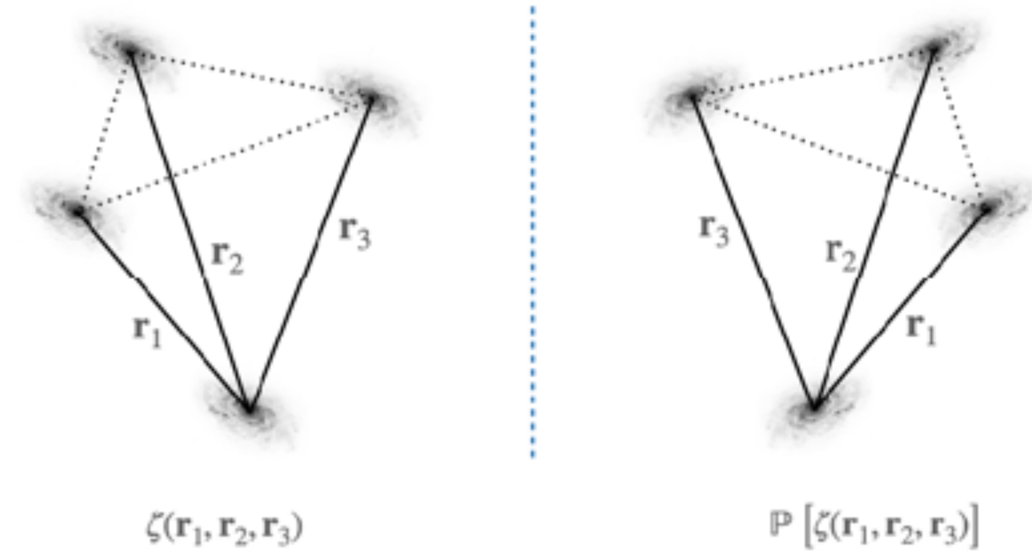
$$B_n^{\text{contact}}(\{k\}; \{\mathbf{k}\}) = - \frac{\psi_n(\{k\}; \{\mathbf{k}\}) + \psi_n^*(\{k\}; -\{\mathbf{k}\})}{\prod_{a=1}^{n} 2 \ \text{Re} \ \psi_2'(k_a)} ,$$

parity odd $\propto \psi_n(\{k\}; \{\mathbf{k}\}) - \psi_n^*(\{k\}; \{\mathbf{k}\})$

scaling $\propto \psi_n(\{k\}; \{\mathbf{k}\}) - (-)^3 \psi_n^*(-\{k\}; -\{\mathbf{k}\})$

cosmo optical theorem $= 0$

No-go for parity odd



- Assuming scale invariance, *unitarity* and a BD initial state, *IR-finite* parity-odd correlators vanish at tree-level for [Liu, Tong, Wang & Xianyu 19; Cabass, Jazayeri, EP & Stefanyszyn '22]

  - Any number of external massless scalars interacting with conformally coupled scalar fields

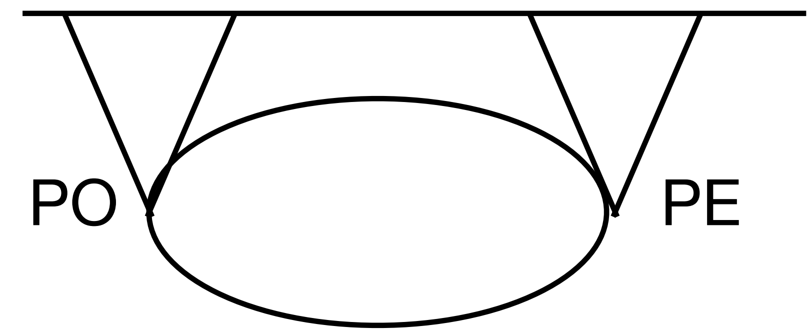
  - 4 external massless scalars interacting with any number of massive scalars, or massless fields of any spin

- Hence B_4^{PO} is an exceptionally sensitive probe of physics beyond vanilla inflation

Yes-go examples

- There are several yes-go example that relax the above assumptions:
    - exchanging massive spinning fields
    - break scale invariance
    - exchange parity-odd massless spinning fields
    - modified dispersions relations (non Bunch-Davies)
    - Beyond tree level

Leading loops



- Because tree-level vanishes, the leading contribution to B_4^{PO} is 1-loop!

- 1-loop 1-vertex vanished in dim reg (no momentum flow)

- Calculation is complicated (d-dimensional mode functions, many derivatives), but answer is simple [Lee, McCulloch, EP 23]

$$B_4^{\mathrm{PO}} = i\,\frac{(\mathbf{k}_1 \times \mathbf{k}_2) \cdot \mathbf{k}_3 \, \mathrm{Poly}_p(\mathbf{k})}{(k_1 k_2 k_3 k_4)^3 k_T^p}$$

- Is this an observable quantum effect?

Poster child N°3: The three parity-odd tensor bispectra

All graviton bispectra

- All tree-level, scale-invariant graviton bispectra on de Sitter must take the form

$$\psi_3^{+++} = \sum \left[e^+(\mathbf{k}_1) e^+(\mathbf{k}_2) e^+(\mathbf{k}_3) \mathbf{k}_1^{\alpha_1} \mathbf{k}_2^{\alpha_2} \mathbf{k}_3^{\alpha_3} \right] \psi_3^{\text{trim}}(k_1, k_2, k_3)\,,$$

- In terms of spinors this is (all other polarizations are fixed by this)

$$\psi_3^{+++} = \frac{[12]^2[23]^2[31]^2}{e_3^2} \sum_{\text{perm's}} h_\alpha(k_1, k_2, k_3) \psi_3^{\text{trim}}(k_1, k_2, k_3)\,,$$

$$h_0 = 1\,, \quad h_1 = ik_1\,, \quad h_2 = k_2 k_3\,, \quad h_3 = iI_1 I_2 I_3\,, \quad h_4 = I_1^2 I_2 I_3\,,$$

$$h_{5a,b} = iI_1^3 I_2 I_3, iI_1 I_2^2 I_3^2\,, \quad h_6 = I_1^2 I_2^2 I_3^2\,, \quad h_7 = iI_1^3 I_2^2 I_3^2\,.$$

- The trimmed wavefunction is the most general solution to the Manifestly Local Test (MLT)

Parity-odd graviton bispectra

- *Parity-odd* graviton non-Gaussianity is a poster child of the *boostless* cosmo bootstrap. The are infinitely many interactions

$$L \supset \cdots + \epsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\lambda\delta} R^{\lambda\delta}{}_{\gamma\theta} R^{\gamma\theta}{}_{\mu\nu} R^n + \dots$$

- Yet, *to all orders in derivatives* there are only *three* possible bispectra (notice no kT poles)!

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{k_T \left(k_T^2 - 2e_2\right)}{e_3^3} \,,$$

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{(-3e_3 + k_T e_2)}{e_3^3} \,,$$

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{(-k_1 + k_2 + k_3)(k_1 - k_2 + k_3)(k_1 + k_2 - k_3)}{e_3^3} \,.$$

Parity-odd mixed bispectra

- Similarly, parity-odd Scalar-Scalar-Tensor and Scalar-Tensor-Tensor bispectra to all orders in derivates can only be:

$$B_3^{00+} = \frac{[13]^2[23]^2}{k_3^2[12]^2}\frac{(k_1+k_2-k_3)^2 k_3}{e_3^3}\,,$$

$$B_3^{0++} = \frac{[23]^4}{k_2^2 k_3^2 e_3^3}[q_{1,1}(k_2+k_3)k_1^2 + q_{1,2,a}(k_2^3+k_3^3) + q_{1,2,b}(k_2k_3^2+k_3k_2^2)]\,,$$

$$B_3^{0+-} = \frac{[12]^4}{[31]^4}\frac{I_2^4}{k_2^2k_3^2e_3^3}\left[q_{1,1}(k_2-k_3)k_1^2 + q_{1,2,a}(k_2^3-k_3^3) + q_{1,2,b}(k_2k_3^2-k_3k_2^2)\right]\,,$$

Phenomenology

Perturbativity and the bounds on the tensor-to-scalar ratio imply various phenomenological constraints:

- Since there are only a few parity-odd shape, they should be a *primary targets* of observations (mostly CMB B-modes)
- Graviton bispectra have a smaller signal-to-noise ratio (S/N) than the graviton power spectrum, so they can be seen only after a detection of primordial tensor modes
- <scalar^2 tensor> has always a larger S/N than <tensor^3> or <scalar tensor^2> by a factor of ϵ^{-1}, so it should be the *first target,* unless one probes only the tensor sector

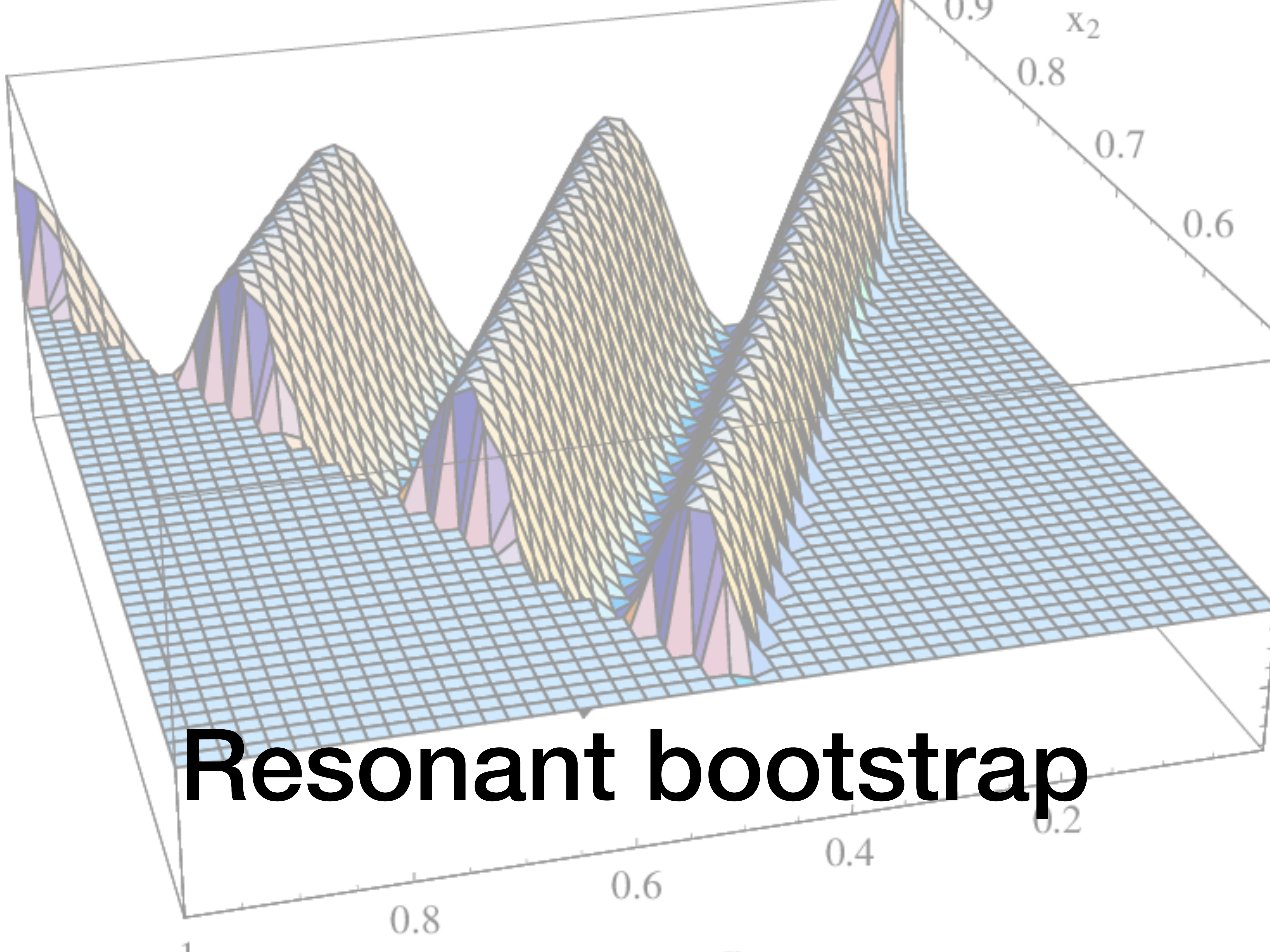
Horizons

- In the past 5 years we have finally understood some general consequences of locality, unitarity and causality for cosmological correlators

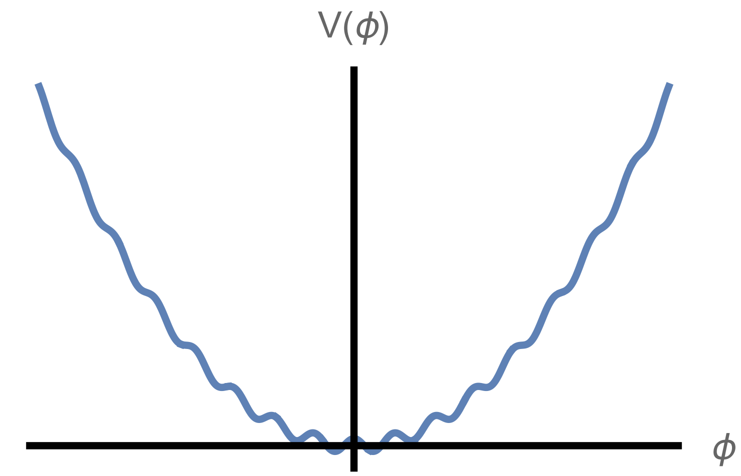
- This enables us to bootstrap many old and new predictions for cosmological observables

- Some results are non transparent from the standard model building point of view

- There are still basic and very general facts about quantum field theory on cosmological spacetimes that are awaiting to be discovered: *it's a wide open field of research*!


Resonant bootstrap
x_2
0.9
0.8
0.7
0.6
0.2
0.4
0.6
0.8
1

Oscillations



- The assumptions of scale invariance greatly simplify the derivation of correlators/wavefunction using the cosmological bootstrap, but it is not necessary
- Example beyond scale invariance: periodic *oscillations* in time, as in axion (monodromy) induced by gauge instantons [Duaso Pueyo & EP to appear]
- The classic example is a potential with periodic modulations $V = V_{\text{SR}}(\phi) + \Lambda^4 \cos \phi / f$
- Because in slow roll $\phi \sim \dot{\phi} t$, this gives periodic oscillations in the background, $\cos(\dot{\phi} t / f)$, which break continuous scale invariance to a discrete scale invariance
- The resulting "resonant non-Gaussianities" were computed [Flauge & Pajer '10; Leblond Pajer '10] and have been searched for in WMAP, Planck, eBOSS and other datasets.
- This was generated to oscillations in the EFT of inflation [Behbahani et al. '11; Behbahani & Green '12]
- Our goal here is to find more general resonant shapes

The resonant ansatz

- Scale invariance is broken to a discrete subgroup

$$\exists\, \lambda \in \mathbb{R}: \quad \psi_n(\lambda^m \omega, \lambda^m \mathbf{k}) = \lambda^{3m} \psi_n(\omega, \mathbf{k}) \quad \forall m \in \mathbb{N}$$

- Most general local, tree-level, on shell ansatz is

$$\psi_3(k_a) = \left[\left(\frac{i\alpha}{k_T} \right)^p \mathrm{Poly}^{(p)}_{p+3}(k_a) + \left(\frac{i\alpha}{k_T} \right)^{p-1} \mathrm{Poly}^{(p-1)}_{p+2}(k_a) \right] e^{-i\alpha \log{(k_T/k_*)}} + O(\alpha^{p-2}$$

$$\alpha = 2\pi / \log \lambda$$

- The factors of i are chosen for later convenience. The bracket has to satisfy the manifestly local test

Resonant bootstrap

- Resonant ψ_3 obeys the cosmo optical theorem (COT), but this requires exponentially suppressed contributions (from the stationary phase solution for the oscillating integral)

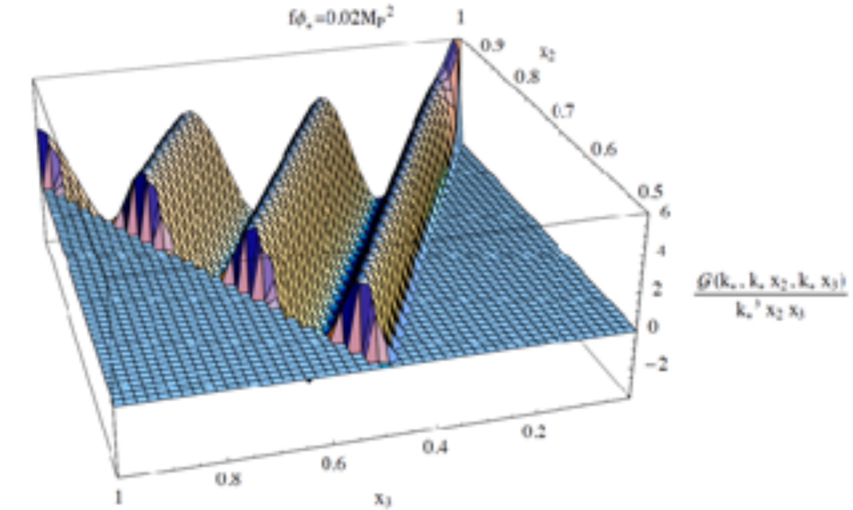
- Instead we use a modified COT for contact diagrams

$$\psi_3(k_a, \alpha) + \psi_3^*(-k_a, -\alpha) = 0$$

- the coefficients of the polynomials are all real by unitarity

- The polynomial are fixed by the manifestly-local test to all orders in derivatives.

Quantum interference



- To leading order we reproduce resonant non-Gaussianity (ϕ^3)

$$\psi_3^{(0)} = C_{0,0}\left(e_3 - i\frac{k_T e_2}{\alpha}\right) e^{-i\alpha\log\left(k_T/k_*\right)} .$$

- The real and imaginary part of the wavefunction interfere with each other when computing the bispectrum

$$B_{\pi^3} \propto \frac{1}{e_3^2}\left[\cos\Big(\alpha\log(k_T/k_*)\Big) - \frac{1}{\alpha}\sum_{i,j}\frac{k_i}{k_j}\,\sin\Big(\alpha\log(k_T/k_*)\Big)\right]$$

- Subleading out-of-phase oscillations are a quantum effect