Functional renormalization for cosmology

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left\{\left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1} \partial_k R_k\right\}$$

Key questions in cosmology

- Inflation
- Dark Energy
- Dark matter (not this talk)
- Beginning of the Universe

Extension of general relativity

Einstein's general relativity is insufficient

Early Universe: cannot explain isotropy of primordial density fluctuations

- Inflationary Universe

Late cosmology: Cannot explain accelerated expansion ?

- (Dynamical ?) dark energy

Metric + scalar field

- Inflation :
  add scalar field (inflaton)

- Dynamical dark energy or quintessence:
  add scalar field (cosmon)

inflaton = cosmon ?

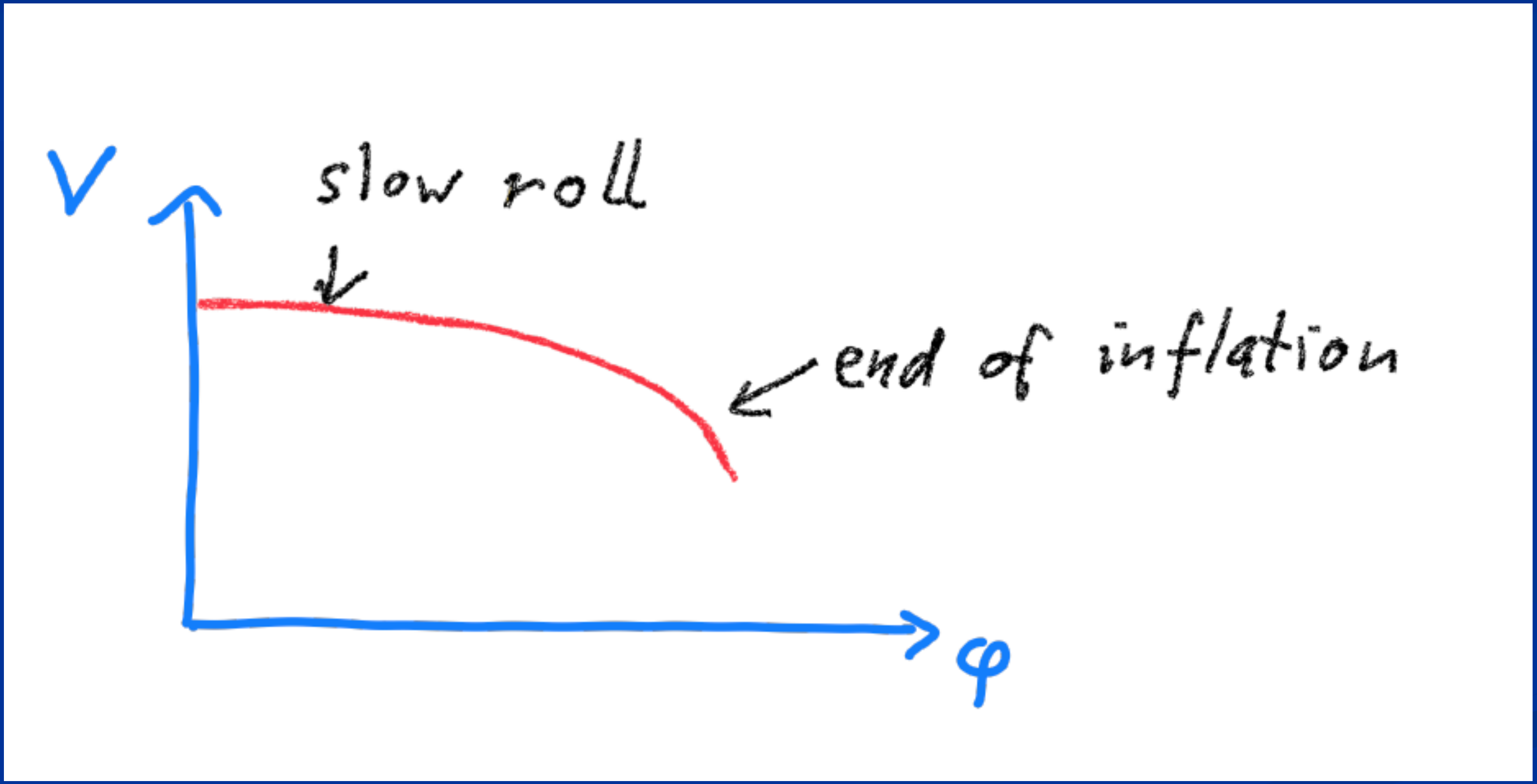
Cosmological equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$$

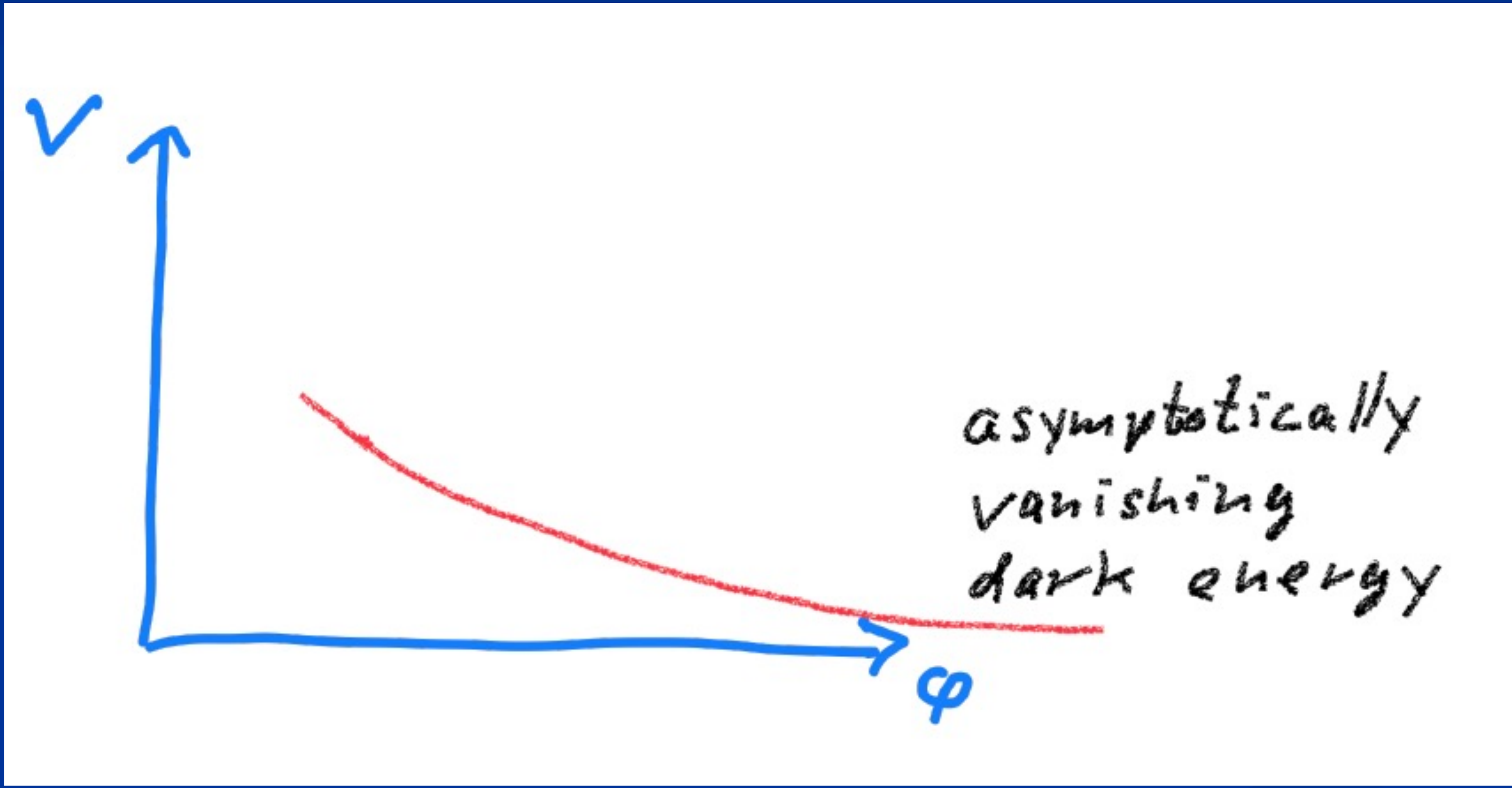
$$3M^2H^2 = V + \frac{1}{2}\dot{\phi}^2 + \rho$$

same equations for inflation and dynamical dark energy

Potential for inflation



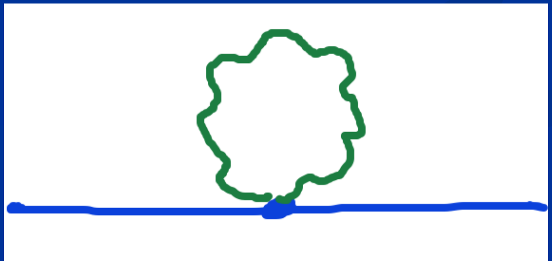
Potential for dynamical dark energy



Can the scalar potential be predicted by functional renormalization for quantum gravity ?

Quantum gravity

- Gravity is field theory. Similar to electrodynamics. Metric field.
- Gravity is gauge theory. Similar to QED or QCD. Gauge symmetry: general coordinate transformations (diffeomorphisms)
- Quantum gravity: include metric fluctuations in functional integral



Quantum gravity

- Quantum gravity is similar to other quantum field theories
- Difference: metric is tensor, gauge bosons are vectors (vierbein, spin connection : vectors)
- Difference: Quantum (Einstein-) gravity is not perturbatively renormalizable
- no small dimensionless coupling constant, effective coupling q^2/M^2

Quantum gravity

Quantum gravity is

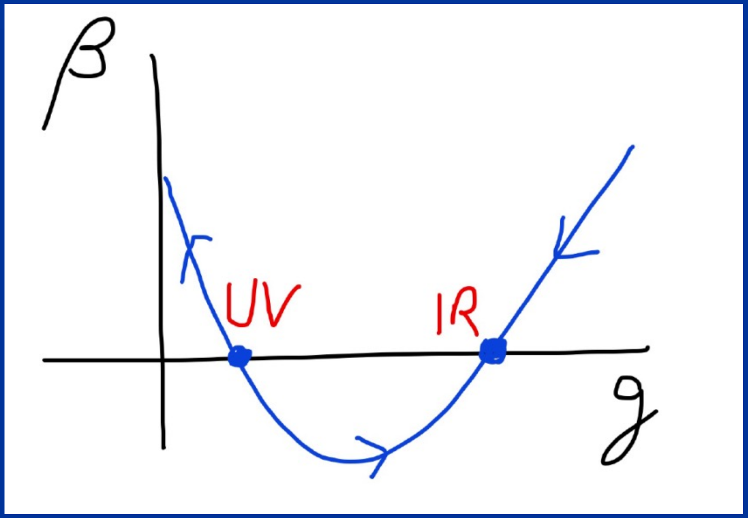
non-perturbatively renormalizable

Asymptotic safety : non-perturbative renormalizabilty

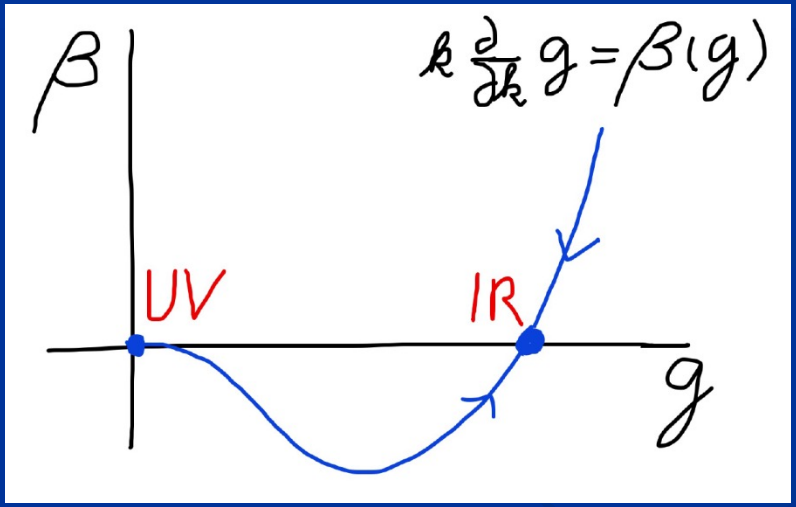
Weinberg, Reuter, …

Use functional renormalization !

Asymptotic safety



Asymptotic freedom



Ultraviolet completeness and renormalizability

Theory can be extrapolated to infinitely small distances or infinitely large energies.

Flowing couplings

Couplings change with renormalization scale k due to (quantum) fluctuations.

Renormalization scale k : Only fluctuations with momenta larger k are included.

Flow of k to zero : all fluctuations included, IR-limit
Flow of k to infinity : UV-limit

k-dependence can differ from momentum dependence

Ultraviolet fixed point

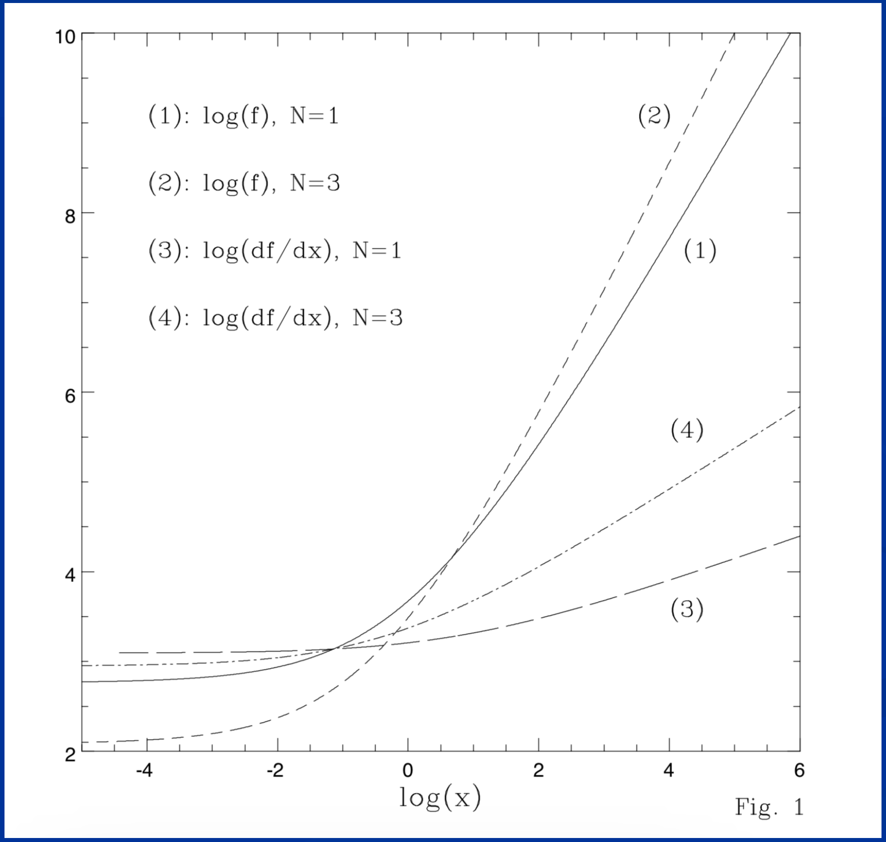
- Flow of dimensionless couplings stops as k increases towards infinity
- Theories with ultraviolet fixed point in the scale dependence of couplings are renormalizable
- Theory can be extrapolated to arbitrarily short distances
- Completeness

Scaling solution

Scaling solutions

- At fixed point: all (infinitely many) dimensionless couplings take fixed values
- Full momentum dependence of graviton propagator (no polynomial expansion)
- Whole scalar potential is fixed, for arbitrary values of scalar field
- Functional flow equations are needed

Pure scalar theory d=3, k=0
Widom scaling function



Berges, Tetradis,… 95

Scaling solutions are restrictive

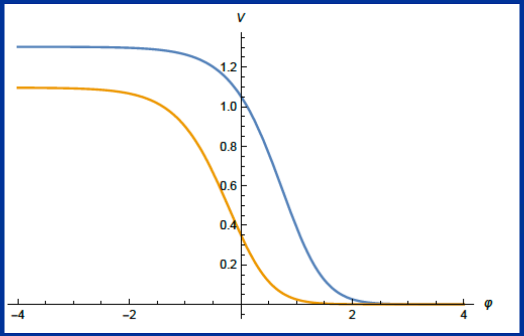
- scaling solutions are particular solutions of non-linear differential equations
- need to extend over whole range of variables
- predictivity !

in presence of gravitational fluctuations:

scalar effective potential no longer approximated by polynomial

Scaling solutions and cosmology

- Cosmology involves scalar potentials over large range of field values



- Inflaton potential
- Higgs potential for Higgs inflation
- Cosmon potential for dynamical dark energy or quintessence

Quantum gravity :
these potentials are not arbitrary

Relevant parameters and relevant functions

- flow away from scaling solution is dominated by relevant functions
- these should not be singular for finite values of variables
- typically only a few, associated to relevant parameters
- Predictivity !

Dominant relevant mass scale

- Flow away from fixed point induces intrinsic mass scales by dimensional transmutation
- Intrinsic violation of quantum scale symmetry
- Can be in different steps (example Planck mass, QCD scale)
- Dominant relevant mass scale is the largest intrinsic scale

Two possibilities

A)

Dominant relevant mass scale is Planck mass

B)

Dominant relevant mass scale is 10^{-3} eV

A

Intrinsic Planck mass

Asymptotically safe standard model

- Standard model seems compatible with asymptotic safety of gravity
- There exists a suitable UV-fixed point for which all observed couplings can be realized
- Non – trivial statement

Dou, Percacci, Daum, Reuter, Eichhorn, Dona, Perrini, Held, Gies, Pawlowski, Reichert, Yamada, Oda, Saueressig, Hamada, Lumma, Pauly, Pastor-Gutierrez and many more

Inflation

minimal setting for standard model + gravity :

- coefficient of R^2 term α is relevant parameter
- can be chosen freely
- large α : Starobinski inflation

Gubitosi, Ooijer, Ripken, Saueressig,

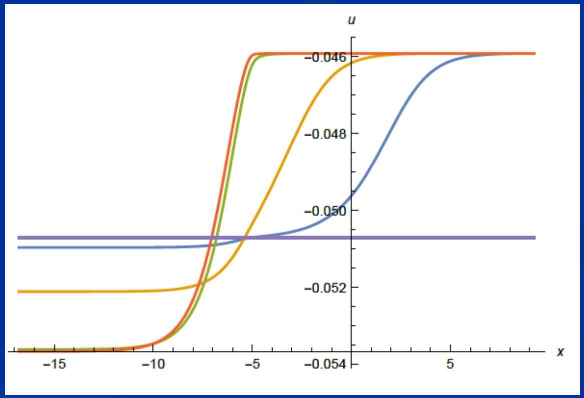
Platania, Vacca, Laporte, Perreira, Wang, Knorr, Bonanno, Falls,…

Inflation in asymptotically safe quantum gravity with intrinsic Planck mass

- use flow away from the scaling solution
- relevant parameters set characteristic mass scales

- free large coefficient of R^2 term α : Starobinski inflation

- Higgs inflation?



FRG landscape

- Beyond Standard Model physics
- similar predictions or restrictions for particles beyond standard model
- not everything goes !
- enhanced predictivity for Dark Matter

Eichhorn, Yamada, Oda, Reichert, Pauly, De Brito, Lino dos Santos, Kowalska, Sessolo, Hamada, Pereira, Miqueleto…

B

Variable Planck mass

Field dependent mass scales

- Scaling solution very good approximation for all momentum scales larger than dominant relevant mass scale
- Planck mass and particle masses are given by expectation value of scalar field
- Spontaneous breaking of scale symmetry

Scale symmetric standard model

- Replace all mass scales by scalar field χ

(1) Higgs potential $U = \frac{\lambda_H}{2}(\varphi^\dagger \varphi - \epsilon \chi^2)^2$ → $\varphi_0^2 = \epsilon \chi^2$

Fujii, Zee, CW

(2) Strong gauge coupling, normalized at $\mu = \chi$, is independent of χ

$g(\chi) = \bar{g}$ → $\Lambda_{QCD} = \chi \exp\left(-\frac{1}{b_0 \bar{g}^2}\right)$ $b_0 = \frac{1}{16\pi^2}\left(22 - \frac{4}{3}N_f\right)$

(3) Similar for all dimensionless couplings

Quantum effective action for standard model does not involve intrinsic mass or length

Quantum scale symmetry

CW'87, Shaposhnikov et al

For $\chi_0 \neq 0$: massless Goldstone boson

FRG prediction for scaling solution

Dilaton quantum gravity

quantum gravity coupled to a scalar field

Henz, Pawlowski, Rodigast, Yamada, Reichert, Eichhorn, Pauly, Laporte, Pereira, Saueressig, Wang, Knorr, …

for low order polynomial expansion of potential : Percacci, Narain, …

Derivative expansion of effective action

$$\Gamma = \int_x \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

variable gravity

Flow equation for scalar potential

$$\mathcal{L} = \sqrt{g}\left\{-\frac{F(\phi_a)}{2}R + U(\phi_a) + \sum_a \frac{Z_a}{2} D^\mu \phi_a D_\mu \phi_a + \ldots\right\}$$

$$u = \frac{U}{k^4}, \quad w = \frac{F}{2k^2}$$

$$v = \frac{2U}{Fk^2} = \frac{u}{w}$$

$$\partial_t u = \beta_u = -4u + 2\tilde{\rho}\,\partial_{\tilde{\rho}} u + 4c_{\mathrm{U}}$$

$$\rho = \chi^2/2$$

$$\tilde{\rho} = \rho/k^2$$

$$c_{\mathrm{U}} = \frac{1}{96\pi^2}\left(\frac{5}{1-v} + \frac{1}{1-v/4}\right) + b_{\mathrm{U}}$$

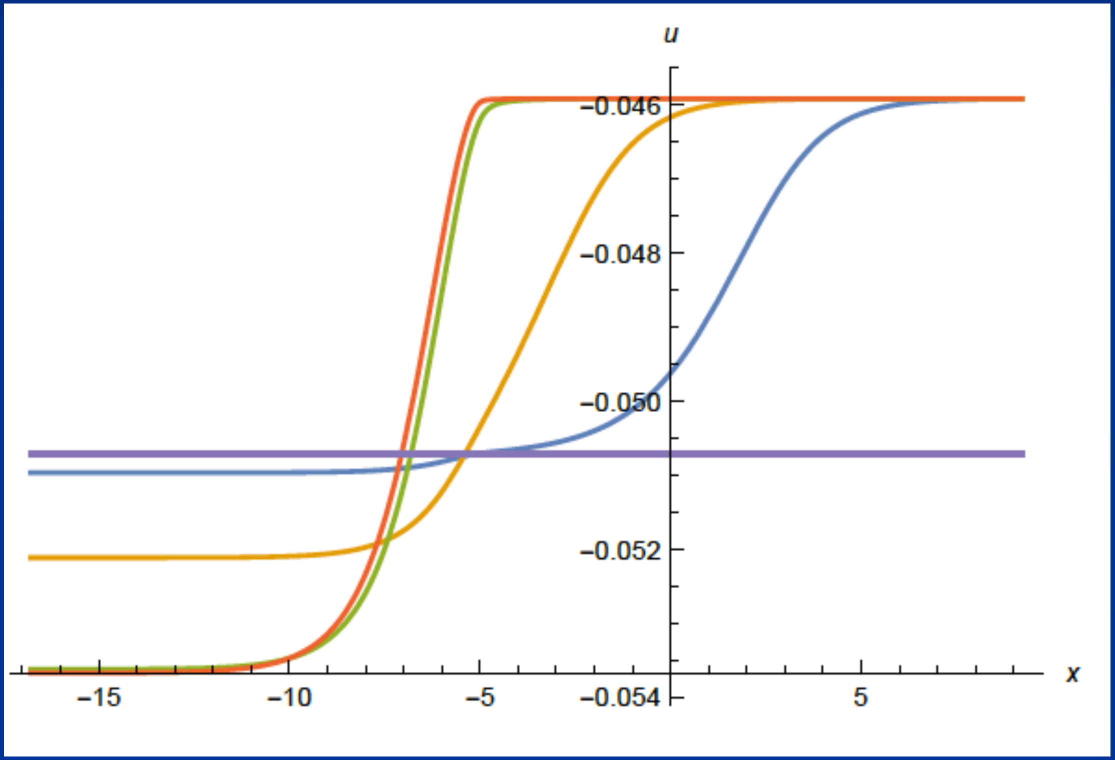
$$b_{\mathrm{U}} = \frac{N-4}{128\pi^2}$$

$$N = N_{\mathrm{S}} + 2N_{\mathrm{V}} - 2N_F$$

Differential equation for scaling solution

$$2\tilde{\rho}\frac{\partial u}{\partial \tilde{\rho}} = 4u - \frac{1}{24\pi^2}\left(\frac{5}{1-u/w} + \frac{1}{1-u/4w}\right) - 4b_{\mathrm{U}}$$

Scaling potential for particles of standard model



u : dimensionless scalar potential

$u = U/k^4$

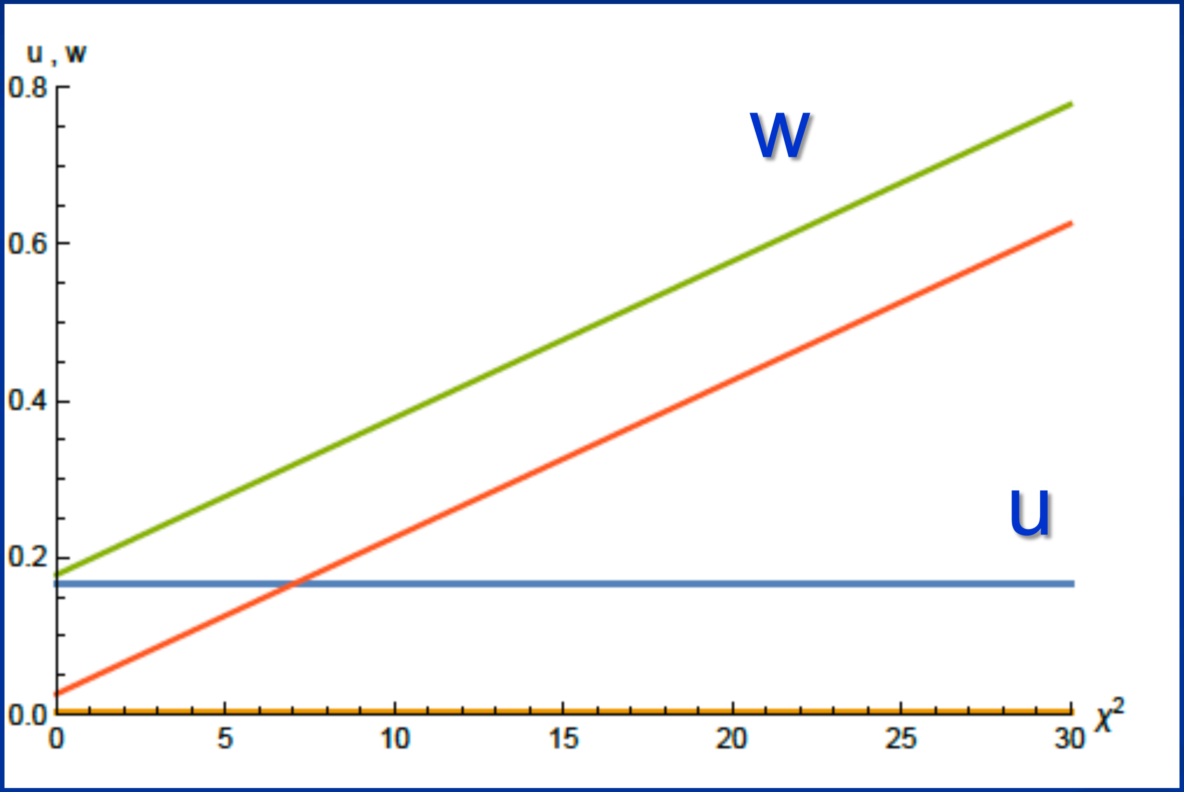
x : logarithm of scalar field value

Generic form of scaling potential

- Interpolates between two plateaus
- Scalar potential =

  field dependent "cosmological constant"
- Effectively massless particles contribute to flow
- Different numbers of massless particles in different regions of field space
- Gravity induced anomalous dimension A describes approach to scaling solution

Scaling solution : flat potential



squared scalar field value χ^2

Differential equation for scaling solution for effective Planck mass

$$2\tilde{\rho}\,\partial_{\tilde{\rho}} w = 2\,(w - c_M)$$

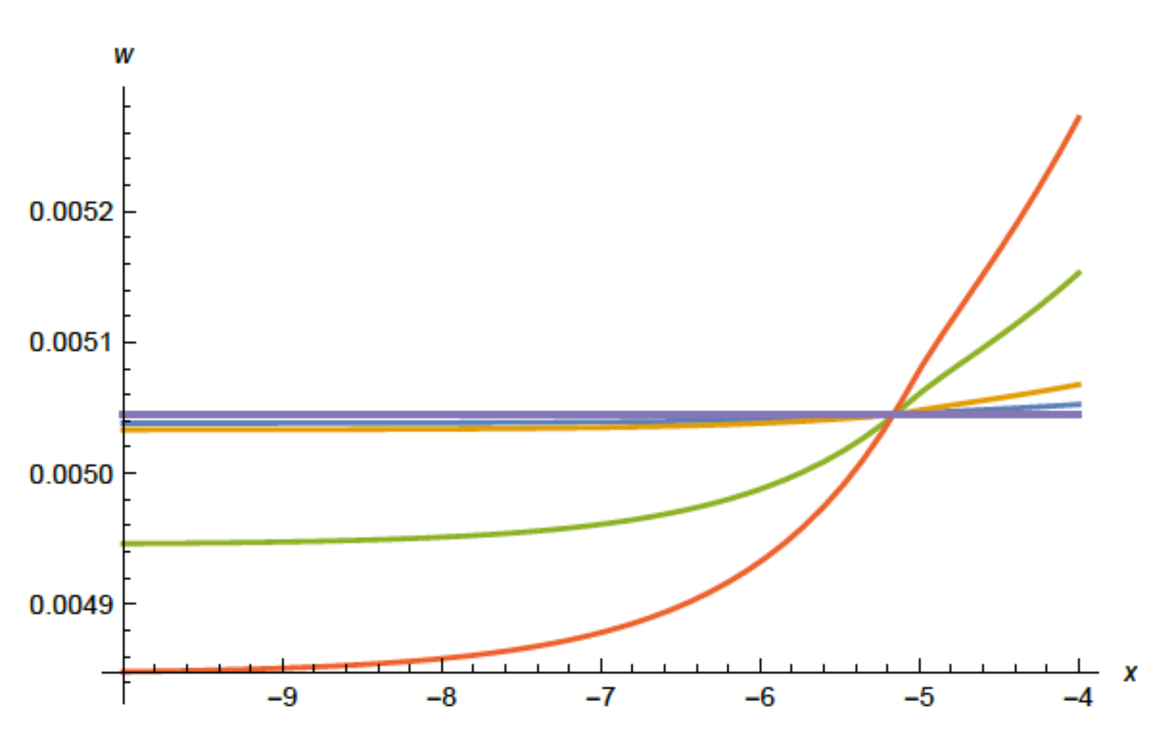
$w = 2\ F/\ k^2$

$$\Gamma = \int_x \sqrt{g}\Big\{ -\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

$$c_M = \frac{25}{128\pi^2}\Big(1 - \frac{u}{w}\Big)^{-1} + \frac{1}{192\pi^2}\Big[- N_S\Big(\frac{1}{1+u'} + \frac{3w'}{(1+u')^2}\Big) + 2N_V\Big(\frac{3}{1+g^2\tilde{\rho}} - 1\Big) - \frac{N_F}{1+y^2\tilde{\rho}} + \frac{43}{6}\Big]$$

M.Yamada, …

Coefficient of curvature scalar in standard model



x : logarithm of scalar field value

w : dimensionless field dependent squared Planck mass

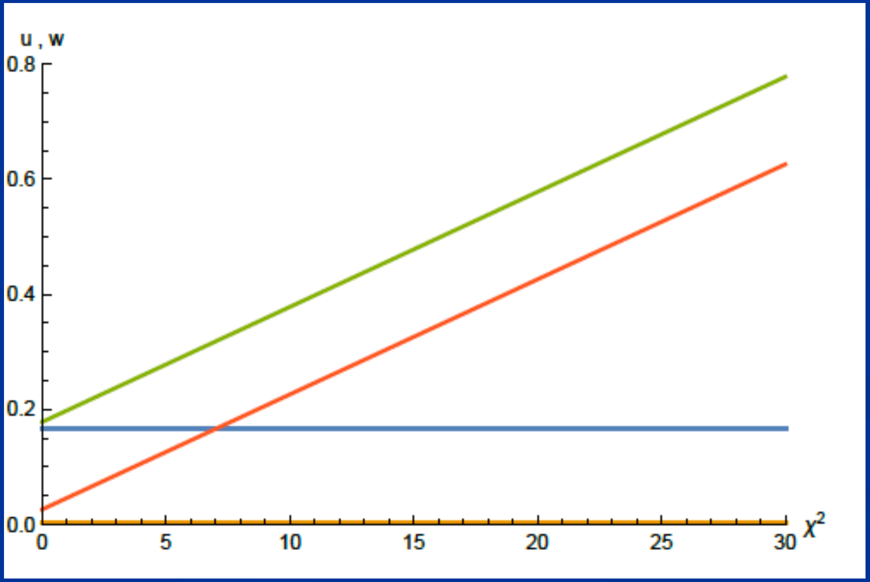
$w = 2\, F / k^2$

non-minimal coupling of scalar field to gravity: $\xi \chi^2 R$

Approximate scaling solution

- flat potential: u constant
- non-minimal scalar- gravity coupling:

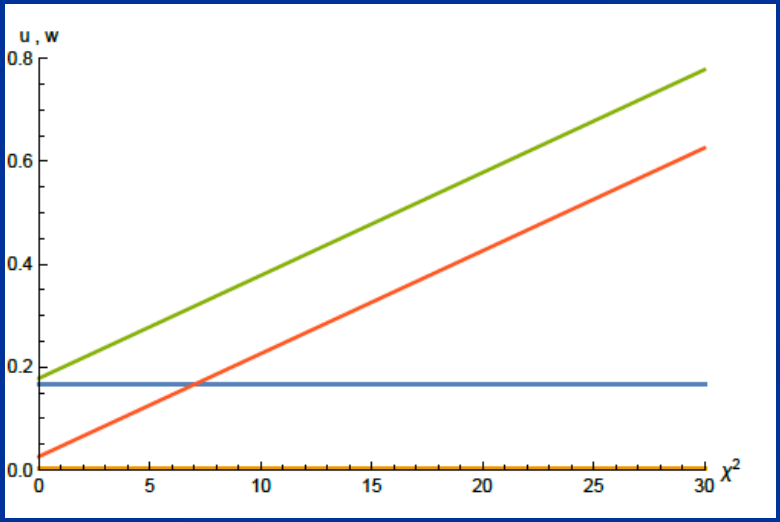
  for large scalar field w increases proportional χ^2



looks natural

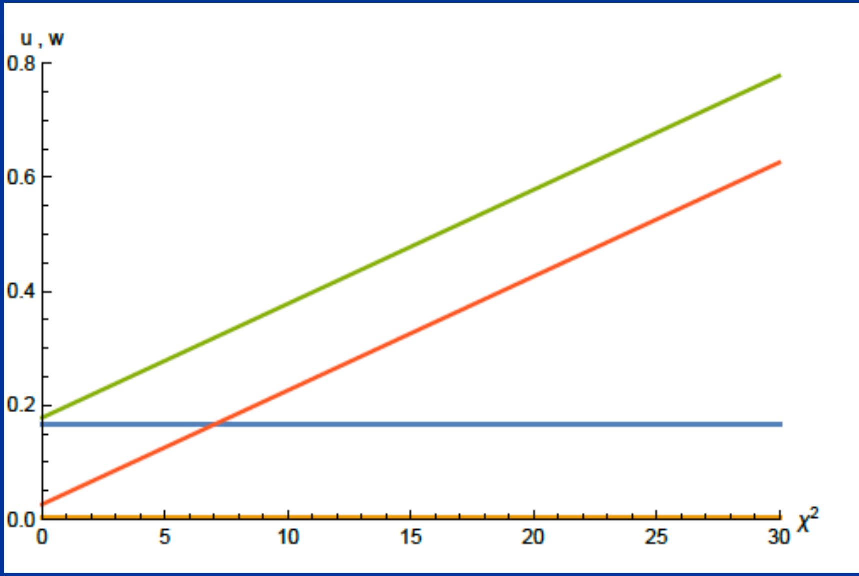
no small parameter

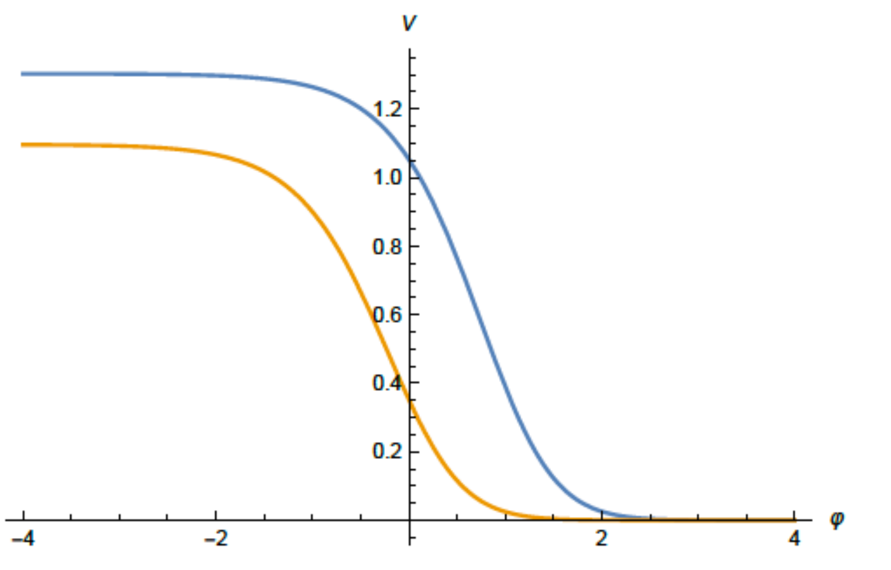
no tuning



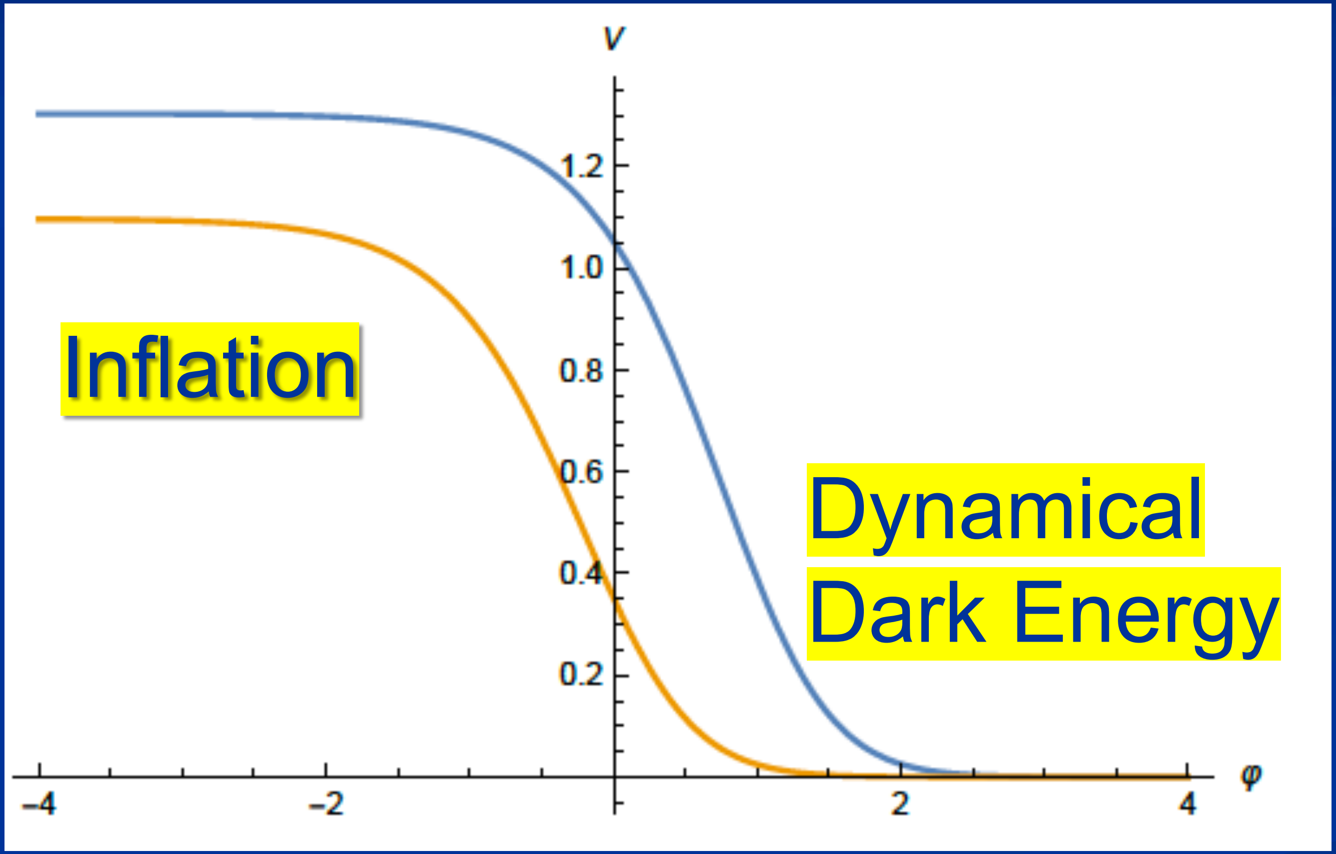
Cosmology

Scaling solution in Einstein frame





Quintessential inflation



Spokoiny, Peebles, Vilenkin, Peloso, Rosati, Dimopoulos, Valle, Giovannini,
Brax, Martin, Hossain, Myrzakulov, Sami, Saridakis, de Haro, Salo, Bettoni, Rubio…

Weyl transformation for variable gravity

$$g_{\mu\nu} = (M^2/F)g'_{\mu\nu}$$

$$\varphi = 4M\ln(\chi/k)$$

$$\Gamma = \int_x \sqrt{g}\Big\{-\frac{1}{2}F(\chi)R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + U(\chi)\Big\}$$

$$\Gamma = \int_x \sqrt{g}\Big\{-\frac{M^2}{2}R' + \frac{1}{2}Z(\varphi)\partial^\mu\varphi\partial_\mu\varphi + V(\varphi)\Big\}$$

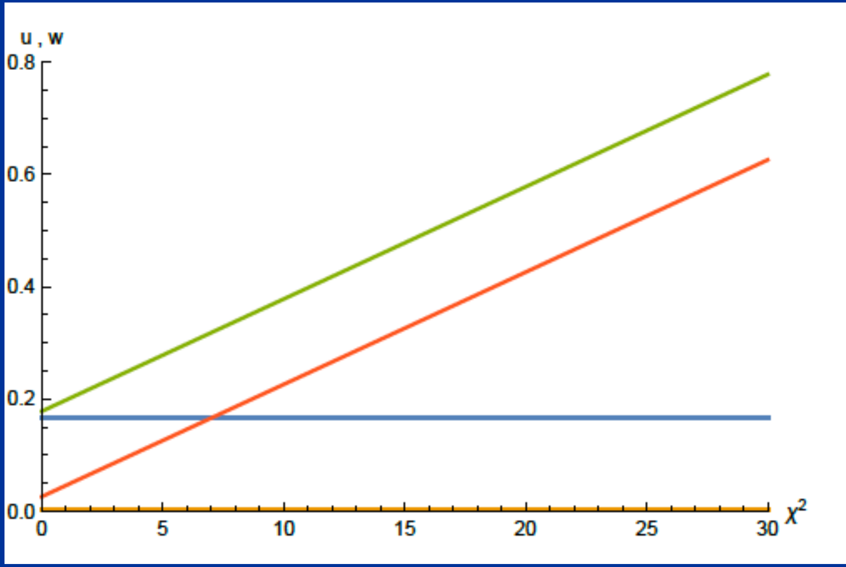
$$V(\varphi) = \frac{UM^4}{F^2}$$

$$Z(\varphi) = \frac{1}{16}\Big\{\frac{\chi^2 K}{F} + \frac{3}{2}\Big(\frac{\partial\ln F}{\partial\ln\chi}\Big)^2\Big\}$$

Scaling solution

$$U = u_0 k^4$$

$$F = 2 w_0 k^2 + \xi \chi^2$$



For low energy

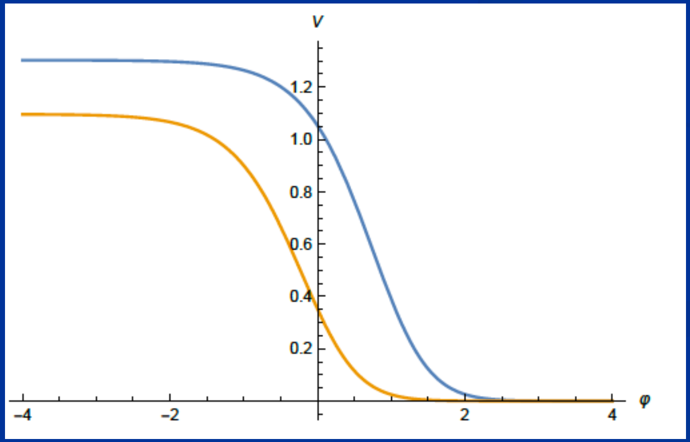
standard model :

$$u_\infty = \frac{7}{256\pi^2}$$

Scaling solution in Einstein frame

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$

$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$

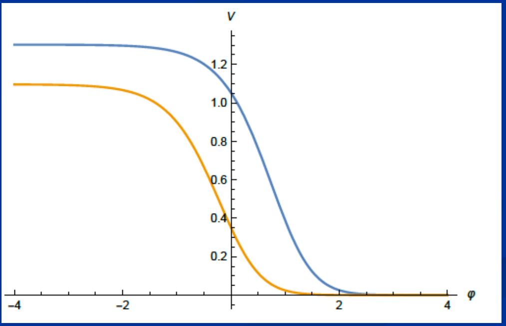


Mass scales in Einstein frame

Renormalization scale k is no longer present

Planck mass M not intrinsic: introduced only by change of variables

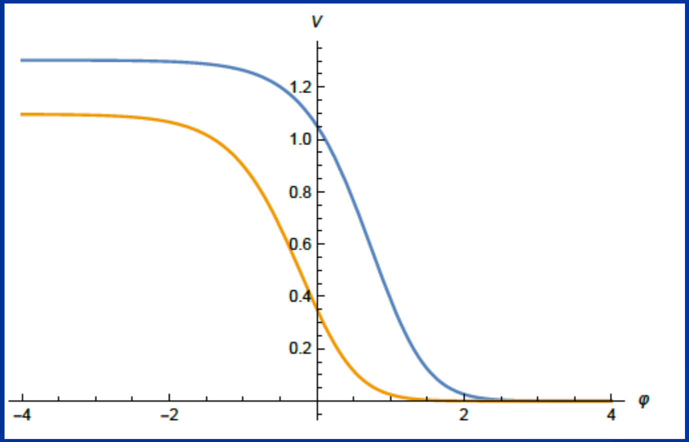
$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2} = \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



Asymptotic solution of cosmological constant problem

$$V = \frac{u_0 M^4}{\left(2w_0 + \xi \exp\left(\frac{\varphi}{2M}\right)\right)^2}$$

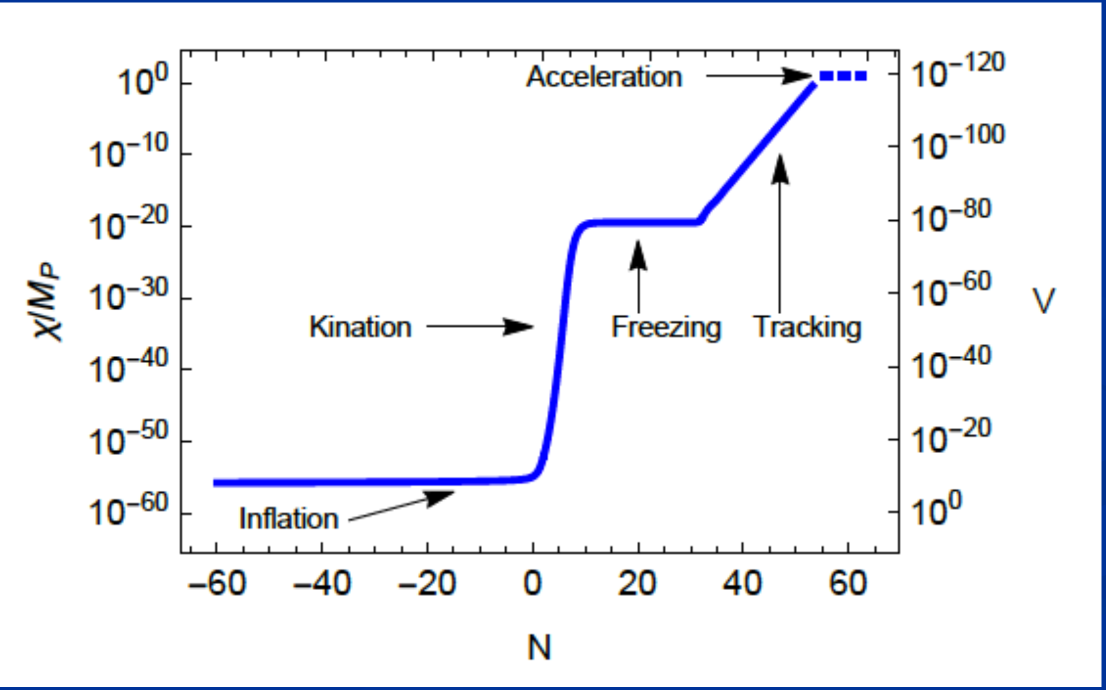
$$= \frac{u_0 M^4}{\xi^2}\left[1 + \frac{2w_0}{\xi}\exp\left(-\frac{\varphi}{2M}\right)\right]^{-2}\exp\left(-\frac{\varphi}{M}\right)$$



no tiny parameter !

Cosmological solution

- scalar field χ vanishes in the infinite past
- scalar field χ diverges in the infinite future



J.Rubio,…

Predictions for primordial cosmic fluctuations

- Depend on form of kinetial K

$$\Gamma = \int_x \sqrt{g} \Big\{ -\frac{1}{2} F(\chi) R + \frac{1}{2} K(\chi) \partial^\mu \chi \partial_\mu \chi + U(\chi) \Big\}$$

- so far form of K assumed assumed :

  realistic cosmology possible for suitable K

- K needs to be computed!

Conclusions

- Fluctuations of the metric matter
- They influence the behavior of scalar potentials for all field values
- Quantum gravity relevant for early cosmology and late cosmology
- Quantum scale symmetry is central ingredient for understanding cosmology
- Fundamental scale invariance is highly predictive
- Understanding of quantum gravity fluctuations still in beginning stage

Conclusion

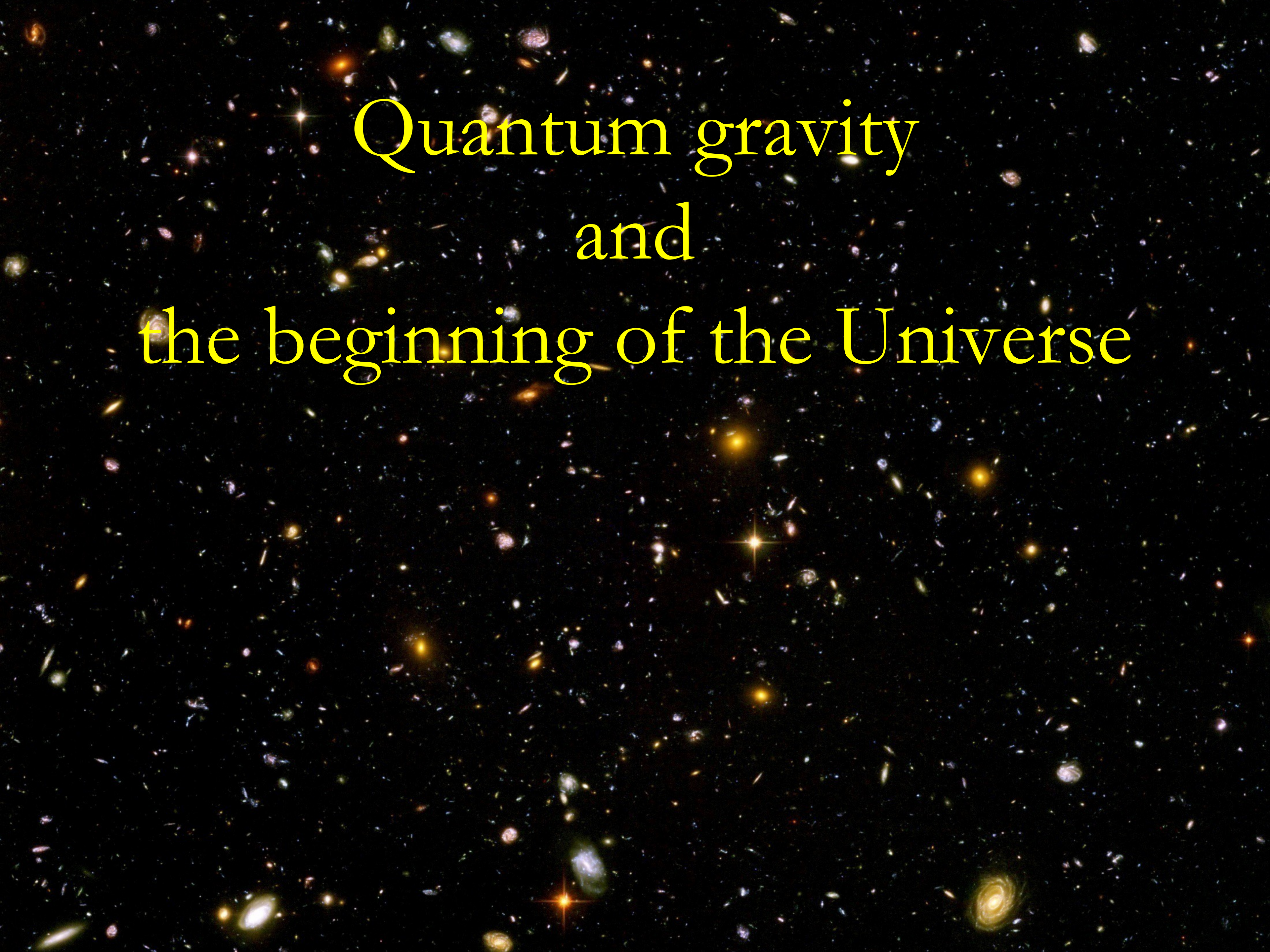
Fixed points of quantum gravity

with associated quantum scale symmetry,

scaling solutions and relevant parameters

are crucial for understanding the

evolution of our Universe


Quantum gravity
and
the beginning of the Universe

Beginning of Universe

Zu Anfang war die Welt öd und leer und währte ewig.

In the beginning the Universe was empty and lasted since ever.

Beginning close to ultraviolet fixed point for vanishing scalar field

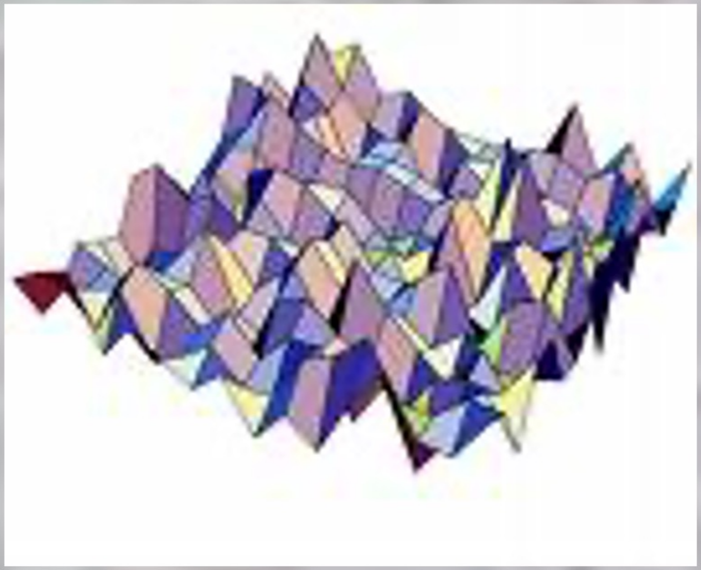
- all particles massless for $\chi = 0$
- fluctuations dominate
- metric field vanishes
- quantum scale symmetry

(equivalent primordial flat frame: initial flat Minkowski space)

Eternal light-vacuum

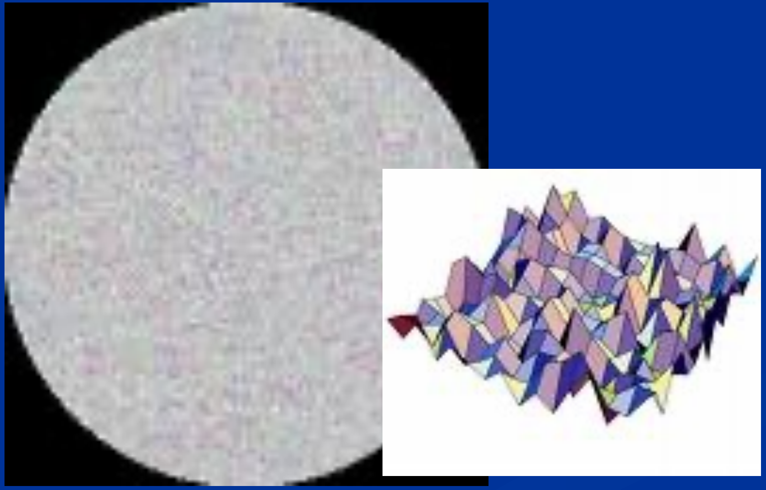
Everywhere almost nothing
only fields and their fluctuations

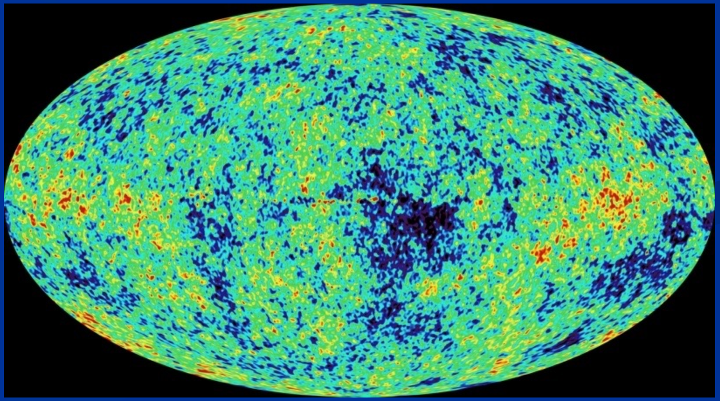
All particles move
with light velocity,
similar to photons



Eternal light-vacuum is unstable

- Slow increase of particle masses
- Only slow change of space-time geometry
- Creation of particles and entropy at crossover away from UV-fixed point
- Consequence for observation : primordial fluctuations become visible in cosmic background radiation
- We see fluctuations in a stage more than 1000 billion years ago.



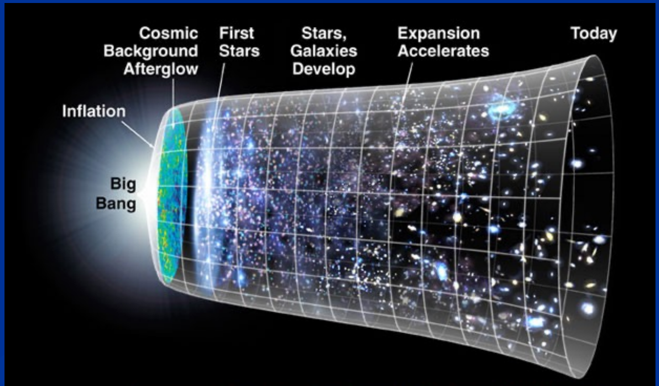


The great emptiness story

In the beginning was light-like emptiness.

The big bang story

- dramatic hot big bang
- started 13.7 billion years ago
- at the beginning extremely short period of cosmic inflation with almost exponential expansion of the Universe, duration around 10^{-40} seconds
- start with singularity : our whole observable Universe evolves from one point



Field relativity

- Both stories are equivalent
- related by field transformation of the metric

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

- different metrics related by Weyl transformation, which depends on scalar field (inflaton)

Field - singularity

- Big Bang is field - singularity
- similar (but not identical with) coordinate - singularity

$$g'_{\mu\nu} = \frac{\chi^2}{M^2} g_{\mu\nu}$$

end

Scale symmetry near UV- fixed point

kinetial : $$\frac{K}{16\xi} = \kappa\left(\frac{k}{\chi}\right)^{\sigma} + \frac{1}{\alpha^2} - \frac{3}{8}$$ $$Z(\varphi) = \kappa \exp\left(-\frac{\sigma\varphi}{4M}\right) + \frac{1}{\alpha^2(\varphi)}$$

scalar anomalous dimension σ , take σ =2

$$\Gamma = \int_x \sqrt{g}\left\{ - w_0 k^2 R + \frac{8\xi\kappa k^2}{\chi^2}\partial^\mu\chi\partial_\mu\chi + u_0 k^4 \right\}$$

Weyl scaling $$g'_{\mu\nu} = \frac{k^2}{\chi^2} g_{\mu\nu}$$

$$\Gamma = \int_x \sqrt{g'}\left\{ - w\chi^2 R' + u\chi^4 + \frac{1}{2}\left(\frac{\chi^2 K}{k^2} - 12w - 12\frac{\partial w}{\partial \ln\chi}\right)\partial^\mu\chi\partial_\mu\chi \right\}$$