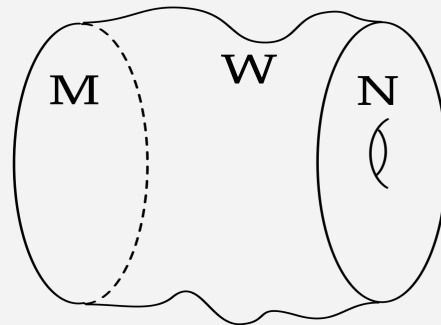


Spacetime topology change & cobordism currents

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Quantum spacetime & topology change



*this conference is about **quantum spacetime**.*

*basic aspect: **topology change***

*this talk's **question**:*

what can we learn from it?



***cobordism
currents***

upcoming work w/ C. Kneissl



Disclaimers

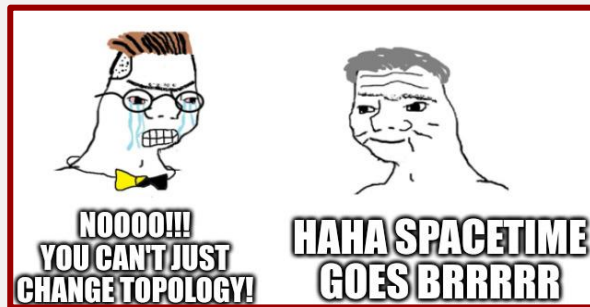


- ❖ *I'll be schematic: algebraic topology is weird. ask me stuff!*
- ❖ *short, “**conceptual**” talk: just kinematics. focus on ideas.*


*also: work **in progress!** :)*



Topology change — why?



a few motivations (see also McNamara's talk @ Swamplandia 2023)

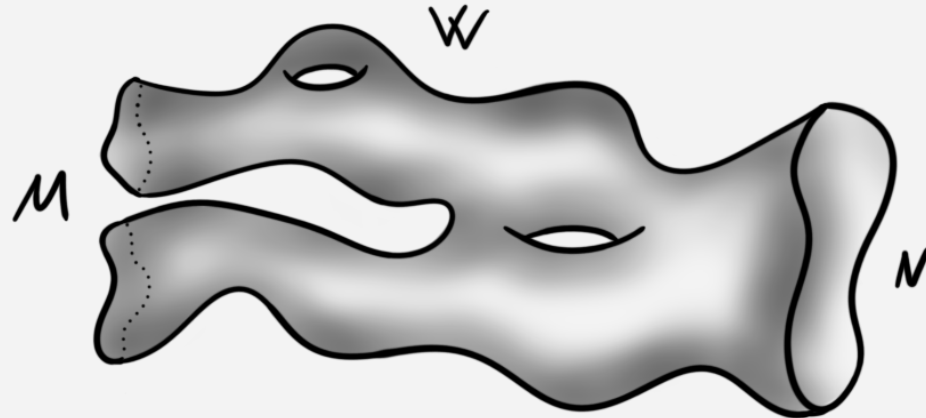
- ❖ ***it is natural: gravity***  ***dynamical spacetime***
 - also “background independence” (Casadio, Kamenshchik, Kuntz, 2022)
- ❖ ***semiclassical path integral: gravitational instantons + locality***
- ❖ ***it works: BH thermodynamics from saddles***
- ❖ ***holographic principle: unitarity ($ER = EPR$)*** (McNamara, 2022)



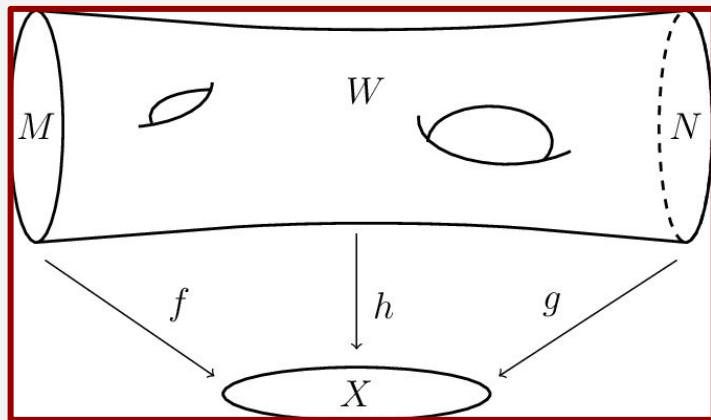
Topology change — how?

basic move: “pinch a wormhole” ($ER = EPR$) \longrightarrow surgery

❖ sequence of surgeries: **cobordism**



Cobordism in a nutshell



equivalence relation for manifolds w/ structure: (un)oriented, (s)pin, gauge, ...



example: spin + G

Abelian group under disjoint union

$$Y_n \sim Y'_n \rightarrow [Y_n] \in \Omega_n^{\text{Spin}}(BG)$$

Topology change — so what?

no obstructions allowed

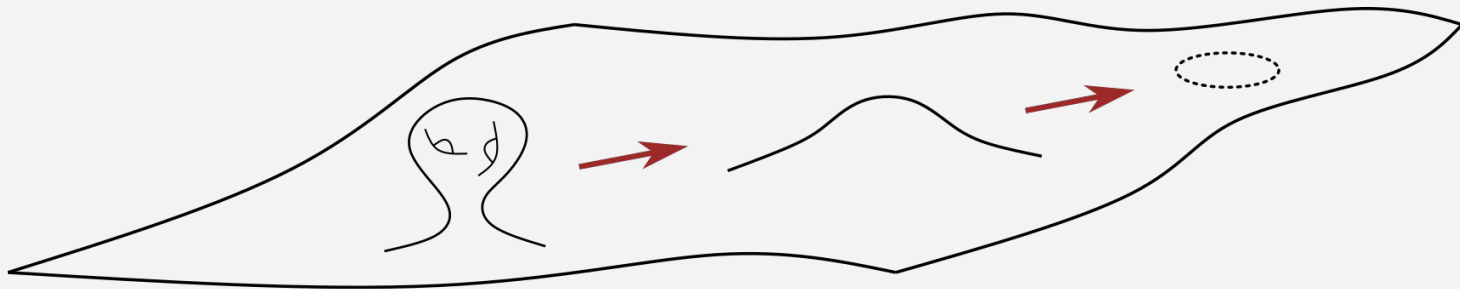


cobordism triviality

(McNamara, Vafa, 2019)

❖ *breaks unitarity via holography*

❖ *gravitational solitons carry global charge* $[M] \in \Omega_n^\xi$



Currents and cohomology

❖ *integer charges* \longrightarrow *Noether current*

$$Q(\Sigma) = \int_{\Sigma} J$$

❖ *local conservation* \longrightarrow *cohomology*

$$dJ = 0 \rightarrow [J] \in H^n(X)$$

*goal of talk:
currents for
topological
symmetries*



A simple example — U(1) gauge theory

$$\Omega_2^{U(1)} = \mathbb{Z}$$

❖ *only one characteristic class:*

$$[M, P] \mapsto c_1(P) = [F]$$


$$J = F$$



A simple example — U(1) gauge theory

$$\Omega_2^{U(1)} = \mathbb{Z}$$

- ❖ *topological charge: magnetic flux*
- ❖ *conserved current: $J = F \longrightarrow dJ = 0$*
- ❖ *breaking the symmetry: **magnetic monopoles!** (elementary carriers)*

$$dJ = j \neq 0 \longrightarrow j = q_{\text{mag}} \delta_{\text{worldline}}$$



A harder example — U(1) gauge theory

$$\Omega_4^{U(1)} = \mathbb{Z} \oplus \mathbb{Z}$$

- ❖ *now two characteristic classes!*

$$[M, P] \mapsto a c_2(P) + b p_1(M)$$

$$J = a F^2 + b \operatorname{tr} R^2$$

- ❖ *can't compute a and b* 😞 *what to do?*

Cobordism currents

$$\Omega_n^\xi \rightarrow H^n(X)$$

$$[M] \mapsto J_M$$

abstract charge

current

spacetime

❖ *how to find?* 😞

➤ **in general:** combination of cobordism invariants (characteristic classes)



Key idea — generalized (co)homology

$$\Omega_n^\xi \rightarrow E^n(X) \rightarrow H^n(X)$$

“generalized cohomology”

game plan:

- ❖ cobordism is a **generalized homology**. *natural map* to $E^n(X)$ (“orientation + duality”)
- ❖ **co**homologies have pairings (“Poincaré duality”) \longrightarrow **isometry** to get J!

“Concrete” example — K-theory

$$\Omega_n^{\text{spin}} \rightarrow KO^{-n}(X) \rightarrow H^n(X)$$

“real K-theory”

spacetime embedding of M in X

$$[M_n] \mapsto f!(\underline{\mathbb{R}}^n) \xrightarrow{\text{iso}} J_M$$


“Gysin map”



Recovering the current via isometry

- ❖ **bilinear pairing** in *K*-theory:

$$\langle x, y \rangle = \int_X \widehat{A}(X) \operatorname{ch}(x \otimes y)$$

- ❖ **Gysin map shenanigans:** $\operatorname{ch} f_!(E) = \frac{\operatorname{ch} E \delta(\nu_M)}{\widehat{A}(\nu_M)}$ 

$$\langle x, y \rangle = \int_X J_M \wedge J_N \longrightarrow$$

$$J_M = \sqrt{\frac{\widehat{A}(M)}{\widehat{A}(\nu_M)}} \delta(\nu_M)$$

cobordism current



So... what do we learn?

simplified for lack of time (extra stuff in upcoming paper!)

- ❖ *include gauge bundle: “cobordism defects” carry gauge fields*

$$J = \text{ch}F \sqrt{\frac{\widehat{A}(M)}{\widehat{A}(\nu_M)}} \delta(\nu_M)$$

- ❖ *anomaly inflow: defects carry **chiral d.o.f.** \longrightarrow **dynamical objects!**
“no probes in QG”*
- ❖ *smells like D-branes: their charges live in K-theory! (Witten, 1998) (Minasian, Moore, 1999)*



Our favorite example — U(1) gauge theory

$$\begin{aligned}\text{ch}(F) \sqrt{\widehat{A}(M)} &= \left(1 - \frac{1}{2} F^2\right) \sqrt{1 + \frac{1}{48} \text{tr} R^2} \\ &= \underbrace{\frac{1}{96} \text{tr} R^2} - \underbrace{\frac{1}{2} F^2}\end{aligned}$$

- ❖ current for *K3 solitons* with a and b (if charges $\in K$ -theory)

punchline: “generalized (co)homology as intermediary”



Bonus round — gimme some phenomenology!

- ❖ *new anomalies from topology change* (Garcia-Etxebarria, Montero, 2020)
 - e.g. MSSM *baryon triality* anomaly \longrightarrow #generations **multiple of 3**
- ❖ *cosmology: parity domain walls are stable* $\Omega_0^{SO} = \mathbb{Z} \rightarrow \Omega_0^O = \mathbb{Z}_2$
 - *strong bounds* on inflation! (McNamara, Reece, 2022)
- ❖ *“magnetic monopoles exist”* $\longleftarrow \Omega_2^{U(1)} = \mathbb{Z}$
 - also from other arguments (completeness, generalized symmetry breaking...)
- ❖ *“end-of-the-world”?* (Antonelli, IB, 2019) (Blumenhagen, Kneissl, Wang, 2023) (...) *more? who knows!*



Outlook

- ❖ ***punchline: topology (change) is important!***
- ❖ ***implications for EFTs & defects (& anomalies!)***
(Garcia-Etxebarria, Montero, 2020)
 - *connections w/ D-branes & O-planes in string theory*
(Scrucca, Serone, 1998) (many others...)
 - *“generalized generalized” cohomology? → “**TMF**”*
(Gaiotto, Johnson-Freyd, Witten, 2019) (Tachikawa, 2021)
- ❖ ***other (co)homologies? dynamics? (hard!)***

thank you!

me checking
cobordism
groups

cobordism
groups are
not
trivialized



credits to “Pauc” for the meme ;)