Corrections to Schwarzschild geometry from the two-loop counterterm

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with
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Outline

- Goal and motivation
- A convenient metric form of the black hole geometry
- Local solutions to the *EH+GS* system
- Global solutions
- Concluding remarks

Motivation

- Goal: change the equations of motion due to higher-order operators

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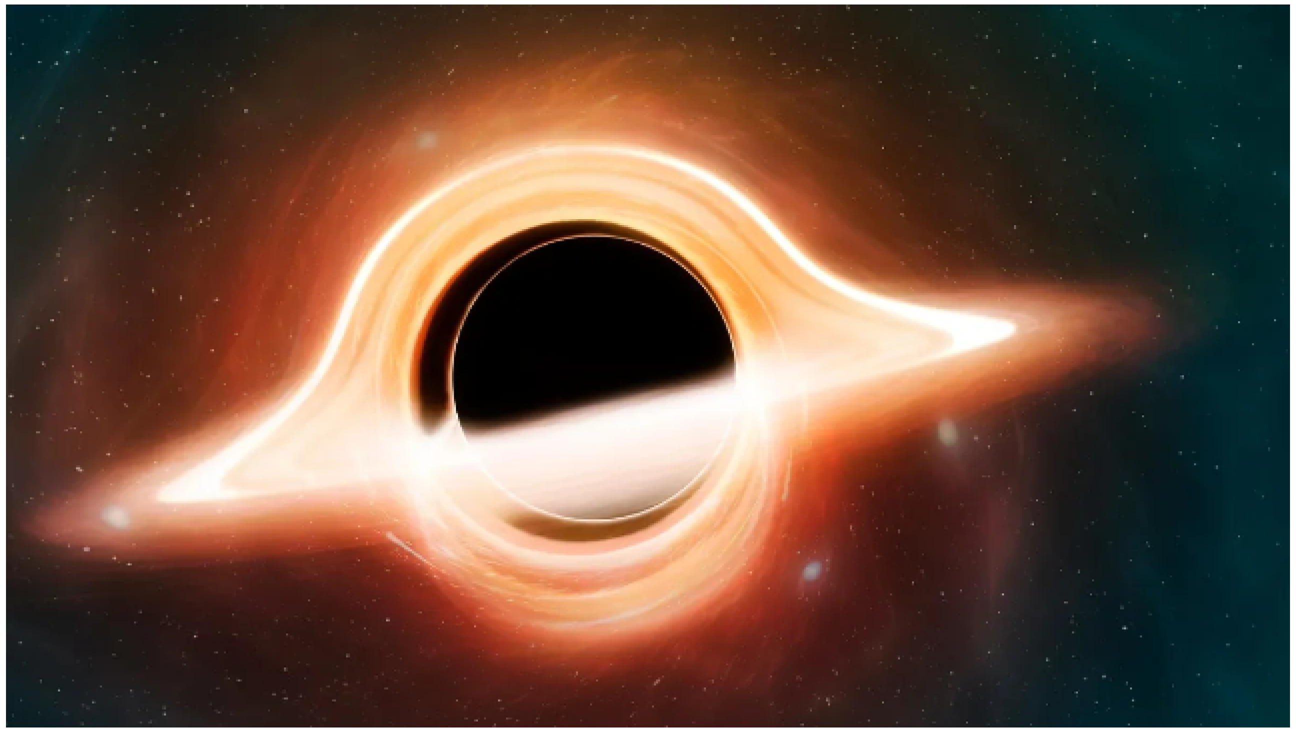
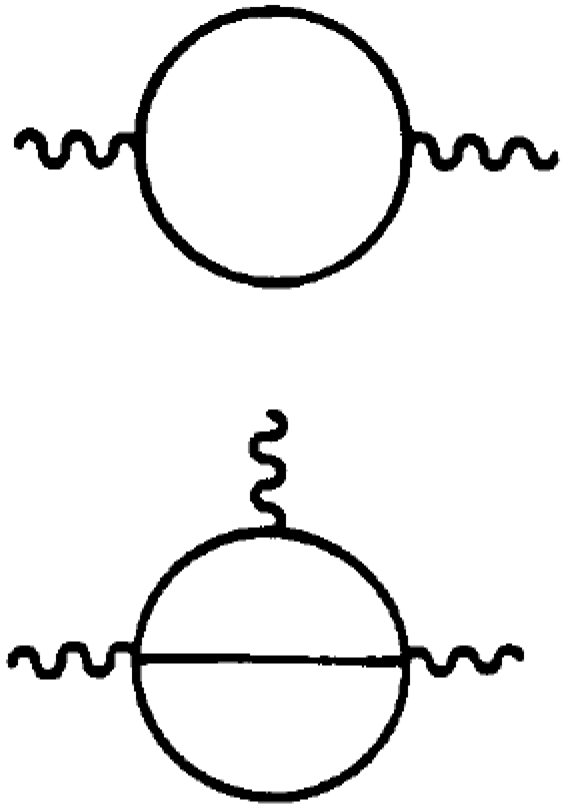
- UV behavior of GR





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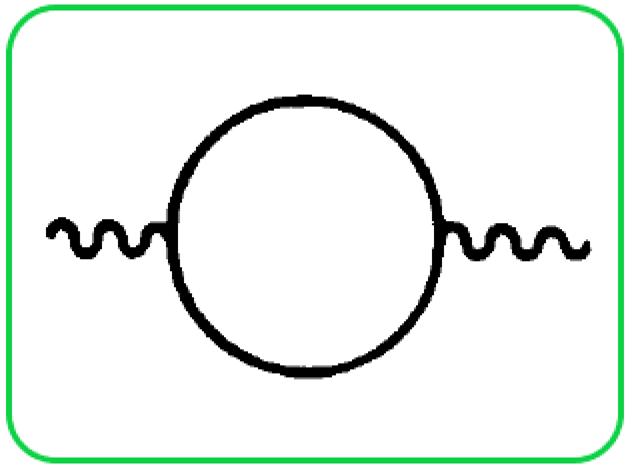

$$\mathcal{L}_{\infty}^{(1)} = \frac{1}{\epsilon}\frac{1}{(4\pi)^2}\int d^4x\,\sqrt{|g|}\left(\frac{R^2}{60}+\frac{7}{10}R^{\mu\nu}R_{\mu\nu}\right)$$

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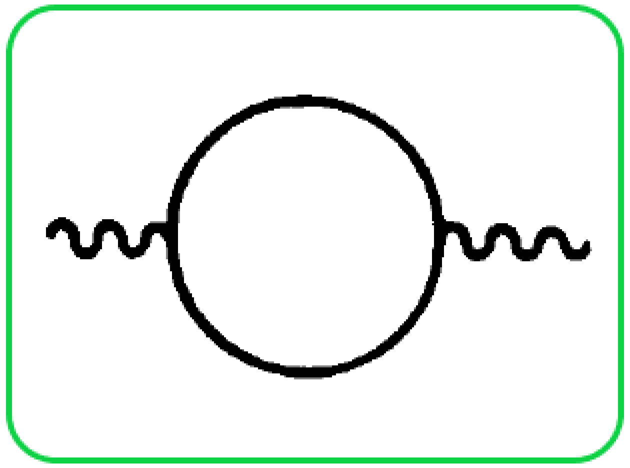
$= 0$



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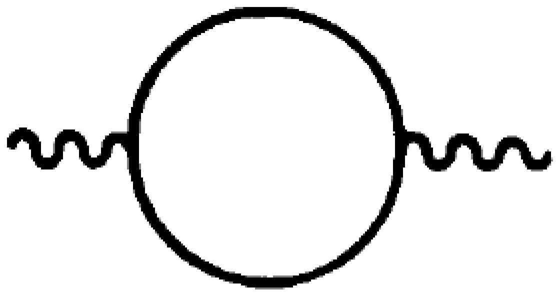

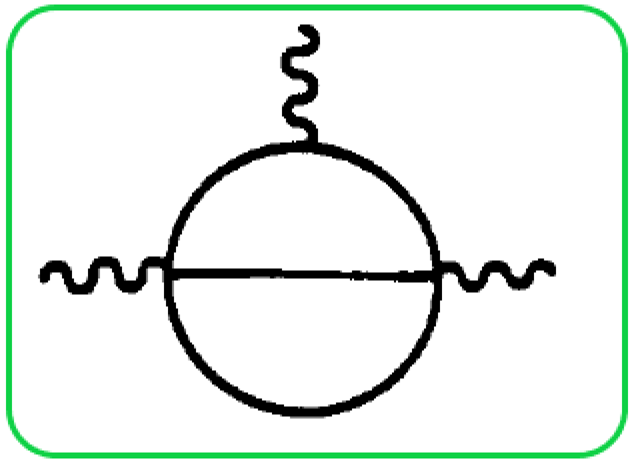

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$$R\,\Box\,R\,,\ R_{\mu\nu}\,\Box\,R^{\mu\nu}\,,\ R^3$$

$$R\,R_{\mu\nu}\,R^{\mu\nu}\,,R_{\mu\nu}\,R^{\nu}_{\alpha}\,R^{\alpha\mu}\,,\ R\,R_{\mu\nu\alpha\beta}\,R^{\mu\nu\alpha\beta}$$

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$$\sqrt{-g}\, R_{\mu\nu}{}^{\alpha\beta}\, R_{\alpha\beta}{}^{\kappa\rho}\, R_{\kappa\rho}{}^{\mu\nu}$$

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$$\mathcal{L}_{\infty}^{(2)} = \frac{1}{\epsilon}\,\frac{209}{2880}\,\frac{1}{(4\pi)^2}\int d^4x\,\sqrt{|g|}\,C_{\mu\nu}{}^{\rho\sigma}\,C_{\rho\sigma}{}^{\alpha\beta}\,C_{\alpha\beta}{}^{\mu\nu}$$

  - Divergence not of EH form
  - Addition to C^3 operator to bare action
    new free parameter
    GR is perturbatively non-renormalizable

Including the two-loop counterterm in the action

Supplemented the Einstein-Hilbert action with the Goroff-Sagnotti term

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\,\sqrt{|g|}\,\left[R + G^2\,\lambda\, C_{\mu\nu}{}^{\kappa\gamma}\, C_{\kappa\gamma}{}^{\rho\sigma}\, C_{\rho\sigma}{}^{\mu\nu}\right]$$

The theory contains:

- 2 gravitational constants $G\,,\lambda$
- Ricci Scalar R
- Weyl tensor $C_{\mu\nu\alpha\beta}$

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The sixth-order field equations following from this action are the following monsters

$$\begin{aligned}
H_{\mu\nu} &= \frac{32\pi}{\sqrt{|g|}\, G}\, \frac{\delta S}{\delta g^{\mu\nu}} \\
&= \frac{(2R_{\mu\nu} - g_{\mu\nu}\, R)}{G^3} + 2R_{\mu\nu} C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} - 2 g_{\mu\nu} R\, C_{\alpha\gamma\beta\delta} C^{\alpha\beta\gamma\delta} + 4R C_{\mu}{}^{\alpha\beta\gamma} C_{\nu\beta\alpha\gamma} \\
&\quad - 7 g_{\mu\nu} C_{\alpha\beta}{}^{\eta\lambda} C^{\alpha\beta\gamma\delta} C_{\gamma\delta\eta\lambda} - 6 g_{\mu\nu} C_{\alpha\beta}{}^{\eta\lambda} C^{\alpha\beta\gamma\delta} C_{\gamma\eta\delta\lambda} - 4 C^{\alpha\beta\gamma\delta} \nabla_\nu \nabla_\mu C_{\alpha\beta\gamma\delta} \\
&\quad - 12 R^{\alpha\beta} \left(C_{\mu\alpha}{}^{\gamma\delta} C_{\nu\gamma\beta\delta} + C_{\mu}{}^{\gamma}{}_{\alpha}{}^{\delta} C_{\nu\beta\gamma\delta} - g_{\mu\nu} C_{\alpha}{}^{\gamma\delta\eta} C_{\beta\gamma\delta\eta} \right) - 6 \nabla^\delta C_{\mu}{}^{\alpha\beta\gamma} \nabla_\nu C_{\alpha\delta\beta\gamma} \\
&\quad - 12 \left(C^{\alpha\beta\gamma\delta} \nabla_\beta \nabla_{(\mu} C_{\nu)\alpha\gamma\delta} - \nabla_\alpha C_{\mu}{}^{\alpha\beta\gamma} \nabla_\delta C_{\nu}{}^{\delta}{}_{\beta\gamma} - C_{\nu}{}^{\alpha\beta\gamma} \nabla_\delta \nabla_\alpha C_{\mu}{}^{\delta}{}_{\beta\gamma} \right) \\
&\quad + 6 g_{\mu\nu} \left(\nabla_\alpha C^{\alpha\beta\gamma\delta} \nabla_\eta C_{\beta}{}^{\eta}{}_{\gamma\delta} - 2 C^{\alpha\beta\gamma\delta} \nabla_\eta \nabla_\delta C_{\alpha\beta\gamma}{}^{\eta} - \nabla_\delta C_{\alpha\beta\gamma\eta} \nabla^\eta C^{\alpha\beta\gamma\delta} \right) \\
&\quad - 6 C_{\nu}{}^{\alpha\beta\gamma} \Box C_{\mu\alpha\beta\gamma} - 6 C_{\nu}{}^{\alpha\beta\gamma} \nabla_\delta \nabla_\mu C_{\alpha}{}^{\delta}{}_{\beta\gamma} - 6 \nabla^\delta C_{\nu}{}^{\alpha\beta\gamma} \nabla_\mu C_{\alpha\delta\beta\gamma} \\
&\quad - 12 \nabla_\delta C_{\alpha}{}^{\delta}{}_{\beta\gamma} \nabla_{(\mu} C_{\nu)}{}^{\alpha\beta\gamma} + 4 g_{\mu\nu} \left(C^{\alpha\beta\gamma\delta} \Box C_{\alpha\beta\gamma\delta} + \nabla_\eta C_{\alpha\beta\gamma\delta} \nabla^\eta C^{\alpha\beta\gamma\delta} \right) \\
&\quad + 12 \nabla_\alpha C_{\nu\delta\beta\gamma} \nabla^\delta C_{\mu}{}^{\alpha\beta\gamma} - 12 \nabla_\delta C_{\nu\alpha\beta\gamma} \nabla^\delta C_{\mu}{}^{\alpha\beta\gamma} - 4 \nabla_\mu C^{\alpha\beta\gamma\delta} \nabla_\nu C_{\alpha\beta\gamma\delta} \\
&\quad + 6 C_{\mu}{}^{\alpha\beta\gamma} \left(C_{\beta\gamma\delta\eta} C_{\nu\alpha}{}^{\delta\eta} + C_{\alpha\gamma\delta\eta} C_{\nu\beta}{}^{\delta\eta} + C_{\beta\gamma\delta\eta} C_{\nu}{}^{\delta}{}_{\alpha}{}^{\eta} - \Box C_{\nu\alpha\beta\gamma} \right) \\
&\quad + 6 C_{\mu}{}^{\alpha\beta\gamma} \left(4 C_{\alpha\delta\gamma\eta} C_{\nu}{}^{\delta}{}_{\beta}{}^{\eta} + 2 \nabla_\delta \nabla_\alpha C_{\nu}{}^{\delta}{}_{\beta\gamma} - \nabla_\delta \nabla_\nu C_{\alpha}{}^{\delta}{}_{\beta\gamma} \right).
\end{aligned}$$

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We will explore static and spherically symmetric spacetimes metrics. In Schwarzschild form

$$ds^2 = -h(\bar{r})\, dt^2 + \frac{d\bar{r}^2}{f(\bar{r})} + \bar{r}^2 \left(d\theta^2 + \sin^2(\theta)\, d\phi^2\right)$$

The H field equation tensor takes the form

$$\bar{H}_{\mu\nu} = \begin{pmatrix} \bar{H}_{tt}(\bar{r}) & 0 & 0 & 0 \\ 0 & \bar{H}_{rr}(\bar{r}) & 0 & 0 \\ 0 & 0 & \bar{H}_{\theta\theta}(\bar{r}) & 0 \\ 0 & 0 & 0 & \bar{H}_{\theta\theta}(\bar{r})\, \sin^2\bar{\theta} \end{pmatrix}$$

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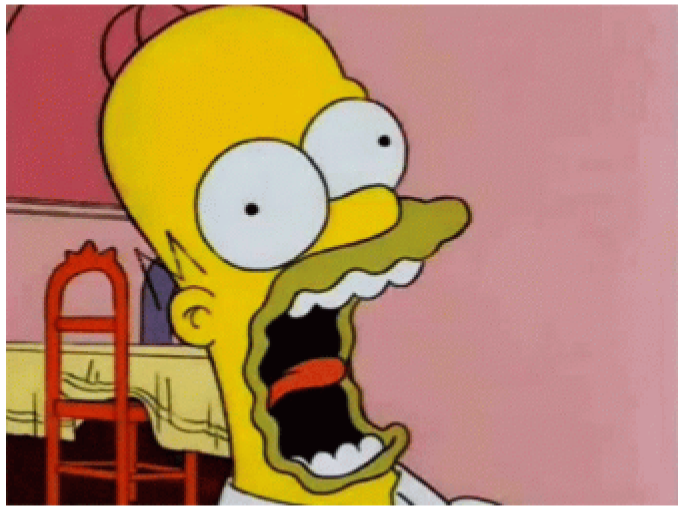

After some derivative reductions, f''(r̄) becomes,

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(2 ⋯2⋯ ( h[rr] f'[rr] / (f[rr] rr^2) + 2 h[rr] (2-2 f[rr]+rr ⋯1⋯'[rr]) / (f[rr] rr^3) + ⋯129⋯ + (⋯1⋯^2 rr^2 ⋯1⋯^2 ⋯1⋯ (⋯1⋯)) / ((2 ⋯1⋯-rr ⋯1⋯) ⋯1⋯ (⋯1⋯)) +
  f[rr]^2 (⋯1⋯) / (12 ⋯1⋯^2 ⋯2⋯ (⋯1⋯+⋯1⋯) (h[rr]^2 (-4+4 f[rr]-2 rr f'[rr]) - f[rr] rr^2 ⋯1⋯^2 + h[rr] rr (rr f'[rr] h'[rr] - 2 f[rr] (h'[rr] - rr ⋯1⋯''[rr])))) )) / (3 ⋯6⋯
  (36 (-1)^(2/3) GG^6 δδ^3 f[rr]^10 h[rr]^20 rr^29 f'[rr] + ⋯4⋯ + ⋯1⋯ - i 3^(1/6) (-i + √3) √(GG^6 ⋯4⋯ (h[rr]^2 (-8 rr^2 + 27 GG^2 δδ ⋯1⋯'[rr]^2) + 54 ⋯5⋯ ⋯1⋯'[rr] + 27 GG^2 δδ ⋯1⋯[rr]^2 ⋯1⋯'[rr]^2)
  (-9 GG^4 δδ^2 f[rr]^6 h[rr]^12 rr^17 f'[rr] - 9 ⋯5⋯ ⋯1⋯ + √3 √(GG^6 ⋯4⋯ (h[rr]^2 (-8 rr^2 + 27 ⋯2⋯ ⋯1⋯^2) + ⋯1⋯ + 27 ⋯4⋯)) )^(2/3)))
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Including the two-loop counterterm in the action

Idea: use Kundt geometry. They include *spherically symmetric BH* which can be written as

$$ds^2 = \Omega^2(r)\left[d\theta^2 + \sin^2(\theta)\, d\phi^2 - 2du\, dr + \mathcal{H}(r)\, du^2\right]$$

Relation to the Schwarzschild form of the BH metric

$$\bar{r} = \Omega(r) \quad , \quad t = u - \int \frac{dr}{\mathcal{H}(r)}.$$

$\Omega^2(r)$ and $\mathcal{H}(r)$ are related to $f(\bar{r})$ and $h(\bar{r})$ via,

$$h(\bar{r}) = -\Omega^2(r)\,\mathcal{H}(r) \quad , \quad f(\bar{r}) = -\left(\frac{\Omega'(r)}{\Omega(r)}\right)^2 \mathcal{H}(r).$$

Advantage: this new form of the BH metric ***simplifies a lot*** the field equations

Equations of motion in Kundt coordinates

We reduce the field equations to a simpler system of *two equations for two metric functions* $\mathcal{H}, \Omega$. Moreover, the *conformal to Kundt* geometry generates an *autonomous system*

$$
\begin{aligned}
H_{rr} \equiv\, & 3\Omega^4 \left(\Omega''\,\Omega - 2\Omega'^2\right) + \frac{G^2\,\lambda\,\Omega'}{2}\left[3\Omega'\,(2+\mathcal{H}'')^2 - 4\Omega\,\mathcal{H}^{(3)}\,(2+\mathcal{H}'')\right] \\
& + \frac{G^2\,\lambda\,\Omega}{2}\left[\Omega\left(H^{(3)^2} + (2+\mathcal{H}'')\,\mathcal{H}^{(4)}\right) - (2+\mathcal{H}'')^2\,\Omega''\right],
\end{aligned}
$$

$$
\begin{aligned}
H_{ru} \equiv\, & \frac{G^2\,\lambda}{2}\left[3\mathcal{H}'\,\Omega'\,(2+\mathcal{H}'') + \left(\mathcal{H}'' + \mathcal{H}''^2 - 3\mathcal{H}'\,\mathcal{H}^{(3)} - 2\right)\,\Omega\right] \\
& + 9\Omega^3\,\left(\Omega^2 + \Omega\,\Omega'\,\mathcal{H}' + 3\mathcal{H}\,\Omega'^2\right)
\end{aligned}
$$

Local solutions to the field equations

We have a relatively simple *autonomous system.* Thus we can find its solutions as an *infinite power series in r* expanded *around any fixed value r_0*

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$$\Omega(r) = \Delta^n \sum_{i=0}^{\infty} a_i \, \Delta^i \quad , \quad \mathcal{H}(r) = \Delta^p \sum_{i=0}^{\infty} b_i \, \Delta^i .$$

where,

$$\Delta = r - r_0, \quad n, p \in \mathbb{R}, \quad a_0 \, , b_0 \neq 0$$

Substituting these series into the field equations, the dominant (lowest) power of Δ put *restrictions on the parameters [n,p]*

- (n,p) = (0,0)
- (n,p) = (-1,2)
- (n,p) = (0,1)
- (n,p) = (0,2)

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The mutual relations between the *Schwarzschild radial coordinate r* and the *Kundt coordinate r* follows from

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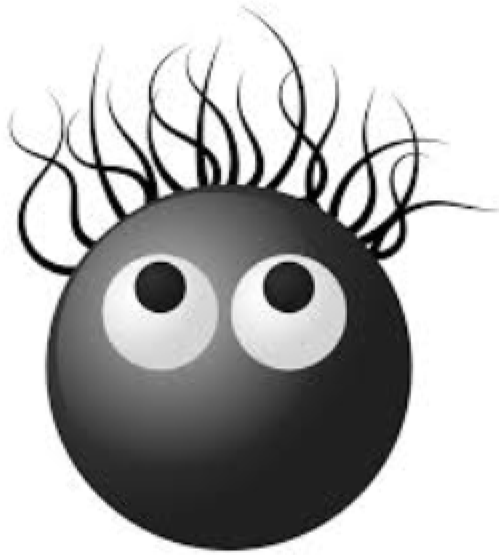
1. $\bar{r} \to 0$ for $r \to r_0 \, , \; n > 0$
2. $\bar{r} \to r_0$ for $r \to r_0 \, , \; n = 0$
3. $\bar{r} \to \infty$ for $r \to r_0 \, , \; n < 0$

Description of solutions in powers of Δ

The (-1,2) family: The asymptotic region

This family dictates the behaviour of the system at $\bar{r} \to \infty$ and gives the *correction to the Schwarzschild metric*, given by,

$$a_0^{\rm Sch} = -1, \qquad a_i^{\rm Sch} = 0 \qquad \forall\, i \geq 1,$$
$$b_0^{\rm Sch} = -1, \qquad b_1^{\rm Sch} = -2M, \qquad b_j^{\rm Sch} = 0 \qquad \forall\, j \geq 2$$



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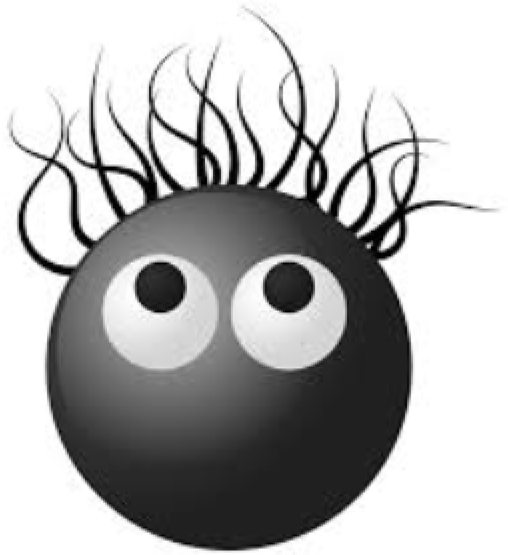
Metric functions in Kundt coordinates

$$\Omega^{\text{Sch}} = \frac{a_0^{\text{Sch}}}{\Delta} \quad , \quad \mathcal{H}^{\text{Sch}} = -\Delta^2 + b_1^{\text{Sch}}\Delta^3$$



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Metric functions in Kundt coordinates ***with the two-loop counterterm***

$$\Omega(r) = \frac{a_0}{\Delta} - \frac{6G^2\,\lambda\, b_1^2\,\Delta^5}{7a_0^3} + \frac{288G^4\,\lambda^2\, b_1^3\,\Delta^{10}}{7007a_0^7}\left[5733 - 6710b_1\,\Delta^2\right] + \mathcal{O}\left(\lambda^3\right)$$

$$\mathcal{H}(r) = -\,\Delta^2 + b_1\,\Delta^3 - \frac{2G^2\,\lambda\, b_1^2\,\Delta^8}{7a_0^8}\left[27a_0^4 - 23a_0^4\, b_1\,\Delta\right]$$
$$+ \frac{24G^4\,\lambda^2\, b_1^3\,\Delta^{13}}{7007a_0^8}\left[504504 - b_1\,\Delta\left(1047978 - 539143b_1\,\Delta\right)\right] + \mathcal{O}\left(\lambda^3\right)$$

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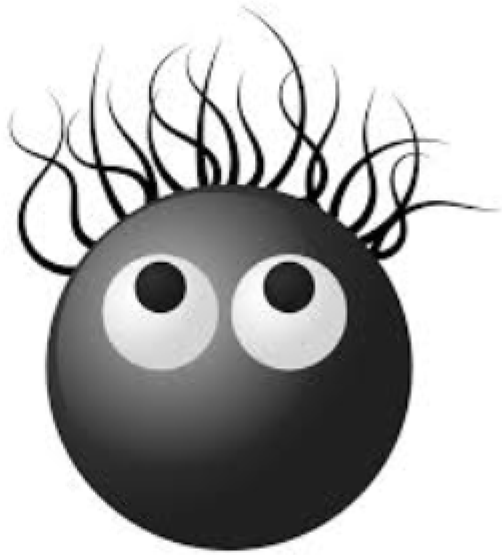

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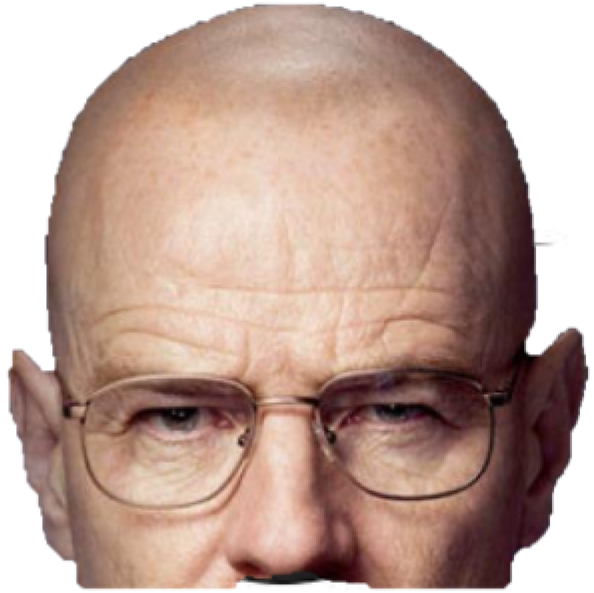
Remarkably, the inclusion of the Goroff-Sagnotti counterterm *does not generate quantum hair -* asymptotically flat, static, spherical solutions are still determined *in terms of a single field parameter*



Description of solutions in powers of Δ

The [n,p] = [0,1] (and [n,p] = [0,2]) family: Near the horizon

From $\mathcal{H}(r) = (r - r_0)^p \, (b_0 + b_1 \, (r - r_0) + \ldots)$ we observe that

the class [n,p] = [0,1] has $\mathcal{H}(r) \sim b_0 \, \Delta + \mathcal{O}(\Delta^2)$ with a single root
the black hole has a horizon at $r_h = r_0$

the class [n,p] = [0,2] has $\mathcal{H}(r) \sim b_0 \, \Delta^2 + \mathcal{O}(\Delta^3)$ with a doble root
the black hole has an extreme horizon
but it has 0 free parameters

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- This class of solutions has four free parameters: a_0, b_0, b_1 and $r_0 = r_h$

$$a_1 = \frac{(1+b_1)\left(2G^2\,\lambda\,(1+b_1^2) - 9a_0^4\right)}{27a_0^3\,b_0},$$

$$a_2 = \frac{4G^2\,\lambda\,(1+b_1)^2\left[a_0^4\,(2+11b_1) - G^2\,\lambda\,(1+b_1)^2\,(2+b_1)\right] - 9a_0^8\,(b_1-2)}{216G^2\,\lambda\,a_0^3\,b_0^2\,(1+b_1)},$$

$$b_2 = \frac{27a_0^8\,(b_1-2) + 2G^2\,\lambda\,(1+b_1)^2\left[2G^2\,\lambda\,(1+b_1)^3 + 3a_0^4\,(2b_1-7)\right]}{162G^2\,\lambda\,a_0^4\,b_0\,(1+b_1)}, \ldots$$

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- This class of solutions has four free parameters: a_0, b_0, b_1 and $r_0 = r_h$

- The Weyl curvature invariant and Kretschmann scalars are finite

$$C_{\mu\nu\alpha\beta}\,C^{\mu\nu\alpha\beta}(r_h) = \frac{4\,(1+b_1)^2}{3a_0^4},$$

$$\mathcal{K}(r_h) = \frac{4\left[36G^2\,\lambda\,a_0^4\,(b_1-2)\,(b_1+1)^3 + 8G^4\,\lambda^2\,(1+b_1)^6 + 81a_0^8\,(11+b_1\,(5b_1-2))\right]}{729a_0^{12}}.$$

Description of solutions in powers of Δ

The [n,p] = [0,0] family: Near a generic point

- Most general class of possible spherically symmetric vacuum solutions in *EH+GS*

$$\Omega(r) = a_0 + a_1\,\Delta + \frac{2G^2\,\lambda\,(1+b_2)^3 - 9a_0^3\,[3a_1\,b_1 + a_0\,(1+b_2)]}{54a_0^3\,b_0}\,\Delta^2 + \ldots$$

$$\mathcal{H}(r) = b_0 + b_1\,\Delta + b_2\,\Delta^2 + b_3\,(a_0, a_1, b_0, b_1, b_2)\,\Delta^3 + \ldots$$

Description of solutions in powers of Δ

The [n,p] = [0,0] family: Near a generic point

- Most general class of possible spherically symmetric vacuum solutions in *EH+GS*

$$\Omega(r) = a_0 + a_1\,\Delta + \frac{2G^2\,\lambda\,(1+b_2)^3 - 9a_0^3\,[3a_1\,b_1 + a_0\,(1+b_2)]}{54a_0^3\,b_0}\,\Delta^2 + \dots$$

$$\mathcal{H}(r) = b_0 + b_1\,\Delta + b_2\,\Delta^2 + b_3\,(a_0, a_1, b_0, b_1, b_2)\,\Delta^3 + \dots$$

- Expansion is considered around *an arbitrary point* which is *different from the BH horizon*

$$r_0 \neq r_h$$

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- It contains <u>*wormhole solutions*</u>

Description of solutions in powers of r^{-1}

Now we investigate spherical solutions in the domain as $r \to \infty$ by expanding the metric functions in *negative powers of r*

$$\Omega(r) = r^N \sum_{i=0}^{\infty} A_i\, r^{-i} \quad , \quad \mathcal{H}(r) = r^P \sum_{i=0}^{\infty} B_i\, r^{-i}$$

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Substituting these series into the field equations, *only 1 class of solution is allowed*

$$[N, P] = [-1, 2]$$

Description of solutions in powers of r^{-1}

The [N,P] = [-1,2] family: Near the core

- *Identication of Minkowski space*. One possible set of coefficients that solve the equations on motion in the [-1,2] class is given by

$$\Omega(r) = \frac{A_0}{r} \sum_{i=0}^{\infty} \left(\frac{B_1}{2r} \right)^i , \quad \mathcal{H}(r) = -\left(r - \frac{B_1}{2} \right)^2$$

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We can set $A_0 = -1,\ B_1 = 0$, and the metric functions become

$$\Omega(r) = \frac{1}{r} \ , \quad \mathcal{H}(r) = -r^2$$

flat Minkowski space

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Invariants

$$C_{\alpha\beta\mu\nu}\,C^{\alpha\beta\mu\nu} = 0,$$
$$\mathcal{K}(r\to\infty) \sim \frac{927B_1^{12}}{256A_0^4\,r^4} \to 0$$

Description of solutions in powers of r^{-1}

The [N,P] = [-1,2] family: Near the core

- *The most general solution*. The metric functions associated with the solution that has the highest number of free parameters (4) produce a finite Kretschmann scalar

$$\Omega(r) = \frac{A_0}{r} + \frac{A_0\, B_1}{2r^2} + \frac{A_2}{r^3} + \frac{A_3}{r^4} + \ldots \quad , \quad \mathcal{H}(r) = -\, r^2 + B_1\, r + B_2 + \frac{B_3}{r} + \ldots$$

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Invariants

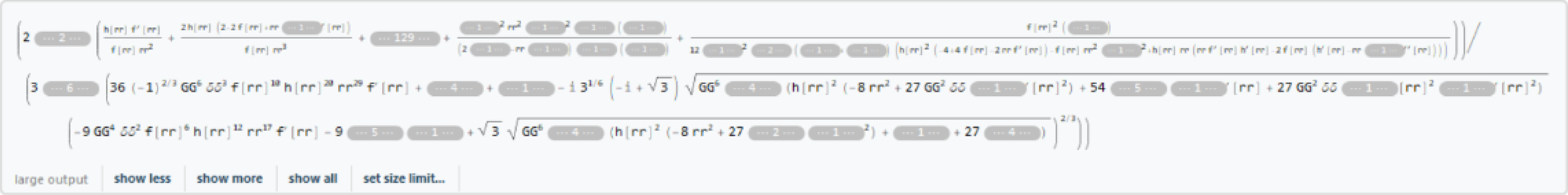
$$C_{\alpha\beta\mu\nu}\,C^{\alpha\beta\mu\nu}\,(r \to \infty) \sim -\frac{32A_2 + 12B_2}{10\lambda\, r^2} \to 0$$

$$\mathcal{K}(r \to \infty) \sim 24\,\left(10A_2^2 + 6A_2\,B_2 + B_2^2\right)$$

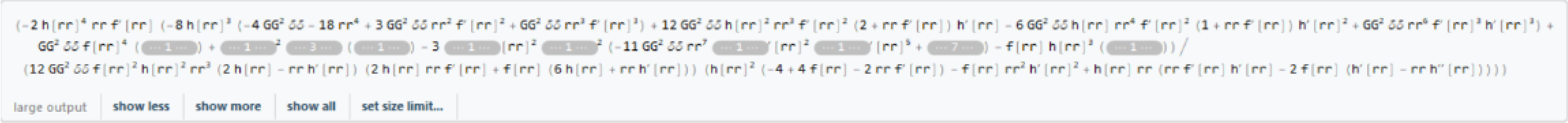
Numerical integration

Numerical integration

$$f''(\bar{r}) =$$



$$h^{(3)}(\bar{r}) =$$



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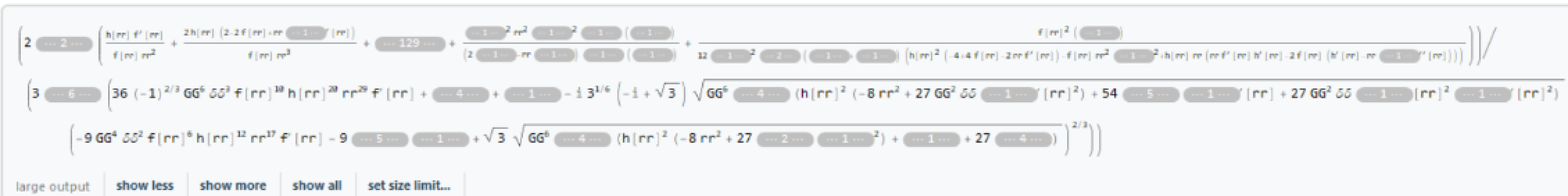

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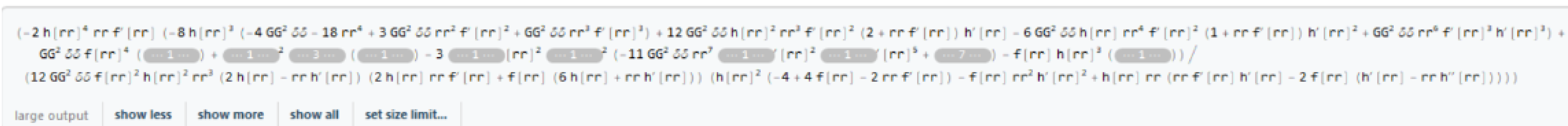

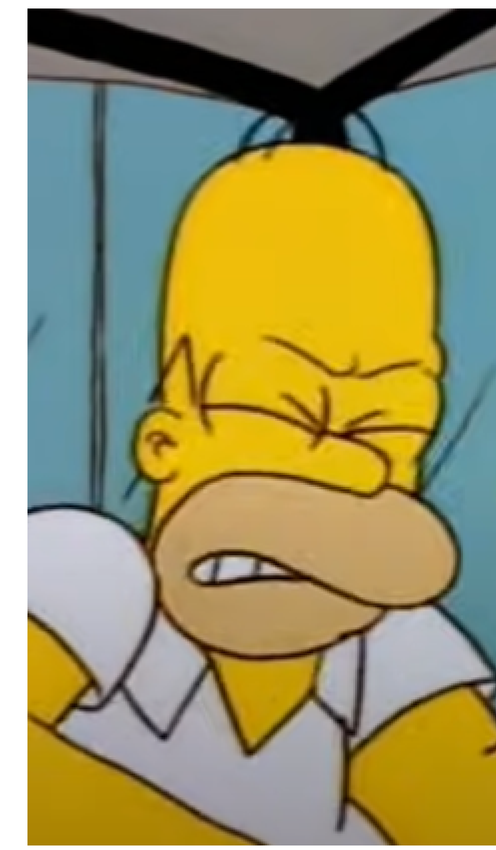

$$\bar{r} \Rightarrow \Omega(r)$$

$$\{h\,,\, f\} \Rightarrow \{\Omega\,,\, \mathcal{H}\}$$

Numerical integration

$$\mathcal{H}^{(3)} = \frac{3G^2\,\lambda\,\mathcal{H}'\,\Omega'(2+\mathcal{H}'')^2 + G^2\,\lambda\,\Omega\,(\mathcal{H}''-1)\,(\mathcal{H}''+2)^2 - 54\mathcal{H}\,\Omega^3\,(\Omega'')^2 - 18\mathcal{H}'\,\Omega^4\,\Omega' - 18\Omega^5}{3G^2\,\lambda\,\Omega\,\mathcal{H}'\,(2+\mathcal{H}'')}$$

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Numerical integration

We set the initial conditions in our numerical integration at infinity

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Numerical integration

We set the initial conditions in our numerical integration at infinity

- There are *no free parameters* other than the *mass*

$$\Omega(r) = \frac{a_0}{\Delta} - \frac{6G^2\,\lambda\, b_1^2\,\Delta^5}{7a_0^3} + \mathcal{O}\left(\lambda^2\right)$$

$$\mathcal{H}(r) = -\,\Delta^2 + b_1\,\Delta^3 - \frac{2G^2\,\lambda\, b_1^2\,\Delta^8}{7a_0^8}\left[27a_0^4 - 23a_0^4\, b_1\,\Delta\right] + \mathcal{O}\left(\lambda^2\right)$$

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We set the initial conditions in our numerical integration at infinity

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- Independent of the initial conditions

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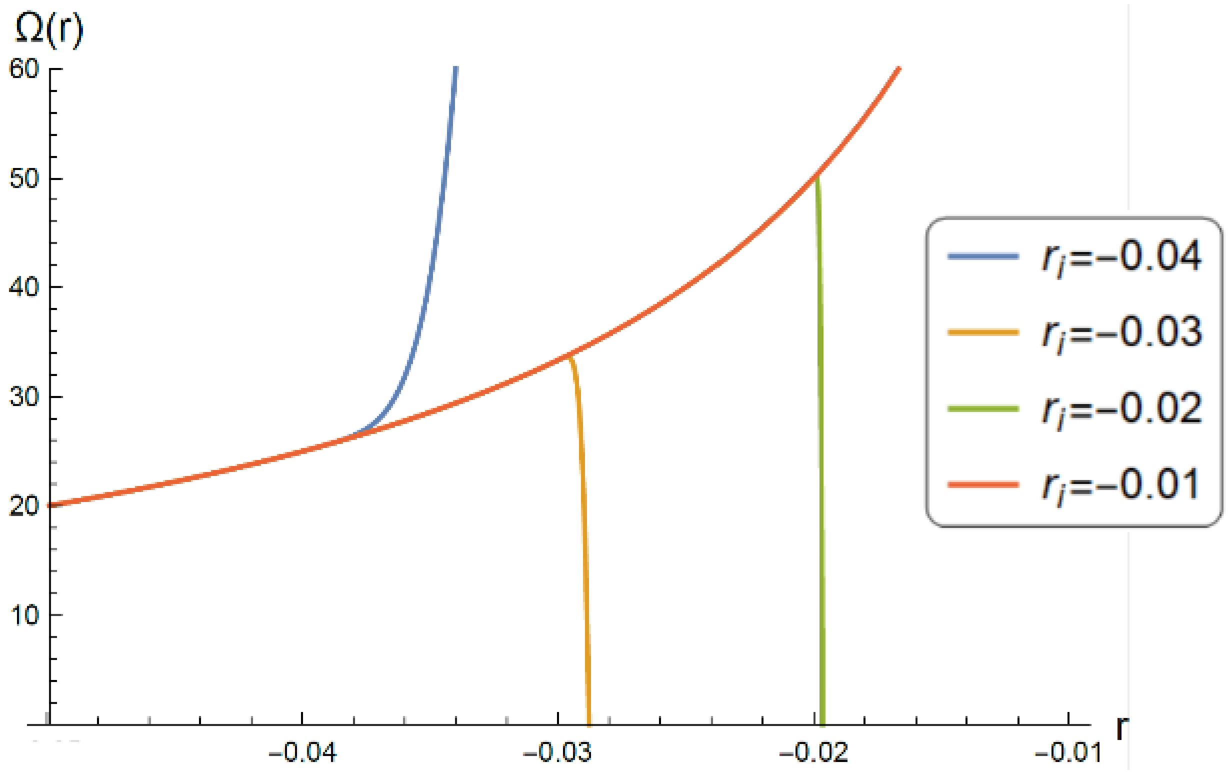

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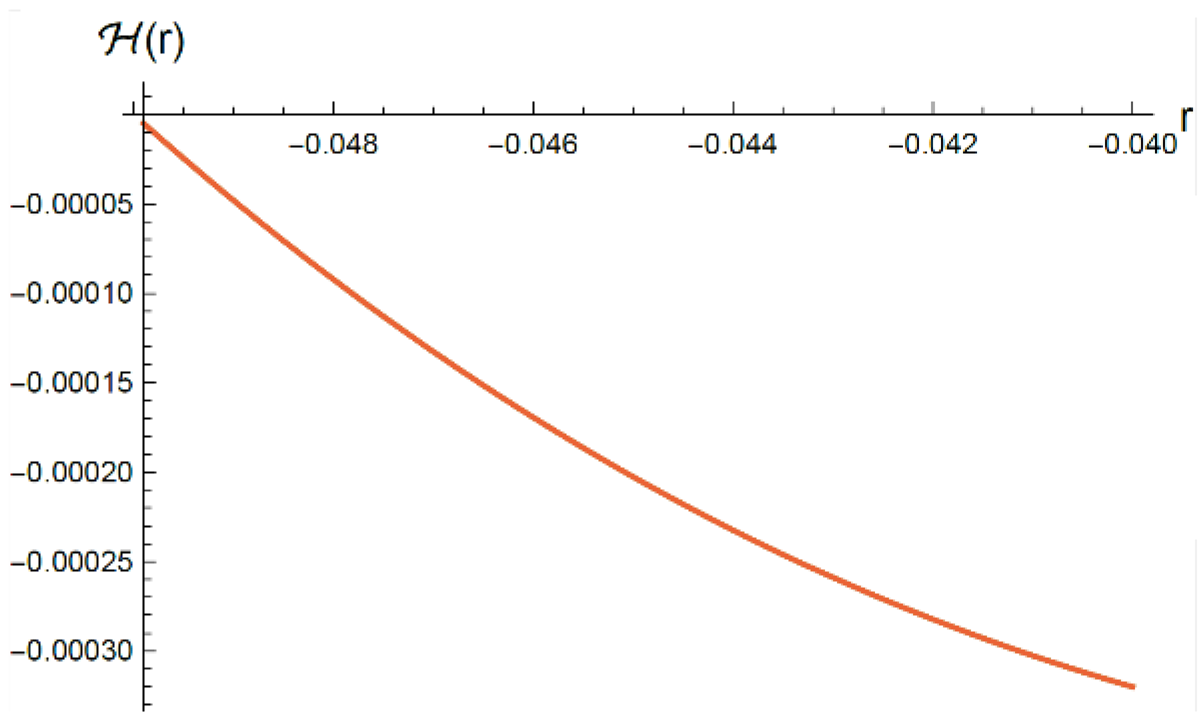
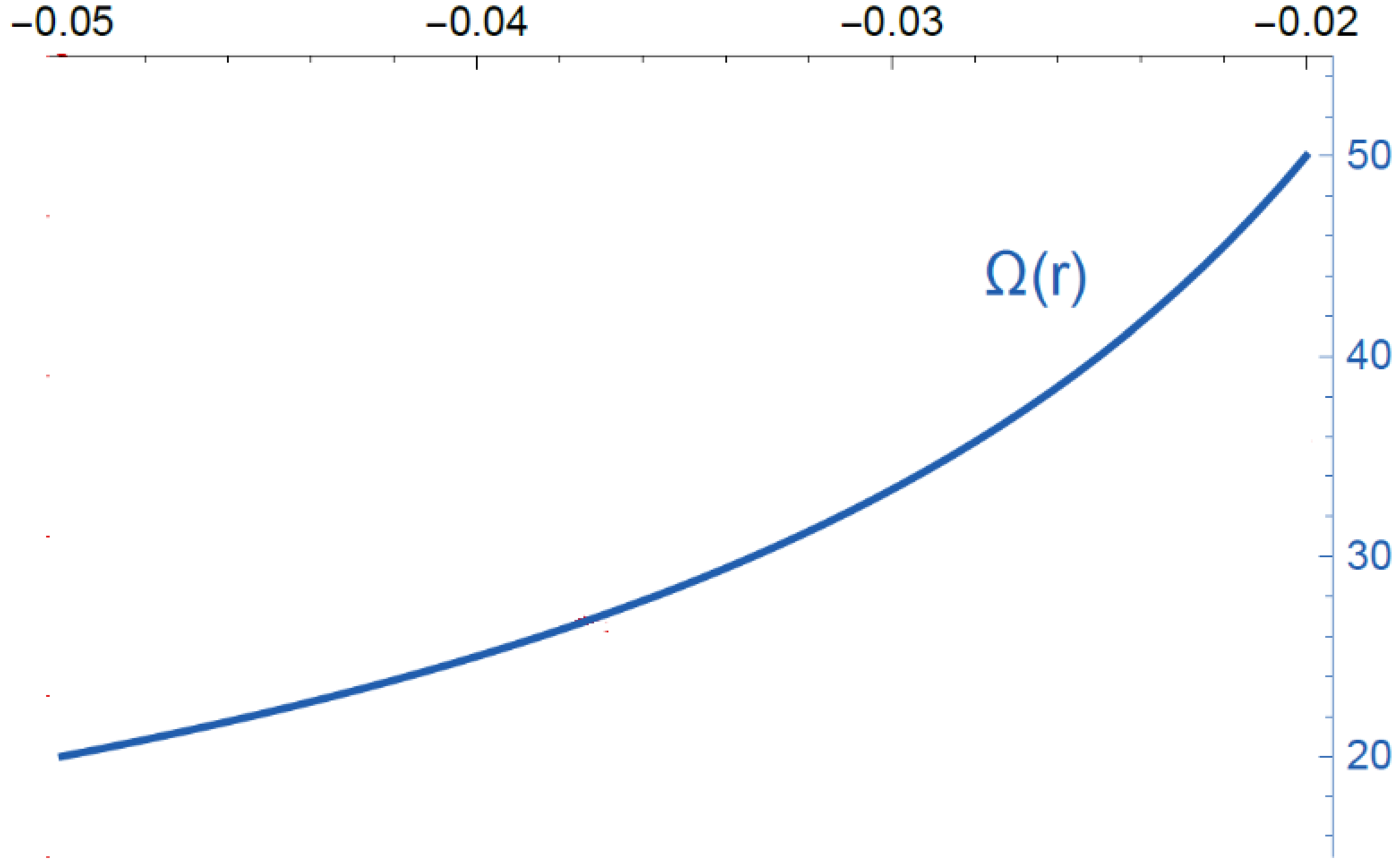


Numerical integration

Behaviour of the metric functions

typical functions Ω(r) and



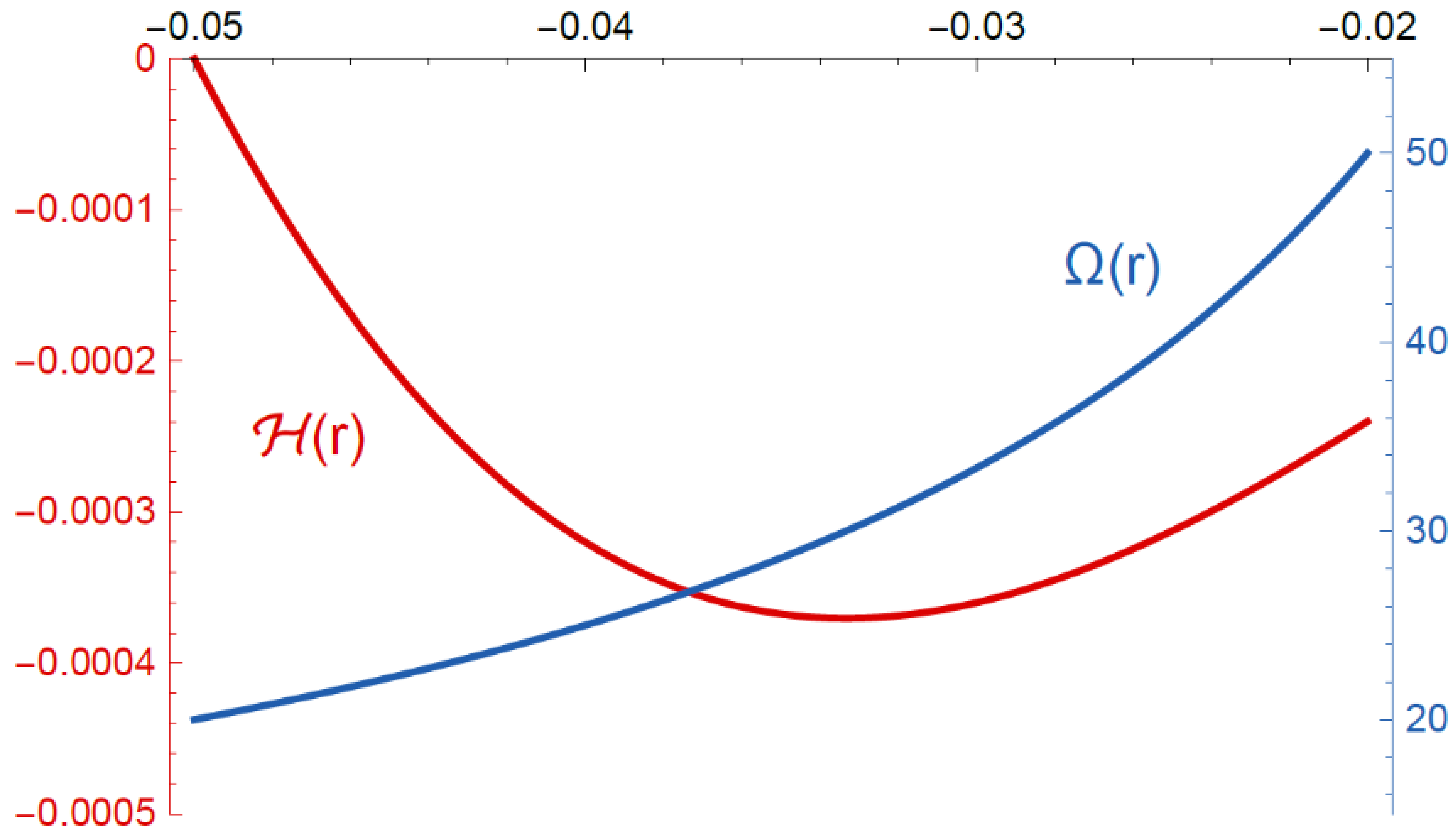
$r_{\mathrm{ini}} = -0.002$
$M = 10$
$\lambda = 0.1$

Recall that $\mathcal{H}$(r) $= 0$ identifies the *black-hole horizon* at r_h

Numerical integration

Behaviour of the metric functions

typical functions $\Omega(r)$ and $\mathcal{H}(r)$



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Recall that $\mathcal{H}(r) = 0$ identifies the *black-hole horizon* at r_h

Conclusions

- We derived a new class of spherically symmetric black holes solutions generated by the inclusion of the two-loop counterterm.
- Using a confomal-to-Kundt metric ansatz, we arrive at a much simpler form of the fields equations in comparison with their counterpart in Schwarzschild coordinates.
- In this new system, asymptotically flat, static and spherically symmetric solutions are still determined in terms of the asymptotic mass.
- Outlook

  Wormholes?
  Numerical integration from the horizon to the core?
  ...

Relations to Schwarzschild coordinates

Different classes of solutions can be denoted by the symbols (s,t) using the standard spherically symmetric form. *Leading terms* of the two metric functions

$$f^{-1}(\bar{r}) \sim \bar{r}^{s}, \quad h(\bar{r}) \sim \bar{r}^{t}$$

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For $\{n, N\} \neq 0$

$$s = \frac{2-p}{n} \quad , \quad t = 2 + \frac{p}{n}$$
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- $(s,t) = (0,0)_0$ corresponds to $[N, P] = [-1, 2]$
- $(s,t) = (0,0)^{\infty}$ corresponds to $[n, p] = [-1, 2]$

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For $n = 0$ and $a_1 \neq 0$

$$s = -p \quad , \quad t = p$$

- $(s,t) = (0,0)_{\bar{r}_0}$ corresponds to $[n,p] = [0,0]$
- $(s,t) = (-1,1)_{\bar{r}_0}$ corresponds to $[n,p] = [0,1]$

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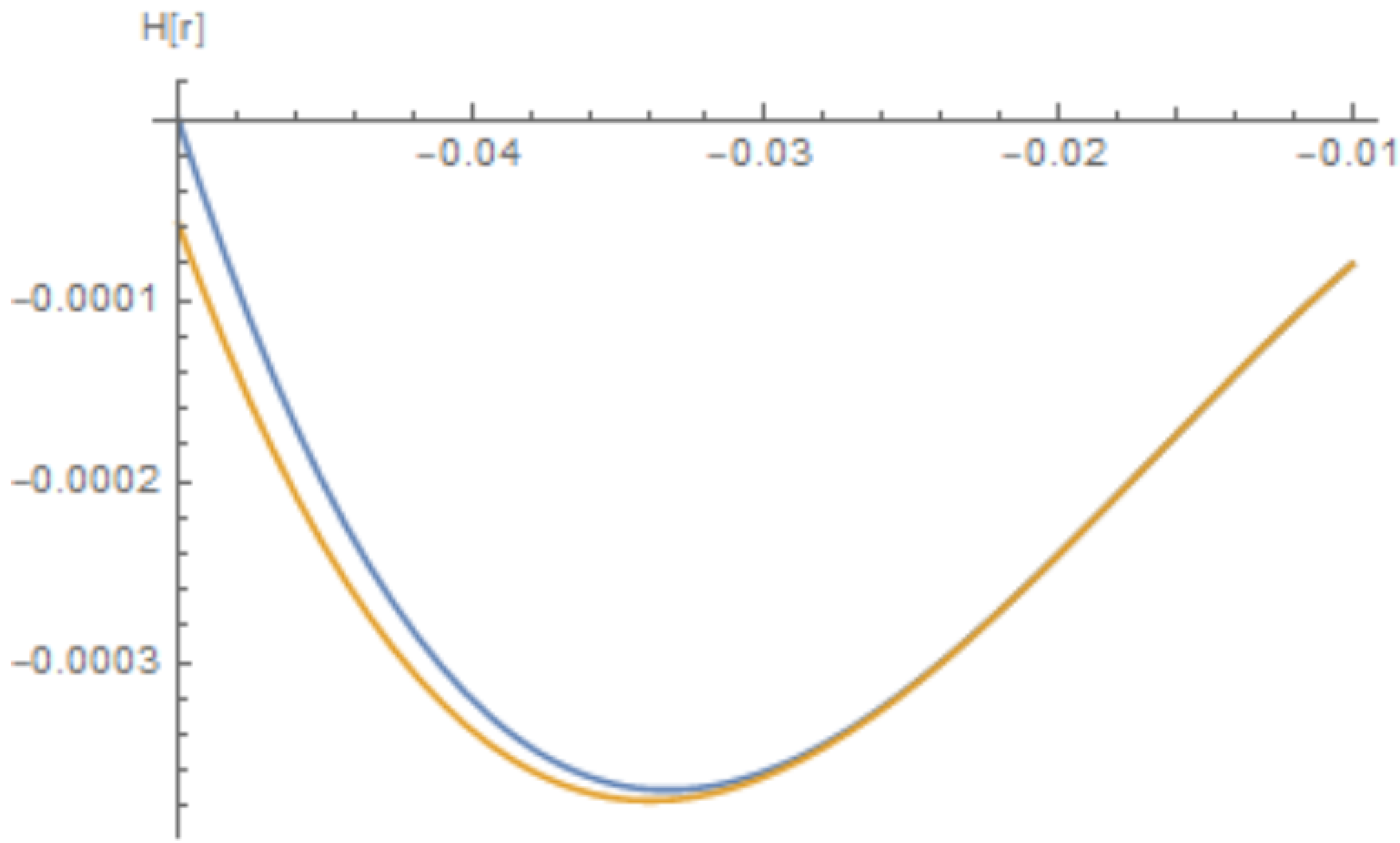
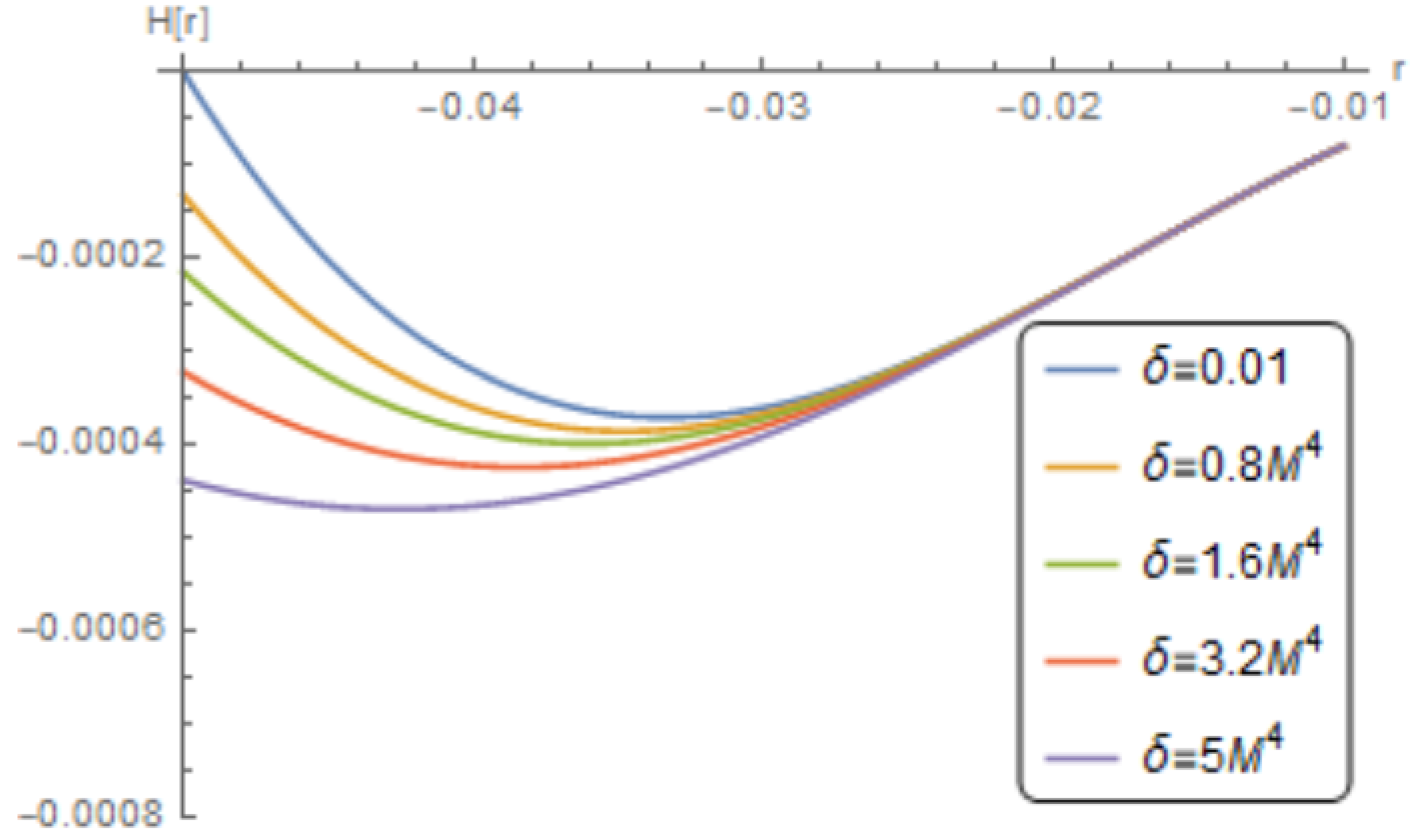
For $n = 0$, $a_1 = 0$ and $a_2 \neq 0$

$$s = -1 - \frac{p}{2} \quad , \quad t = \frac{p}{2}$$

- $(s,t) = (-\frac{3}{2}, \frac{1}{2})_{\bar{r}_0, 1/2}$ corresponds to $[n,p] = [0,0]$
- $(s,t) = (-1, 0)_{\bar{r}_0, 1/2}$ corresponds to $[n,p] = [0,0]_{a_1=0}$


H[r]
r
−0.04
−0.03
−0.02
−0.01
−0.0001
−0.0002
−0.0003


H[r]
r
-0.04
-0.03
-0.02
-0.01
-0.0002
-0.0004
-0.0006
-0.0008
δ=0.01
δ=0.8M4
δ=1.6M4
δ=3.2M4
δ=5M4