What are degrees of freedom in gravity?

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in collaborations with J. Haruna, Y. Maitiniyazi, J. Liu, K. Oda, S, Matsuzaki

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Quantum spacetime and the renormalization group@ Sant'Elmo Beach Hotel

Is metric field fundamental?

- We start with an assumption that the metric field $g_{\mu\nu}$ (spin 2 symmetric tensor) is fundamental degrees of freedom in gravity. (Physically 2 d.o.f.).
- In low energy, the metric field $g_{\mu\nu}$ well-describes gravitational interactions.
- Indeed, gravitational waves are discovered.
- But, is it true even in high energy?

Is metric field fundamental?

- Standard procedure in computations:
  - expansion of metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
  - obtain inverse metric from $g_{\mu\rho} g^{\rho\nu} = \delta^\nu_\mu$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - h^{\mu\nu} + \frac{1}{2} h^\mu{}_\rho h^{\rho\nu} + \cdots$$

  - This series gives an infinite number of vertices.
  - It is natural for metric theories to be perturbatively non-renormalizable.
  - This may indicate new degrees of freedom.

Contents

- Lesson from pion (non-linear sigma model)
- Example: SO(1,3)×diff model.
- Metric as composite matter field?

Pion

- lightest particle in QCD (1936) $m_\pi = 135\,\mathrm{MeV}$
  - Before 1960, QCD was not known.
  - Quark model was proposed in 1960.
- Low energy theorems was known.
  - e.g. Goldberger-Treiman relation (1958)
- Pion dynamics is well-described by O(N) non-linear sigma model in low energy.

Non-linear sigma model

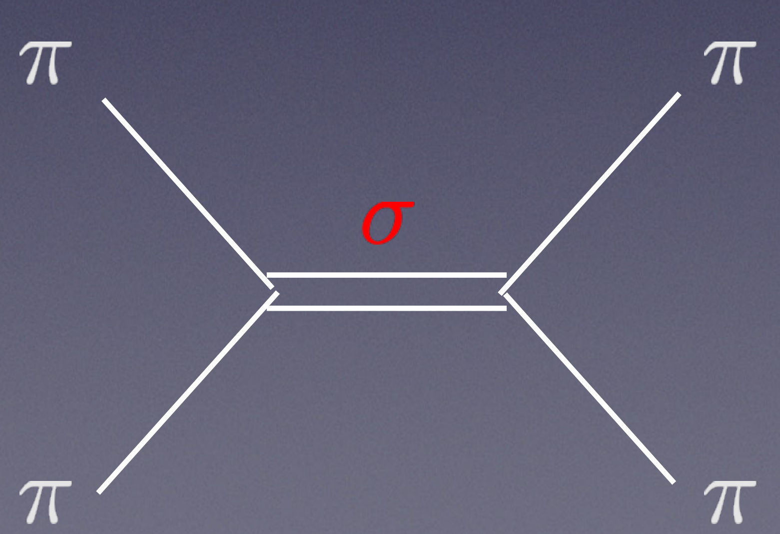
- O(N) non-linear sigma model: *N-1* d.o.f. $\pi^i = (\pi^1, \cdots, \pi^{N-1})$

$$S_{\rm NLS} = \frac{1}{2}\int \mathrm{d}^D x \left[\partial_\mu \pi^i \partial^\mu \pi^i - \frac{1}{f_\pi^2}(\pi^i \partial^2 \pi^i)^2 + \cdots\right]$$

  - Perturbatively non-renormalizable.
  - Breaks unitarity for $|p| > f_\pi$
- Existence of massive sigma meson (~f0(500)) $m_\sigma \sim 500\,\mathrm{MeV}$
- O(N) linear sigma model: *N* d.o.f.

$$\phi^i = (\pi^1, \cdots, \pi^{N-1}, \sigma)$$

  - Perturbatively renormalizable and unitary



Where does the nonlinearity come from?

- Spontaneous symmetry braking O(N)→O(N-1)

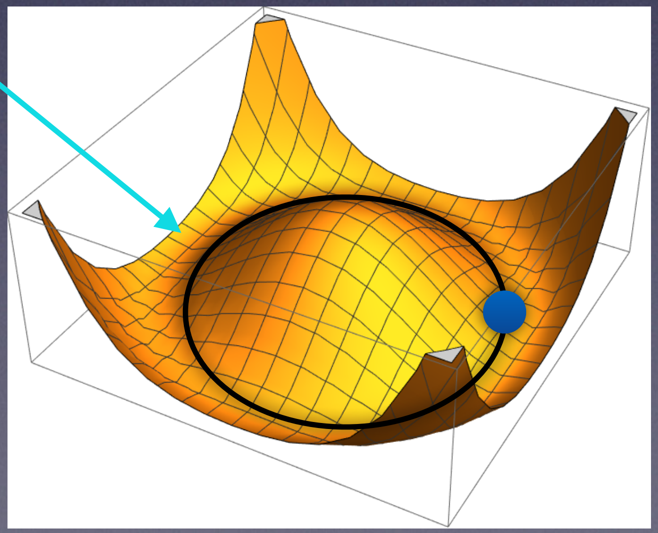
- Vacuum condition gives a constraint:

$$\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$$

$$V = -\frac{m^2}{2}(\phi^i)^2 + \frac{\lambda}{4}(\phi^i)^2$$

$$f_\pi^2 = \frac{2m^2}{\lambda}$$

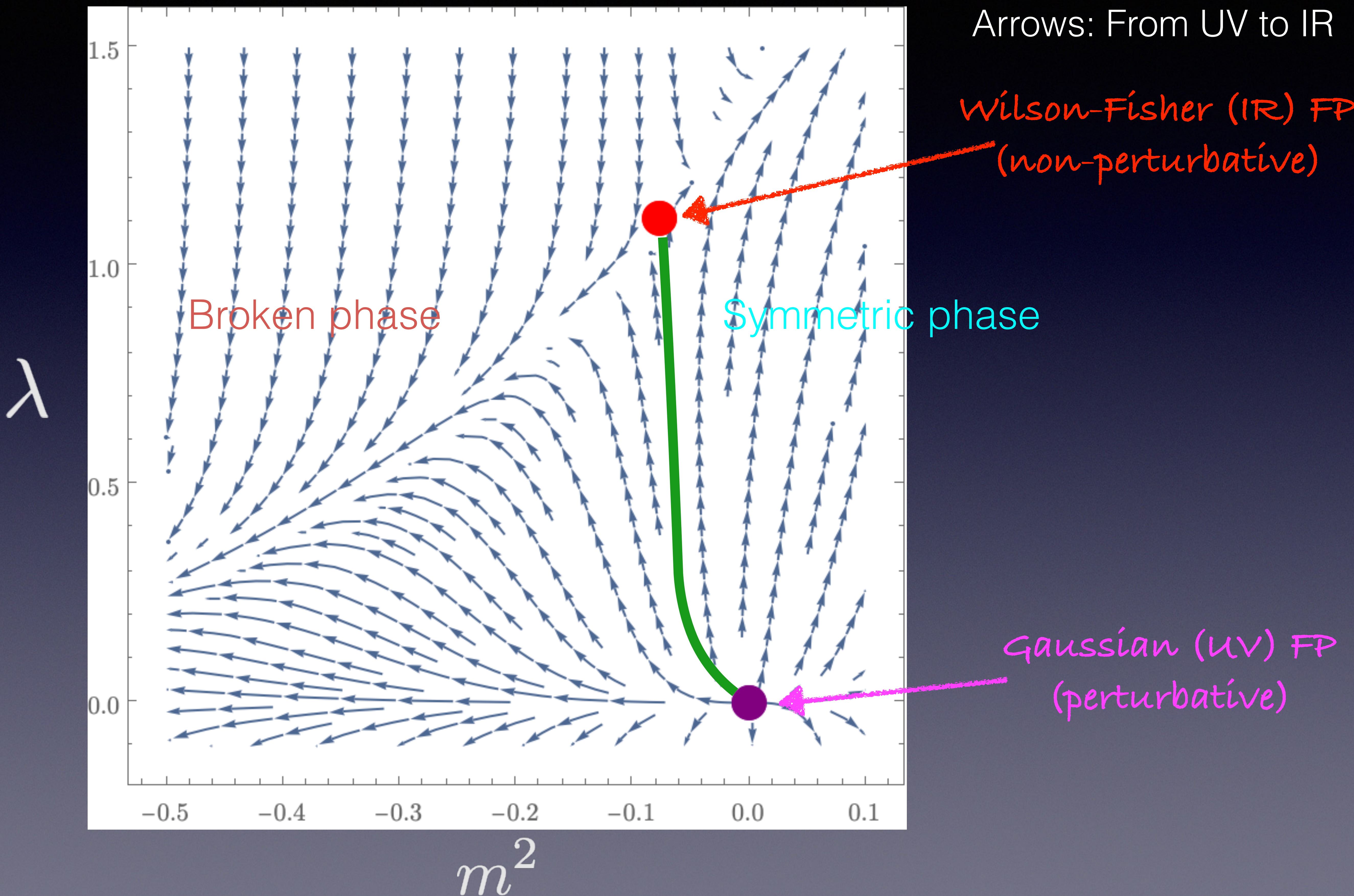
$$\phi^i = (\pi^1, \cdots, \pi^{N-1}, \sigma)$$



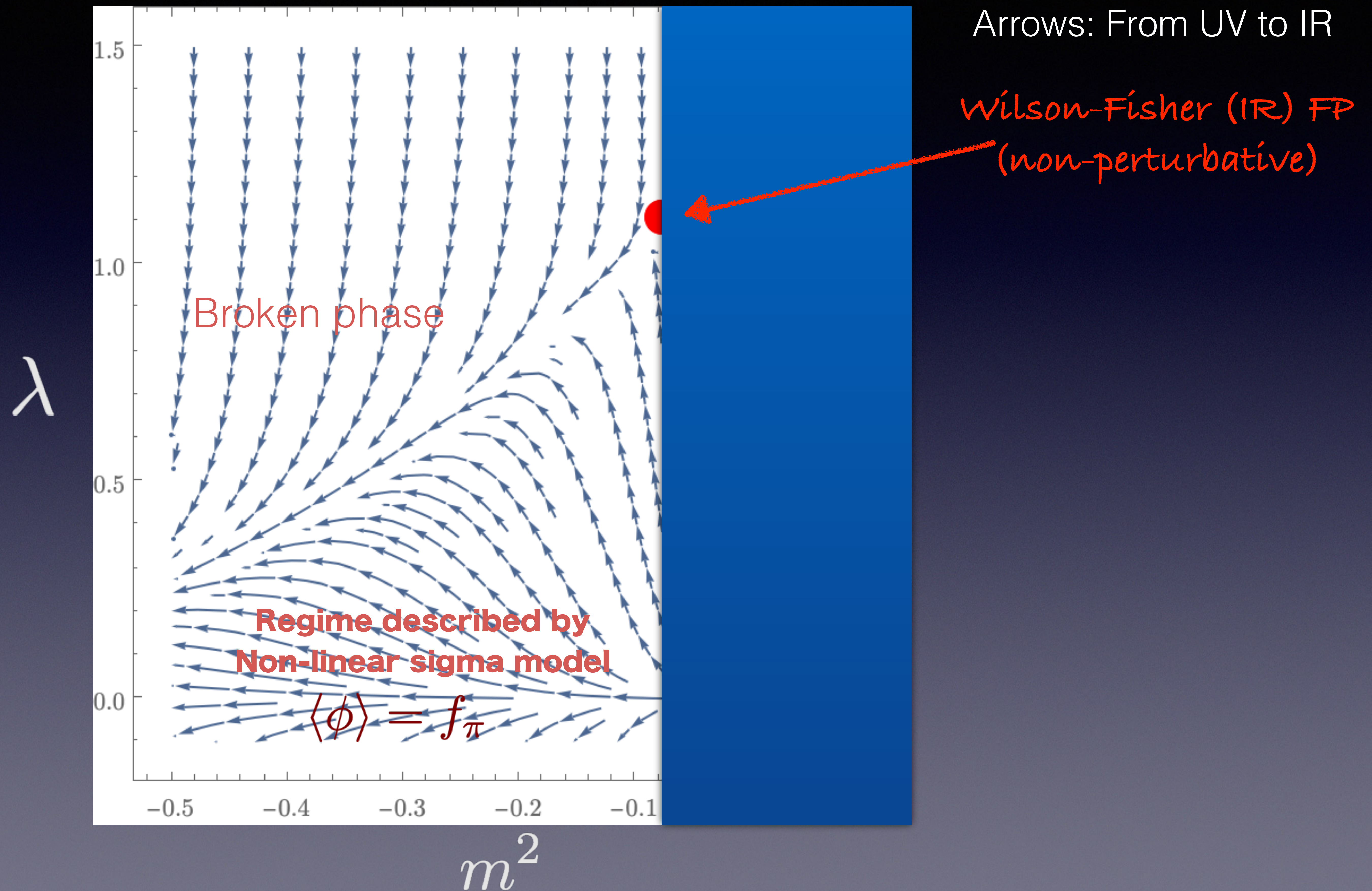
$$\sigma = \sqrt{f_\pi^2 - (\pi^i)^2} = f_\pi^2 - \frac{1}{2}(\pi^i)^2 + \cdots$$

$$\sigma^2 + (\pi^i)^2 = f_\pi^2$$

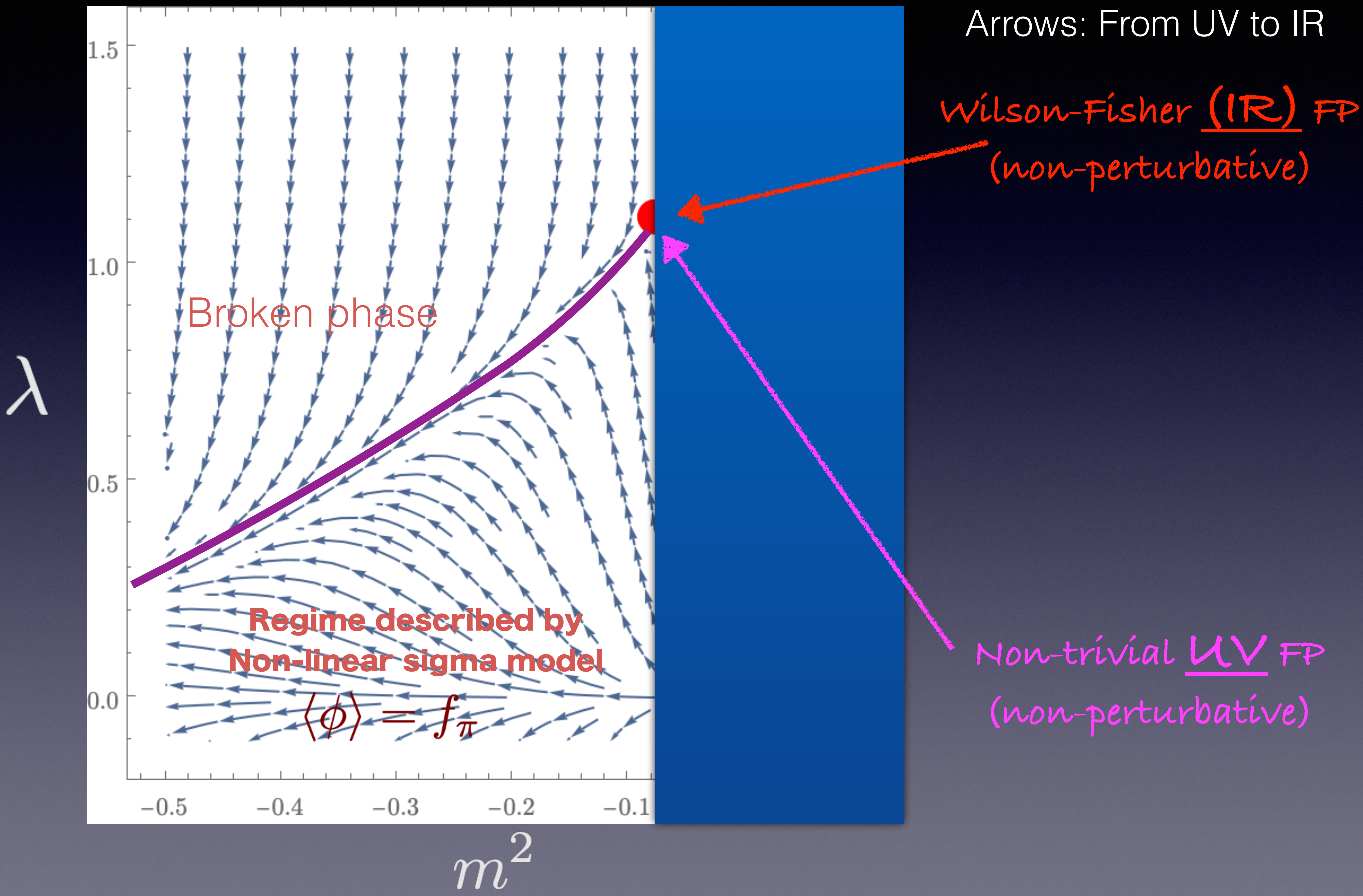
Phase diagram in 3 dim linear σ model



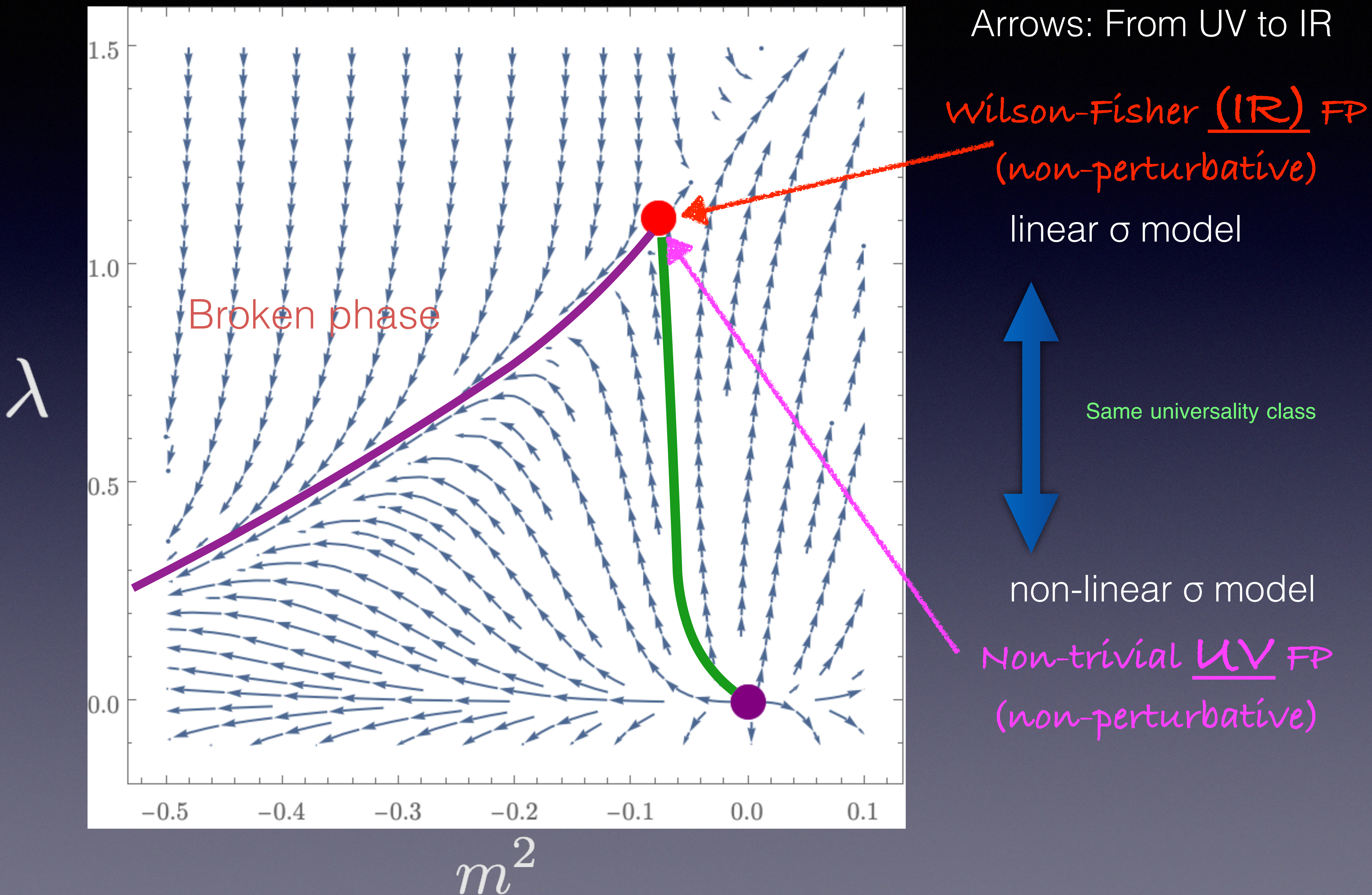
Phase diagram in 3 dim linear σ model



Phase diagram in 3 dim linear σ model



Phase diagram in 3 dim linear σ model



“Duality” at fixed point

Asymptotically safe theory	Asymptotically free theory
established	
3 dim non-linear σ model	3 dim linear σ model
3 dim Gross-Neveu model	3 dim Higgs-Yukawa model
Asymptotically safe gravity	?

What can we learn?

- In low energy, we can observe only light d.o.f.
- New d.o.f. associated to bigger symmetry group may define a theory to be renormalizable and unitary.
- Symmetry spontaneously breaks into its subgroup.
  - Some d.o.f. become massive.
  - Massless (light) modes become effective d.o.f. in low energy.

Bigger symmetry broken into smaller symmetry

- Non-linear sigma model

O(N) $\xrightarrow{\text{SSB}}$ O(N-1)

$\langle\sigma\rangle = f_\pi \neq 0$

π^i is fluctuation around f_π.

- Gravity

$h_{\mu\nu}$ is fluctuation around $\bar{g}_{\mu\nu}$.

$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

? $\xrightarrow{\text{SSB}}$ Diffeomorphism

$\bar{g}_{\mu\nu} = \langle g_{\mu\nu}\rangle \neq 0$

(Einstein phase)

Earlier Attempts

? —SSB→ Diffeomorphism

$$\bar{g}_{\mu\nu} = \langle g_{\mu\nu} \rangle \neq 0$$

- GL(4)

N. Nakanishi and I. Ojima, Phys. Rev. Lett. 43 (1979) 91.
R. Floreanini and R. Percacci, Phys. Lett. B 379 (1996) 87 [hep-th/9508157].
R. Percacci, Nucl. Phys. B 353 (1991) 271 [0712.3545].
R. Percacci, PoS ISFTG (2009) 011 [0910.5167].

"hidden" local-Lorentz symmetry

- $SO(1,3)_{\text{local}} \times$ diff.

S. Matsuzaki, S. Miyawaki, K. Oda and **M. Yamada**, Phys. Lett. B 813 (2021) 135975 [2003.07126].
C. Wetterich, Nucl. Phys. B 971 (2021) 115526 [2101.07849].
Y. Maitiniyazi, S. Matsuzaki, K. Oda, and **M. Yamada**, [2309.16230]

Contents

- Lesson from pion (non-linear sigma model)
- Example: SO(1,3)×diff model.

  detail: See poster by Y. Maitiniyazi
- Metric as composite matter field?

First-order formalism

- Based on SO(1,3) local Lorentz symmetry (and diff.)
  - Vierbein $e^{\mathbf{a}}{}_{\mu}$ → $g_{\mu\nu} = \eta_{\mathbf{ab}} e^{\mathbf{a}}{}_{\mu} e^{\mathbf{b}}{}_{\nu}$
  - Local-Lorentz (LL) gauge field $(A_\mu)^{\mathbf{a}}{}_{\mathbf{b}} = A^i_\mu (\sigma^i)^{\mathbf{a}}{}_{\mathbf{b}}$
- Vierbein is in fundamental rep. of SO(1,3)

cf. $e^{\mathbf{a}}{}_{\mu} \leftrightarrow \phi^a \qquad g_{\mu\nu} \leftrightarrow \phi_a \phi^a$

- Action under SO(1,3)×diff.

$$S = \int \mathrm{d}^4x |e(x)| \left[-\Lambda_{\mathrm{cc}} + \frac{M_P^2}{2} e_{\mathbf{a}}{}^{\mu} e_{\mathbf{b}}{}^{\nu} F^{\mathbf{ab}}{}_{\mu\nu} + \frac{1}{4g_{\mathrm{L}}^2} e_{\mathbf{a}}{}^{\mu} e_{\mathbf{b}}{}^{\nu} e_{\mathbf{c}}{}^{\rho} e_{\mathbf{d}}{}^{\sigma} F^{\mathbf{ab}}{}_{\mu\nu} F^{\mathbf{cd}}{}_{\rho\sigma} + \cdots \right]$$

$$F^{\mathbf{a}}{}_{\mathbf{b}\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^{\mathbf{a}}{}_{\mathbf{b}}$$

Degenerate limit
and
Irreversible vierbein postulate

- How to define "symmetric phase"?

$$\bar{e}^{\mathbf{a}}{}_{\mu} = \langle e^{\mathbf{a}}{}_{\mu} \rangle = 0$$

- Degenerate limit: Some eigenvalues of vierbein take zero; $\lambda_a \to 0$.

$$e^{\mathbf{a}}{}_{\mu} \sim \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix}$$

In this limit, volume element becomes zero.

$$|e(x)| = \det_{\mathbf{a}\mu} e^{\mathbf{a}}{}_{\mu} = 0$$

- Irreversible vierbein postulate
  - The tree level action for gravity does not contain divergent terms for $\lambda_a \to 0$.

Example

- Scalar kinetic term

$$|e|g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = |e|\eta^{\mathbf{ab}}e_{\mathbf{a}}{}^{\mu}e_{\mathbf{b}}{}^{\nu}\partial_\mu\phi\partial_\nu\phi$$

$$|e|\eta^{\mathbf{ab}}e_{\mathbf{a}}{}^{\mu}e_{\mathbf{b}}{}^{\nu} = \lambda_1\lambda_2\lambda_3\lambda_4\begin{pmatrix}\lambda_1^{-2} & & & \\ & \lambda_2^{-2} & & \\ & & \lambda_3^{-2} & \\ & & & \lambda_4^{-2}\end{pmatrix}$$

$$= \begin{pmatrix}\frac{\lambda_2\lambda_3\lambda_4}{\lambda_1} & & & \\ & \frac{\lambda_1\lambda_3\lambda_4}{\lambda_2} & & \\ & & \frac{\lambda_1\lambda_2\lambda_4}{\lambda_3} & \\ & & & \frac{\lambda_1\lambda_2\lambda_3}{\lambda_4}\end{pmatrix}$$

This term diverges for $\lambda_a \to 0$

Why degenerate limit?

- Scalar potential is usually assumed to be

$$V(\phi) = \frac{m^2}{2}\phi^i\phi^i + \frac{\lambda}{4}(\phi^i\phi^i)^2$$

- But, in terms of symmetry, we can write

$$V(\phi) = \frac{m^2}{2}\phi^i\phi^i + \frac{\lambda}{4}(\phi^i\phi^i)^2 + \frac{g_1}{\phi^i\phi^i} + \frac{g_2}{(\phi^i\phi^i)^2} + \cdots$$

- In weak field limit ($\phi^i \to 0$), these terms diverge.

- Symmetric phase becomes ill-defined.

$$\langle\phi^i\rangle = 0$$

Why degenerate limit?

Topology change in classical and quantum gravity

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Received 7 November 1990

Abstract. In a first-order formulation, the equations of general relativity remain well defined even in the limit that the metric becomes degenerate. It is shown that there exist smooth solutions to these equations on manifolds in which the topology of space changes. The metric becomes degenerate on a set of measure zero, but the curvature remains bounded. Thus if degenerate metrics play any role in quantum gravity, topology change is unavoidable.

- Symmetric phase becomes ill-defined.

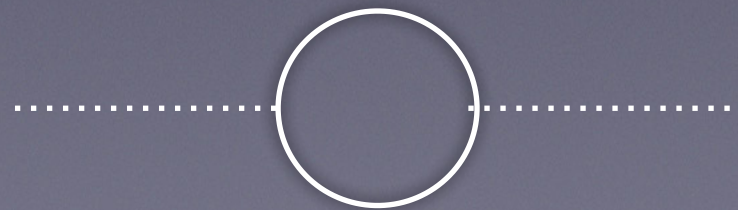
$$\langle \phi^i \rangle = 0$$

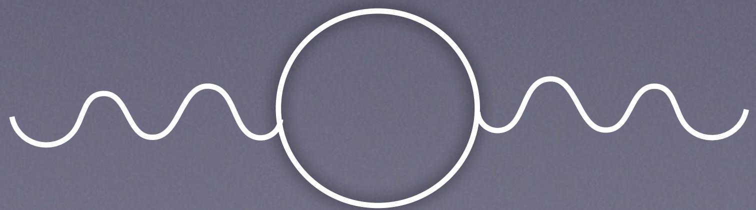
SO(1,3)×diff. model in degenerate limit

- Including matters, at a certain scale $\Lambda_{\rm G}$,

$$S_{\Lambda_{\rm G}} = \int \mathrm{d}^4x|e|\left[-\Lambda_{\rm cc} + \frac{M_{\rm P}^2}{2} e_{[\mathbf{a}}{}^{\mu} e_{\mathbf{b}]}{}^{\nu} F^{\mathbf{ab}}{}_{\mu\nu} - \bar{\psi} e_{\mathbf{a}}{}^{\mu}\gamma^{\mathbf{a}}\left(\partial_\mu + \frac{1}{2}A_{\mathbf{bc}\mu}\sigma^{\mathbf{bc}} + m_f\right)\psi\right]$$

  - $\Lambda_{\rm cc}$ can contains ϕ and $\bar{\psi}\psi$.
  - Only fermions are dynamical!
  - No kinetic terms of vierbein, gauge and scalar fields.
  - These fields would be dynamical via fermion quantum corrections.





Spontaneous local Lorentz symmetry breaking

- Generation of expectation value of vierbein

$$\bar{e}^{\mathbf{a}}{}_{\mu} = \langle e^{\mathbf{a}}{}_{\mu} \rangle = 0 \quad \longrightarrow \quad \bar{e}^{\mathbf{a}}{}_{\mu} = \langle e^{\mathbf{a}}{}_{\mu} \rangle \neq 0$$

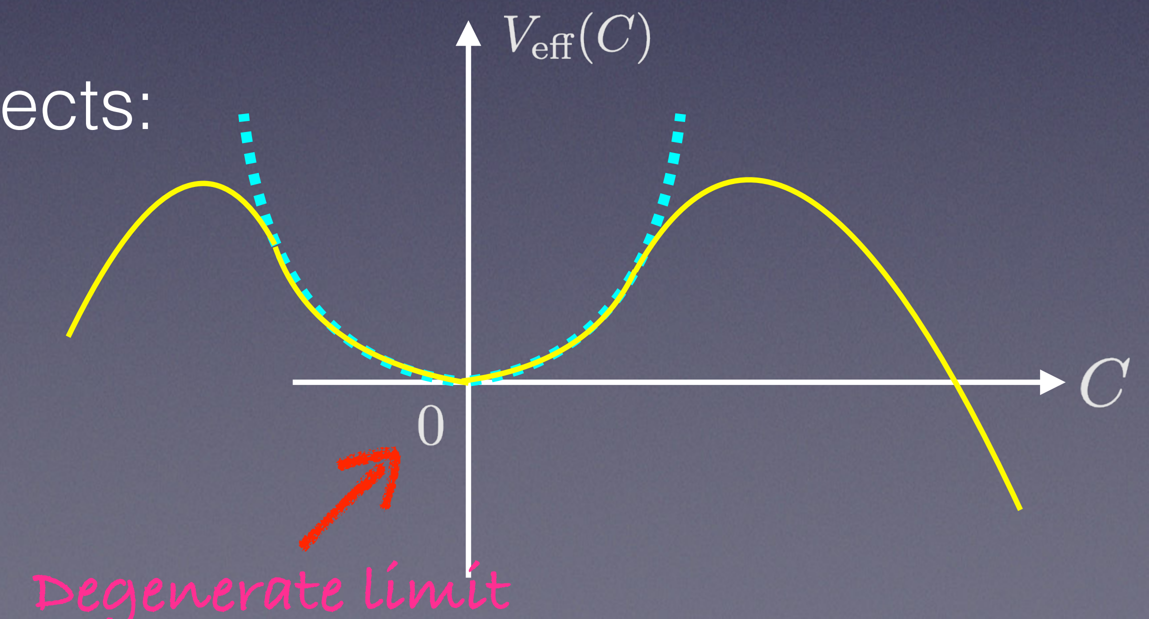
- A possible solution would be a flat spacetime.

$$\langle e^{a}{}_{\mu} \rangle = C\delta^{a}_{\mu} \qquad \lambda_1 = \cdots = \lambda_4 = C$$

  - Effective potential from spinor loop effects:

$$V_{\rm eff}(C) = \Lambda_{\rm cc} C^4 - \frac{m_f^4}{2(4\pi)^2} C^4 \log\left(\frac{m_f^2 C^2}{\Lambda_{\rm G}^2}\right)$$

tree level — spinor loop effect



Spontaneous local Lorentz symmetry breaking

- Local Lorentz gauge symmetry is broken. $\langle e^{\mathbf{a}}{}_{\mu} \rangle \neq 0$

  - Degrees of freedom (d.o.f.):

    Symmetric part (metric) (radial modes)

    Anti-symmetric part (NG modes)

    - Vierbein $e^{\mathbf{a}}{}_{\mu}$: 16 d.o.f. = 10 + 6 d.o.f.

      eaten

    - LL gauge field $(A_\mu)^a{}_b$: 4 d.o.f × 6 d.o.f : massive vector

  - LL gauge bosons become massive and decouple.

  - The symmetry parts (radial modes) are still massless thanks to diff.

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detail: See poster by J. Liu

Gravitational d.o.f. are composite fields of matter?

- Composite of fermions: "Spinor gravity"

$$e^{\mathbf{a}}{}_{\mu} \sim \langle \bar{\psi} \gamma^{\mathbf{a}} \partial_\mu \psi \rangle \qquad g_{\mu\nu} \sim \langle \bar{\psi} \gamma_\mu \partial_\nu \psi \rangle$$

H. C. Ohanian, Phys. Rev. 184 (1969) 1305.
H. Terazawa, Y. Chikashige, K. Akama and T. Matsuki, J. Phys. Soc. Jap. 43 (1977) 5.
A. Hebecker and C. Wetterich, Phys. Lett. B 574 (2003) 269 [hep-th/0307109].
C. Wetterich, Phys. Rev. D 70 (2004) 105004 [hep-th/0307145].

- Composite of scalar fields $\quad g_{\mu\nu} \sim \langle \partial_\mu \phi \partial_\nu \phi \rangle$

C. D. Carone, J. Erlich and D. Vaman, JHEP 03 (2017), 134, [arXiv:1610.08521 [hep-th]].
J. Liu, J. Haruna and **M. Yamada**, [arXiv:2309.15584 [hep-th]]. (two-dimensional)

TT-deformed O(N) scalar theory in d=2

- Action

A. B. Zamolodchikov arXiv:hep-th/0401146

$$S = \int \mathrm{d}^2 x \left[\frac{1}{2} (\partial_\mu \vec{\phi})^2 + \frac{m^2}{2} \vec{\phi}^2 + \alpha \det(T_{\mu\nu}) \right]$$

$T_{\mu\nu}$: energy-momentum tensor

α :deformation parameter

- Introducing auxiliary field (kind of bosonization)

$$\alpha \det(T_{\mu\nu}) = -\frac{1}{2} T_{\mu\nu} C^{\mu\nu} + \frac{1}{8\alpha} \det(C_{\mu\nu})$$

Attractive features of TT-deformed O(N) scalar theory in d=2

- Relate to the Nambu-Goto string action in a N+2 dimensional target space $\alpha \to \infty$

$$\mathcal{L}_{\text{Nambu-Goto}} = \frac{1}{2\alpha}\sqrt{\det\left(\partial_\alpha X \cdot \partial_\beta X\right)}$$

$$X^1 = x,\ X^2 = y,\ X^3 = \sqrt{\alpha}\phi/2$$

- Scalar theory coupled to gravity

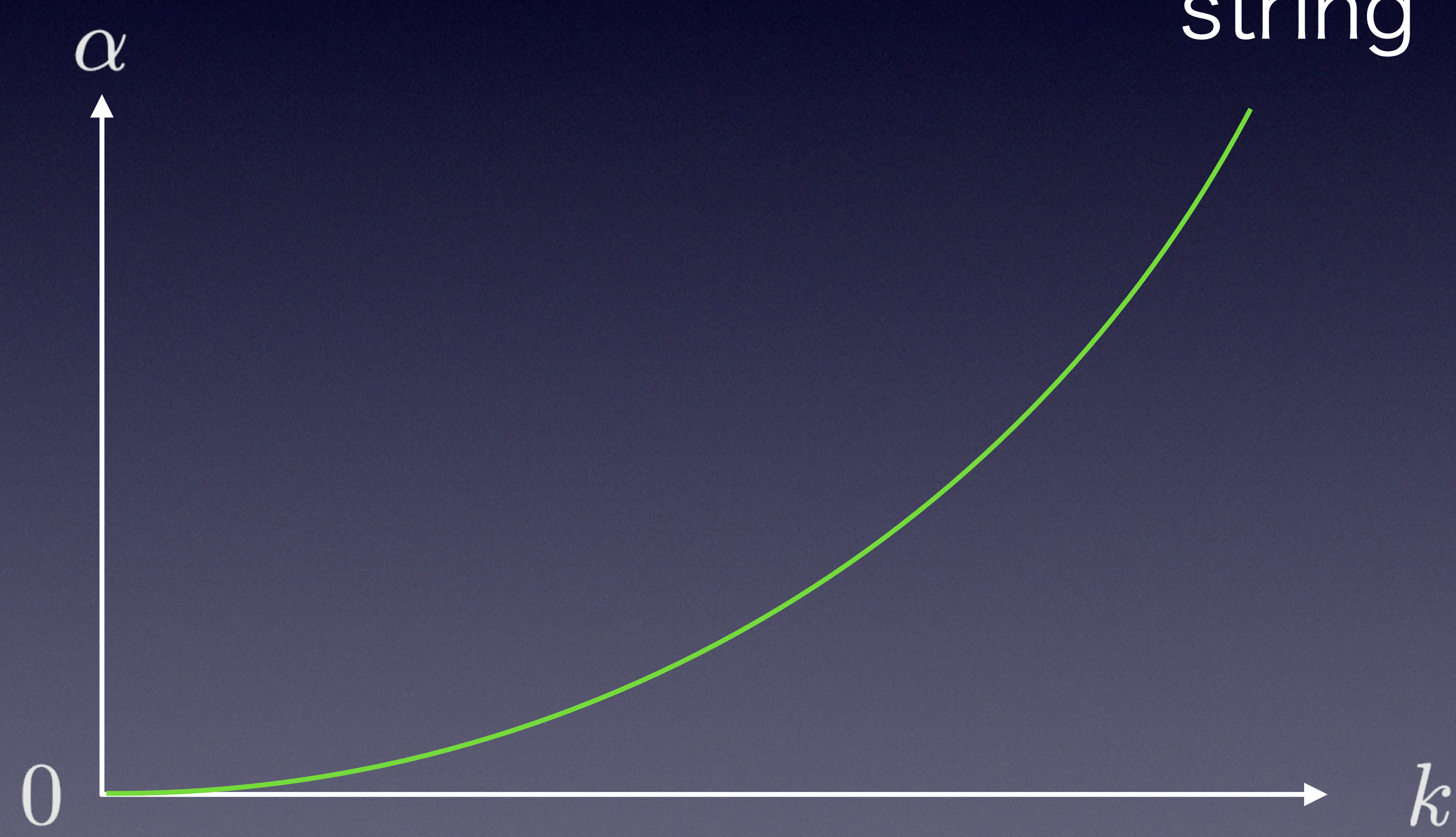
$$S = \int \mathrm{d}^2 x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\vec{\phi}\cdot\partial_\nu\vec{\phi} - \frac{m^2}{2}\vec{\phi}^2 + \frac{1}{8\alpha}\det(C_{\mu\nu})\right]$$

$$C_{\mu\nu} = \gamma^{\mu\nu} + \delta^{\mu\nu} C \qquad g^{\mu\nu} \equiv (\delta^{\mu\nu} - \gamma^{\mu\nu})/(1+C)$$

$$C \sim \vec{\phi}^2 \qquad \gamma_{\mu\nu} \sim \partial_\mu\vec{\phi}\cdot\partial_\nu\vec{\phi} - \frac{\delta_{\mu\nu}}{2}\partial_\rho\vec{\phi}\cdot\partial_\rho\vec{\phi}$$

Naive perturbative picture

- α is irrelevant.



FRG analysis

- Effective action

$$\Gamma_k = \int \mathrm{d}^2x \left[\frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{m_k^2}{2}\vec{\phi}^2 + \frac{\kappa_k}{2} T_{\mu\nu}C^{\mu\nu} + \Lambda_k + \lambda_k C + \frac{Z_{C,k}}{2}(\partial_\rho C^{\mu\nu})^2 - \frac{1}{8\alpha_k}\det(C^{\mu\nu}) + \beta_k C_{\mu\nu}C^{\mu\nu} \right]$$

- Non-trivial fixed point exists

	$\tilde{\Lambda}_k^*$	$\tilde{\lambda}_k^*$	$\tilde{m}_k^{2*}$	$\tilde{\kappa}_k^*$	$\tilde{\alpha}_k^*$	$\tilde{\beta}_k^*$	η_C
$N=1$ (LPA)	0.243	∓ 0.183	-1.25	± 0.471	0.328	-0.236	0
(w/η_C)	0.246	∓ 0.181	-1.26	± 0.462	0.323	-0.239	0.249
$N=2$ (LPA)	0.324	∓ 0.354	-1.15	± 0.174	0.303	-0.266	0
(w/η_C)	0.336	∓ 0.356	-1.15	± 0.167	0.302	-0.267	0.101
$N=3$ (LPA)	0.405	∓ 0.669	-1.07	± 0.045	0.302	-0.281	0
(w/η_C)	0.421	∓ 0.677	-1.06	± 0.043	0.299	-0.282	0.036

FRG analysis

- Effective action

$$\Gamma_k = \int \mathrm{d}^2 x \left[\frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{m_k^2}{2}\vec{\phi}^2 + \frac{\kappa_k}{2} T_{\mu\nu} C^{\mu\nu} + \Lambda_k + \lambda_k C + \frac{Z_{C,k}}{2}(\partial_\rho C^{\mu\nu})^2 - \frac{1}{8\alpha_k}\det(C^{\mu\nu}) + \beta_k C_{\mu\nu} C^{\mu\nu} \right]$$

- Critical exponents

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
$N=1$ (LPA)	2	2	$-3.22+36.6i$	$-3.22-36.6i$	4.37	1.91
(w/η_C)	2	1.88	$-6.20+37.3i$	$-6.20-37.3i$	4.02	1.68
$N=2$ (LPA)	2	2	$-2.69+80.6i$	$-2.69-80.6i$	3.40	1.91
(w/η_C)	2	1.94	$-4.51+83.2i$	$-4.51-83.2i$	3.34	1.82
$N=3$ (LPA)	2	2	$-2.41+211i$	$-2.41-211i$	2.84	1.94
(w/η_C)	2	1.98	$-3.73+218i$	$-3.73-218i$	2.88	1.91

FRG analysis

- Effective action

$$\Gamma_k = \int d^2x \left[\frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{m_k^2}{2}\vec{\phi}^2 + \frac{\kappa_k}{2} T_{\mu\nu} C^{\mu\nu} + \Lambda_k + \lambda_k C + \frac{Z_{C,k}}{2}(\partial_\rho C^{\mu\nu})^2 - \frac{1}{8\alpha_k}\det(C^{\mu\nu}) + \beta_k C_{\mu\nu} C^{\mu\nu} \right]$$

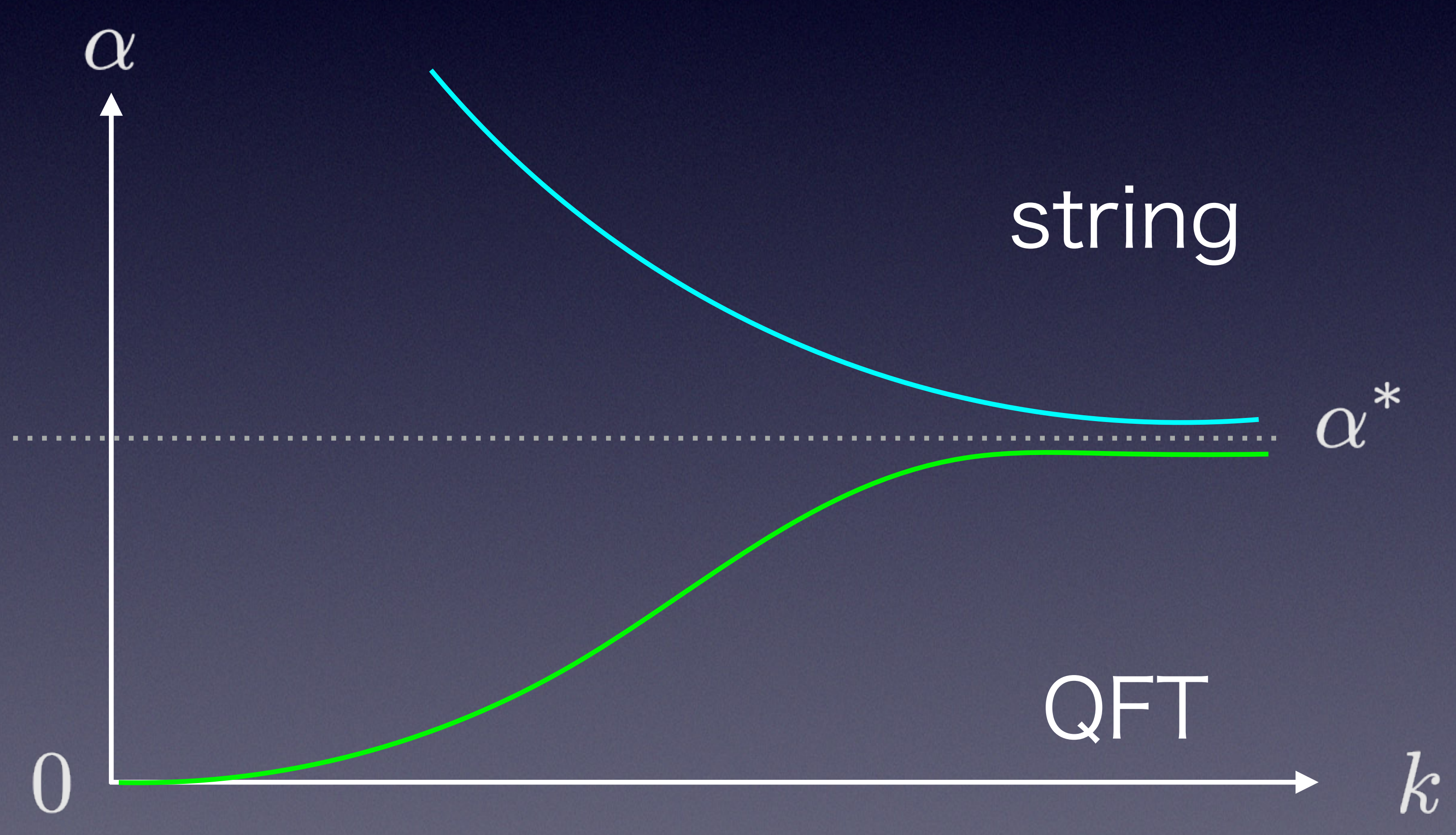
- Critical exponents

$$\theta_\alpha = -2$$

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
$N = 1$ (LPA)	2	2	$-3.22 + 36.6i$	$-3.22 - 36.6i$	4.37	1.91
(w/η_C)	2	1.88	$-6.20 + 37.3i$	$-6.20 - 37.3i$	4.02	1.68
$N = 2$ (LPA)	2	2	$-2.69 + 80.6i$	$-2.69 - 80.6i$	3.40	1.91
(w/η_C)	2	1.94	$-4.51 + 83.2i$	$-4.51 - 83.2i$	3.34	1.82
$N = 3$ (LPA)	2	2	$-2.41 + 211i$	$-2.41 - 211i$	2.84	1.94
(w/η_C)	2	1.98	$-3.73 + 218i$	$-3.73 - 218i$	2.88	1.91

Non-perturbative picture

- α is relevant.



Summary

- Metric d.o.f. may be not enough to describe quantum gravity in high energy.
- Bigger symmetry including diff. may exist.
- Spontaneous symmetry breaking gives massive modes and massless modes.
- Duality at fixed points becomes a hit for such a high energy theory of asymptotically safe gravity.

Appendix

Formula

$$|e(x)|\ \epsilon_{[\mu\nu\rho\sigma]} = \epsilon_{[\mathbf{abcd}]}\, e^{\mathbf{a}}{}_{\mu}(x)\, e^{\mathbf{b}}{}_{\nu}(x)\, e^{\mathbf{c}}{}_{\rho}(x)\, e^{\mathbf{d}}{}_{\sigma}(x)\,,$$

$$\begin{aligned}
|e(x)|\, e_{\mathbf{a}}{}^{\mu}(x) &= \frac{1}{3!}\, \epsilon_{[\mathbf{abcd}]}\ \epsilon_{[\mu\nu\rho\sigma]}\, e^{\mathbf{b}}{}_{\nu}(x)\, e^{\mathbf{c}}{}_{\rho}(x)\, e^{\mathbf{d}}{}_{\sigma}(x)\,, \\
|e(x)|\, e_{[\mathbf{a}}{}^{\mu}(x)\, e_{\mathbf{b}]}{}^{\nu}(x) &= \frac{1}{2}\, \epsilon_{[\mathbf{abcd}]}\ \epsilon_{[\mu\nu\rho\sigma]}\, e^{\mathbf{c}}{}_{\rho}(x)\, e^{\mathbf{d}}{}_{\sigma}(x)\,, \\
|e(x)|\, e_{[\mathbf{a}}{}^{\mu}(x)\, e_{\mathbf{b}}{}^{\nu}(x)\, e_{\mathbf{c}]}{}^{\rho}(x) &= \epsilon_{[\mathbf{abcd}]}\ \epsilon_{[\mu\nu\rho\sigma]}\, e^{\mathbf{d}}{}_{\sigma}(x)\,, \\
|e(x)|\, e_{[\mathbf{a}}{}^{\mu}(x)\, e_{\mathbf{b}}{}^{\nu}(x)\, e_{\mathbf{c}}{}^{\rho}(x)\, e_{\mathbf{d}]}{}^{\sigma}(x) &= \epsilon_{[\mu\nu\rho\sigma]}\,.
\end{aligned}$$

Lesson 2:
Fermi's weak theory

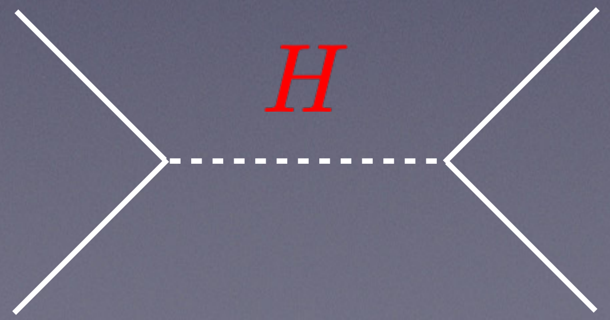
- Fermi's weak theory (1930)

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu$$

$$J_\mu = \bar{e}\gamma_\mu(1-\gamma^5)\nu_e + \cdots$$

  - Perturbatively non-renormalizable.
  - Breaks unitarity for $|p| > G_F^{-1/2}$

- Discovery of W boson and Higgs boson
- Standard model SU(2)×U(1)
  - Perturbatively renormalizable and unitary



First-order formalism

$$S = \int \mathrm{d}^4x\, e \left[-\Lambda + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} \right]$$

- Equation of motion $\quad (A_\mu)^a{}_b = e_\nu{}^a D_\mu e^\nu{}_b$
  - Obtain the EH action in the vierbein formalism
  - Introducing inverse vierbein breaks $SO(1,3)_{\text{local}}$ symmetry.
- Kinetic term of LL gauge field

$$\frac{1}{4} F^{ab}{}_{\mu\nu} F_{ab}{}^{\mu\nu} + \cdots \qquad \to R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \cdots$$

Degenerate limit

- Non-linear σ model: O*(N-1)* invariant
  - Constraint on fields $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
  - $f_\pi^2 \to 0$: symmetric phase (O*(N)* invariant)
- Gravity in first-order formalism
  - Constrain on metric $g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$
  - $C \to 0$: symmetric phase (SO(1,3) invariant).

$$\langle e^a{}_\mu \rangle = C \delta_\mu^a$$

$$\bar{g}_{\mu\nu} \propto C^2$$

$$\bar{g}^{\mu\nu} \propto C^{-2}$$

GL(4)

- GL(4) gauge field $\Upsilon^{\alpha}_{\beta\mu}(x)$

$$\Upsilon^{\alpha}_{\beta\mu}(x) \overset{\mathrm{GL}(4)}{\to} \Upsilon'^{\alpha}{}_{\beta\mu}(x') = \left(M^{\alpha}_{\gamma}(x)\Upsilon^{\gamma}_{\delta\nu}(x)\left(M^{-1}\right)^{\delta}_{\beta}(x) - \partial_{\nu}M^{\alpha}_{\gamma}(x)\left(M^{-1}\right)^{\gamma}{}_{\beta}(x)\right)\left(M^{-1}\right)^{\nu}_{\mu}(x)$$

$$M^{\mu}{}_{\nu}(x) := \frac{\partial x'^{\mu}}{\partial x^{\nu}}$$

$$\Upsilon^{\alpha}_{\beta\mu}(x) = \Upsilon^{\alpha}_{(\beta\mu)}(x) + \Upsilon^{\alpha}_{[\beta\mu]}(x)$$

Christoffel symbols — Torsion

$D_{\mu}g_{\rho\sigma} = 0$ — 40 d.o.f. — 24 d.o.f. — in 4 dimensions

- GL(4) → diff. by the condition $\partial_{[\lambda}M^{\mu}{}_{\nu]}(x) = 0$

24 conditions in 4 dimensions