Essential Quantum Einstein Gravity

Talk @ Quantum Spacetime and the Renormalization Group 2023

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Introduction

- Based on arXiv:2105.11482 (A. Baldazzi, R. Ben Alì Zinati and KF) ; arXiv:2107.00671 (A. Baldazzi and KF and *to appear* (A. Baldazzi, KF, Y. Kluth and B. Knorr) ,
- The key idea: Physics is invariant under a change of variables so we can use this to our advantage to remove inessential couplings from the set of running couplings.
- Exploiting these ideas we come closer to Weinberg's original idea of Asymptotic Safety since we only computing the running of essential couplings: those that enter into observables.
- The RG flow of quantum gravity has now been studied up to sixth order in derivatives (in the background field approximation) using this approach.
- A new picture of the asymptotic safety scenario emerges from the results.

Introduction

- Asymptotic Safety is becoming a more mature approach to quantum gravity.
- Provides a potentially consistent ultraviolet (UV) completion of Quantum Gravity known as Quantum Einstein Gravity.
- However, there remain many open questions:

  —How many free parameters are there?

  — What are the fundamental degrees of freedom?

  —Is the theory Unitary?

Introduction

- Number of free parameters = number of relevant directions (minus one).
- Weinberg '79: However we should disregard inessential couplings since these do not enter expressions of the physical observables.
- Wegner '74: inessential couplings have scheme dependent scaling exponents. So there is no sense in calling an inessential coupling relevant or irrelevant.
- Although evidence suggested that the Reuter fixed point has three relevant directions in pure gravity, we can question this.

Introduction

- Adding higher derivative terms to the Einstein-Hilbert action e.g.

$$R \to R + c_1 G_N R^2 + c_2 G_N R_{\mu\nu} R^{\mu\nu} \tag{1}$$

  is usually associated to introducing new degrees of freedom other than the massless graviton of Einstein gravity.

- At the level of the propagator higher order terms introduce new poles.

- However, using field redefinitions we can remove such terms from the propagator evaluated on any conformally flat background.

- Technical advantage: calculations are more manageable.

- Conceptual advantage: Will not find fixed points without additional poles e.g. Stelle's higher derivative theory.

Frames

- In quantum field theory we are interested in computing expectation values of operators by averaging over all field configuartions

$$\langle \hat{\mathcal{O}} \rangle \equiv \mathcal{N} \int_{\mathcal{M}} (\mathrm{d}\hat{\chi}) e^{-S_{\hat{\chi}}[\hat{\chi}]} \hat{\mathcal{O}}_{\hat{\chi}}[\hat{\chi}] \,, \tag{2}$$

where $\mathcal{N}^{-1} = \int (\mathrm{d}\hat{\chi}) e^{-S_{\hat{\chi}}[\hat{\chi}]}$.

- But why do we have to use $\hat{\chi}$?

- We could write observables in terms of other field variables $\hat{\phi}[\hat{\chi}]$ provided they provide another coordinate system on $\mathcal{M}$.
- Observables transform as scalars

$$\hat{\mathcal{O}}_{\hat{\phi}}[\hat{\phi}] = \hat{\mathcal{O}}_{\hat{\chi}}[\hat{\chi}] \tag{3}$$

Frames

- We can compute observables using a generating functional of the connected correlation functions of $\hat{\phi}$ namely

$$\mathrm{e}^{W_{\hat{\phi}}[J]} = \int_{\mathcal{M}} (\mathrm{d}\hat{\chi}) \mathrm{e}^{-S_{\hat{\chi}}[\hat{\chi}]} \mathrm{e}^{J\cdot\hat{\phi}[\hat{\chi}]} \,, \tag{4}$$

where $J \cdot \hat{\phi} = \int \mathrm{d}^4x \, J(x)\hat{\phi}(x)$.

- Observables are given by

$$\langle \hat{\mathcal{O}} \rangle = \mathrm{e}^{-W_{\hat{\phi}}[J]} \mathcal{O}_{\hat{\phi}} \left[\frac{\delta}{\delta J} \right] \mathrm{e}^{W_{\hat{\phi}}[J]} \Big|_{J=0} \tag{5}$$

which are invariant under a change of frame i.e. the coordinate system $\hat{\phi}$.

Inessential couplings

- In general we can consider the generating functionals $W_{\hat{\phi}}[J]$ as depending on a set of couplings g_i.
- If the change in a combination of couplings ζ is equivalent to a change in $\hat{\phi}$ we call ζ an inessential coupling
- Under infinitesimal frame transformations $\hat{\phi}[\hat{\chi}] \to \hat{\phi}[\hat{\chi}] - \hat{\xi}[\hat{\chi}]$ we have

$$W_{\hat{\phi}}[J] \to W_{\hat{\phi}-\hat{\xi}}[J] = W_{\hat{\phi}}[J] - J \cdot \xi[J] + \dots , \tag{6}$$

where $\xi[J] = \langle \hat{\xi} \rangle_J$ is the J dependent expectation value.

So an inessential coupling is one for which

$$\frac{\partial}{\partial \zeta} W_{\hat{\phi}}[J] = -J \cdot \Phi[J] \tag{7}$$

For the effective action $\Gamma_{\hat{\phi}}[\phi] = -W_{\hat{\phi}}[J] + J \cdot \phi$ we have (Weinberg '79)

$$\frac{\partial}{\partial \zeta} \Gamma_{\hat{\phi}}[\phi] = \frac{\delta \Gamma_{\hat{\phi}}[\phi]}{\delta \phi} \cdot \Phi[\phi] \tag{8}$$

the RHS is the *redundant operator* conjugate to ζ.

Essential Renormalisation Group

- The essential RG is a method to eventually compute observables that makes use of the generalised exact RG equation for the effective average action EAA $\Gamma_k[\phi]$, which depends on the RG scale k.
- The EAA obtains a dependence on the RG scale k from two origins: by decreasing $k \to k - \delta k$ we integrate out modes in a small shell in momentum space while additionally we make a k-dependent change of frame.
- The essential RG uses the change of frame along the flow to fix the values of the inessential couplings ζ_α such that they are independent of k.
- Only compute the running of the remaining essential couplings λ_a.

Essential Renormalisation Group

- The generalised EAA is given by the solution to

$$e^{-\Gamma_k[\phi]} := \int (\mathrm{d}\hat{\chi})\, e^{-S[\hat{\chi}]+(\hat{\phi}_k[\hat{\chi}]-\phi)\cdot\frac{\delta}{\delta\phi}\Gamma_k[\phi]-\frac{1}{2}(\hat{\phi}_k[\hat{\chi}]-\phi)\cdot\mathcal{R}_k\cdot(\hat{\phi}_k[\hat{\chi}]-\phi)}\,, \tag{9}$$

from which it follows that

$$\phi = \langle\hat{\phi}_k\rangle_{\phi,k}\,, \tag{10}$$

where

$$\langle\hat{\mathcal{O}}\rangle_{\phi,k} := e^{\Gamma_k[\phi]}\int (\mathrm{d}\hat{\chi})\, e^{-S[\hat{\chi}]+(\hat{\phi}_k[\hat{\chi}]-\phi)\cdot\frac{\delta}{\delta\phi}\Gamma_k[\phi]-\frac{1}{2}(\hat{\phi}_k[\hat{\chi}]-\phi)\cdot\mathcal{R}_k\cdot(\hat{\phi}_k[\hat{\chi}]-\phi)}\hat{\mathcal{O}}[\hat{\chi}] \tag{11}$$

is the ϕ and k dependent expectation value.

Pawlowski's generalised Wetterich-Morris equation

- The generalised flow equation satisfied by $\Gamma_k[\phi]$ is given by (Pawlowski 2005)

$$\left(\partial_t + \Psi_k[\phi] \cdot \frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\mathcal{G}_k[\phi]\left(\partial_t + 2 \cdot \frac{\delta}{\delta\phi}\Psi_k[\phi]\right) \cdot \mathcal{R}_k\,, \tag{12}$$

where $t := \log(k/k_0)$, with k_0 some physical reference scale, under the trace appearing in the RHS

$$\mathcal{G}_k[\phi] := (\Gamma_k^{(2)}[\phi] + \mathcal{R}_k)^{-1} \tag{13}$$

is the IR regularised propagator, with $\Gamma_k^{(2)}[\phi]$ denoting the hessian of the EAA with respect to the field $\phi(x)$, and

$$\Psi_k[\phi] := \langle \partial_t \hat{\phi}_k[\hat{\chi}] \rangle_{\phi,k} \tag{14}$$

is the RG kernel which takes into account the k-dependent field reparameterisations.

Pawlowski's generalised Wetterich-Morris equation

- The form of the flow equation

$$\left(\partial_t + \Psi_k[\phi] \cdot \frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\mathcal{G}_k[\phi]\left(\partial_t + 2 \cdot \frac{\delta}{\delta\phi}\Psi_k[\phi]\right) \cdot \mathcal{R}_k\,, \tag{15}$$

can be compared to the form of a redundant operator which has an extra regulator dependent term

$$\zeta\frac{\partial}{\partial\zeta}\Gamma_k[\phi] = \Phi[\phi] \cdot \frac{\delta}{\delta\phi}\Gamma_k[\phi] - \mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + \mathcal{R}_k} \cdot \frac{\delta}{\delta\phi}\Phi[\phi] \cdot \mathcal{R}_k \tag{16}$$

- One then sees that the form of Ψ_k dictates the flow of the inessential couplings.
- Thus we can conclude that applying RG conditions that fix the values of inessential couplings we can solve the flow equation for $\Gamma_k[\phi]$ under these constraints and for $\Psi_k[\phi]$.
- Thus the RG conditions implicitly define $\hat{\phi}_k[\hat{\chi}]$.

Minimal essential scheme for a scalar field theory

- The form of redundant operators depends on the form of the action and therefore where we are in theory space.
- However, one can identify the essential couplings at a point in theory space and they will remain essential a finite distance way.
- The simplest point is the Gaussian Fixed Point (GFP) $\Gamma_k = \frac{1}{2}\int_x \partial_\mu\phi\partial_\mu\phi$ then all the essential couplings can be identified as those that do not vanish (up to boundary terms) when $-\partial^2\phi = 0$.

Minimal essential scheme for a scalar field theory

- In the minimal essential scheme we keep only the essential couplings identified at the GFP in the action and self consistently solve for the flow of the essential couplings and the parameters inside Ψ_k.

- For example the most general action with four derivatives is:

$$\Gamma_k = \int_x \Big\{ V_k(\phi) + \frac{1}{2} Z_k(\phi) \left(\partial_\mu \phi \, \partial_\mu \phi\right) + \\ + W^a_k(\phi) \left(\partial^2 \phi\right)^2 + W^b_k(\phi) \phi \partial^2 \phi \left(\partial_\mu \phi \, \partial_\mu \phi\right) + W^c_k(\rho) \left(\partial_\mu \phi \, \partial_\mu \phi\right)^2 \Big\} \,, \tag{17}$$

- In the minimal essential scheme we instead have:

$$\Gamma_k = \int_x \Big\{ V_k(\phi) + \frac{1}{2} \left(\partial_\mu \phi \, \partial_\mu \phi\right) + W^c_k(\rho) \left(\partial_\mu \phi \, \partial_\mu \phi\right)^2 \Big\} \,, \tag{18}$$

Quantum Einstein Gravity

- We will apply the Essential RG to gravity in the background field approximation

- The diffeomorphism invariant action has the derivative expansion

$$\bar{\Gamma}_k[g] = \int d^4x \sqrt{\det g} \left\{ \frac{\rho_k}{8\pi} - \frac{1}{16\pi G_k} R + a_k R^2 + b_k R_{\mu\nu} R^{\mu\nu} + c_k E + O(\partial^6) \right\} . \quad (19)$$

  Here G_k and ρ_k are the running Newton's constant and vacuum energy, respectively, and a_k, b_k and c_k multiply the $O(\partial^4)$ terms with $E = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$.

- The minimal essential scheme consists of identifying the inessential couplings at the Gaussian fixed point $G_k = \rho_k = a_k = b_k = 0$ and maintaining the values of the inessential couplings along the RG flow.

$$\bar{\Gamma}_{\mathrm{GFP}}[g] \sim \int d^4x \sqrt{\det g} R \quad (20)$$

Quantum Einstein Gravity

- We use the Feynman-Dedonder gauge

$$F^{\mu} = \frac{\sqrt{2}}{\kappa_k}\left(\bar{g}^{\mu\lambda}\bar{g}^{\nu\rho} - \frac{1}{2}\bar{g}^{\nu\mu}\bar{g}^{\rho\lambda}\right)\bar{\nabla}_{\nu} g_{\lambda\rho}\,, \tag{21}$$

and κ_k denotes the dimensionful coupling

$$\kappa_k \equiv \sqrt{32\pi G_k}\ . \tag{22}$$

- For the regulator we use for the metric

$$\mathcal{R}_k^{\mu\nu,\alpha\beta} = \frac{1}{2\kappa_k^2}\sqrt{\det g}\left(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta}\right)R_k(\Delta) \tag{23}$$

the Litim cutoff function $R_k(p^2) = (k^2 - p^2)\Theta(k^2 - p^2)$.

Quantum Einstein Gravity

- The redundant operators for $\bar{\Gamma}_k[g]$ are given by

$$\mathcal{T}[\Phi^g] := \Phi^g \cdot \frac{\delta}{\delta g}\bar{\Gamma}_k - \mathrm{Tr}\,\mathcal{G}_k^{gg} \cdot \frac{\delta}{\delta g}\Phi^g \cdot \mathcal{R}_k^{gg}\,, \tag{24}$$

  where Φ^g are symmetric covariant tensors which appear in Ψ i.e. $\Phi^g_{\mu\nu} = g_{\mu\nu}, Rg_{\mu\nu}, R_{\mu\nu}$.
- At the Gaussian fixed point the first term of the redundant dominates.
- The RG kernel

$$\Psi^g_{\mu\nu}[g] = \gamma_g g_{\mu\nu} + \gamma_R R g_{\mu\nu} + \gamma_{Ricci} R_{\mu\nu} + O(\partial^4)\,, \tag{25}$$

- We have four couplings (ignoring the topological term) and three gamma functions.
- Thus we can expect three inessential couplings leaving only one essential coupling.

Fourth order operators are inessential

- It is straight forward to see that both higher derivative coupling a_k and b_k are inessential at the Gaussian fixed point where $\tilde{G} = k^2 G_k$ goes to zero. This follows since the operators $R_{\mu\nu}$ and R are proportional to the Einstein Equations.
- The freedom to pick $\gamma_R R g_{\mu\nu} + \gamma_{Ricci} R_{\mu\nu}$ then allows us to set the RG conditions $a_k = 0 = b_k$ along the flow.
- It is possible that away from the Gaussian fixed point these RG conditions can fail. This happens if $\mathcal{T}[R g_{\mu\nu}]$ or $\mathcal{T}[R_{\mu\nu}]$ are no long linearly independent of the Einstein-Hilbert action. This typically lead to a singularity in the beta functions.

Higher orders and matter

- At all higher orders in perturbation theory all terms that vanish which $R_{\mu\nu} = 0$ can be removed by a field redefinition.
- Consequence: on any conformally flat spacetime space time the propagator is just that of GR.
- The minimal essential scheme for QEG is a non-perturbative form of this scheme.
- This is still the case with matter: the one-loop counter term that makes GR + scalar matter perturbatively renormalisable is

$$\int \mathrm{d}^4 x \sqrt{\det g} (\nabla_\mu \phi \nabla^\mu \phi)^2 \,, \tag{26}$$

which does not upset the form of propagator (see the paper by Benjamin Knorr arXiv:2204.08564).

The vacuum energy is inessential

- The role of γ_R and γ_{Ricci} is to remove the higher order curvature terms.
- It is less clear what the role of γ_g is.
- Evidently it can be used to apply some RG condition on some combination of G_k and ρ_k... but which combination and which condition?
- To answer these questions we have to study the Gaussian fixed point to know form of γ_g is consistent with the Gaussian fixed point and precisely identify the inessential coupling there.

The vacuum energy is inessential

- To properly understand the GFP corresponding to linearised Einstein Gravity we must work in terms of the graviton field $\hat{\phi}_{\mu\nu}$ defined by

$$\hat{g}_{\mu\nu} = \mathfrak{g}_{\mu\nu} + \kappa_k \hat{\phi}_{\mu\nu} \,, \tag{27}$$

where $\mathfrak{g}_{\mu\nu}$ is a flat metric, then

$$\Psi^{\phi}_{\mu\nu} \equiv \langle \partial_t \hat{\phi}_{\mu\nu}(k) \rangle = \gamma_{\text{shift}} \mathfrak{g}_{\mu\nu} - \frac{1}{2}\eta_\phi \phi_{\mu\nu} + O(\kappa_k) \,, \tag{28}$$

where $\phi_{\mu\nu} = \langle \hat{\phi}_{\mu\nu} \rangle$ and

$$\eta_\phi := \eta_N - 2\gamma_g \,, \qquad \gamma_{\text{shift}} := \frac{\gamma_g}{\kappa_k} \tag{29}$$

- Imposing that γ_{shift} is finite when $\kappa_k = 0$, we deduce that $\gamma_g = 0$ at the GFP.

The vacuum energy is inessential

- The GFP the EAA has the form

$$\bar{\Gamma}_k^{\rm GFP} := \frac{1}{2}\phi \cdot K_{\phi\phi}[\mathfrak{g}] \cdot (\Delta[\mathfrak{g}] - \Delta_{\rm gf}[\mathfrak{g}]) \cdot \phi + k^4 \frac{1}{8\pi} \int {\rm d}^4 x \sqrt{\det \mathfrak{g}}\,\tilde{\rho}_{\rm GFP}\,, \qquad (30)$$

  where we anticipate that for $\kappa_k = 0$ where $\tilde{\rho}_{\rm GFP}$ is the dimensionless fixed point value for the vacuum energy. From the LHS of the flow equation we have

$$\left(\partial_t|_\phi + \Psi_k^\phi \cdot \frac{\delta}{\delta\phi}\right)\bar{\Gamma}_{GFP} = \frac{k^4}{2\pi} \int {\rm d}^4 x \sqrt{\det \mathfrak{g}}\,\tilde{\rho}_{\rm GFP} - \frac{1}{2}\eta_\phi \phi \cdot K[\mathfrak{g}] \cdot (\Delta[\mathfrak{g}] - \Delta_{\rm gf}[\mathfrak{g}]) \cdot \phi\,, \qquad (31)$$

  while on the RHS we have, using that $\gamma_g = 0$,

$$\frac{1}{2}{\rm Tr}\,\mathcal{G}_k^{\phi\phi}\left(\partial_t + 2 \cdot \frac{\delta}{\delta\phi}\Psi_k^\phi\right) \cdot \mathcal{R}_k^{\phi\phi} - {\rm Tr}\,\mathcal{G}_k^{\bar{c}c} \cdot \partial_t \mathcal{R}_k^{\bar{c}c} = \int_0^\infty {\rm d}z\, z \frac{-3\eta_\phi R_k(z) + \partial_t R_k(z)}{16\pi^2 (R_k(z) + z)}\,, \qquad (32)$$

- We find that $\eta_\phi = 0$ at the GFP which together with $\gamma_g = 0$ implies $\eta_N = 0$.
- Computing the integral with the Litim cutoff we have that

$$\tilde{\rho}_{\rm GFP} = \frac{1}{8\pi} \qquad (33)$$

The vacuum energy is inessential

- Having found the GFP we can now look at the linearised beta functions keeping γ_g arbitrary such that

$$\gamma_g = w_1\left(\tilde{\rho} - \frac{1}{8\pi}\right) + w_2\tilde{G} + \dots\,, \tag{34}$$

- The linearised beta functions are given by

$$\partial_t\tilde{G} = 2\tilde{G} + \dots\,, \tag{35}$$

$$\partial_t\tilde{\rho} = \left(\frac{w_1}{3\pi} - 4\right)\left(\tilde{\rho} - \frac{1}{8\pi}\right) + \left(\frac{w_2}{3\pi} + \frac{38}{24\pi^2}\right)\tilde{G} + \dots \tag{36}$$

- From which we see that we can choose the flow of $\tilde{\rho}$ which is the hallmark of an inessential coupling!
- The usual assignment of the scaling exponent 4 to the vacuum energy is a scheme dependent!

Minimal essential scheme at orders ∂^2 and ∂^4

- The RG condition for γ_g in the minimal essential scheme is to set

$$\rho(k) = k^4 \tilde{\rho}_{\rm GFP} \tag{37}$$

for all scales.

- Thus our ansatz for the EAA at order $O(\partial^4)$ is

$$\bar{\Gamma}_k[g] = \int d^4x \sqrt{\det g} \left\{ k^4 \frac{\tilde{\rho}_{\rm GFP}}{8\pi} - \frac{1}{16\pi G_k} R + c_k E + O(\partial^6) \right\}, \tag{38}$$

- We can study the flow of Newton's constant at orders $O(\partial^2)$ and $O(\partial^4)$ using heat kernel techniques to compute the traces
- This is much similar in the minimal essential scheme than in a standard scheme!
- Drawback or advantage?: We can't find fixed points associated to higher derivative theories.

Minimal essential scheme at orders $O(\partial^2)$ and $O(\partial^4)$

- At order $O(\partial^2)$ we obtain one beta function and one gamma function by expanding the flow equation

$$\gamma_g = \gamma_g(\tilde{G}) , \qquad \partial_t \tilde{G} = \beta_{\tilde{G}}(\tilde{G}) ,$$

- Keeping all terms of order $O(\partial^4)$ we find the beta and (dimensionless) gamma functions

$$\gamma_g = \gamma_g(\tilde{G}) , \quad \partial_t \tilde{G} = \beta_{\tilde{G}}(\tilde{G}) ,$$
$$\tilde{\gamma}_R = \tilde{\gamma}_R(\tilde{G}) , \quad \tilde{\gamma}_{Ricci} = \tilde{\gamma}_{Ricci}(\tilde{G}) , \quad \partial_t c_k = \beta_c(\tilde{G}) .$$

These are all only functions of $\tilde{G}$!

Perturbation theory

- Check perturbation theory: one-loop divergencies of GR (in our gauge)

$$\Gamma_{\rm div} = \frac{1}{d-4}\frac{1}{(4\pi)^2}\int {\rm d}^d x\sqrt{\det g}\left[\frac{1}{60}R^2 + \frac{7}{10}R_{\mu\nu}R^{\mu\nu} + \frac{53}{45}E\right]. \qquad (39)$$

- In the MES these induce the field redefinitions and the running of the coupling c_k for the topological term

$$\gamma_R = -\frac{11}{30\pi}G_k + O(G_k^2)\,, \quad \gamma_{Ricci} = \frac{7}{10\pi}G_k + O(G_k^2)\,, \quad \beta_c = \frac{1}{(4\pi)^2}\frac{53}{45} + O(G_k)\,. \qquad (40)$$

Non-perturbative beta and gamma functions

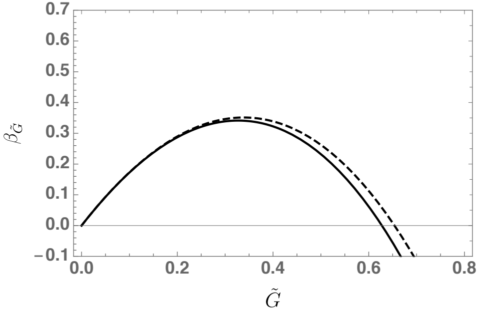

Figure: The beta function for Newton's constant in the Einstein–Hilbert approximation (dashed line) and the order ∂^4 approximation (solid line).

Non-perturbative beta and gamma functions

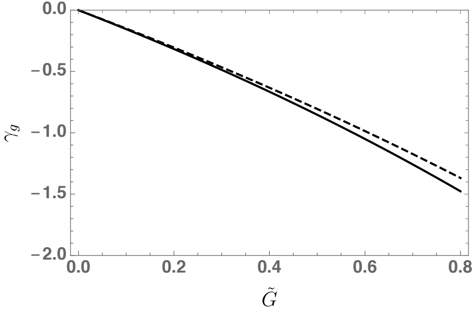

Figure: The gamma function γ_g in the Einstein–Hilbert approximation (dashed line) and the order ∂^4 approximation (solid line).

Non-perturbative beta and gamma functions

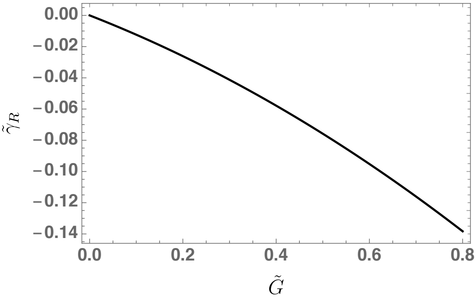

Figure: The gamma function $\tilde{\gamma}_R$ in the order ∂^4 approximation.

Non-perturbative beta and gamma functions

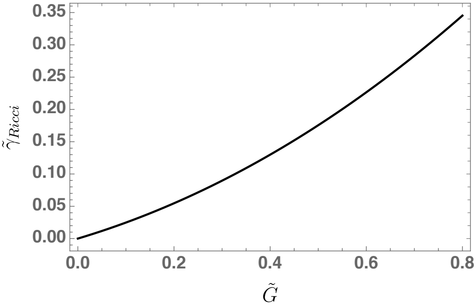

Figure: The gamma function $\tilde{\gamma}_{Ricci}$ in the order ∂^4 approximation.

Reuter fixed point

- In both approximations find the Reuter fixed point

$$\tilde{G} = \tilde{G}_\star = 0.6538 \quad \text{at } O(\partial^2)\,,$$
$$\tilde{G} = \tilde{G}_\star = 0.6275 \quad \text{at } O(\partial^4)\,,$$

- The critical exponent at the Reuter fixed point

$$\theta = -\frac{\partial \beta_{\tilde{G}}}{\partial \tilde{G}}(\tilde{G}_\star) \tag{41}$$

is given by

$$\theta = 2.3129 \quad \text{at } O(\partial^2)\,, \tag{42}$$
$$\theta = 2.3709 \quad \text{at } O(\partial^4)\,, \tag{43}$$

which can be compared to the canonical scaling dimension of $\theta_{\text{can}} = 2$ which is obtained at one-loop, and, therefore, receives a small correction.

An apparent conundrum?

- Exploiting the minimal essential scheme we have identified the vacuum energy as an inessential coupling and stopped it flowing.

- We should not lose any physics in doing this. However, in standard schemes there are trajectories which allow for a non-zero physical cosmological constant in Planck units $\tau_k = G_k^2 \rho_k$ at $k = 0$. So what gives?

- If we do not choose γ_g to stop the flow of $\tilde{\rho}$ but only assume that it vanishes at $\tilde{G} = 0$ then at order ∂^2

$$\partial_t \tau_k = -\frac{14 \tilde{G} \tau_k}{3\pi} + O(G^2)\,, \tag{44}$$

  which implies that τ_0 can be non-zero.

- However at order ∂^4 we instead find

$$\partial_t \tau_k = -\frac{328 \tau_k^2}{3(20\pi - 7\tau_k)} + O(G^2)\,, \tag{45}$$

  such that $\tau_0 = 0$ is the only allowed value. Thus we find a consistent picture at order ∂^4.

Sixth order in derivatives

- In recent work with Alessio Baldazzi, Yannick Kluth and Benjamin Knorr we have extended the study of quantum gravity to sixth order.
- Made an extensive study of the regulator dependence e.g. $R_k \to \alpha R_k$ and $R_k(-\nabla^2 + \beta\,\text{curvature terms})$.
- At order ∂^6 there is one further essential coupling at the GFP, the infamous Goroff-Sagnotti term:

$$\Gamma_k = \int d^4x\sqrt{g}\left[\rho - \frac{1}{16\pi G_N}R + \sigma_E E + G_{C^3} C^{\rho\sigma}{}_{\mu\nu} C^{\mu\nu}{}_{\alpha\beta} C^{\alpha\beta}{}_{\rho\sigma}\right].$$

To ensure no inessential couplings are needed we need the RG kernel:

$$\begin{aligned}\Psi^g_{\mu\nu} =& \gamma_g g_{\mu\nu} + \gamma_R R g_{\mu\nu} + \gamma_S S_{\mu\nu} \\ &+ \gamma_{R^2} R^2 g_{\mu\nu} + \gamma_{C^2}\left(C_{\rho\sigma\alpha\beta}C^{\rho\sigma\alpha\beta} g_{\mu\nu} - 8\Lambda S_{\mu\nu}\right) \\ &+ \gamma_{SSTL}\left(S_{\mu\rho}S^{\rho}_{\nu} - \frac{g_{\mu\nu}}{4} S^{\rho\sigma} S_{\rho\sigma}\right) + \gamma_{CS} C_{\mu\rho\nu\sigma} S^{\rho\sigma} \\ &+ \gamma_{\Delta R} g_{\mu\nu}\Delta R + \gamma_{\Delta S}\Delta S_{\mu\nu} \mathcal{O}\left(\partial^6\right).\end{aligned} \tag{46}$$

Sixth order in derivatives

- The Goroff-Sagnotti term is found to be irrelevant at the Reuter fixed point

$$\theta_1 = 2.22519, \qquad \theta_2 = -3.84962\,.$$

- Using a natural type II cutoff ($\beta = 1$), where "momentum squared" p^2 refers to the eigenvalues of the hessian of the Einstein-Hilbert action, the value of the Goroff-Sagnotti is particularly small:

$$g_N^\star = 0.36419, \qquad g_{C^3}^\star = 4.49003 \times 10^{-7}\,.$$

- Furthermore apart from γ_g the gamma functions remain small at the fixed point:

$$\begin{aligned} \gamma_g &= -0.99629, & \gamma_R &= 0.01117, \\ \gamma_S &= 0.05585, & \gamma_{R^2} &= 0.00796, \\ \gamma_{C^2} &= -0.00474, & \gamma_{RS} &= -0.01209, \\ \gamma_{SSTL} &= -0.02386, & \gamma_{CS} &= 0.02484, \\ \gamma_{\Delta R} &= -0.01063, & \gamma_{\Delta S} &= 0.00690\,. \end{aligned} \tag{47}$$

Sixth order in derivatives

- Phase diagram

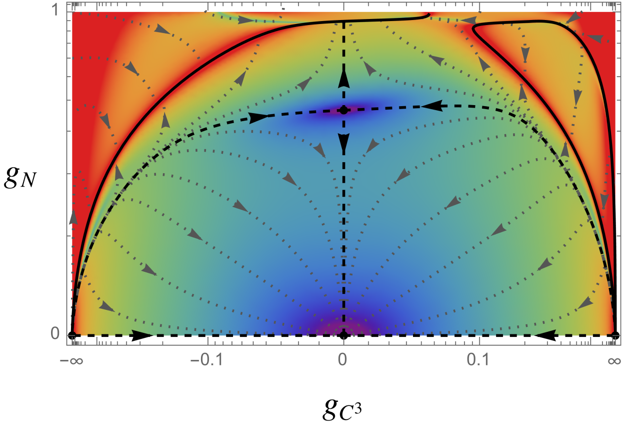

Figure: Phase diagram with natural endomorphism ($\alpha = \beta = 1$). Solid black lines indicate divergences of the β-functions, and the dashed black lines show the separatrices.

Is it GR after all?

- Using the minimal essential scheme and going to sixth order the action remains very close to the Einstein-Hilbert action.
- It seems not implausible that there exists a scheme where one is just “quantising GR” at the Reuter fixed point.
- I think this scenario deserves more focused attention using different methods e.g. lattice, tensor models, perturbation theory in $d = 2 + \varepsilon$ dimensions ... .

Conclusions

- Able to removed inessential couplings to simplify investigations of asymptotically safe quantum gravity.
- Applying the minimal essential scheme means we check for fixed points with the propagator of the graviton: no additional poles which can lead to unitary problems.
- Thus we only look for interacting fixed points for the couplings of the massless graviton.
- The Reuter fixed point is found and is stable between approximations.
- Very encouraging evidence that the Reuter fixed point is physical and has only one relevant direction.
- What are the consequences for the cosmological constant problem? How does matter change the picture?