Modelling GWs from compact binary coalescence

Sascha Husa, Institute of Space Sciences (ICE-CSIC) & University of the Balearic Islands
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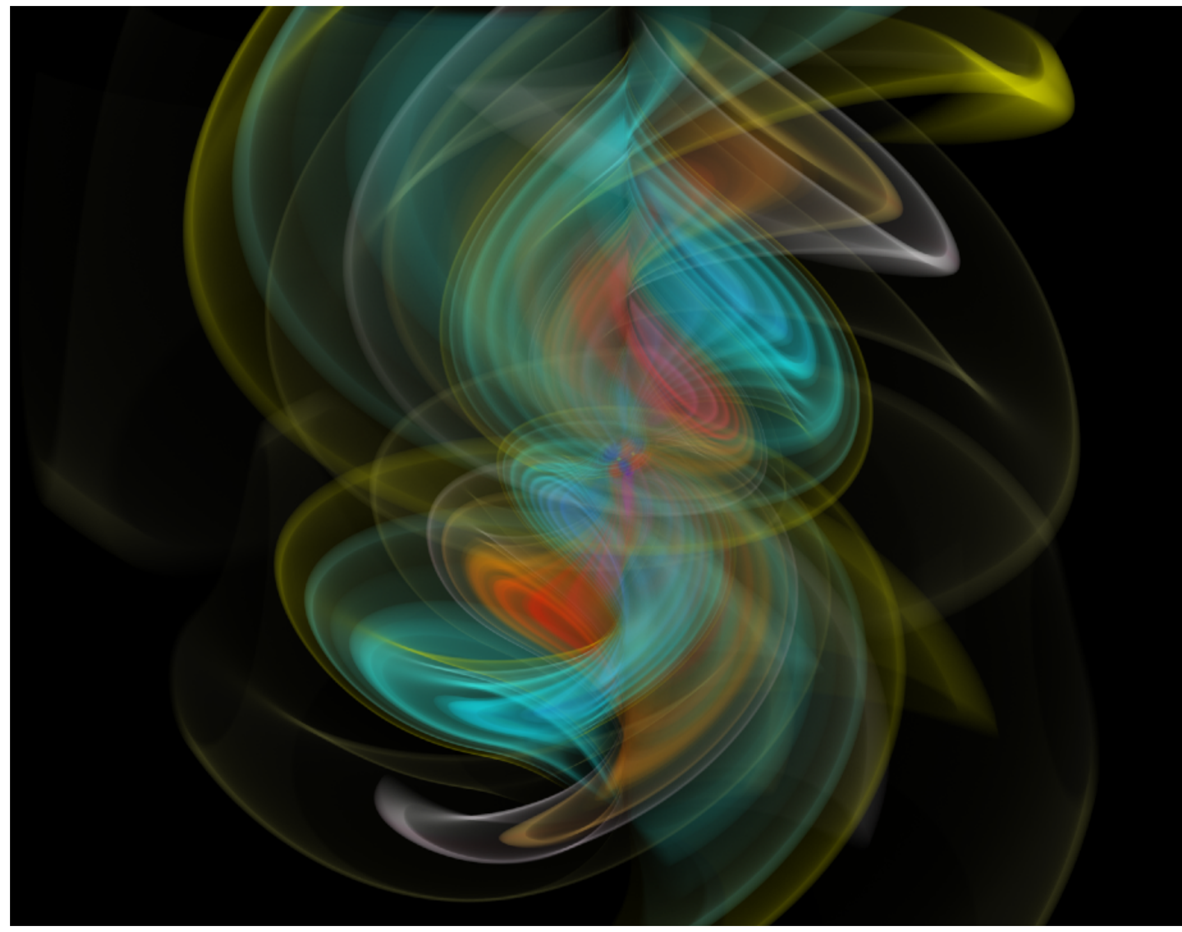
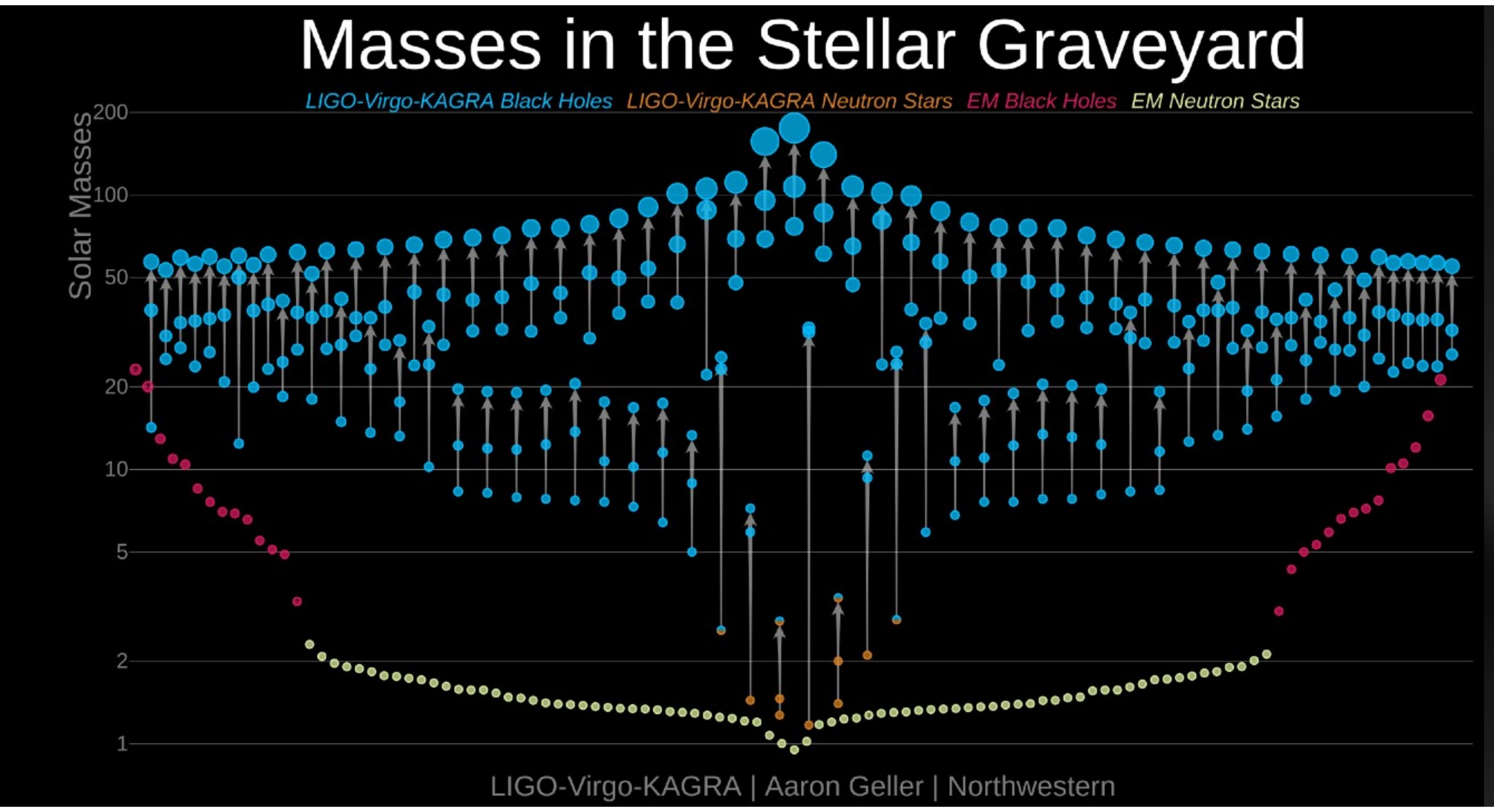
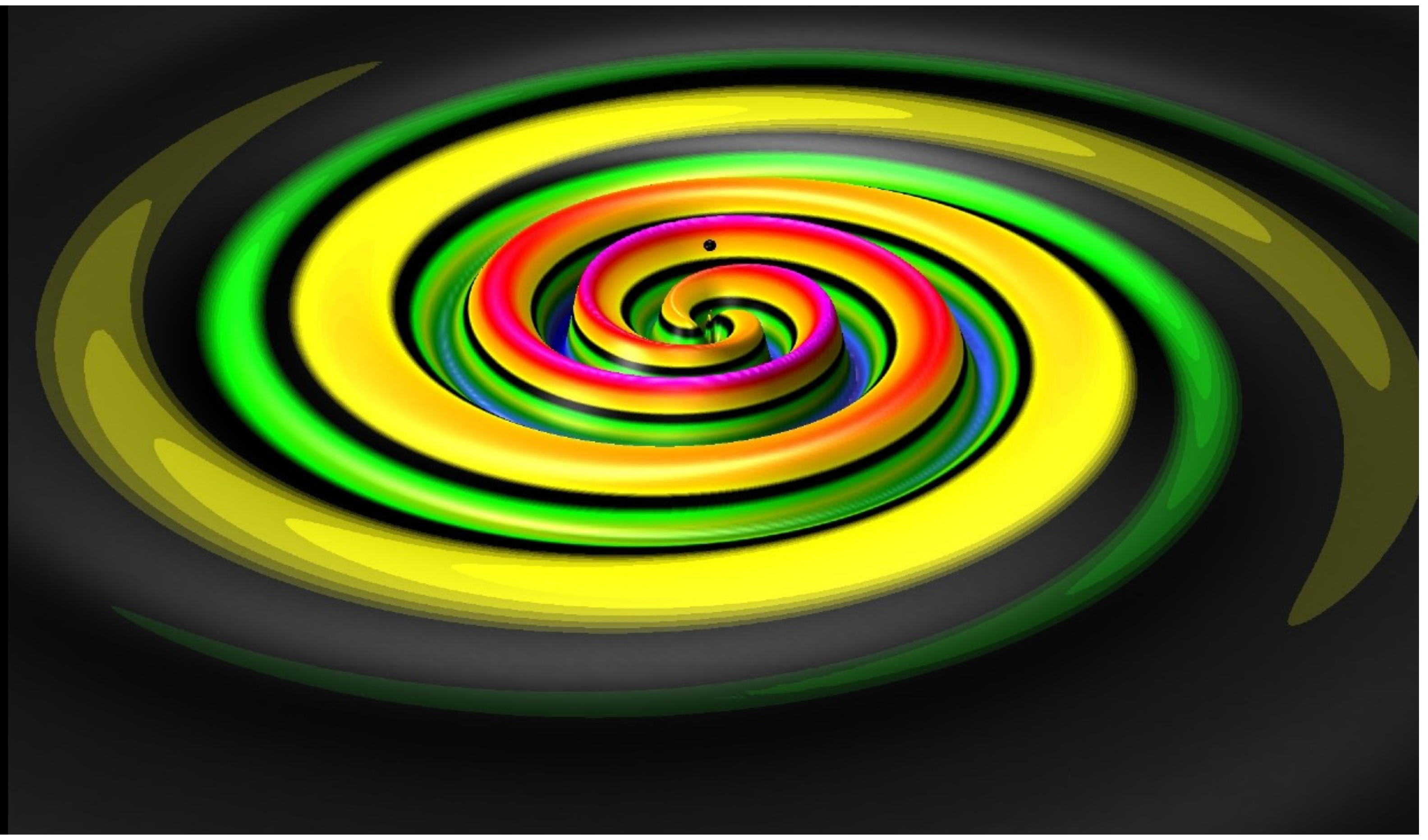






Focus on comparable mass black hole coalescence

Prague, 5th EPS Conference
on Gravitation
11/12/2024

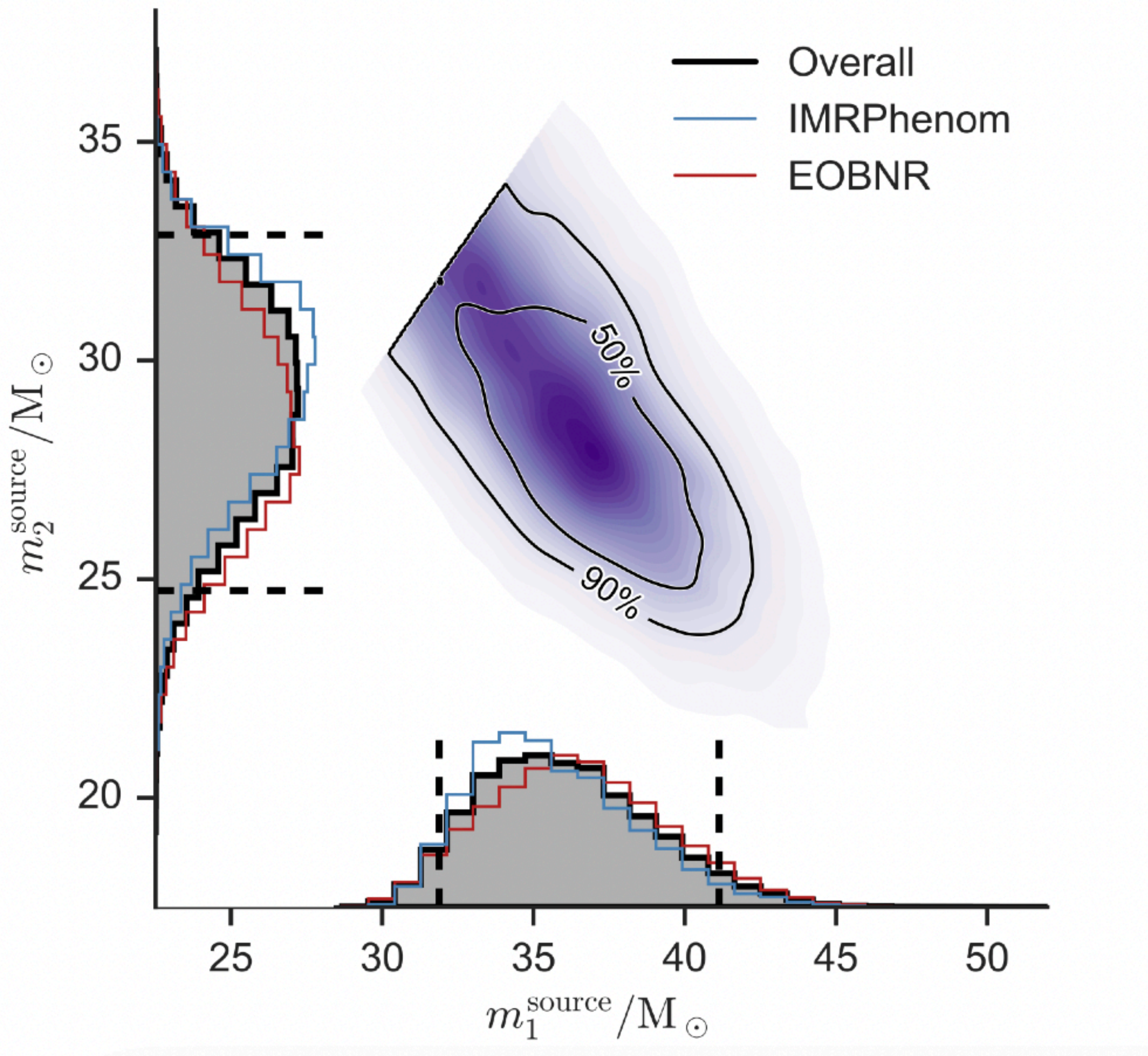
Matter: Tanja Hinderer, Agnieszka Janiuk, Maria J. Rodriguez
EMRIs: Niels Warburton, **Hyperbolic encounters:** Jan Plefka

GW Data Analysis Methods: matched filtering

Optimal strategy for stationary Gaussian noise: perform searches & parameter estimation (PE) by comparing models and data via scalar product:

$$\langle h_1, h_2 \rangle = \max_{\phi_0, t_0} 4\Re \int_{f_1}^{f_2} \frac{\tilde{h}_1(f)\, \tilde{h}_2^*(f)}{S_n(f)}\, df$$

LIGO+Virgo, PRL2016: GW150914



$\mathrm{SNR}^2 = \langle h|d \rangle$ signal-to-noise ratio, high SNR for LVK ≳ 20, LISA 10 000?

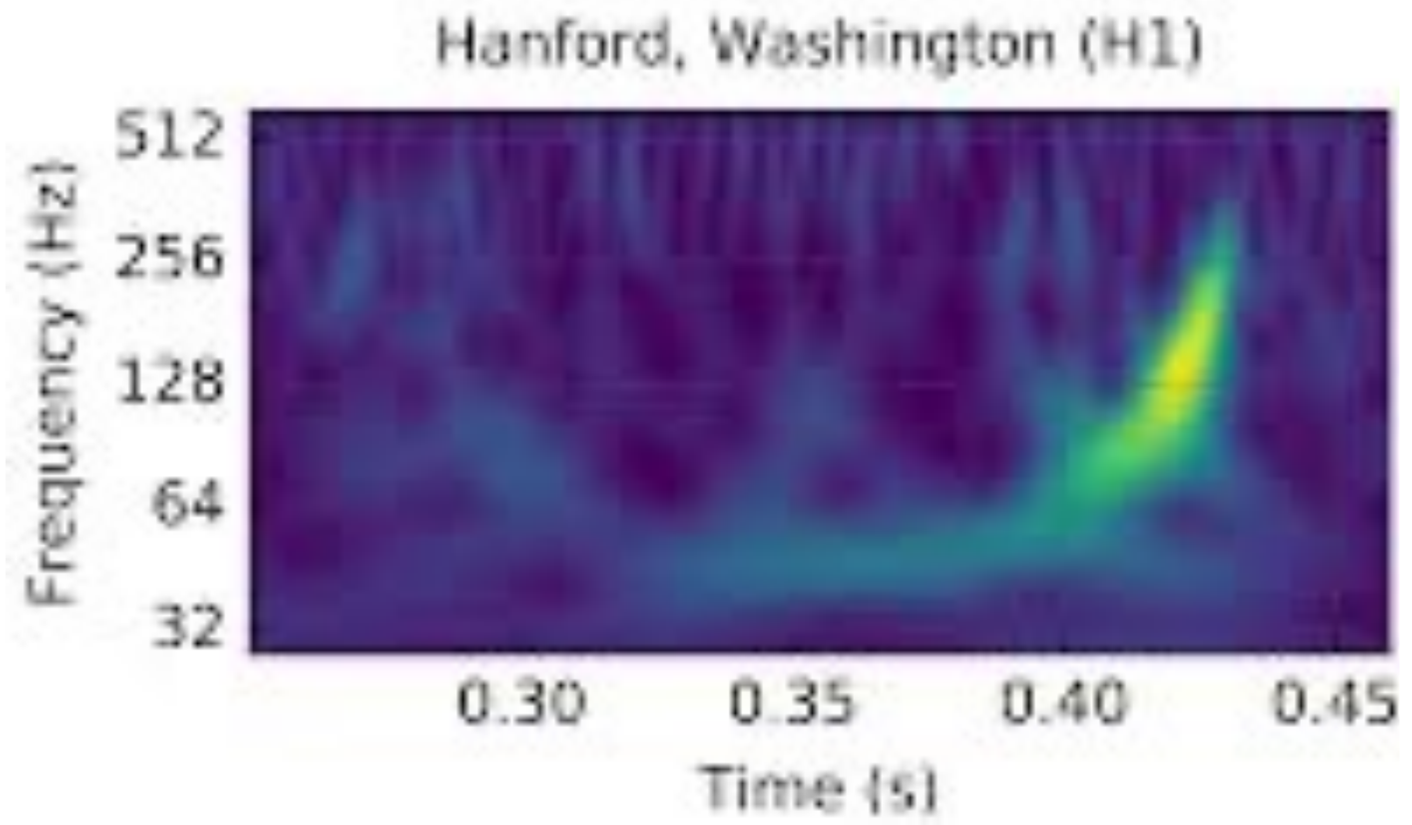
Posterior probability distribution p for signal (model) with parameters θ, given prior π:

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)\, \pi(\theta)}{\mathcal{Z}}$$

Likelihood for data d, given signal with parameters θ:

$$\mathcal{L}(d|\theta) \propto \exp\left(-\frac{1}{2} \langle d - h(\theta) | d - h(\theta) \rangle\right)$$

- Identification of sources is limited by waveform accuracy.
- Also: test pipeline sensitivity -> rates; develop science case, …
- **=> need accurate waveform models across plausible parameter space.**

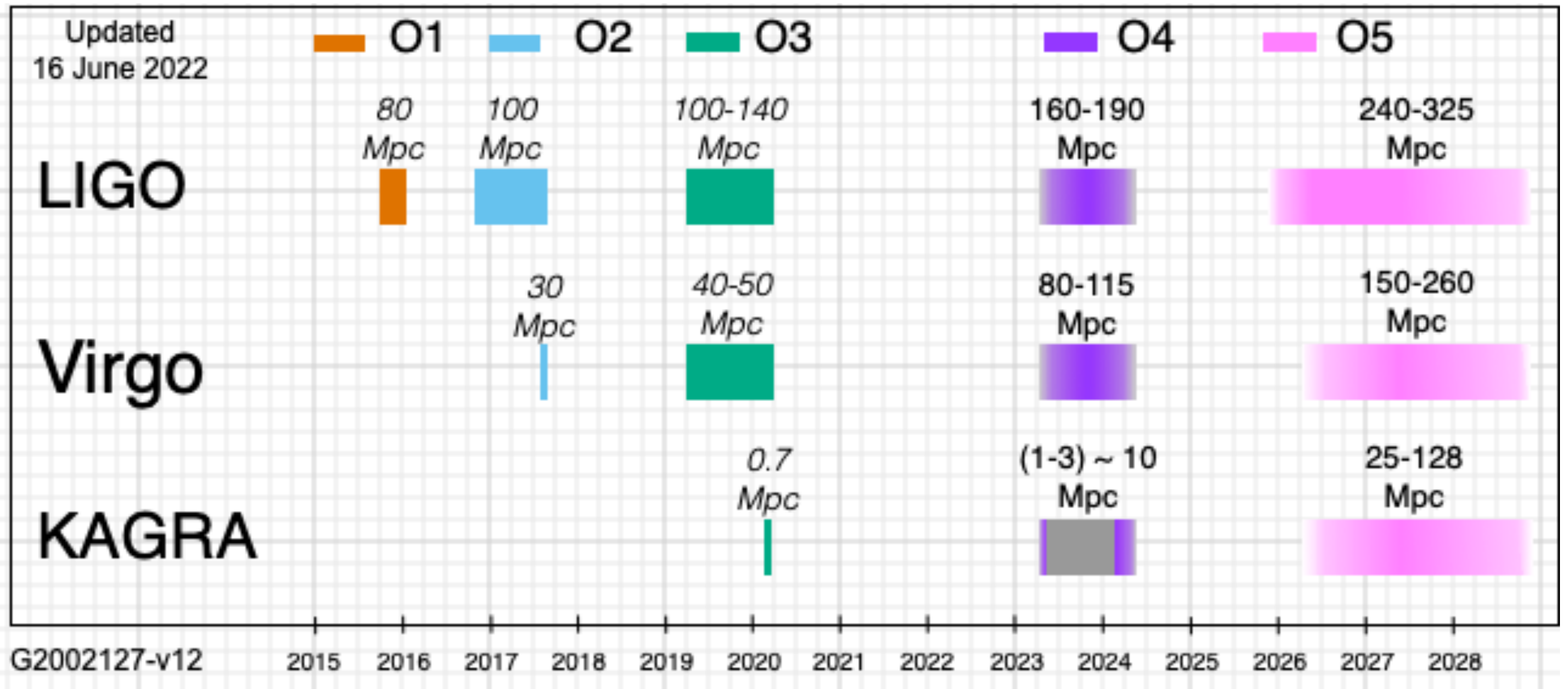


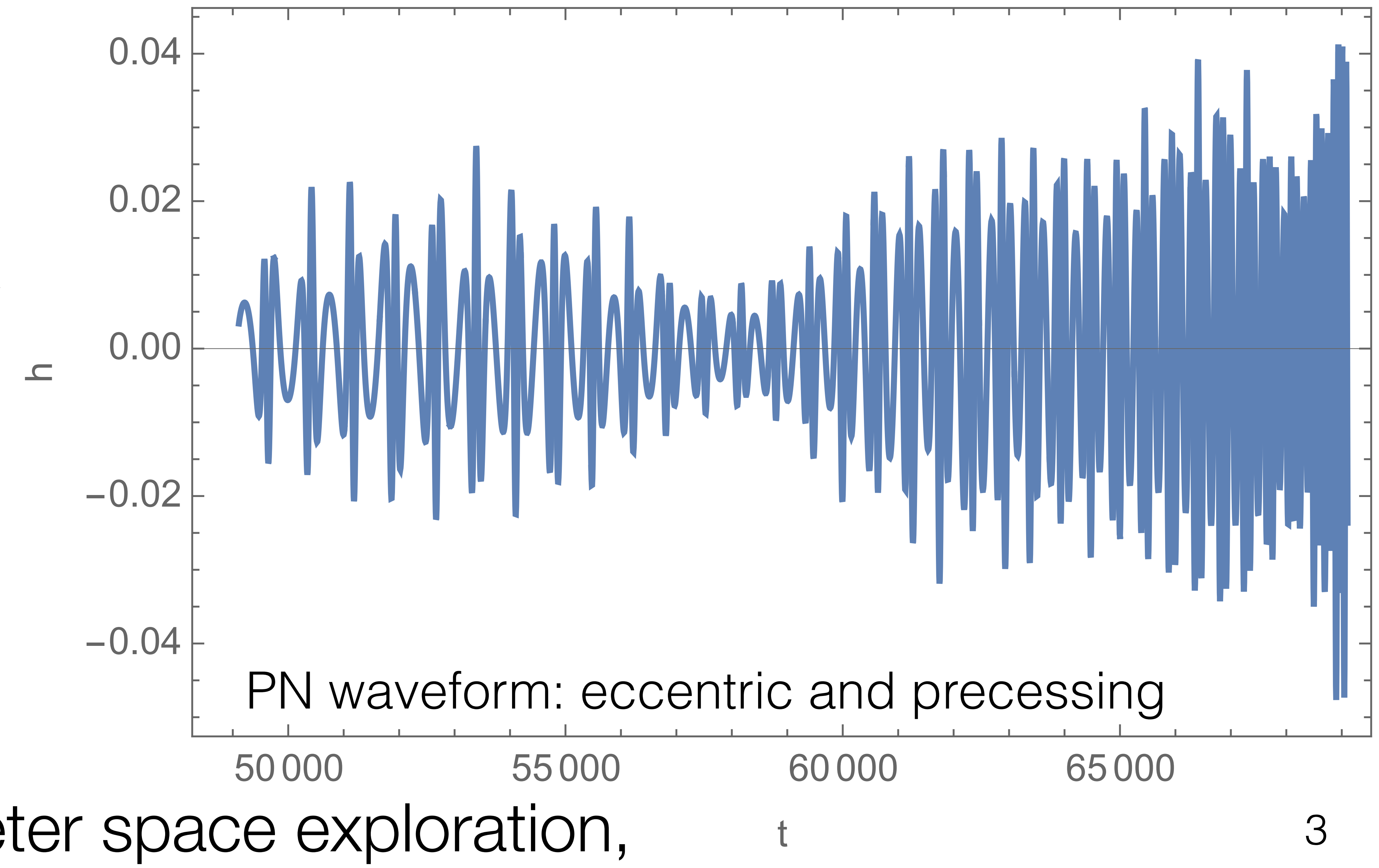
Alternative:

Un-modeled searches and feature extraction: Time-frequency pattern recognition-

Scientific and other challenges

- Different models may give different results -> systematic errors?
- **Keep up with detector sensitivity - upgrade models.**
  - **Development, implementation and reviews are costly**
    - New model generation ~ several people for several years, use for observation not guaranteed.
    - Reviews take months, also costly computationally.
  - **Complexity of waveforms as functions of time/frequency and as a function of intrinsic parameters** increasing as more effects are included (precession, eccentricity, memory, flip-flop instability, “21-mode anomaly”, …
  - **Lacking people/cooperation**
    - Distribution of credit? New methods, NR codes, NR parameter space exploration, “production level” waveform models ….





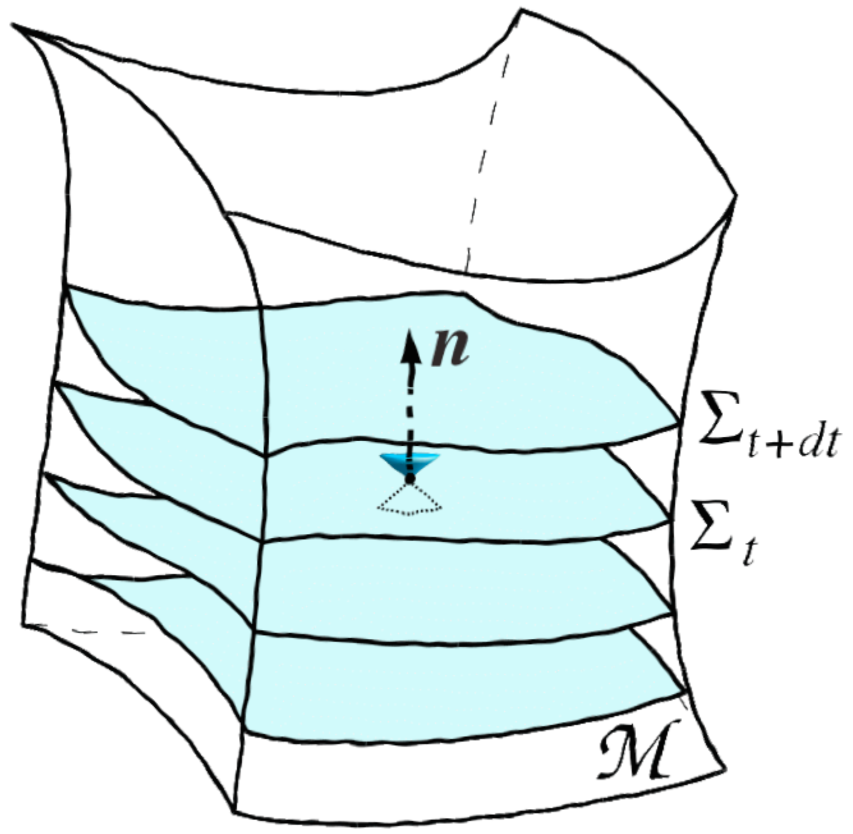
LISA prospects+challenges: waveforms white paper (arXiv:2311.01300); soon ET bluebook.

First we have to solve the Einstein equations!

$$G_{ab} = \frac{8\pi G}{c^4} T_{ab}$$ -— starting 1950's -—>

IBM 7090: first installation in December 1959

NNALS OF PHYSICS: **29**, 304–331 (1964)



Y. Choquet-Bruhat proves well-posedness of harmonic formulation.

etrodynamics

w York, New York

York

Weber bar GW searches:

- J. Weber PRL 1967, "GW observation can't be completely ruled out".
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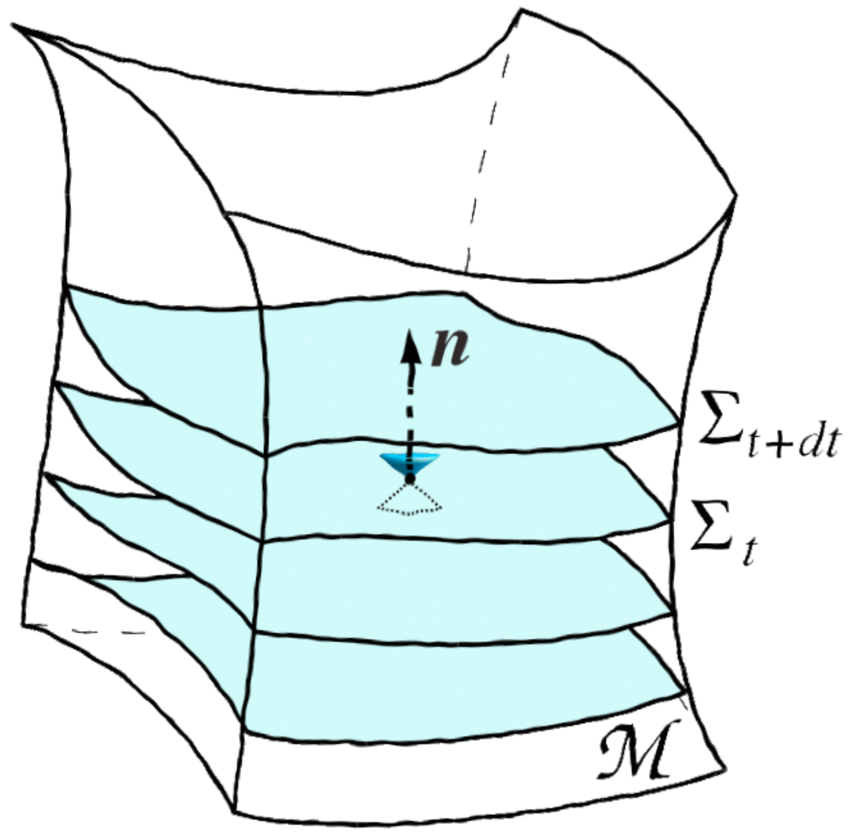

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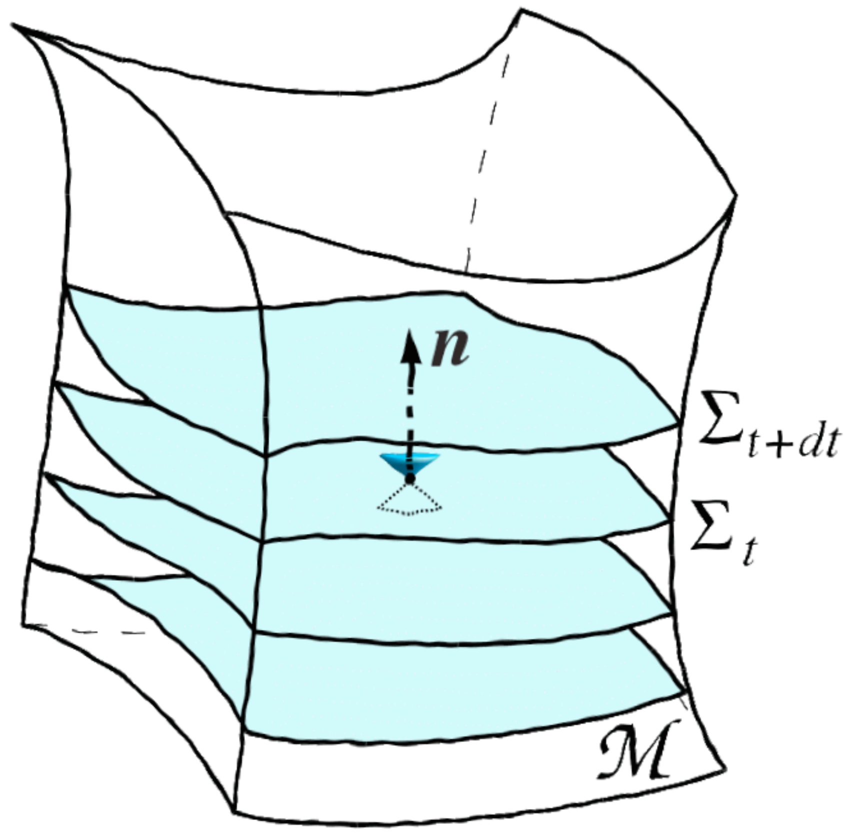

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The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

International Business Machines Corporation, New York, New York

AND

RICHARD W. LINDQUIST

Adelphi University, Garden City, New York

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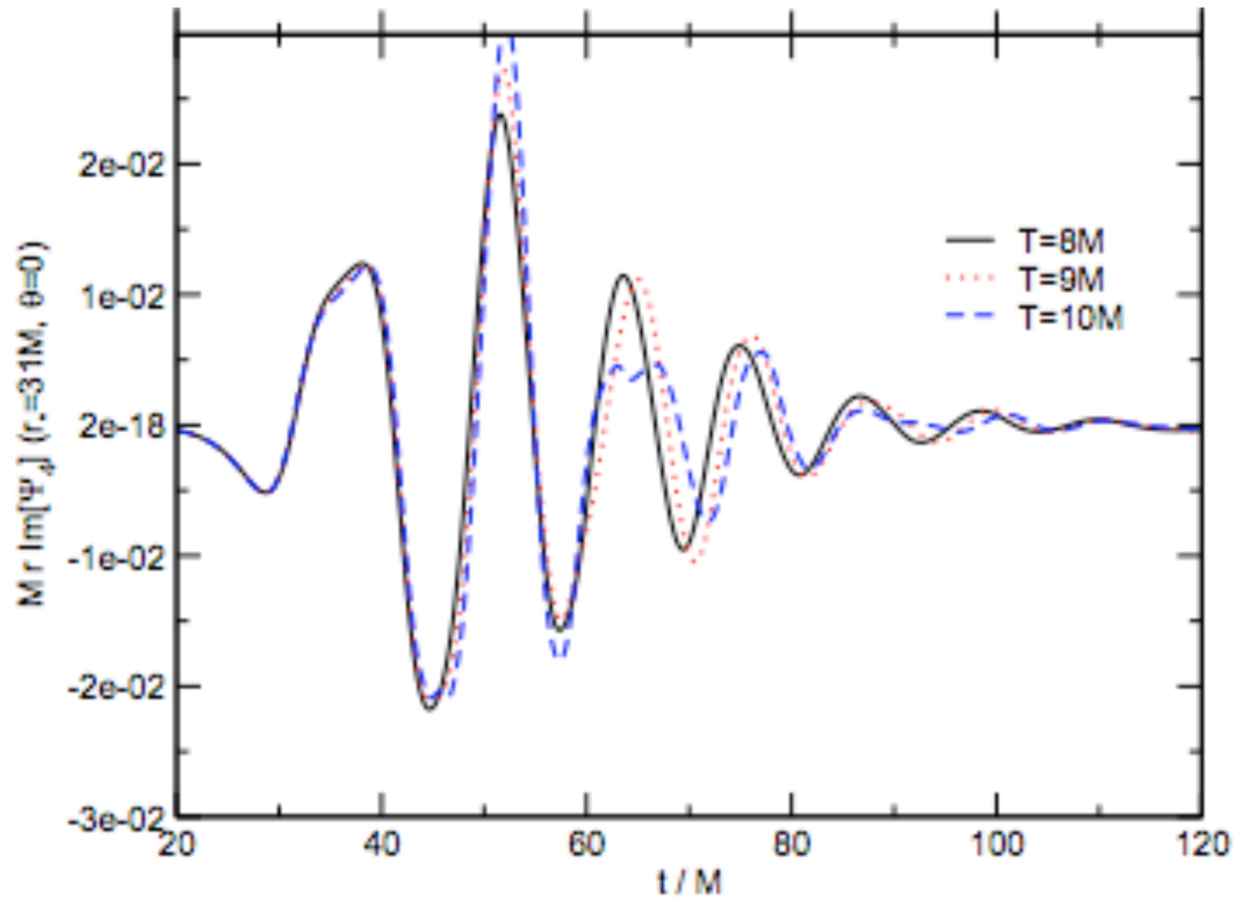
Numerical relativity is hard:

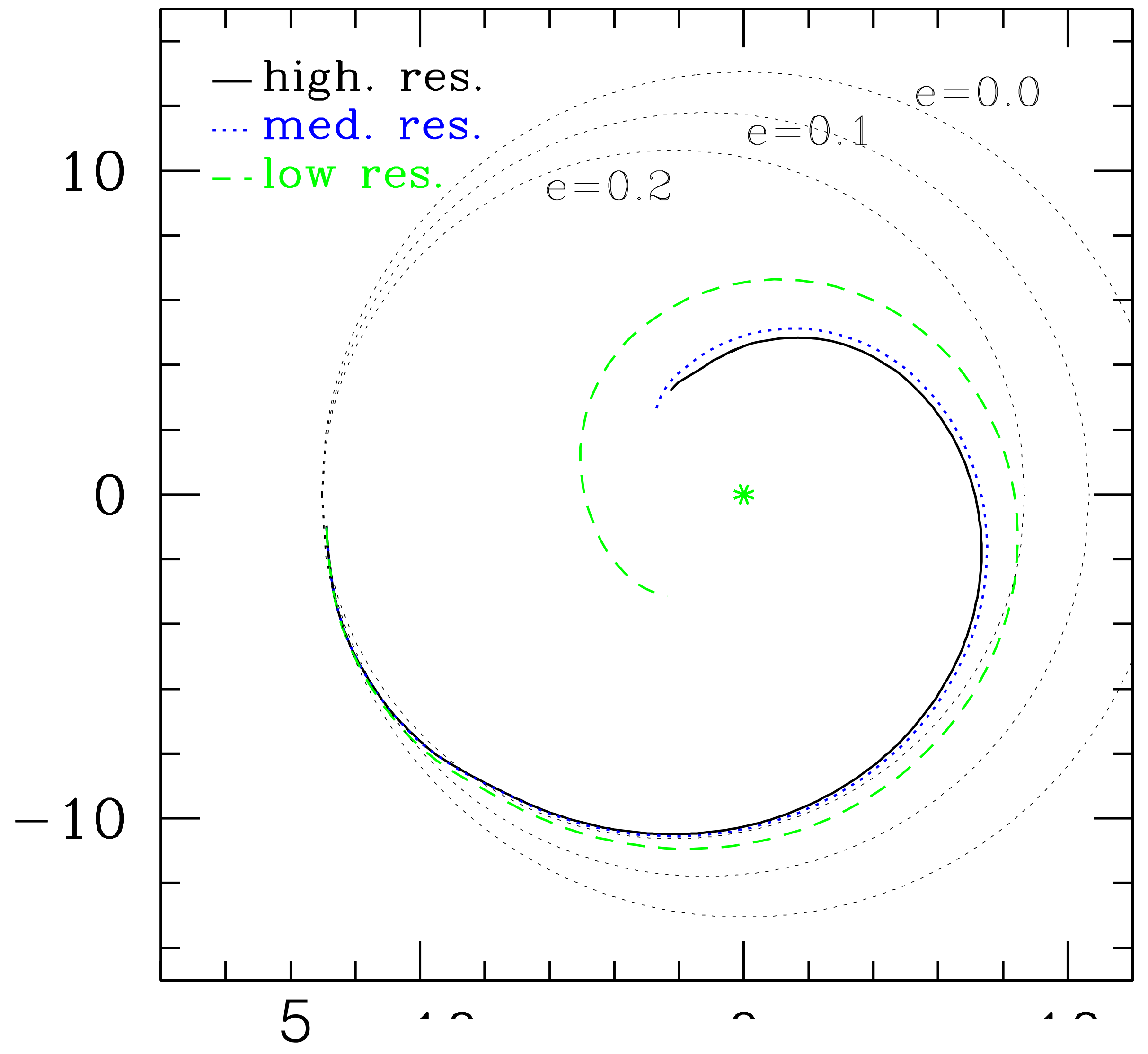
- complex equations without natural IVP,
- need to solve and preserve nasty constraints,
- adapt gauge to underlying simplicity, e.g. approximate Killing vectors, in dynamical strong field regime.

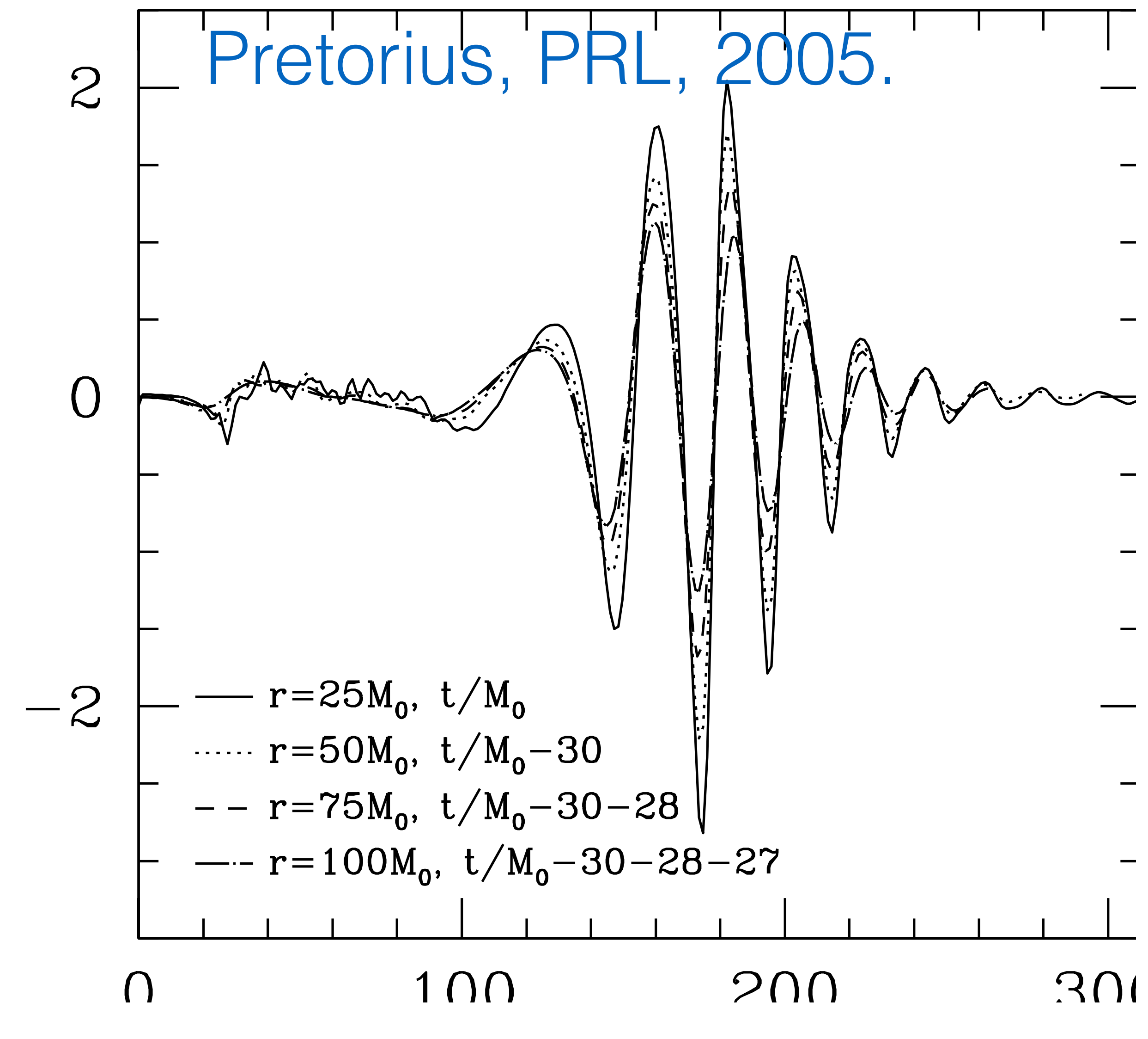
Numerical evolutions of black holes - a “holy grail” problem until 2005

- “Typing things up” required major improvements in the understanding of the Einstein equations.
- 1992: LIGO project founded.
- 1993: Choptuik - critical collapse (PRL 1993)
- 1994-1998: BBH Grand Challenge fails to provide waveforms
- 2002: First LIGO run & energy loss in equal mass merger ~ 3% (Lazarus).
- Will NR have much to contribute to GW detection? [Baker+. PRD65 124012]
  - Expected binary masses small - SNR dominated by inspiral -> use PN!



- **First orbit + GWs: Pretorius PRL 2005**
- Surprise breakthrough after 4 decades of unstable formulations.
- => Gold-rush of improved methods and results, first in NR, then data analysis.
- **Detection of first GW with inspiral-merger-ringdown waveform models 10 years later.**





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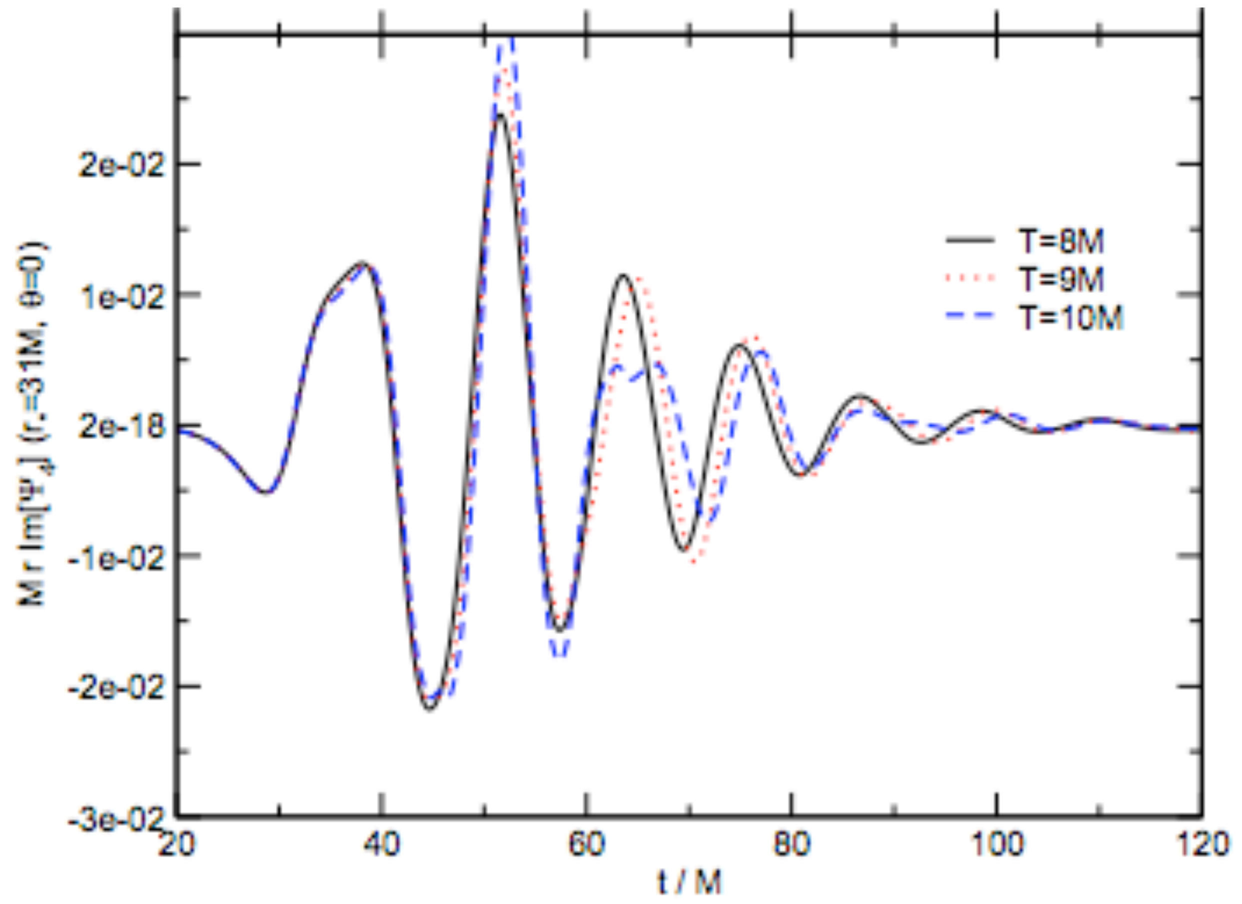
The hardest problem is usually sociology.



[Baker+. PRD65 124012]

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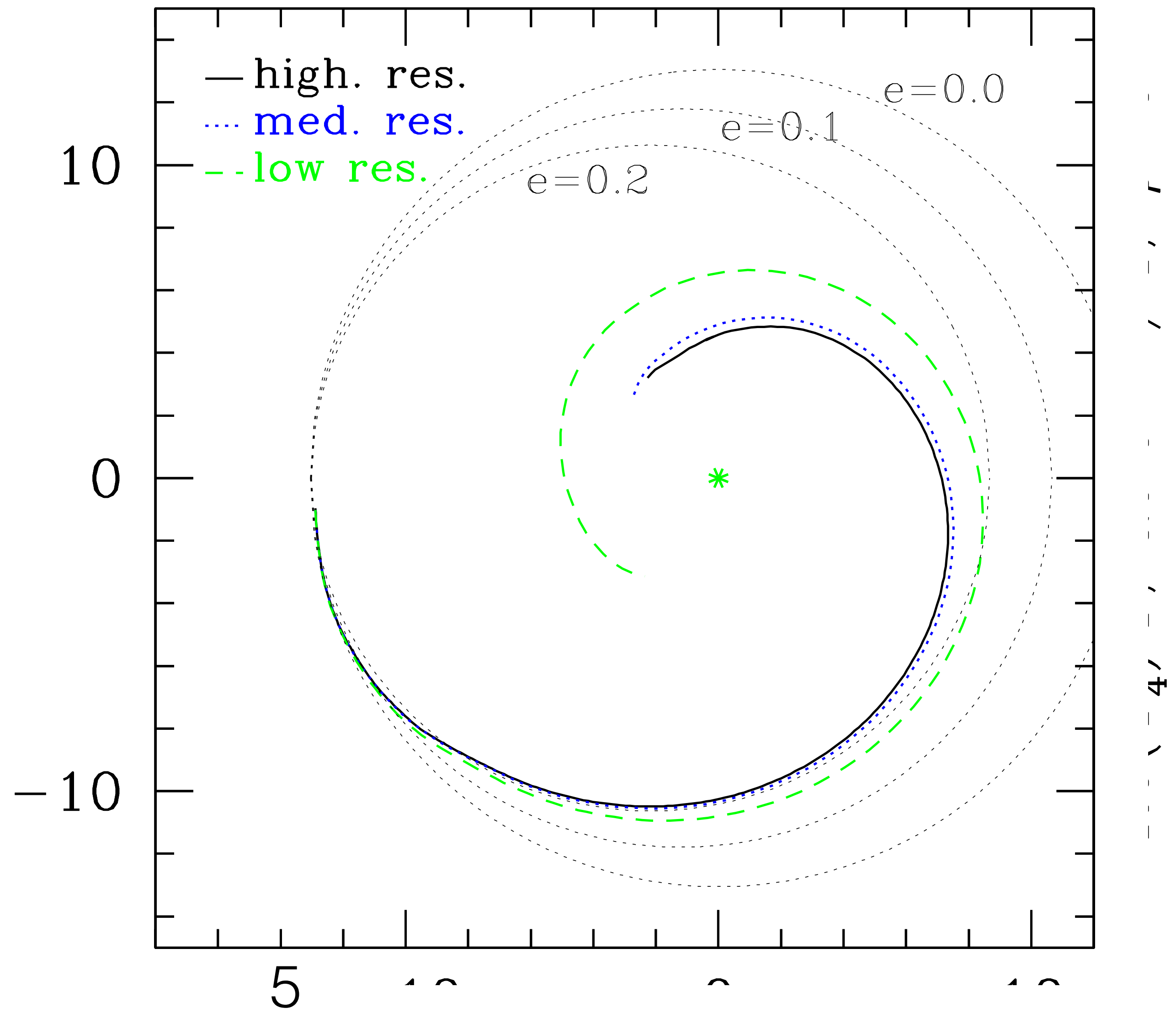
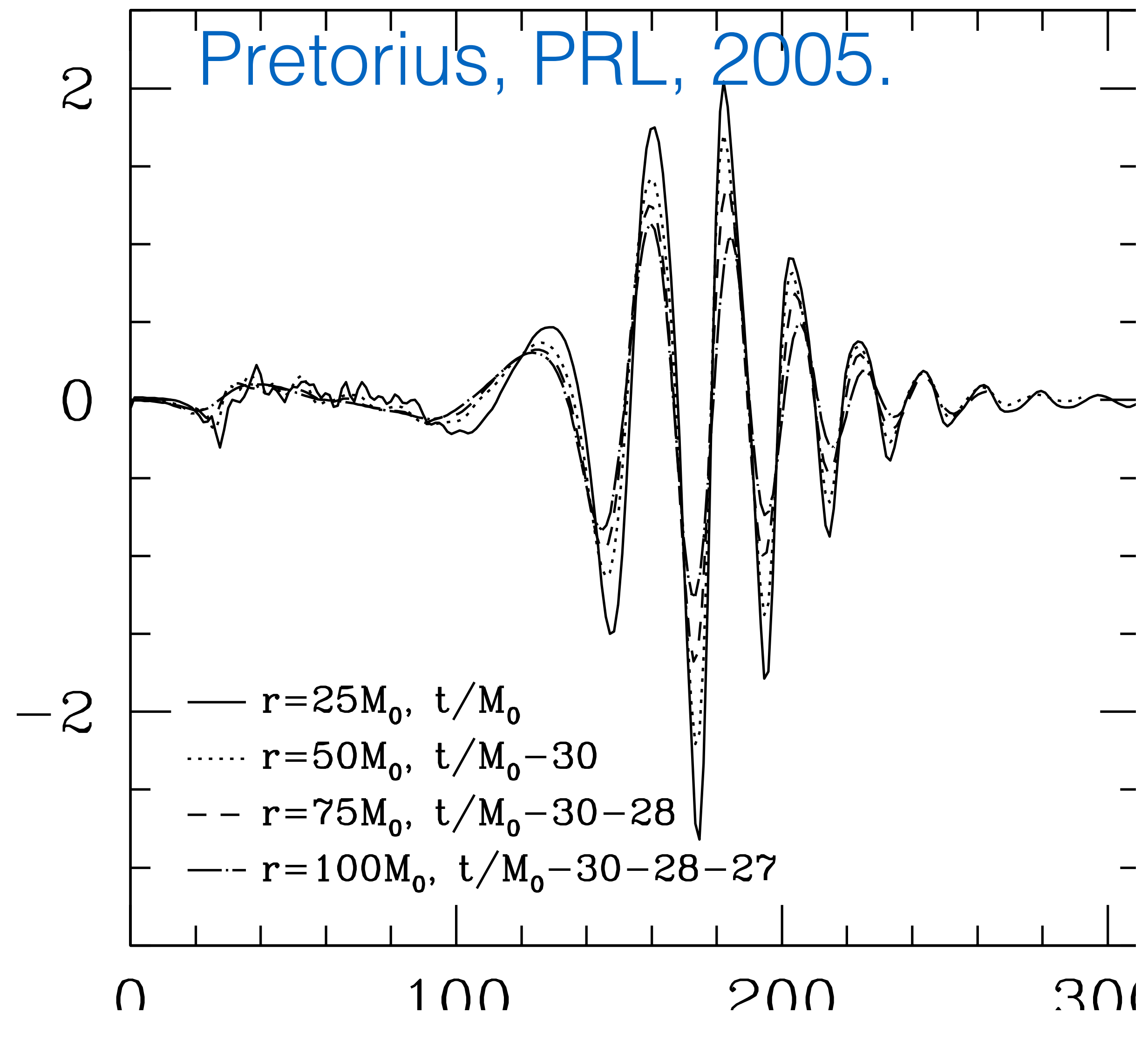
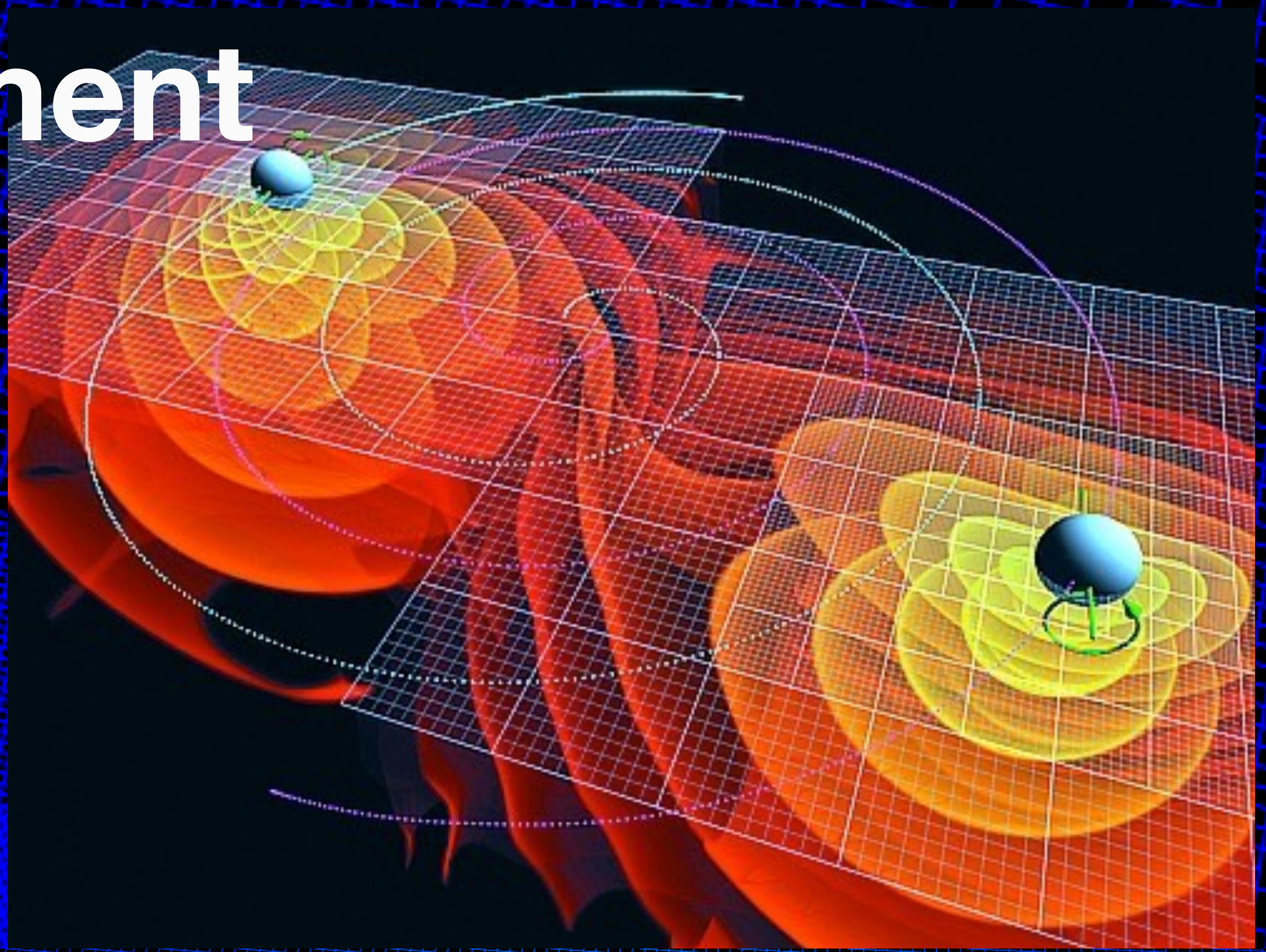
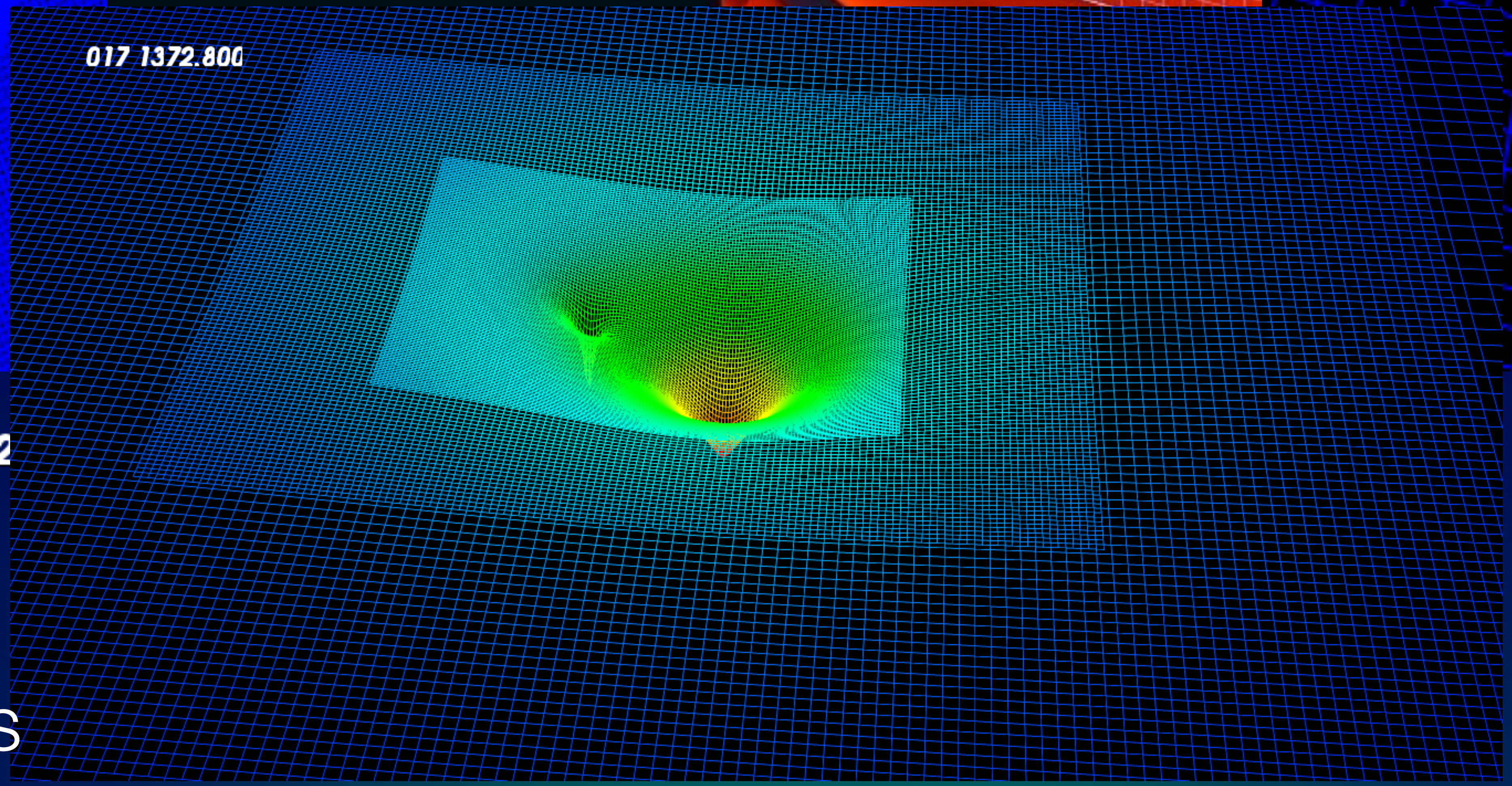




Numerics, Scales & mesh refinement

- **Solutions are smooth without matter: high order (6-8) finite differencing or spectral methods.**
- **Several length & time scales:**
  - individual compact objects
  - orbital scale
  - wave frequency increases ~ factor ≳ 10
  - causally isolate boundaries

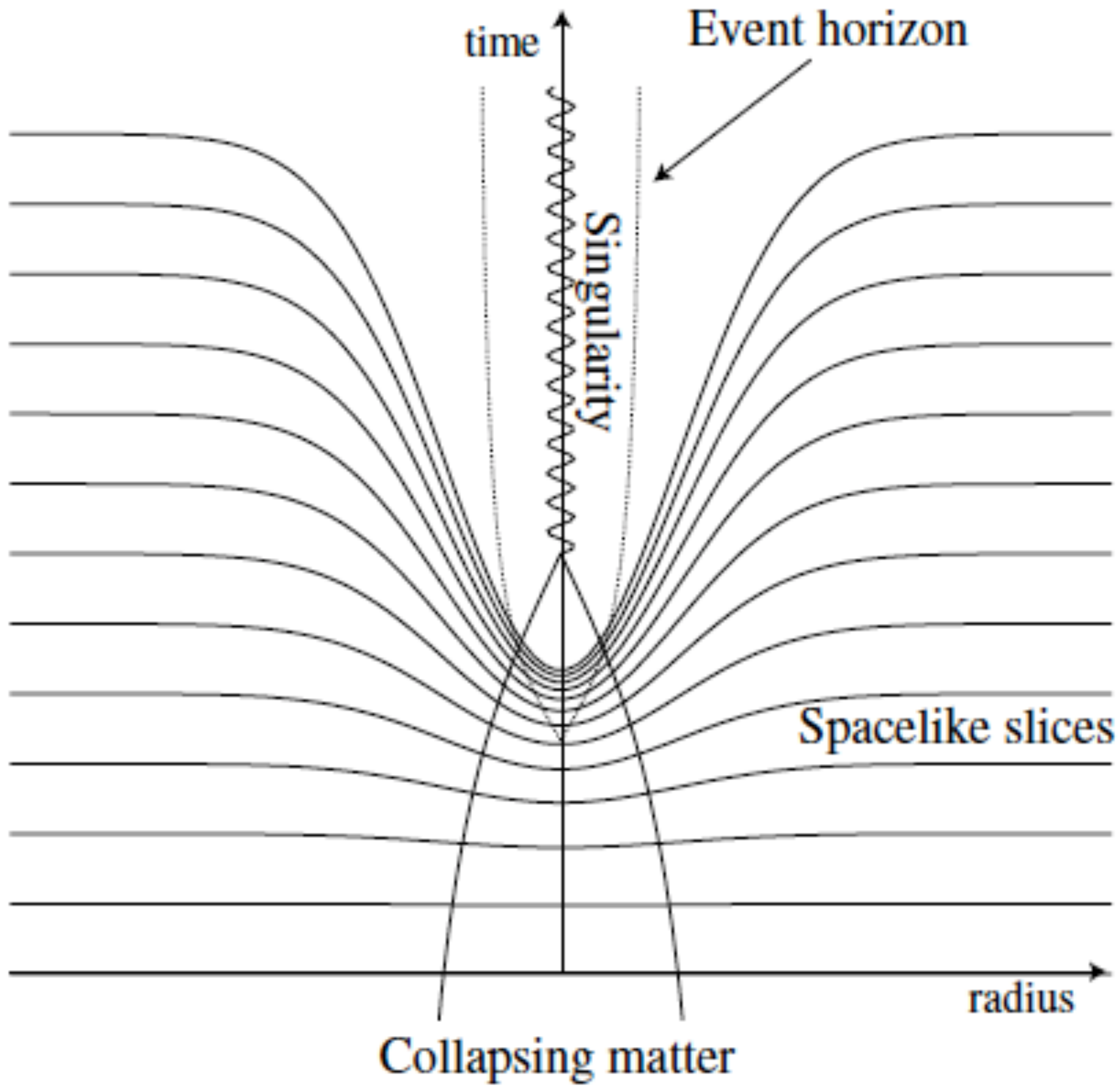




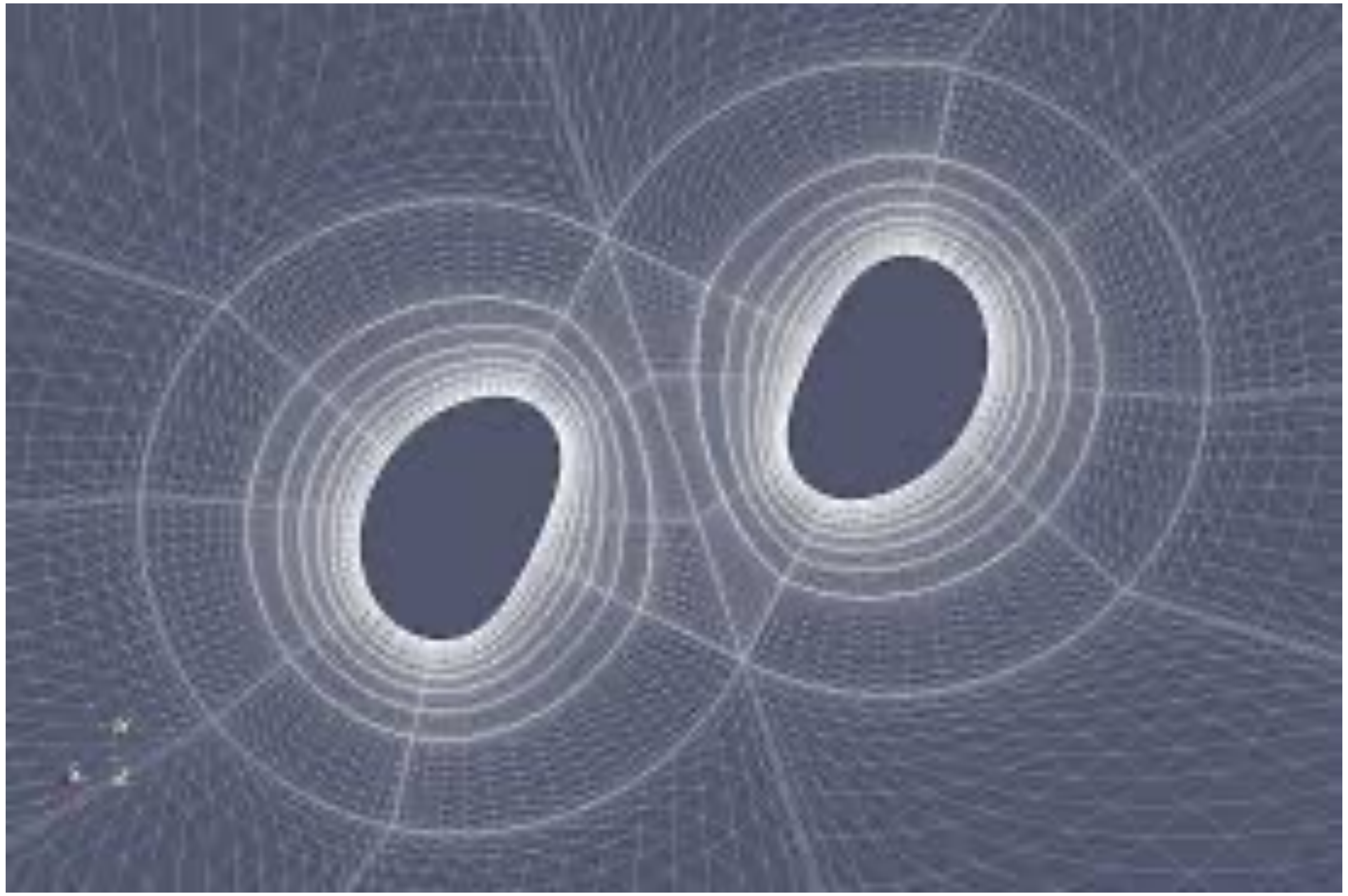
- computational cost in 3+1 D $\propto \Delta x^{-3} \Delta t^{-1}$
- **Need aggressive spatial and temporal mesh refinement**
  -> strong scaling is challenging for Berger-Oliger type algorithms
- BBH simulations ~ 10^5 - 10^6 core hours, > 10^9 core hours in total so far
- BBH: ~ 10^4 simulations available for 9-dimensional parameter space < 3 points/dimension
  (mass ratio, 2 spin vectors, 2 parameters for eccentricity)

How do current NR codes work? - black hole treatment

“moving puncture codes” - temporal excision



Spatial excision closely inside AH



How do current NR codes work? - black hole treatment

“moving puncture codes” - temporal excision

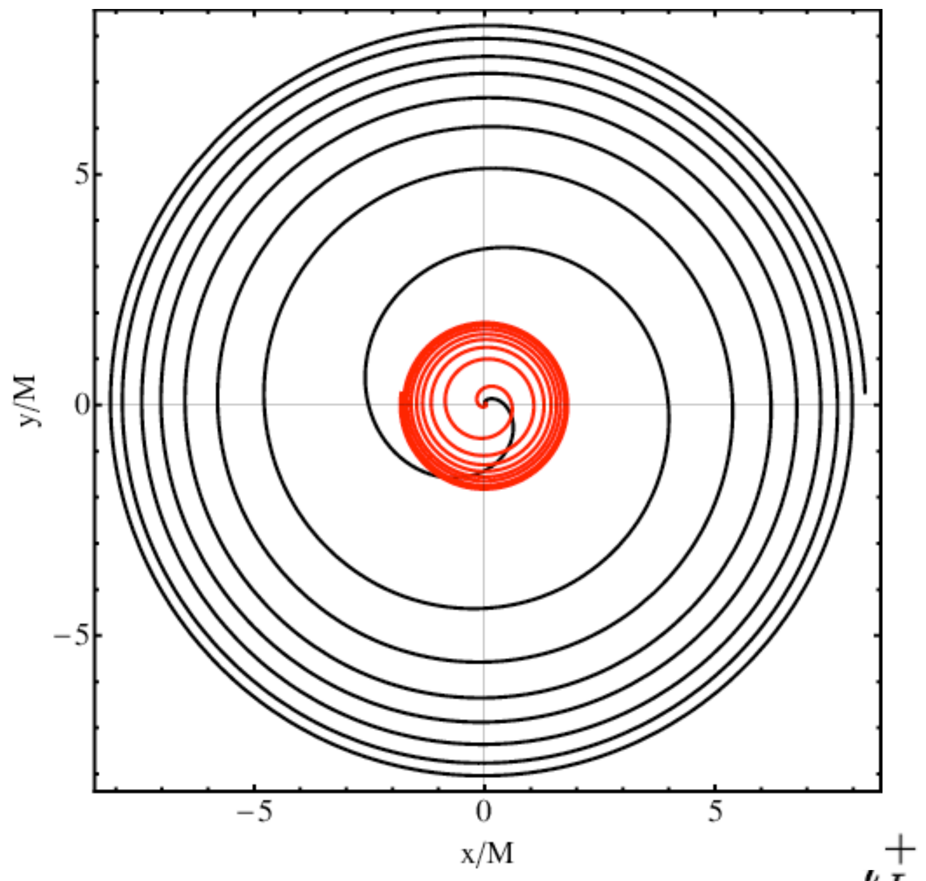
- Several independent codes, including public.
- Small groups; Also: “Einstein Toolkit”, GRChombo.
- Coord. singularity inside AH
    - high order FD
    - High flexibility + robustness
- BSSNOK, Z4c+ “puncture lapse”
    - Typical: second order in space
    - (A)MR regions communicate via buffer points
- Simple mesh refinement geometry
    - Spatial and temporal mesh refinement
    - Communicate between meshes with buffer points
- Dual coordinates unstable for BBH with BSSN (via Jacobians or shift), ok for single BH.

Spatial excision closely inside AH

- SpEC + BAMPS, new: public SpECTRE
- SXS: 1 large collaboration
- Excision surface - is hard to control.
    - Smooth solution - pseudospectral
    - Feedback system for AH location, less robust?
- Generalised harmonic formulation
    - First order reduction
    - Information exchange via characteristic fields
    - Complex domain decomposition
    - Communicate between grids by exchanging characteristic information (1st order formulation)
    - Dual coordinates to evolve in co-rotating frame, factor out BH motion for better accuracy.

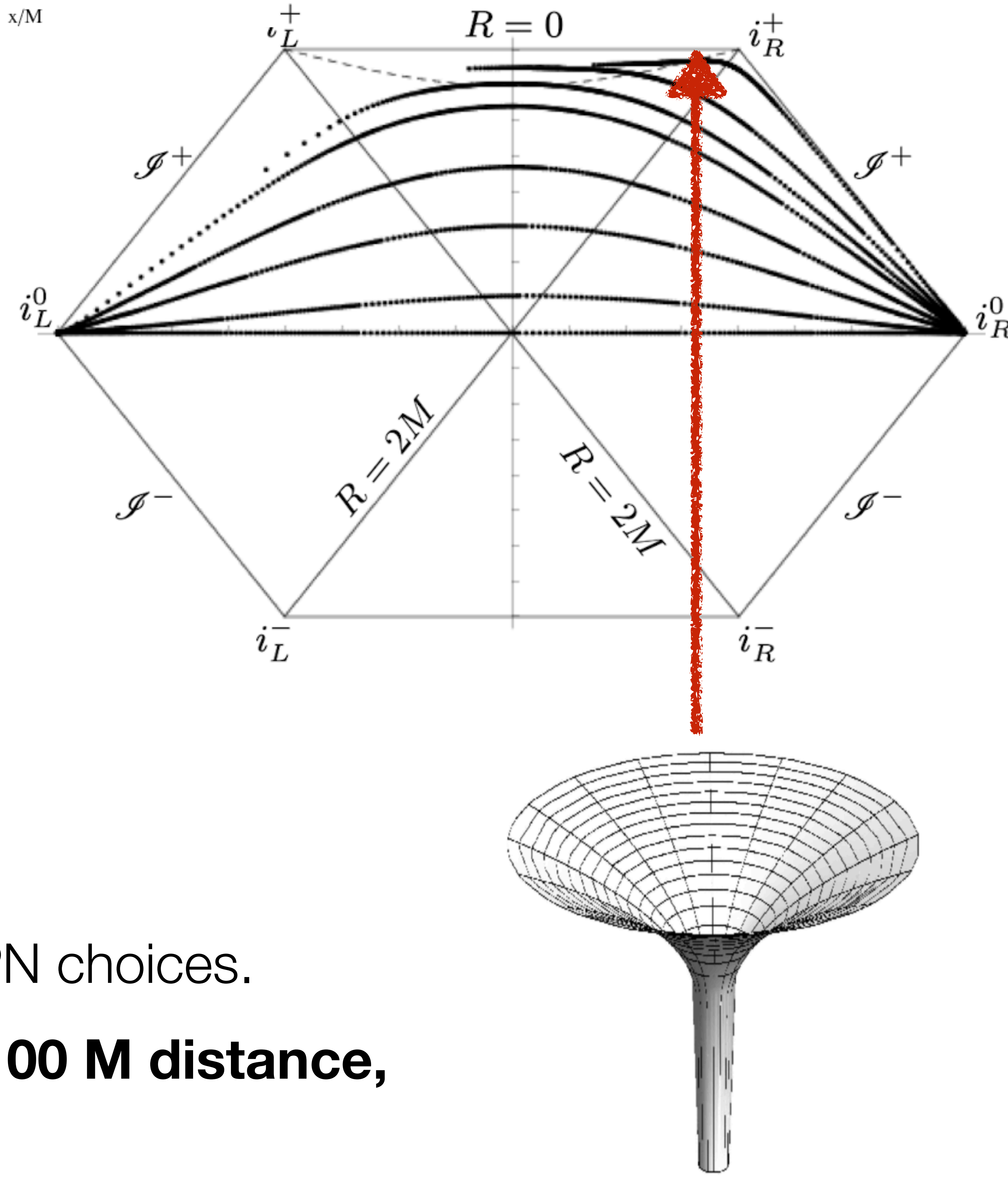
Why does this work?

- Well-posed IVP, theory for FD/MoL.
- Reasonably physical initial data.
- **Gauge conditions are effective at finding (approximate) symmetries.**
  - Time independence of BH geometries (advected along orbits)
  - Helical symmetry - BHs move with the orbital angular velocity that corresponds to half the 22-wave frequency.
  - Axial symmetry of the BHs.
- For **moving punctures**: loss of resolution + dissipation deals with blue-shifted signals traveling toward “throat”.
  -> converge to the convenient symmetric BH solution
- **Constraints don’t grow uncontrollably** even without explicit damping (BSSNOK)
  - Luck, depends on the “background solution” <- only complicated near BHs
  - background changes for hyperboloidal!
- Controlling eccentricity in initial data is simplified by using coordinates close to standard PN choices.
- **Wave extraction: works surprisingly well for the last orbits and merger, even at ~ 100 M distance, except for memory!**



2 x orbital frequency
= wave frequency
(l=2, m=2 mode)

Hannam, SH, Pollney, Brügmann, O´Muchadha, PRL 99:241102,2007



Status: numerical relativity for BBH

- Methods studies (convergence, impact of gauge parameters, …) very expensive!
- **What can be done:**
  - few simulations ~ 100s of orbits
  - routine simulations of ~ 10 orbits
  - systematic exploration up to mass ratio 18, [Planas+, PRD (2024)] short simulations of higher mass ratios (RIT)
  - high spins with non-conformally flat data (SXS, RIT).

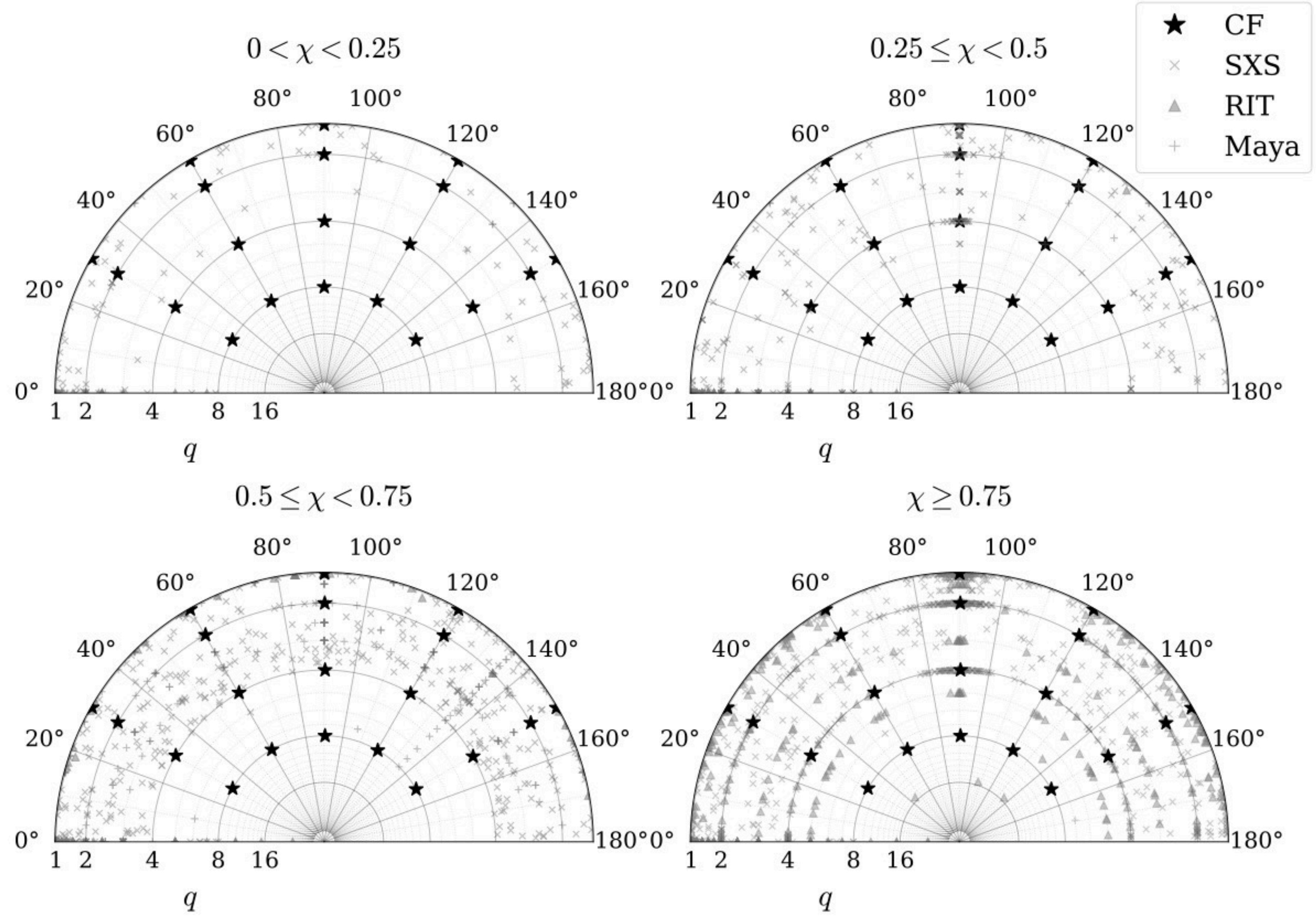


FIG. 16. Comparison between the parameters of the new BBH simulations presented here (CF) and the existing BBH simulations in the the SXS, RIT and Maya catalogues. The *Top left* spin disk shows simulations with spin on the larger black hole $0 < \chi < 0.25$, *Top right* $0.25 \leq \chi < 0.5$, *Bottom left* $0.5 \leq \chi < 0.75$ and *Bottom right* $\chi \geq 0.75$. The radius of each disk shows the mass ratio of the binary and the orientation shows the spin tilt angle of the larger black hole. Spin tilt angles of 90° means that the spin vector lies in the binary's orbital plane.

Cardiff group, arXiv:2303.05419

- Catalogues: SXS (SpEC), Maya, RIT, BAM - need more WFs, minimise time for new NR methods to impact WF models, better metadata!

- **Small selection of new codes and methods:**
  - GPUs start to become mainstream, e.g. CarpetX for Einstein Toolkit (but: cores/GPU?)
  - GR-Athena++ - block based AMR instead of Berger-Oliger
  - Public pseudo spectral (discontinuous Galerkin) code from SXS: Spectre
  - **Hyperboloidal:** "gauge is important" program - [Peterson+, arXiv:2409.02994]

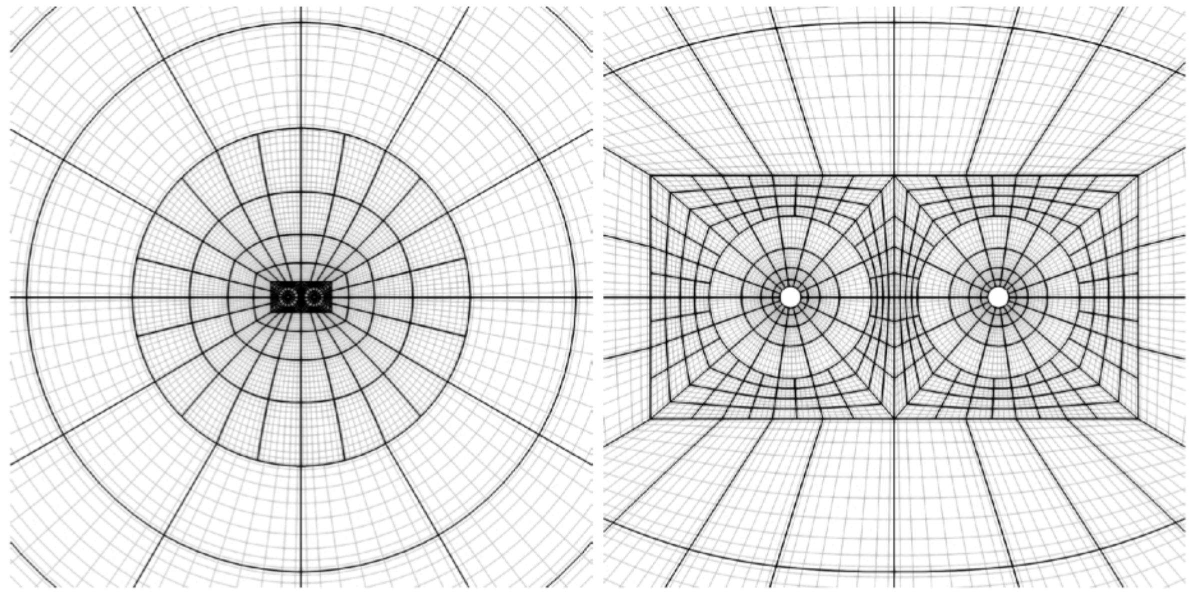
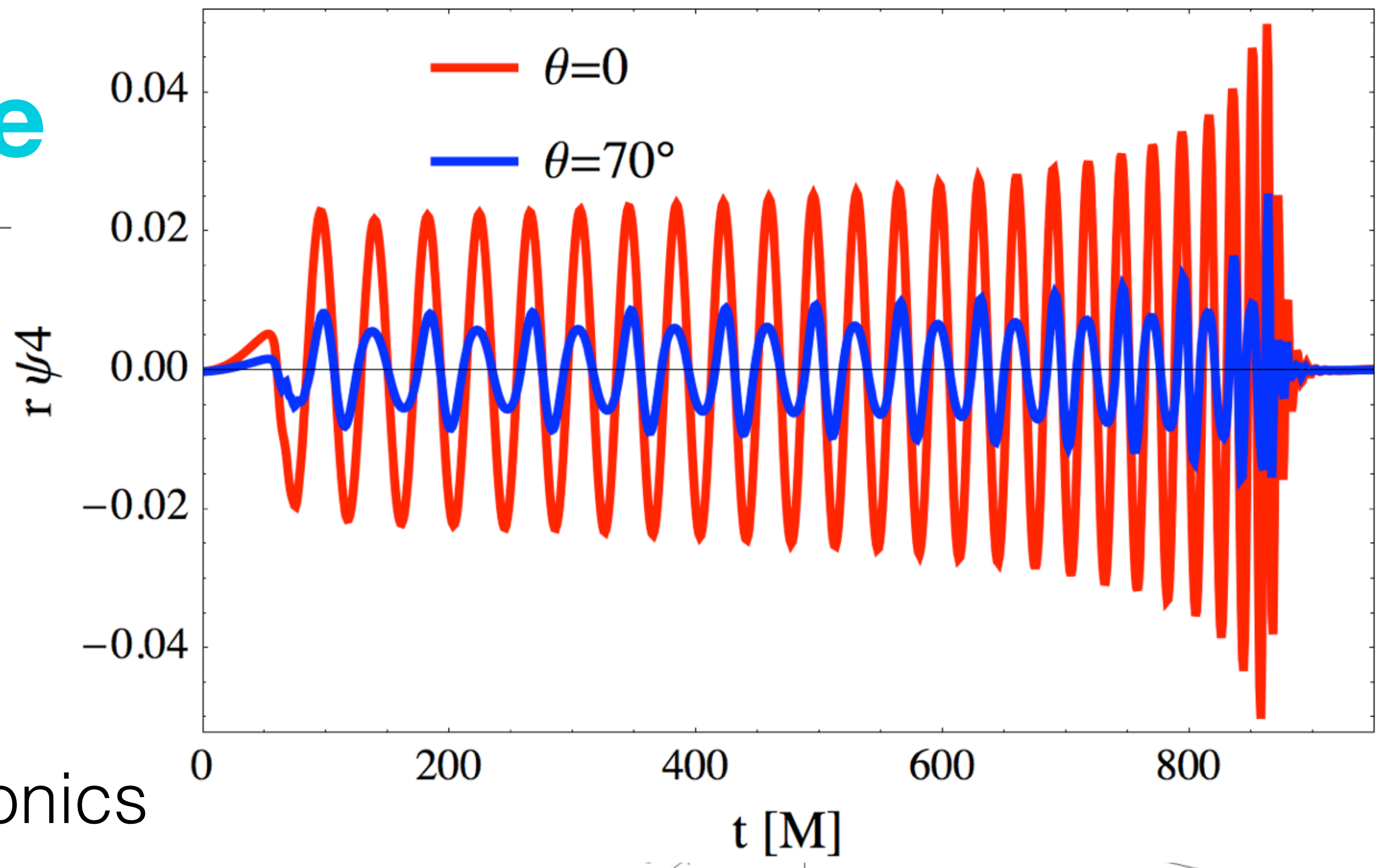


Figure 8. An illustration of the computational grid used during the inspiral. We make use of two excision regions, each region lying inside a black hole's apparent horizon. Each excision is surrounded by a spherical shell partitioned into six deformed cubes as in figure 2. Each spherical shell is then surrounded by another shell of six deformed cubes that transition to a cubical boundary. Then the two cubes themselves are surrounded by a transitionary envelope which becomes spherical. *Left:* The transitionary envelope. *Right:* A close-up of the domain structure around the excisions. The center of each excision is offset from the center of the cube.

SpECTRE (SXS), arXiv:2410.00265

Phenomenology of compact binary coalescence



No hair theorem => BHs are simple

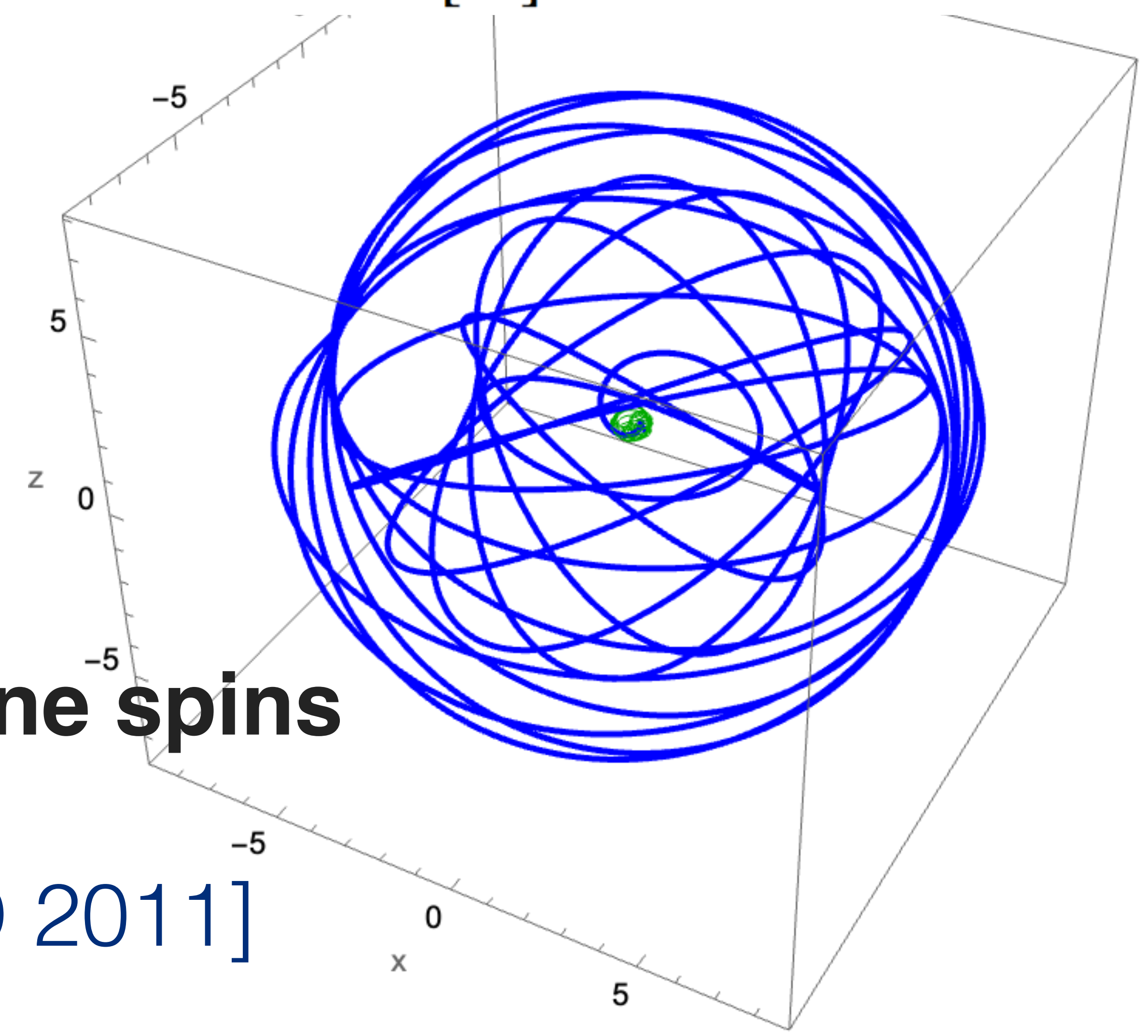
9 intrinsic parameters = m_1/m_2 (1) , spin vectors (6), eccentricity (2)

But: beyond GR, boson stars, environmental effects…

$$h(t,r,\theta,\phi) = \frac{1}{r} \sum_{\ell=2,m=-\ell}^{\ell_{max},\ell} h_{\ell m}(t)\, Y_{\ell m}(\theta,\phi)$$

How many spherical harmonics do we have to include?

Not all CBC waveforms are simple.

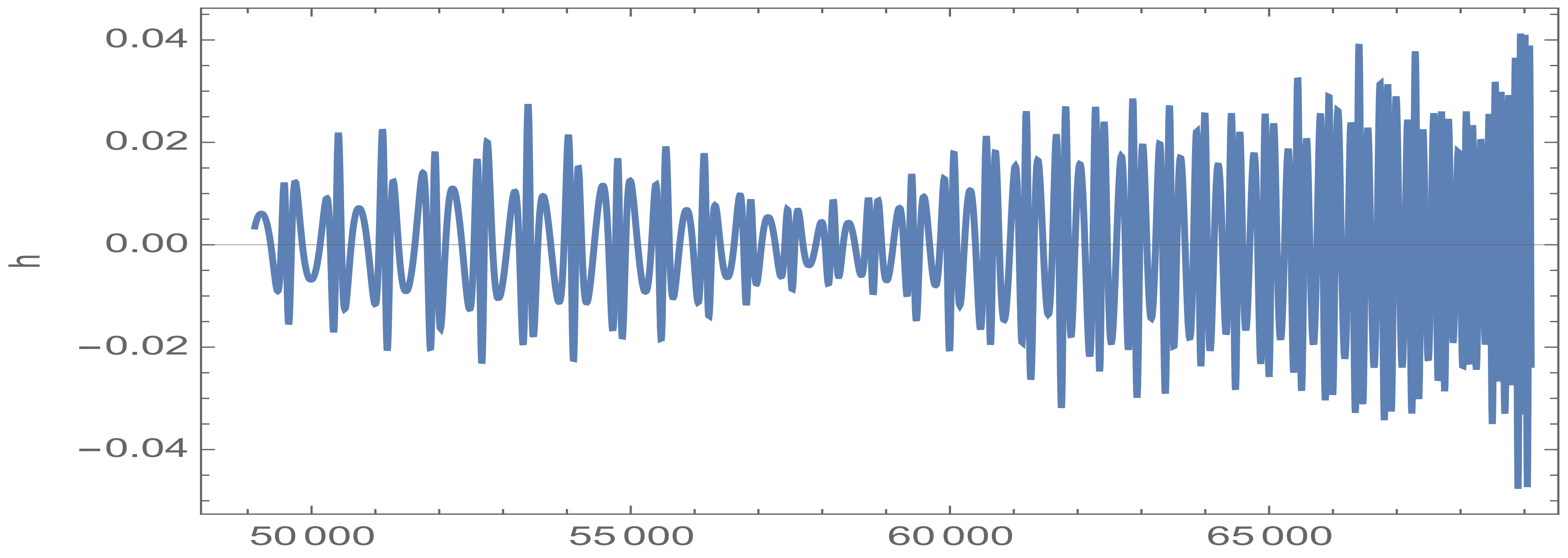


Spins orthogonal to orbital plane: plane and spins are preserved (drop 4 dimensions).

- **Leading order PN spin effect: spin-orbit => amplitude modulations driven by in-plane spins**

  orbital time scale << precession time scale => **"twisting up paradigm"** [Schmidt+ PRD 2011]
  approximate map between precessing and non-precessing systems.
  Wise choice of which quantities to model is more important than the modelling technique.

- Eccentricity: radiated away rapidly, but complex phenomenology and large parameter space - 9D.



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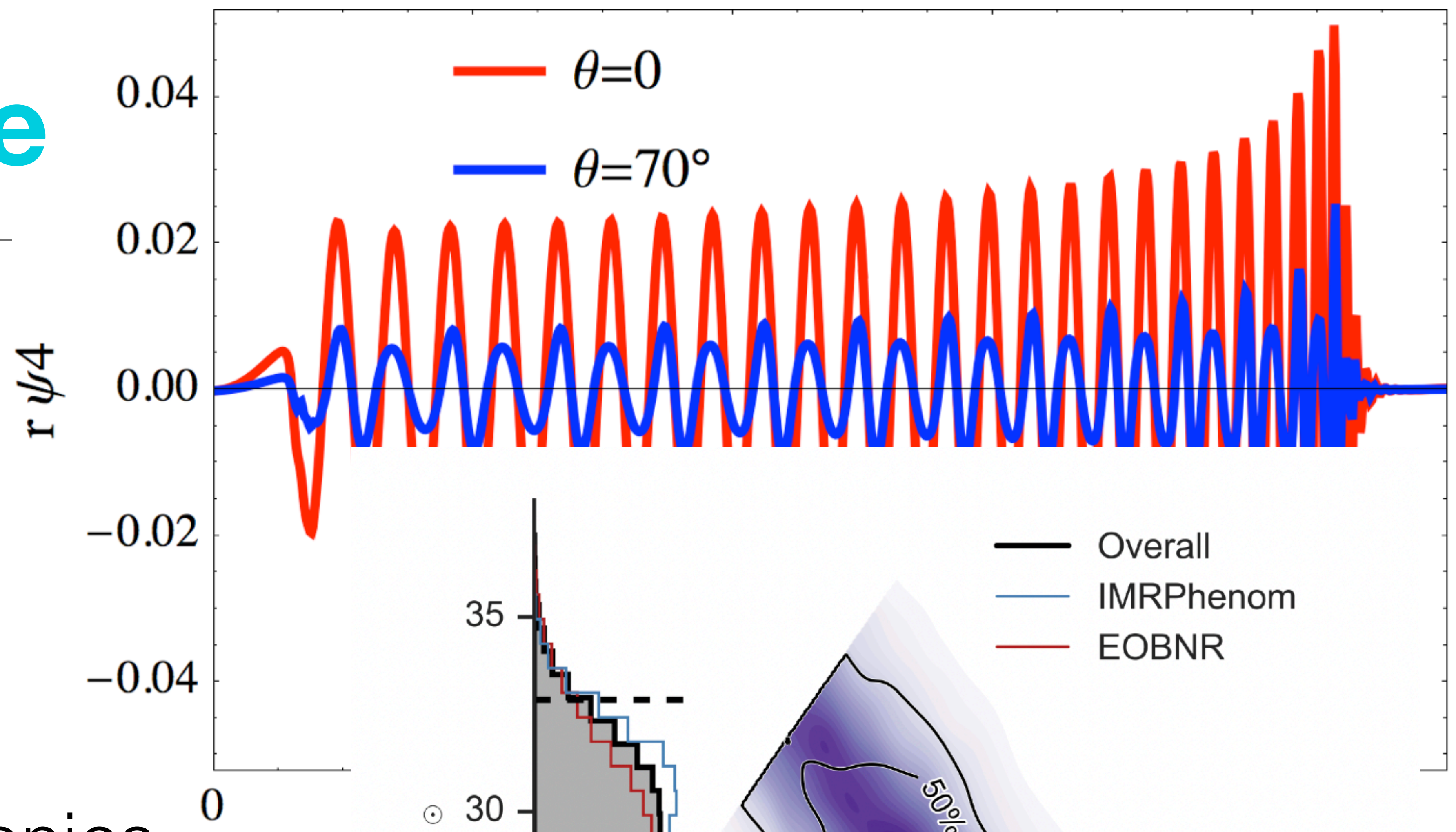

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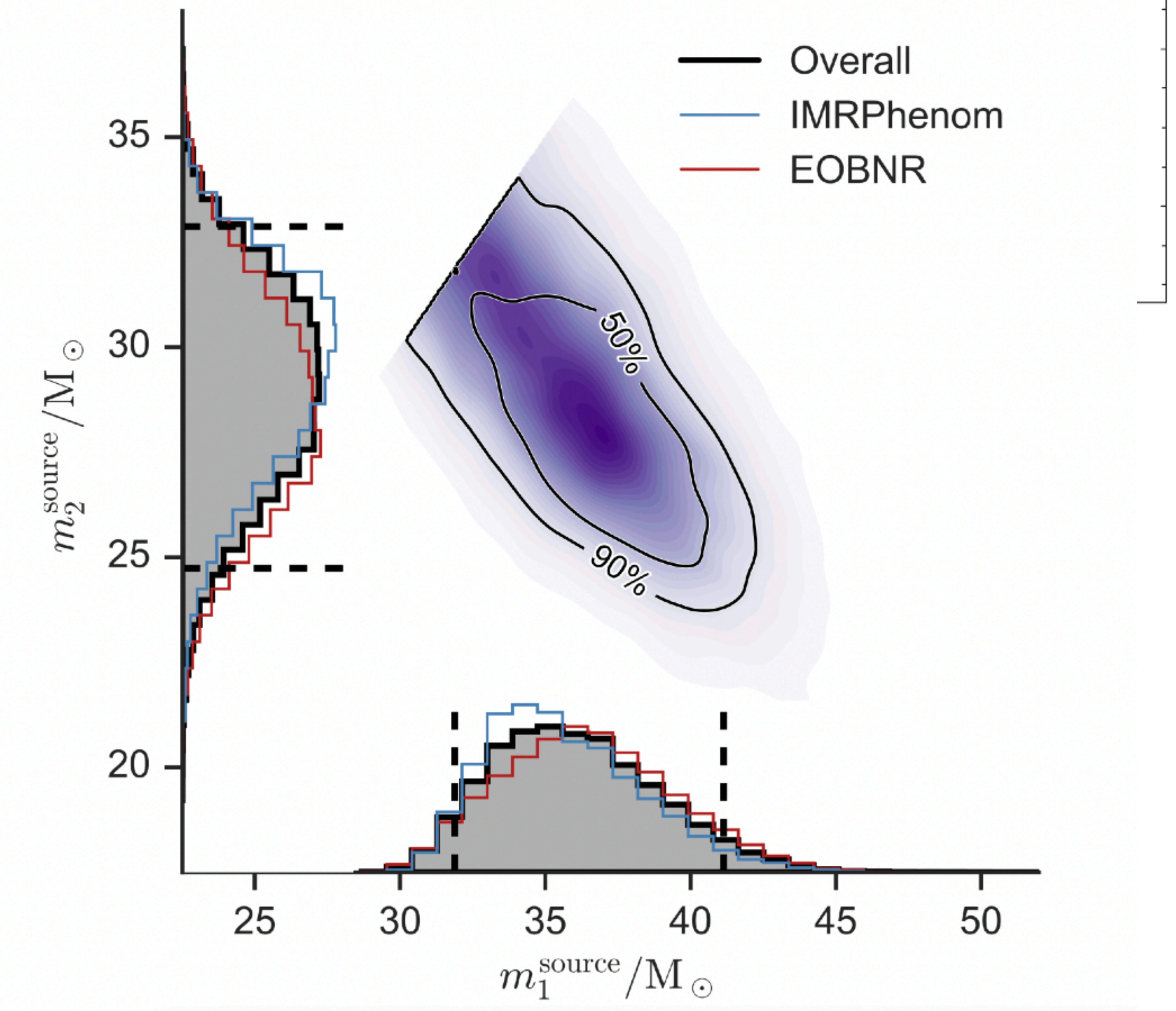

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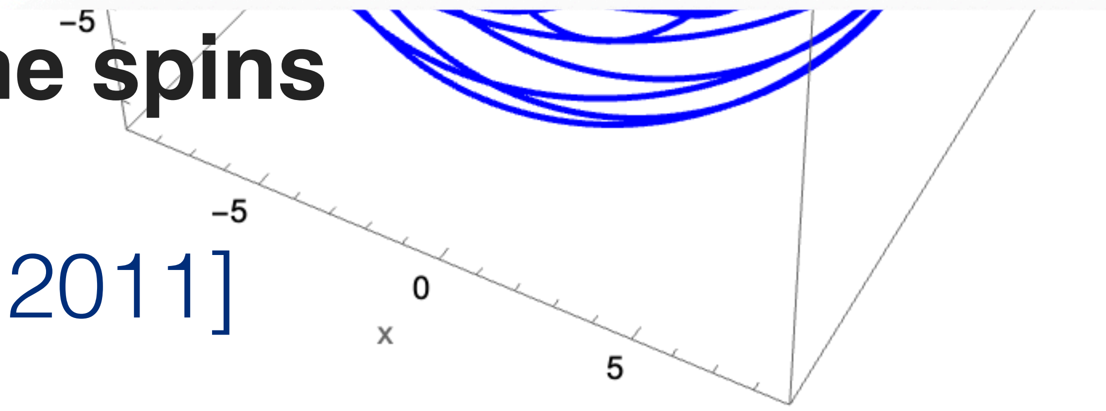

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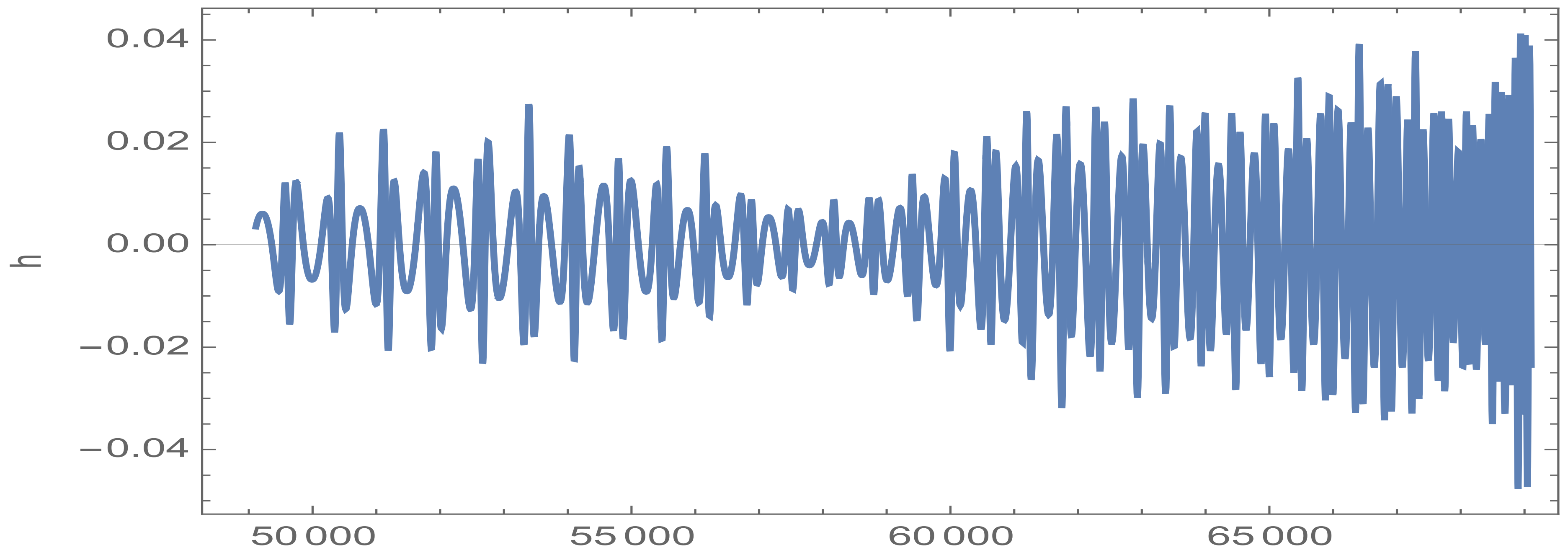


Comparable mass waveforms - Can we always get what we want?

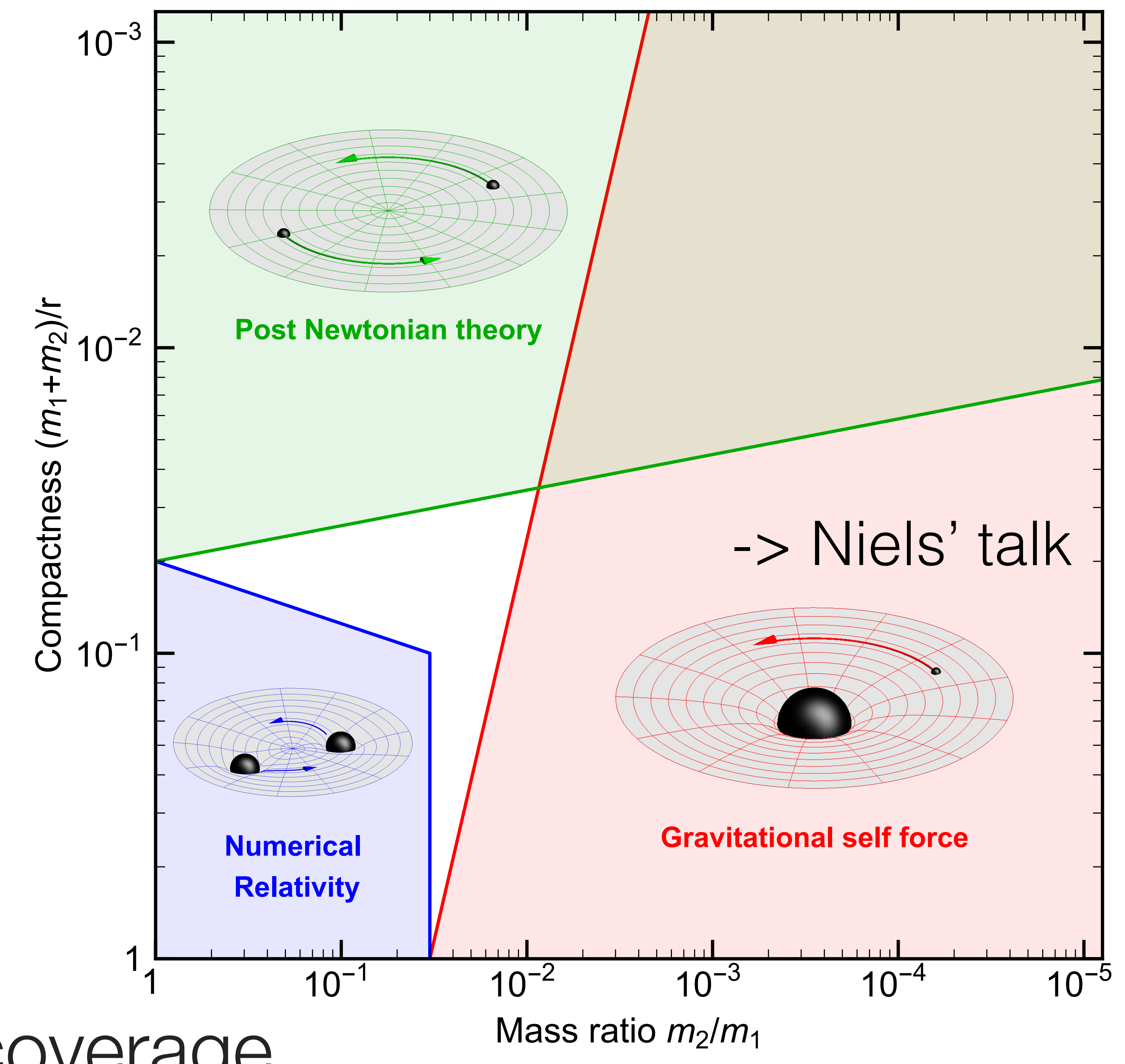
- **Complete waveforms: I**nspiral-**M**erger-**R**ingdown
  - span “entire” frequency band, cover a large mass range.

$$T_{coalescence} \approx \eta^{-1} f_{initial}^{-8/3} M \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

=> extending NR to low frequencies is expensive!
NR WFs are sparse, long ones even more.

  - synthesise from **NR**, PN, EOB, BH perturb., self-force, …

- **Large parameter space coverage**
  - Model the GW signal for astrophysically plausible events:
  - High mass ratio, high spin/strong precession, eccentricity, edge-on, …



- **NO: Make tradeoffs to provide timely models for data analysis.**
  - => vary degree of NR calibration:
    - Full NR calibration: serious restrictions in WF length / parameter space coverage
    - Less NR: more coverage, hopefully not more loss in accuracy than is tolerable.

Can we get what we need?

- Cross-pollination and competing ideas:
  - EOB (SEOBNR, TEOBResumS), IMRPhenom, ROM/surrogates!
  - Development of main “current” model families has become part of the LVK, based on **NRDA meeting series** and **Ninja-project**.
  - “Theoretical development”, (open source) code implementation, testing, review, maintenance & interpretation of parameter estimation results.
- Address trade-offs in different ways - 3 main strategies with different emphasis.
- e**ffective one body (EOB)** - analytical methods to compute waves from dynamics
  - model energy + flux/wave amplitude of a particle in effective metric => integrate ODEs numerically.
  - Slow - need a fast model of the phenomenological EOB model, or fast PE, e.g. with ML
- **“surrogate models”** - algorithms to interpolate large parameter spaces
  - Fast evaluation of EOB or NR data directly.
- **phenomenological models** - model waveform directly
  - piecewise closed form - **extreme compression of information**, fastest, parallelizable - GPUs! **used by LIGO-Virgo for all events to date.**

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Waveform Modelling/Calibration Basics

- **Model simple functions:**
  split waveform modes into amplitude & phase of spherical harmonics, energy & radiated energy, …

- **Carefully choose which pieces of information to model!**

- Hybrid waveforms to capture entire WF: PN/EOB, GSF.

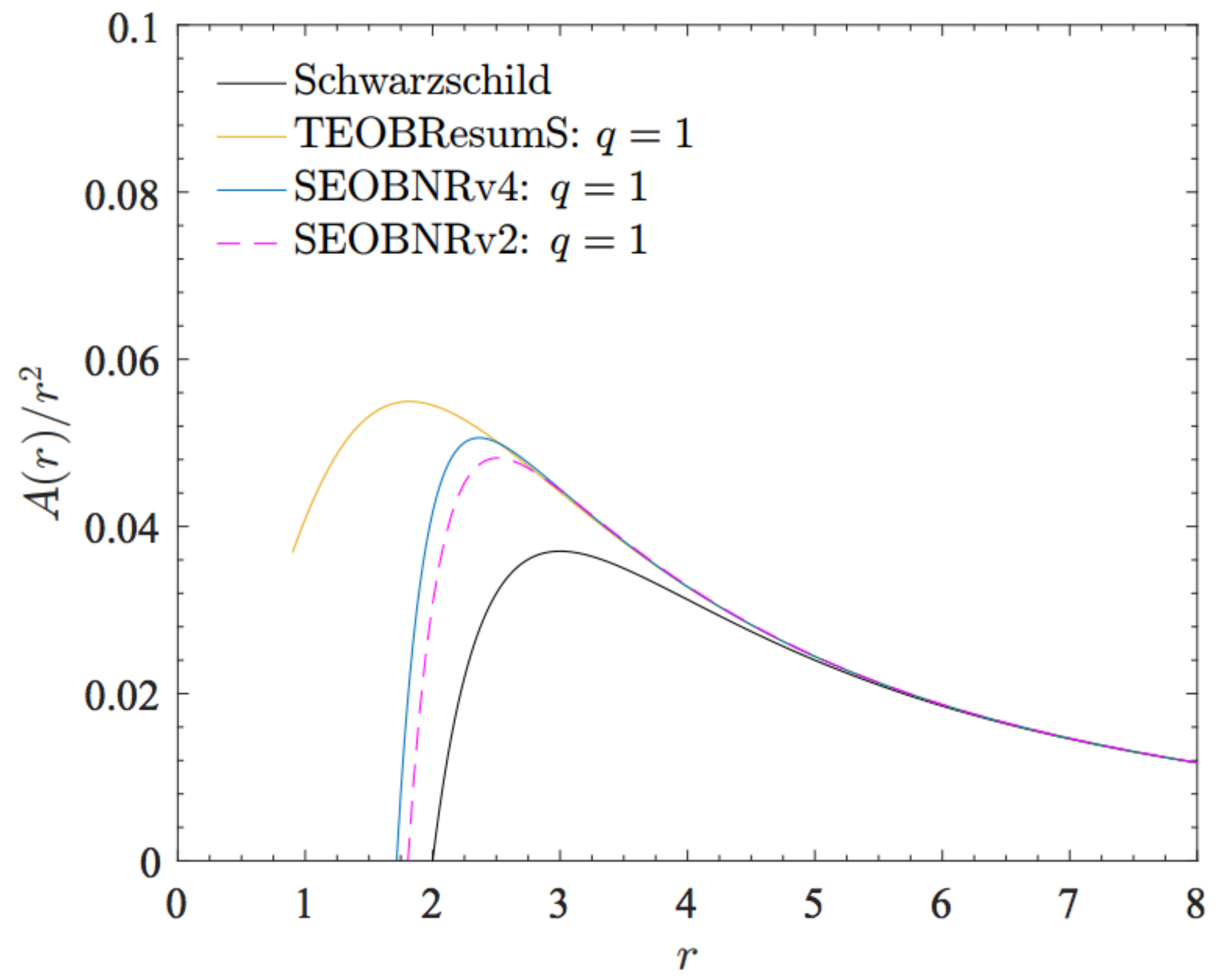
- **Frequency or time domain** (also: Astone+, PRD 2018, TF for SNs):
  - FD natural choice for matched filter <= PSD(f)
  - TD naturally suited for modelling dynamics.

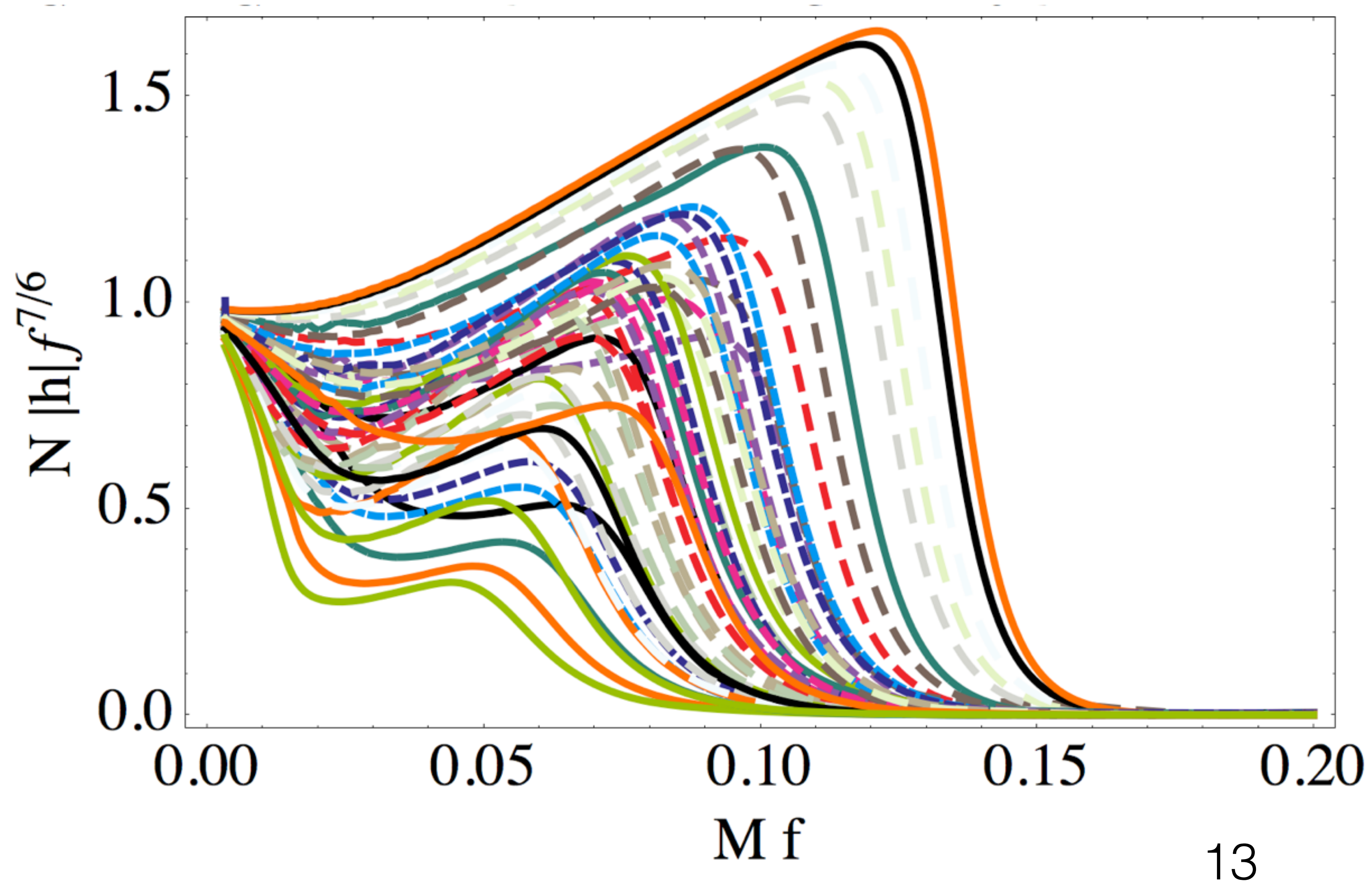
- **Calibrate discretized functions: "reduced order models" - ROM**
  reduce to coefficients in some phenomenological ansatz, grid up, construct basis functions from waveforms.

  1. Fit ansatz to each waveform in a catalogue ("direct fit").

  2. Fit coefficients across parameter space ("parameter space fit").

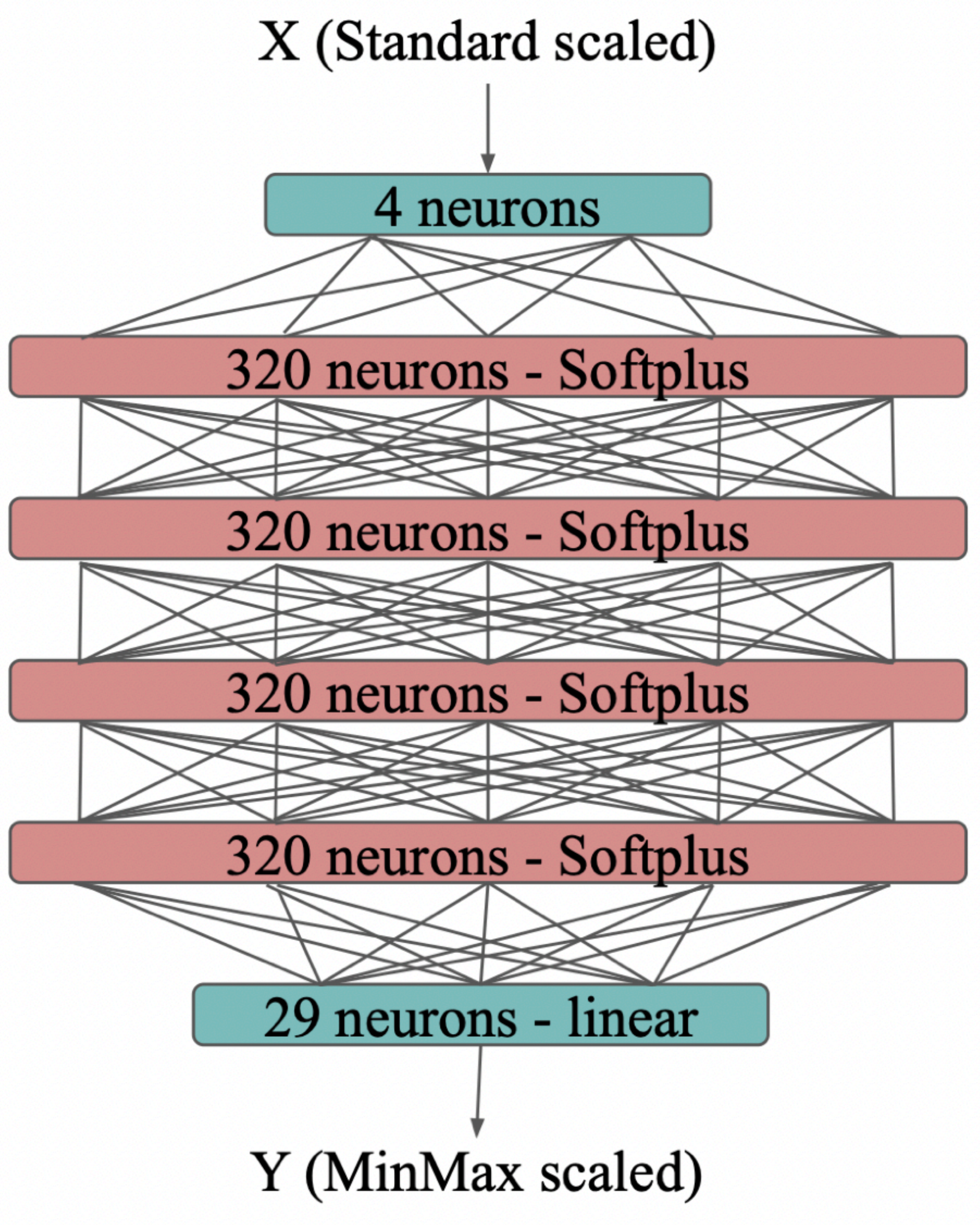
- **Avoid underfitting + overfitting to noise & systematic errors.**





Neural networks for waveform modelling

- Typical strategy: new tool to for parameter space fits,
  e.g. after a "surrogate-model"-style basis decomposition.
    - **Toy model:** precessing final mass and spin (Haegel+SH, CQG 37 (2020) 13, 135005)
    - **Speed up SEOBNR models on GPUs:**
        - 22 mode, no precession (SEOBNRv4): Khan and Green, PRD 103, 064015 (2021)
        - Several modes, no precession (SEOBNRv4HM): Grimbergen+, PRD 109 (2024) 10, 104065.
        - Several modes, precession (SEOBNRv4PHM): Thomas, Pratten, Schmidt, PRD 106, 104029 (2022)
- **Open questions:** how will this approach "scale" when applied to the "entire" parameter space:
    - very long waveforms?
    - Large spins, mass ratio, eccentricity+precession?
    - Guess: it will be important to choose the quantities that NNs are built for.



Example: Co-precessing 22-mode

- **Open question:** how much will future Bayesian inference & "global fit" pipelines benefit from computing many waveforms at once?

Effective One Body approach (EOB)

- Describe **dynamics in terms of Hamiltonian dynamics of a particle in a deformed Kerr metric**.
  - Resum PN expression for better convergence, consistency with EMRI limit, can calibrate to NR.

$$H_{\rm EOB} = M\sqrt{1+2\nu(\hat{H}_{\rm eff}-1)} \qquad {\rm d}s^2_{\rm eff} = -A{\rm d}t^2 + \frac{{\rm d}r^2}{AD} + r^2{\rm d}\phi^2 \qquad \hat{H}^2_{\rm eff} = A\left[1+\frac{P_\phi^2}{\mu^2 r^2}+\frac{ADP_r^2}{\mu^2}+\frac{Q_4(r)\,M^2\,P_r^4}{r^2\,\mu^4}+O(P_r^6)\right]$$

- **Building blocks**
  1. Hamiltonian describes conservative dynamics.
  2. Radiation flux: loss of energy + angular momentum.
  3. Prescription for computing waveform from dynamics.
  4. Phenomenological attachment of merger/ringdown.
- Significant increase in accuracy over PN, especially for small separation, larger mass ratios.
- Including eccentricity requires modifications to blocks (2) and (3) with respect to QC case.
- **2 branches**
  - Latest: SEOBNRv5 & TEOBResumS
  - Differ especially for conservative dynamics, spins [comparison: Rettegno+ PRD 101, 104027 (2020)].

ROM - Reduced Order Models

- ROM: **model based on compressing the number of degrees of freedom**
  - goal: build “surrogate models” that can replace solving a costly PDE, e.g. NR.

- Narrow sense: methods that are based on general algorithms, e.g.
  - create reduced basis with **SVD or greedy** (locally optimizing) algorithms, following Field, Galley, Cañizares, Pürrer, …
  - **Expensive offline “training”, fast “online” evaluation.**
  - Interpolate the projection coefficients/values at specified time steps/frequencies over the parameter space (q, spins) e.g. using tensor product spline interpolation.
  - **ROQ**: fast evaluation of likelihood based on pre-computation with reduced basis.

- **Current models**
  - NRSur7dq4: precessing [Blackman+ PRD 95, 104023 (2017)]
  - NRHybSur3dq8: long non-precessing [Varma+ PRD 99, 064045 (2019)]
  - NRSur2dq1Ecc: eccentric non-spinning [Islam+ PRD 103, 064022 (2021)]
  - Sur[SEOBNRv4PHM]: precessing EOB [Gadre+, arXiv:2203.00381]:
    - $\Delta T = 5000M$, $Q \leq 20$, spins ≤ 0.8, 14 overlapping parameter space regions.

Field+, PRX 4, 031006 (2014)

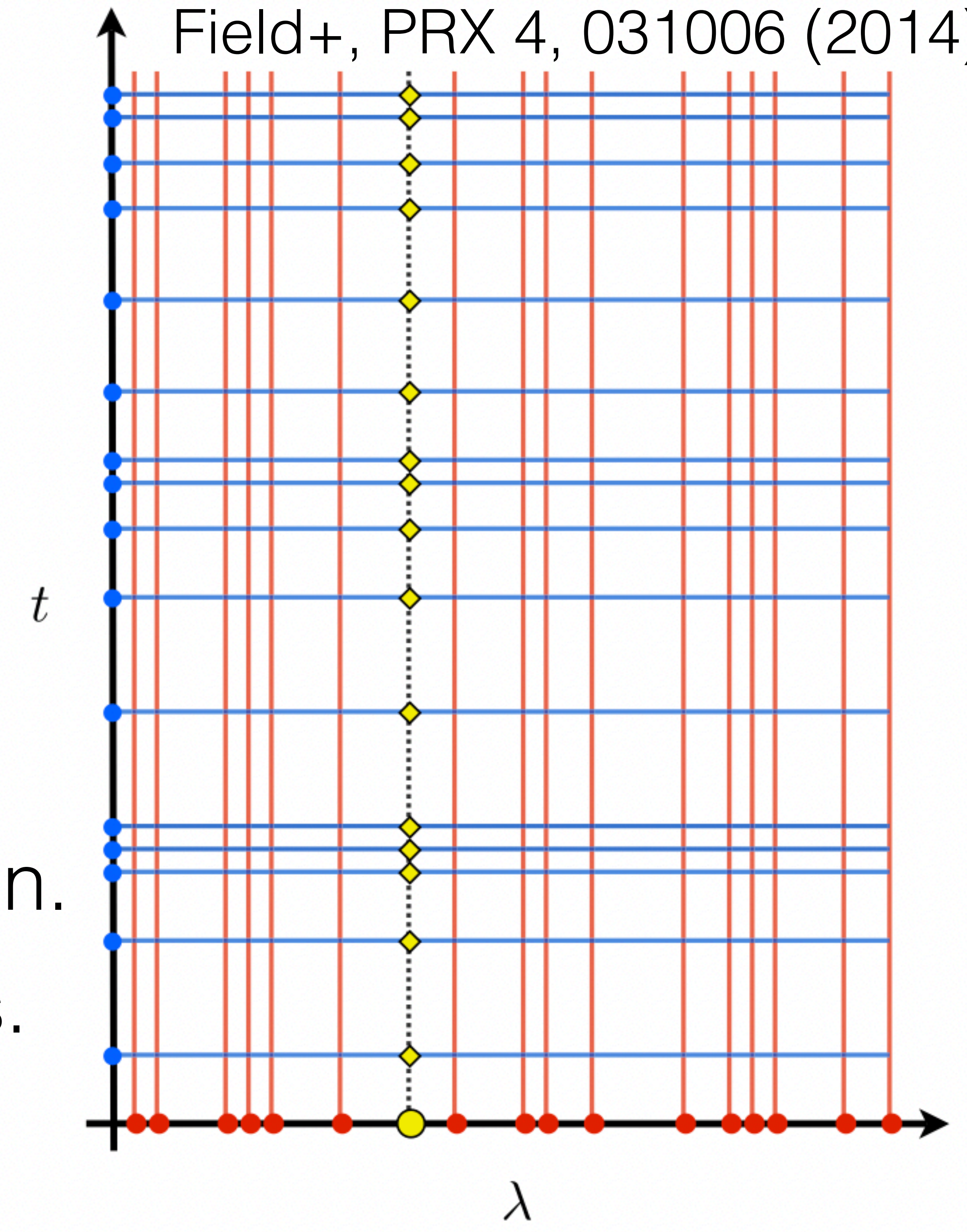


FIG. 1. A schematic of the method for building and evaluating the surrogate model. The red dots show the greedy selection of parameter points for building the reduced basis (step 1, offline), the blue dots (step 2, offline) show the associated empirical nodes in time from which a waveform can be reconstructed by interpolation with high accuracy, and the blue lines (step 3, offline) indicate a fit for the waveform’s parametric dependence at each empirical time. The yellow dot shows a generic parameter, which is predicted at the yellow diamonds and filled in between for arbitrary times using the empirical interpolant, represented as a dotted black line (step 4, online).

IMRPhenom

- **Split into N regions, connected C^k (C^1 or C^2):**
  more regions => ansatz in each region becomes simpler!
  closed form expressions in each region => compress information

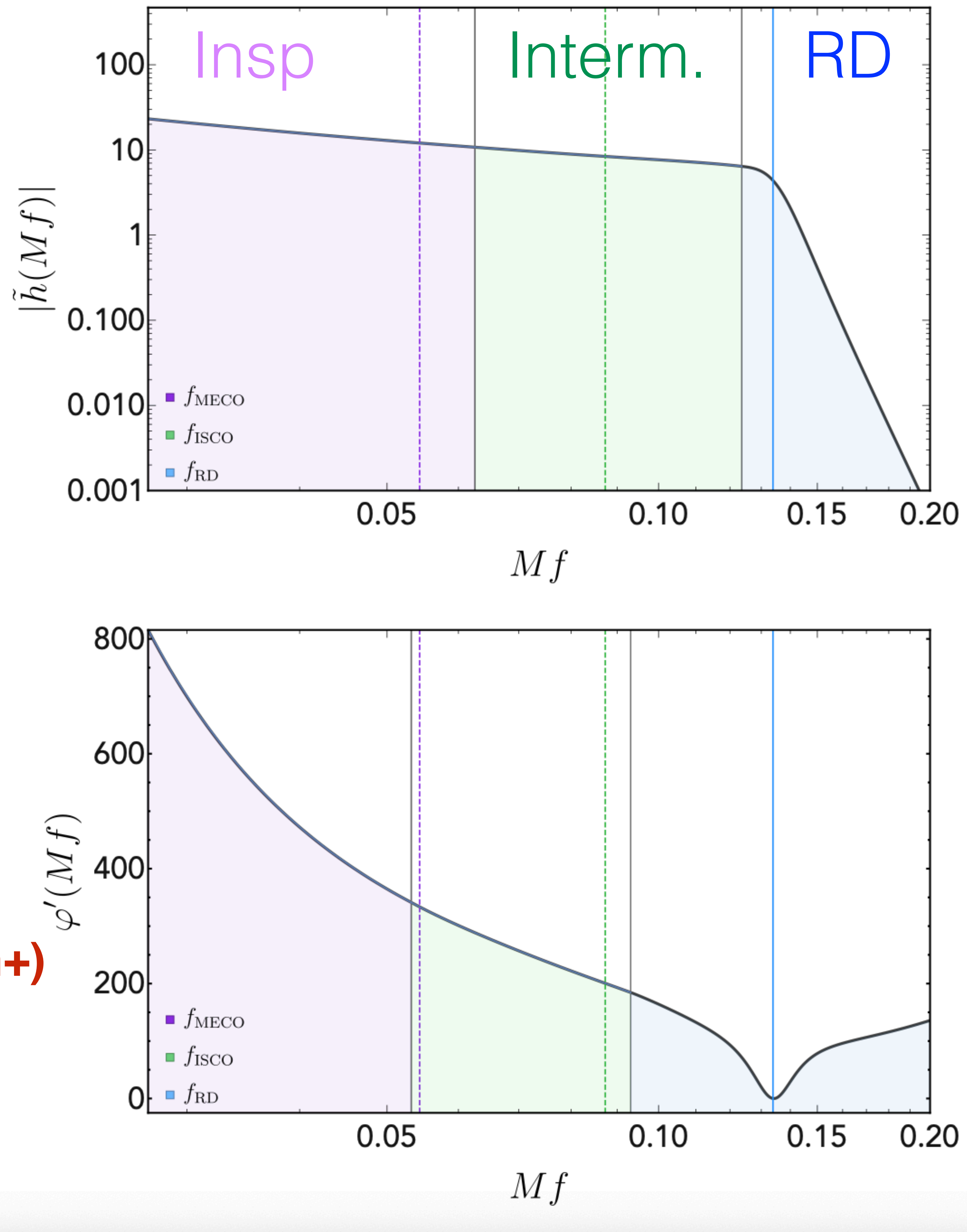
- Use **at least 3 regions**, benefitting from physical intuition:
  Inspiral / intermediate / ringdown

- Inspiral: PN extended to fit EOB
  Ringdown: BH perturbation theory
  Intermediate: more complicated, but short!

- **Currently 4th generation!**
  PhenomXPHM [García-Quirós+ 20; Pratten+ 20]
  New NR-calibrated model under LVK review (Colleoni+Hamilton+)

  PhenomTPHM (TD) [Estellés+ 2021]

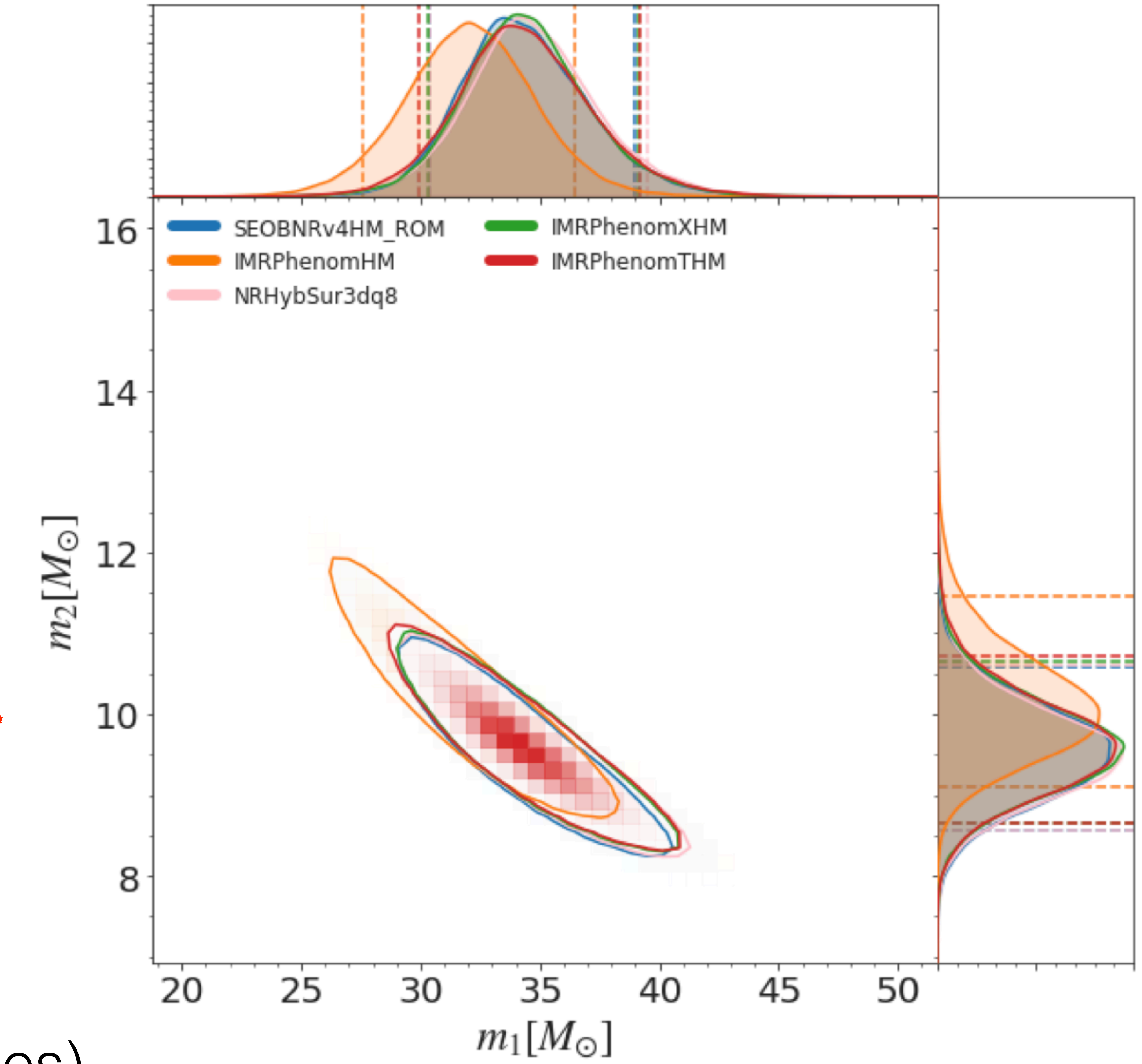
- Code development now focuses on phenomxpy code base (C. García-Quirós) with GPU support.



How well does this work?

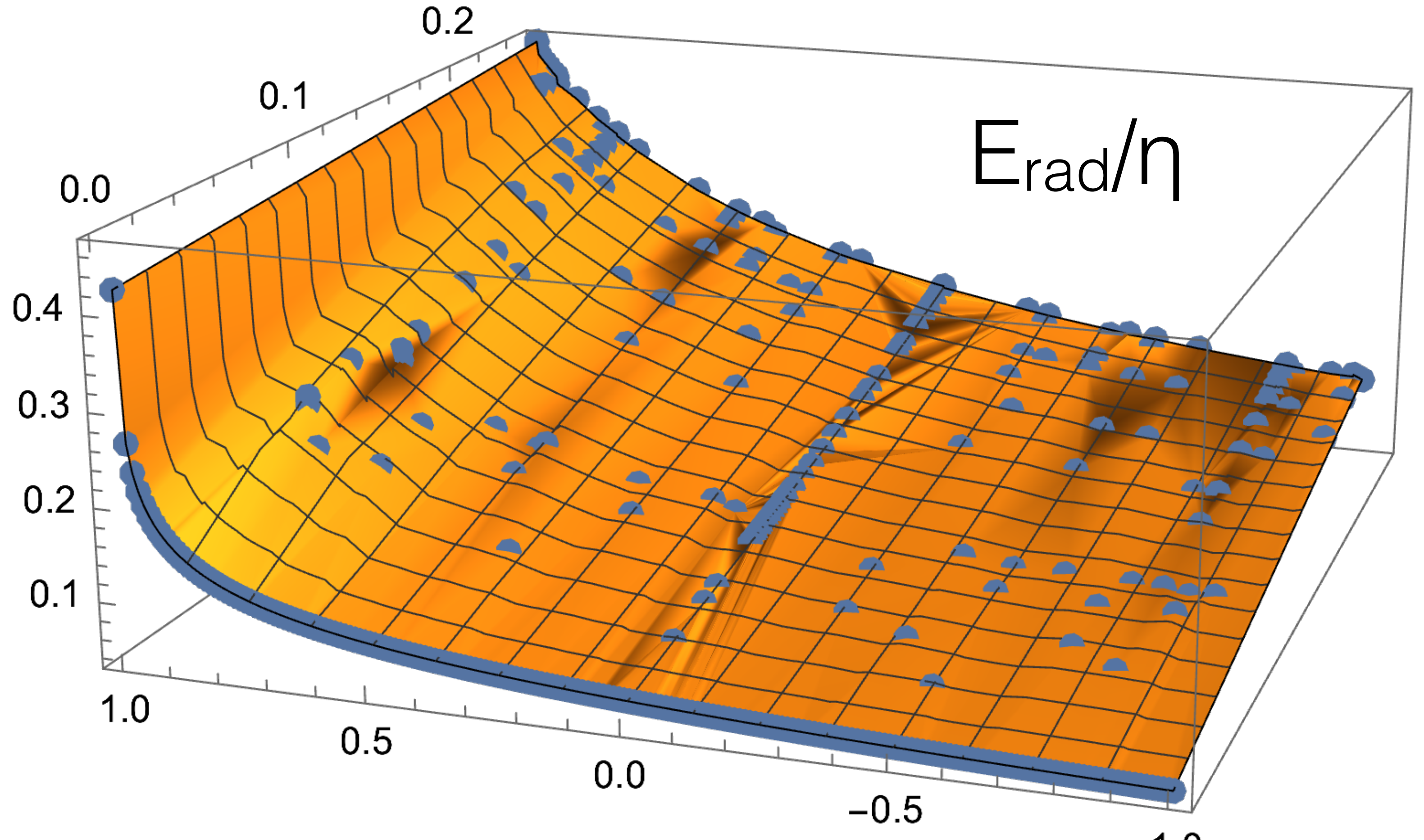
- **Works well were parameter space is well covered by NR simulations:**
  - No eccentricity, precession: 3D (2 spin projections + mass ratio)

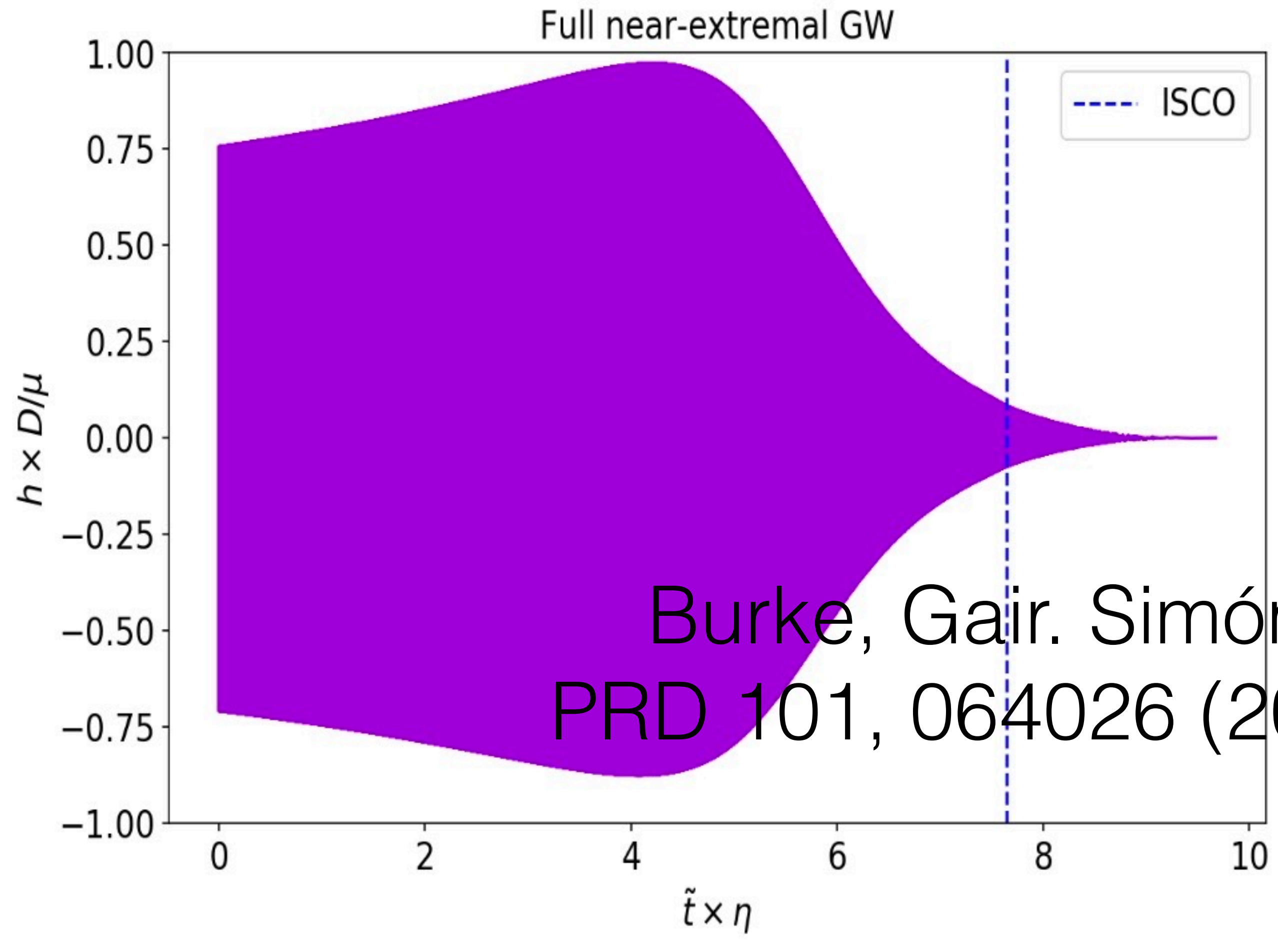
Find **good agreement** between current generation of waveform models, significant deviations for IMRPhenomHM (previous generation, HM not calibrated to NR).



Example: PE results for GW190412
[Colleoni+, PRD 103, 024029 (2021)]

BUT: high q - EMRI transition, extreme spins, edge-on, memory (m=0 modes) …



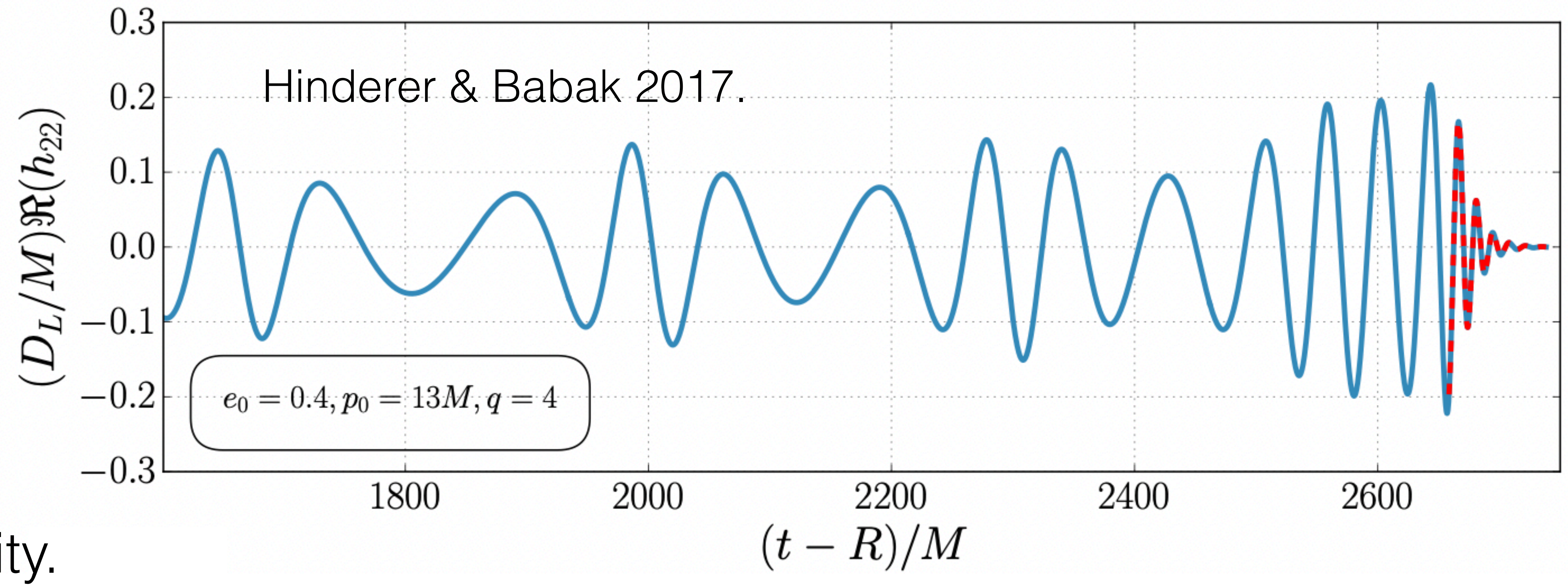


Burke, Gair. Simón,
PRD 101, 064026 (2020)

Also getting there for precession - more models getting calibrated to precessing NR:

- NRSur7dq4
- IMRPhenom under LVK review

Eccentricity & Generic Waveforms



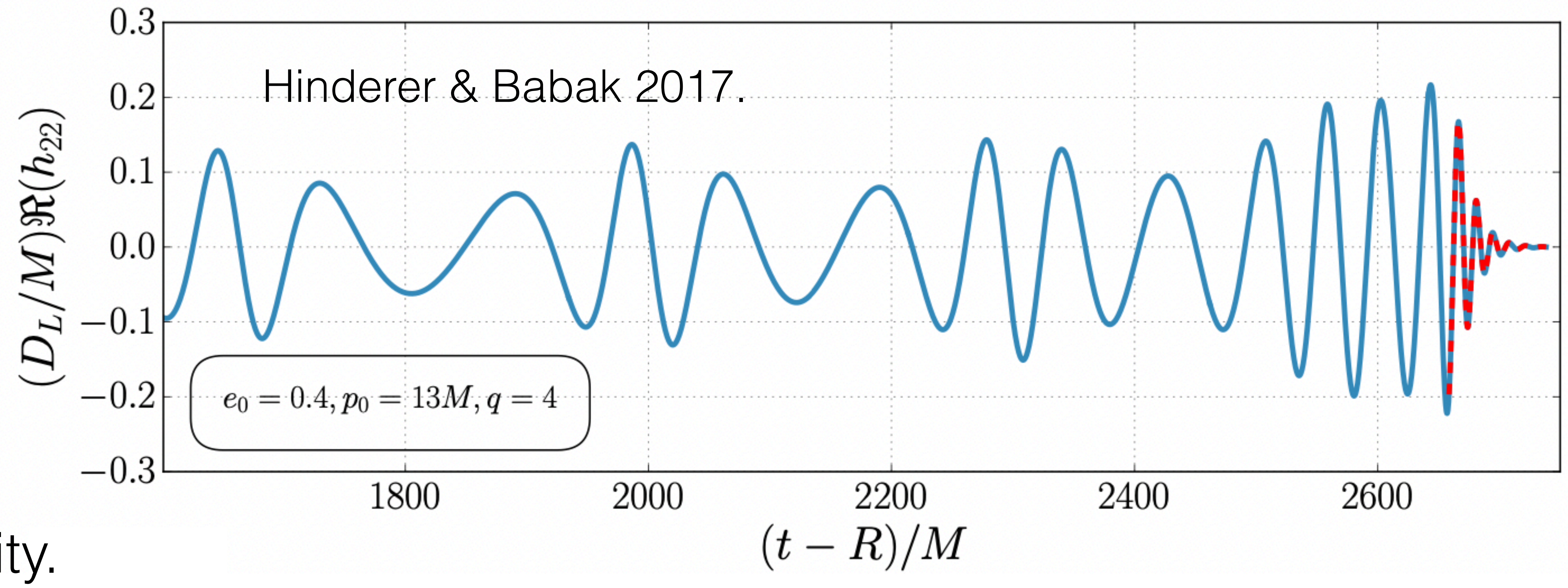
- **Eccentricity is rapidly radiated away**
  - Standard formation scenario: **dynamical capture**
  - **Typically: spin => precession -> need generic WFs**
  - merger/RD only weakly affected for small/moderate eccentricity.
    - **"quick strategy" => glue PN** eccentric inspiral to existing **IMR** models (Huerta+18, Hinder+18: TD non-spinning, calibrated to NR)
- **EOB**
  - Foundations for extending EOB to eccentricity: T. Hinderer and S. Babak, PRD96, 104048 (2017)
  - **Same Hamiltonian, modified waves -> modified flux**
  - **Eccentric models:** SEOBNRv4EHM (Ramos-Buades+, PRD 105, 2022)
    TEOBResumS-Dalí (see e.g. Albertini+, PRD 109, 2024)

e=0.24, m_1/m_2=2, no spins

- **IMRPhenom: fast time domain model being finalised, lead by Lluc Planas, applying for postdocs**

- **Generic waveforms:** precession + eccentricity.
  Very few NR WFs, fast PN WFs to describe inspiral not yet well developed for spin and precession
  => very challenging to develop models that are computationally efficient and calibrated to NR.

Eccentricity & Generic Waveforms



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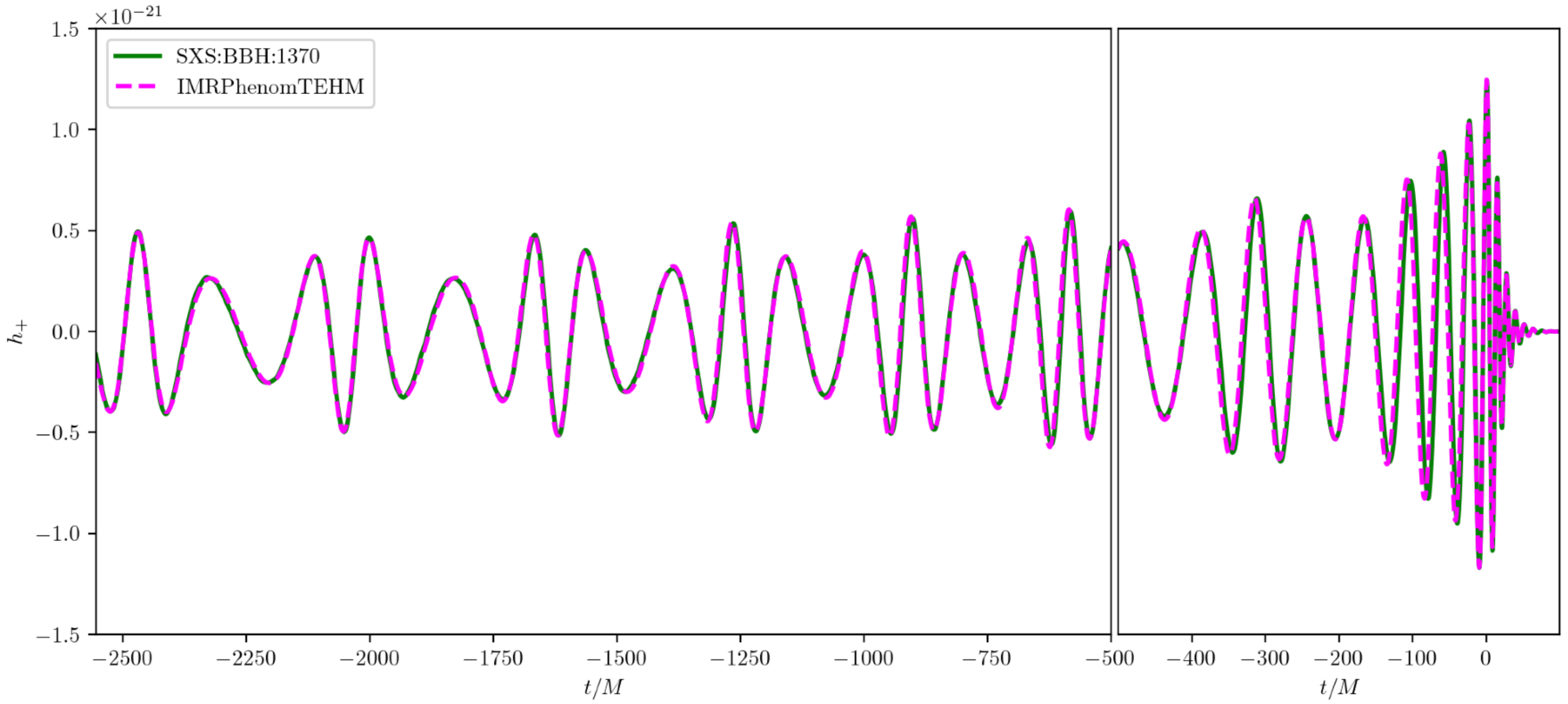

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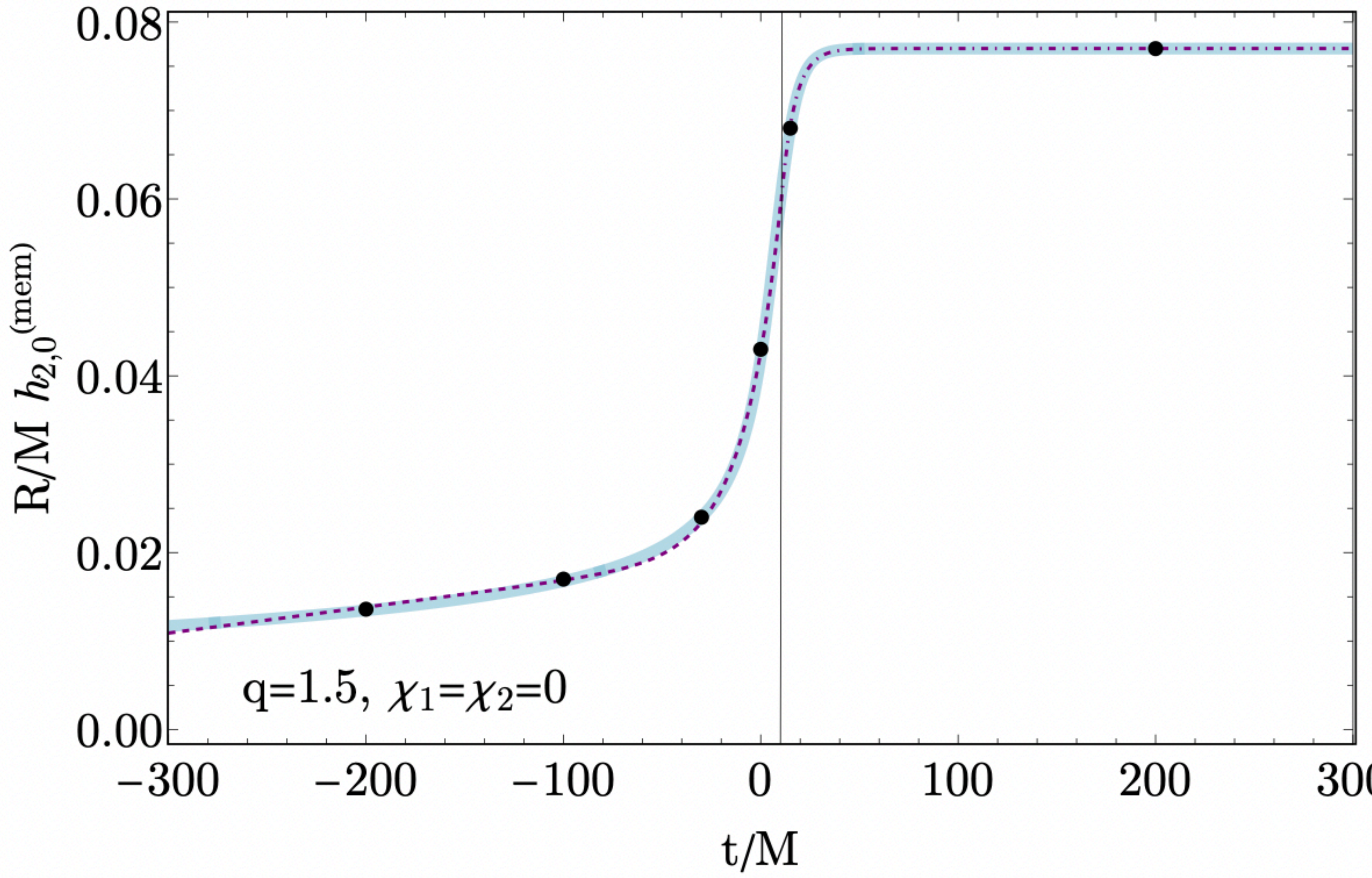
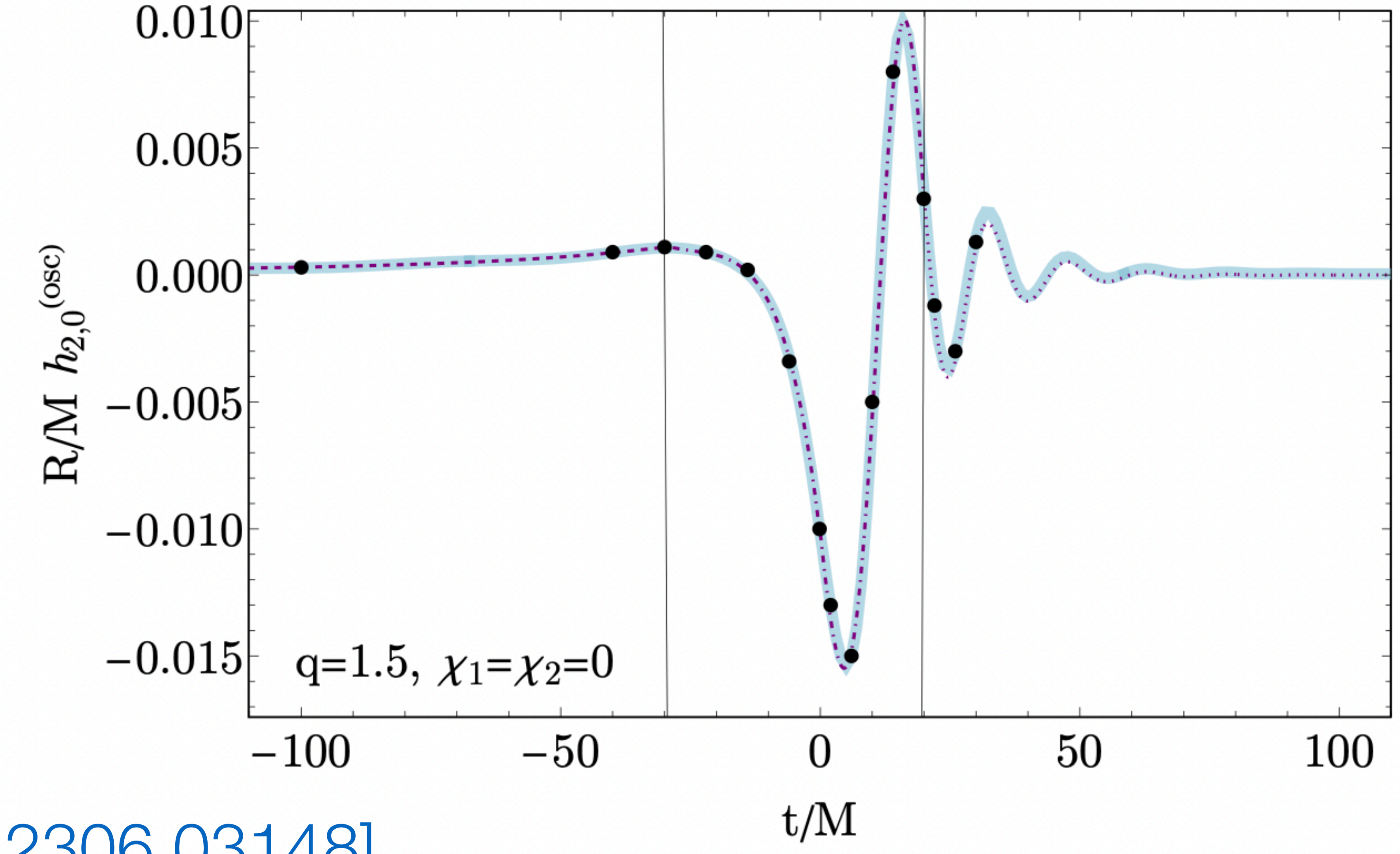
Memory & the missing quadrupole mode



M. Rosselló-Sastre, SH, S. Bera, PRD 110, 084074 (2024)



- Early 1970s: GWs generated by unbound binary creates persistent physical change to metric -> linear/ordinary memory
  Christodoulou 91: nonlinear memory from unbound radiation pulse.

  main effect: l=2, m=0 harmonic (m=0: non-oscillatory, except RD)
- Simple approximation:

$$h_+^{\text{mem}}(u) = \frac{r}{192\pi} \sin^2 \iota (17 + \cos^2 \iota) \int_{-\infty}^{u} |\dot{h}_{22}|^2 du'$$

  - Can also be computed from BMS group - interest from BH information paradox, structure of null infinity.

- To compute memory in NR => need null infinity (CCE)
  [Pollney + Reisswig CQG, 2010; Mitman+ PRD 102, 104007 (2020)]

- Surrogate model based on CCE WFs catalog: NRHybSur3dq8_CCE [Yoo, arXiv:2306.03148]
- IMRPhenom model lead by Maria Rosselló [Rosselló-Sastre, SH, Bera, PRD 110, 084074 (2024)]
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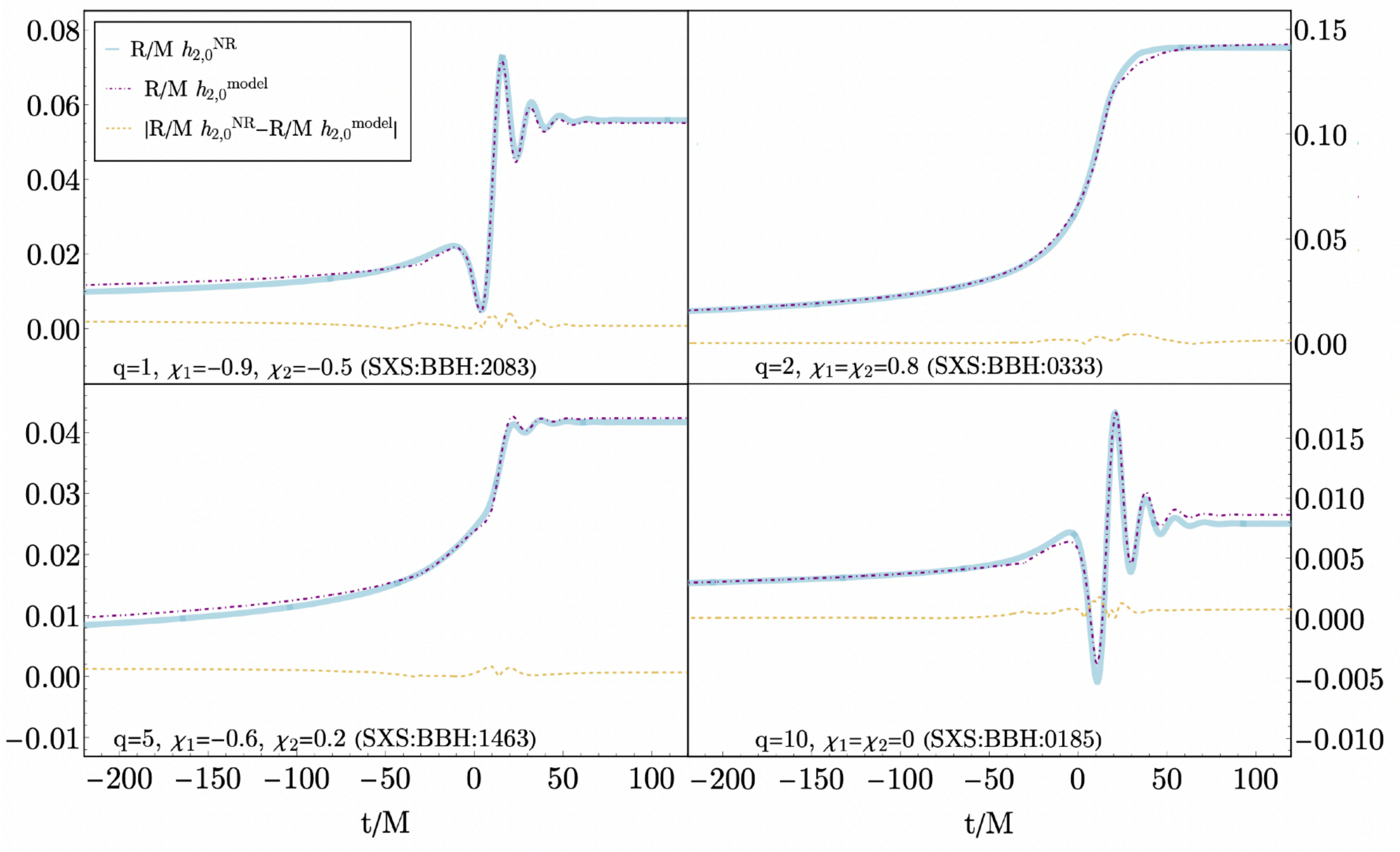

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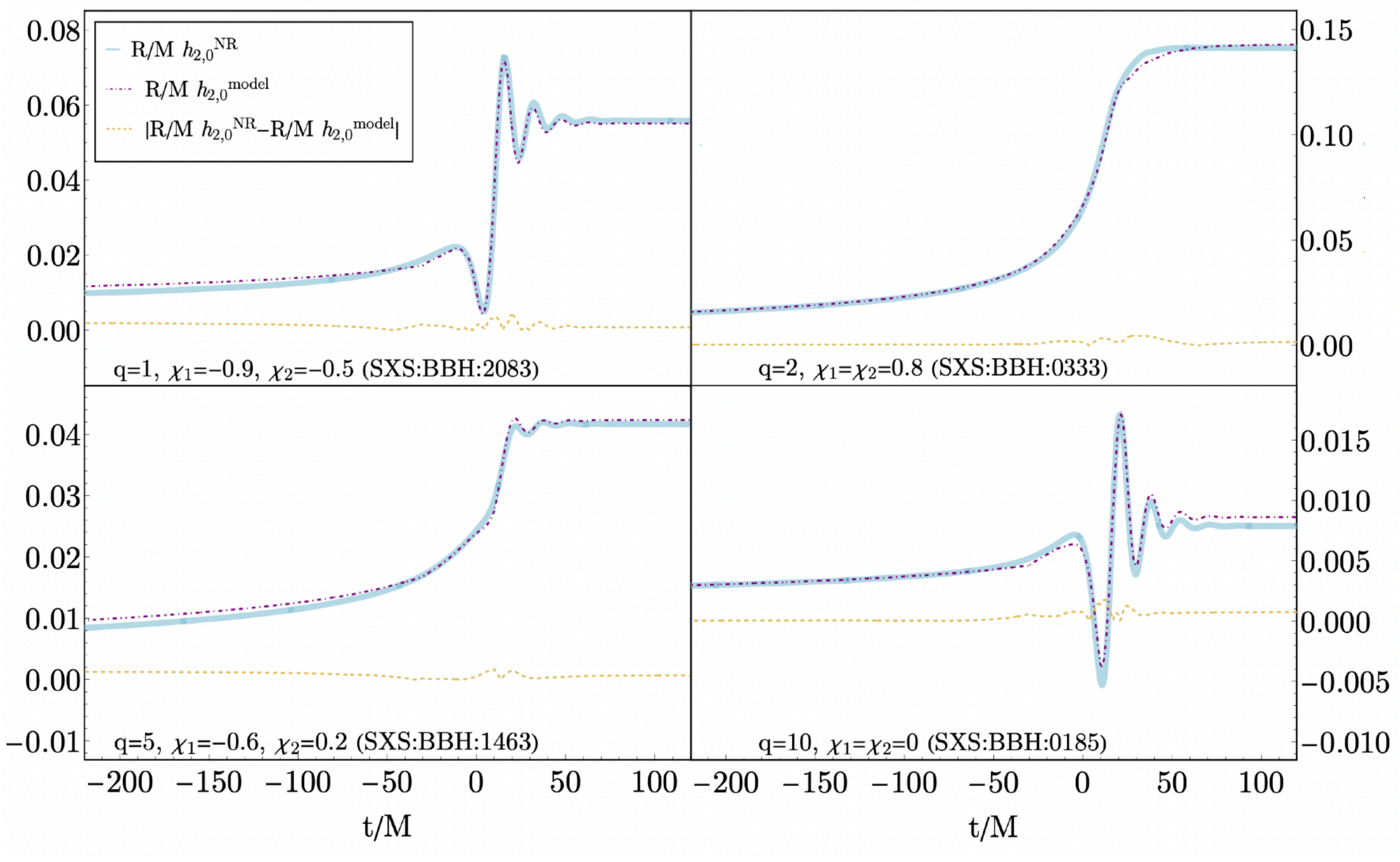

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Conclusions

Several independent approaches (EOB*, NRSur, IMRPhenom) have developed complex models that describe the GW signals of compact binary systems.

- Tradeoff between parameter space coverage and NR calibration.
  Where different models are calibrated to NR they tend to agree very well.
- Challenges: high mass ratios/extreme spins, complex subdominant effects, large parameter spaces (precession, eccentric).
  - Often models can simply be improved by calibration to more NR WFs.
- Getting involved in actual observations and the interaction between different communities is very rewarding.

- **Challenge: Meet requirements for computational efficiency+accuracy within ~ next decade (LISA+3G)!º**
  - Much work is required to develop data analysis strategies (e.g. for LISA global fit) in collaboration between data analysis and waveform modellers.
- **Can we repeat the success of the decade from 2005 - 2015?**

Acknowledgements

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