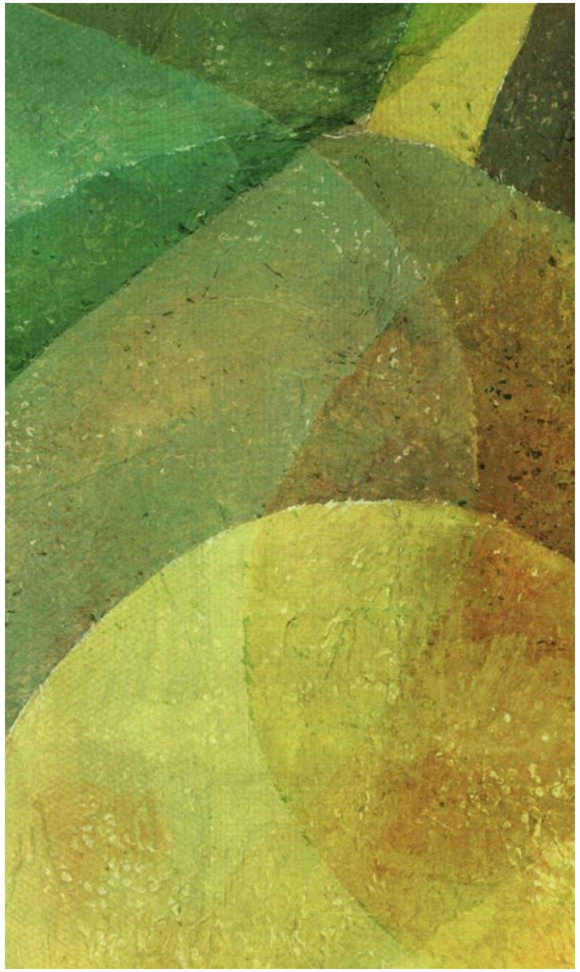

Revisiting Tidal Deformations in Black Holes

Maria J Rodriguez

5^{th} EPS Conference on Gravitation
Czech Academy of Sciences, Prague, Dec, 10th, 2024

Tidal Deformations of Black Holes

<u>Outline</u>

1) Motivation

2) Review: **Static** Love Numbers for Black Holes (BHs)

3) **Dynamical** Tidal Coefficients for BHs

“Dynamical Tidal Love Numbers for Kerr Black Holes” by Malcolm Perry and M.J.R. arXiv: 2310.03660 [gr-qc]
“New structures of Love Numbers for Kerr Black Holes” by Malcolm Perry and M.J.R. [to appear]
”TBA” Glazer, Joyce, MJR, Santoni, Solomon, Temoche arxiv [to appear]

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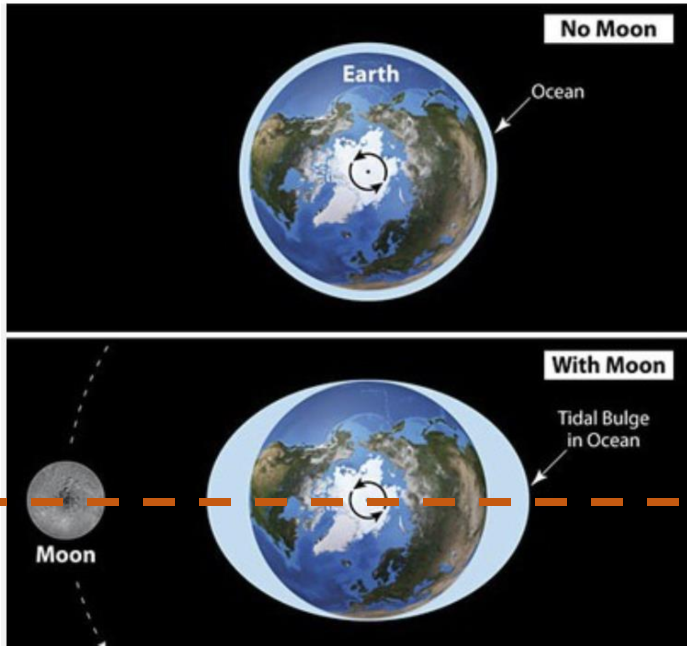
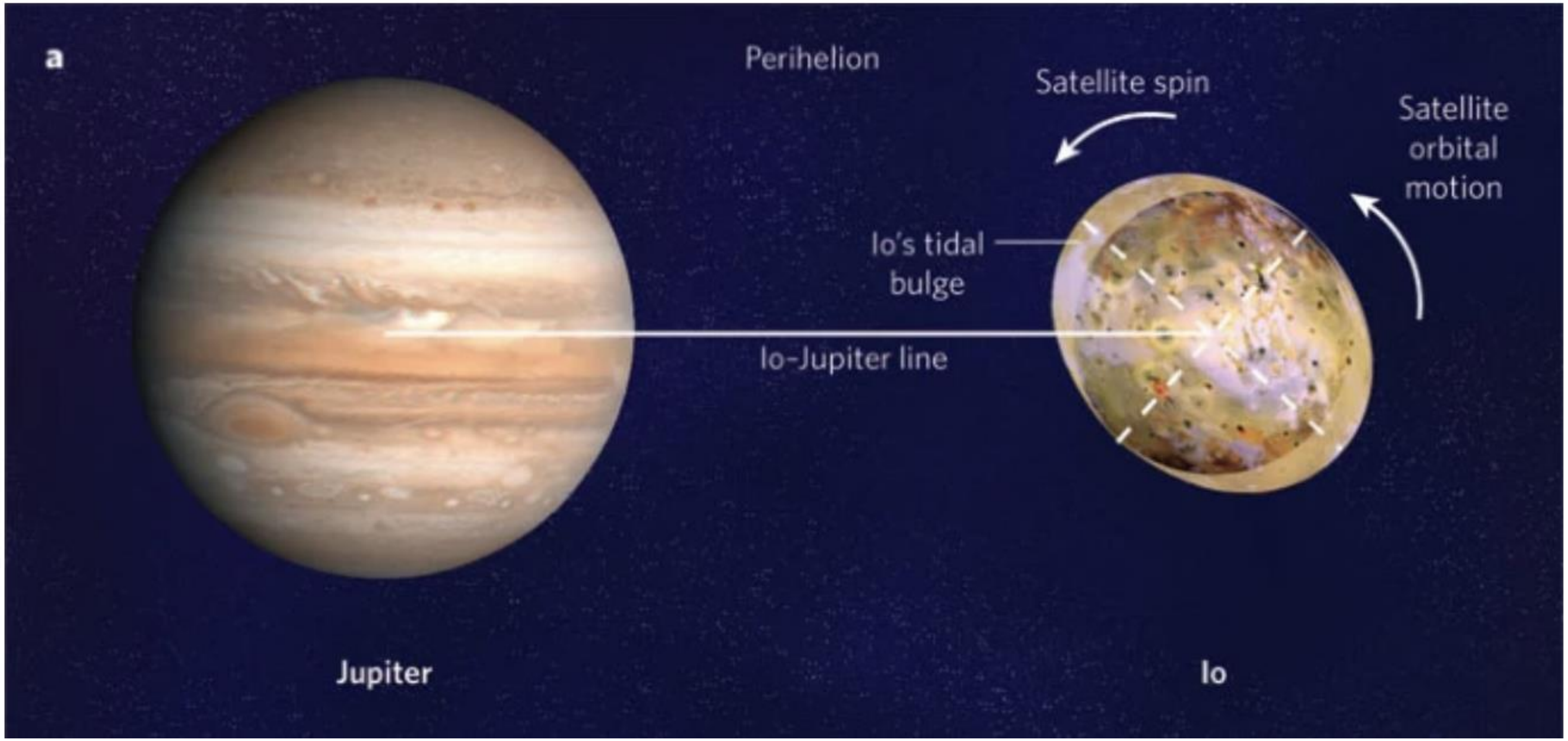
Tidal deformations are a gravitational phenomenon that causes a body to stretch along the line pointing towards and away from the center of mass of another compact object.



This is a result of spatial variations in the gravitational field exerted on one body by another, that is not constant across its parts.

Tidal Squeezing in the Solar System

Figure 1: Jupiter–Io tidal interaction.



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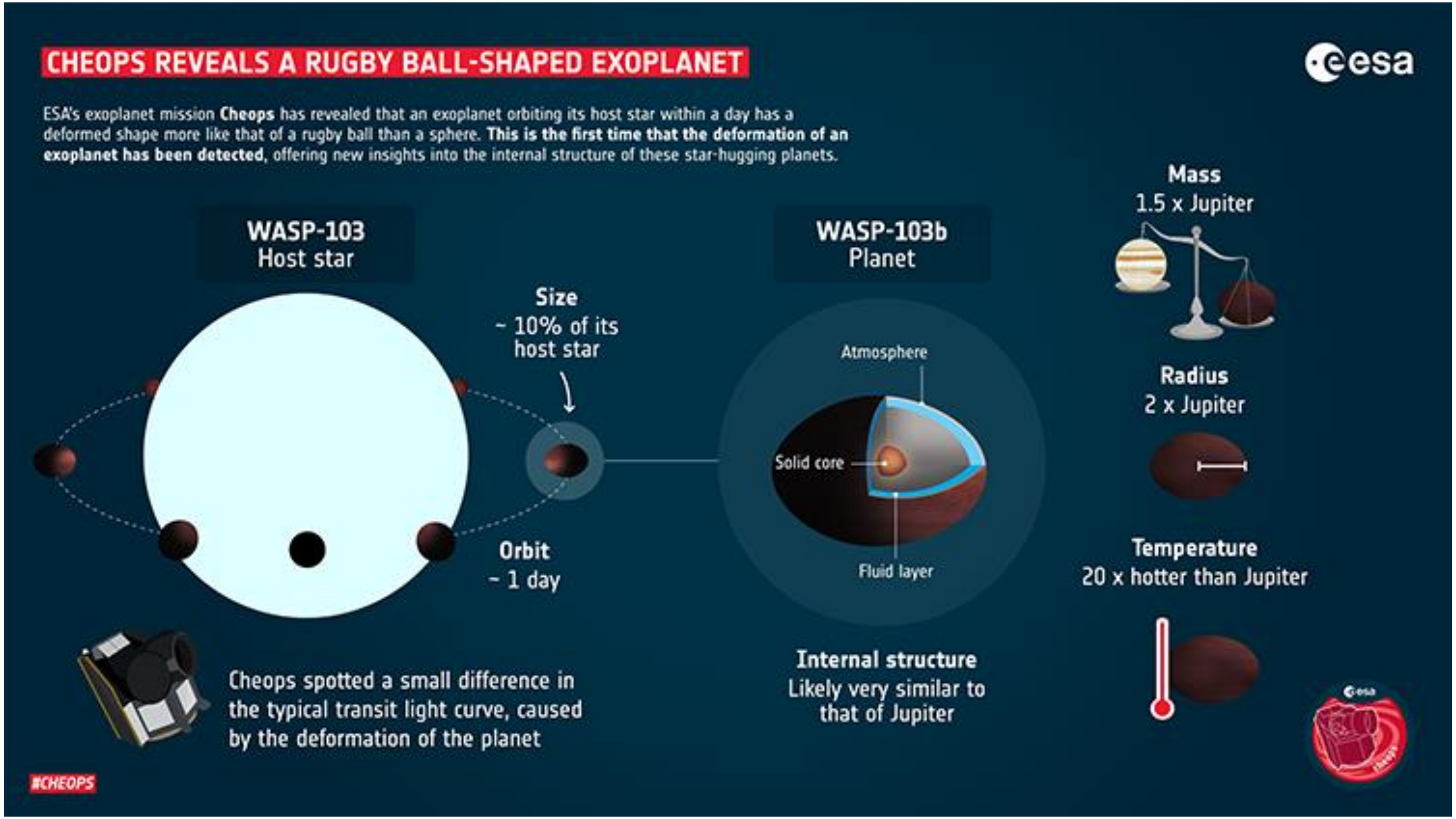
Tidal Squeezing in Exoplanets



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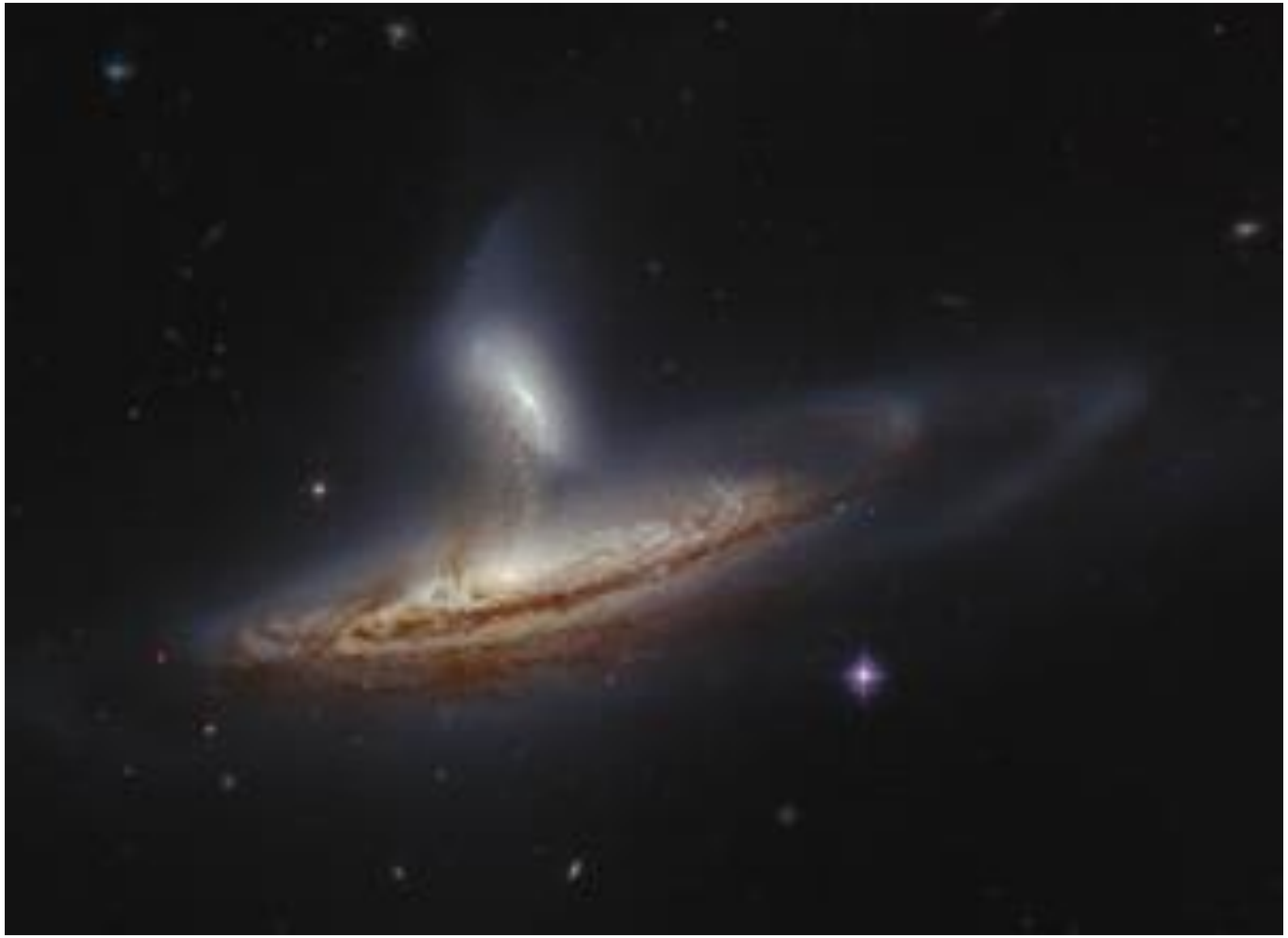
Galactic Tidal Deformations



Hubble Space Telescope
Seyfert galaxy NGC 169 (bottom) and the galaxy IC 1559 (top)

Significance: compact objects with distinct internal compositions undergo distinct deformations.

Tidal squeezing in the farm.

Black Holes

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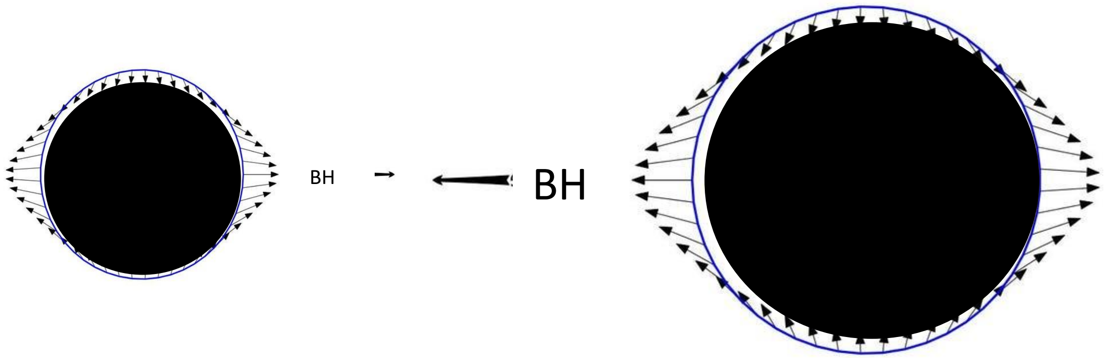
Tidal Deformation of Black Holes

Fundamental Idea:



Black Holes are nothing, simply boundaries of space-time.

Can we tidally squeeze BHs?

How are tidal deformations for BHs characterized?

Can we explain universal features of tidal deformations of black holes?

What can we learn from BH tidal deformations?

Tidal deformations are a gravitational phenomenon that causes a body to stretch along the line pointing towards and away from the center of mass of another compact object.

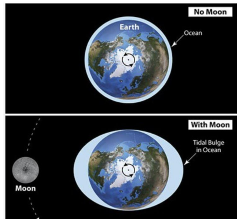
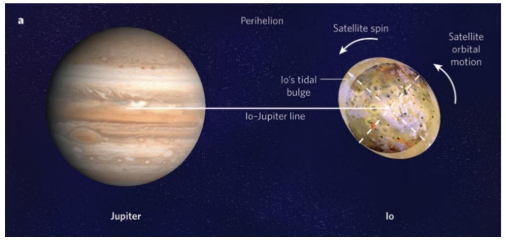
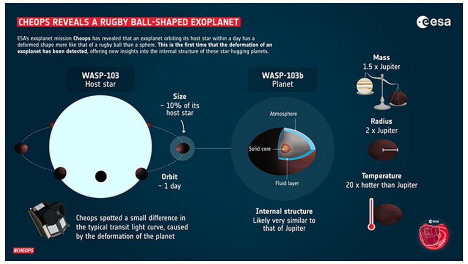


Figure 1: Jupiter–Io tidal interaction.



Tidal Squeezing in the Solar System



Tidal Squeezing beyond the Solar System

Galactic Tidal Deformations

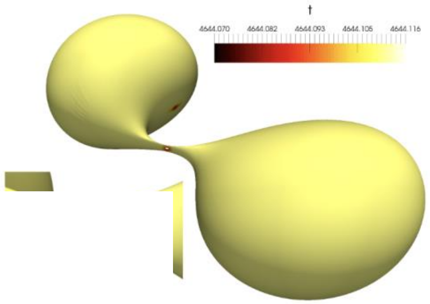


FIG. 1. Event horizon with a toroidal topology, shown in a different time slicing than the one used in the SpEC simulation.

Bohn, Kidder Teukolsky

Tidal Squeezing in Binary Black Hole Mergers

(a) $416.800M$ (b) $417.460M$ (c) $418.000M$

FIG. 8. Event horizon generator surfaces for the equal mass head-on binary. The t slicing in the top row is almost identical to

Bohn, Kidder Teukolsky

Tidal Squeezing in Head-on Black Hole Collisions

Significance: compact objects with distinct internal compositions undergo distinct deformations.

Binary Black Holes

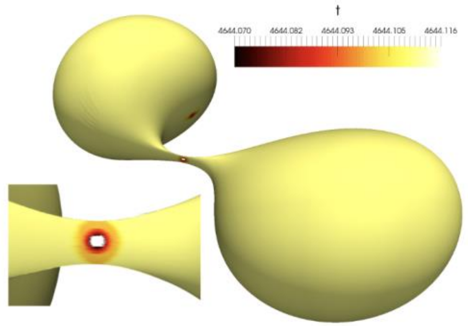


FIG. 1. Event horizon with a toroidal topology, shown in a different time slicing than the one used in the SpEC simulation.

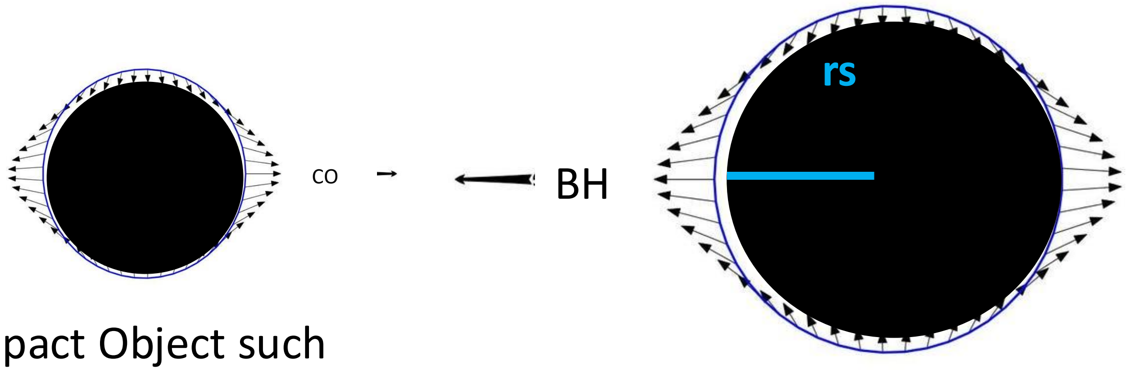
Therefore, the extent of the tidal deformation should be discernible in the gravitational wave and in turn be intricately linked to the inner structure of the entity.

GW: The manifestation of tidal deformation in a body, $\kappa_{\ell m}$, becomes evident at the 5PN order in the phase of a binary waveform. On the other hand, the onset of tidal dissipation in a rotating body, encoded in $\nu_{\ell m}$, is observed for the first time at the 2.5PN order for Kerr and 4PN order for Schwarzschild.

The internal structure of certain objects is governed by the poorly understood nuclear matter in e.g. **NS and new unexpected effects in black holes.**

Tidal deformations in General Relativity (GR)

An important observation is that the tidal response coefficients, first identified by Love, k_{lm} can be extracted directly from the solutions of the wave equation for all fields (integer spin fields):



CO = Compact Object such as another BH or NS

Gravitational external potential

$$\Phi = -\frac{M}{r} + \frac{(\ell-2)!}{\ell!} \sum_{\ell=2} \sum_{m=-\ell}^{\ell} Y_{\ell m} \mathcal{E}_{\ell m} r^{\ell} \left[1 + k_{\ell m} \left(\frac{r}{r_s} \right)^{-2\ell-1} \right],$$

r/rs the dimensionless distance to the body

E_{lm} multipole moment

Tidal Deformations in GR

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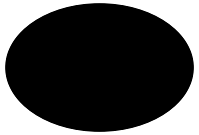
The problem of tidal deformations of Kerr BHs



$$\Phi = -\frac{1}{2} h_{tt} ,$$

Reduces to solving the massless scalar wave-equation equation $\nabla \Phi_s = 0 , \quad s = 0, \pm 1, \pm 2,$

$$\Phi_s(t, r, \theta, \phi) = e^{-i\omega t + im\phi} R_s(r) S_s(\theta) \; , \quad \text{with} \quad \omega \in \mathbb{C} \quad \text{and} \quad m \in \mathbb{Z} \, .$$

Boundary conditions. The radial functions must meet the following ingoing boundary conditions at the horizon

$$\hat{R}_s(r) = const \times (r - r_+)^{-i\alpha_+} \, , \text{ with } \alpha_+ > 0 \qquad \text{as} \qquad r \to r_+ \, .$$

$$\Omega = a/(2Mr_+)$$

where we defined the coefficient

$$T_+ = (r_+ - r_-)/(8\pi M r_+).$$

$$\alpha_+ \equiv \frac{(\omega - m\,\Omega)}{4\pi\, T_+} \pm \frac{is}{2}$$

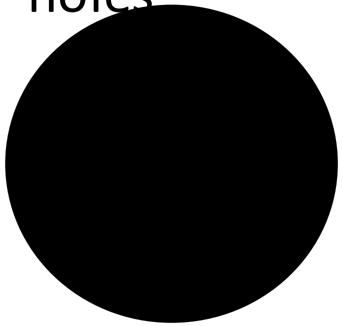
$$r_\pm = M \pm \sqrt{M^2 - a^2},$$

Analytic Continuation

$$\hat{R}_s(r) \xrightarrow[r \to \infty]{} \tilde{c_1}\, r^\ell \left(1 + \left(\frac{r}{r_s}\right)^{-(1+2\ell)} k_{\ell m}\right)$$

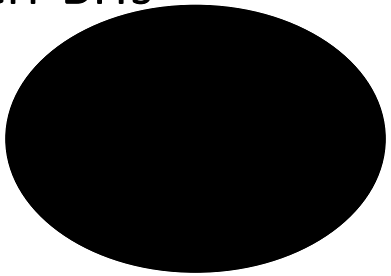
Tidal (Love) Coefficients

Kerr black holes



Characterized by
mass M and spin parameter a

Tidally Deformed Kerr BHs



Characterized by
mass M and spin parameter a and perturbations $h_{\mu\nu}$

Love Numbers for Kerr Black Holes (BHs)

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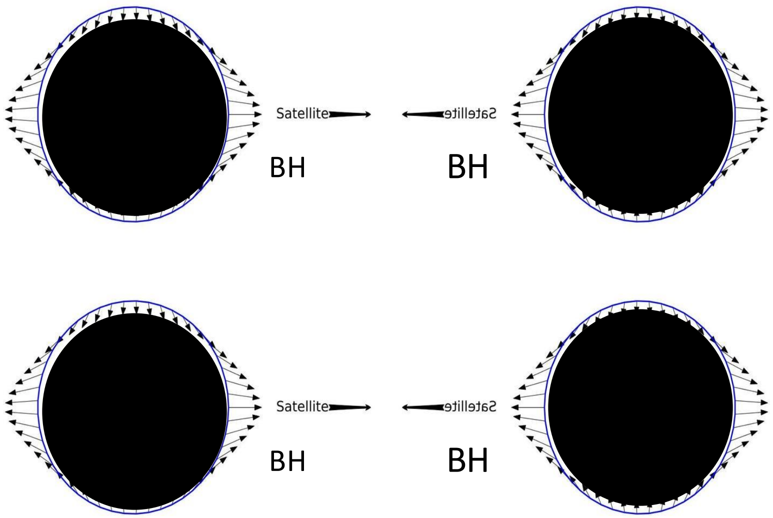
Tidal deformations of BHs



Static tidal Love numbers , $k_{lm} = 0$ for static gravitational deformations ($\omega = 0$)

Dynamical tidal Love numbers , $k_{lm} \neq 0$ for dynamical gravitational deformations ($\omega \neq 0$)

An important observation is that the tidal response coefficients, first identified by Love, k_{lm} can be extracted directly from the solutions of the wave equation for all fields (integer spin fields) to all orders in the frequency, including static ($\omega = 0$) and dynamical ($\omega \neq 0$) responses.

The gravitational tidal coefficients, k_{lm}, describe the tidal response of a rigid object e.g. star, planet or black hole.

$$k_{\ell m}(\omega) = \underbrace{\kappa_{\ell m}(\omega)}_{\text{Conservative effects or Love numbers}} + i\, \underbrace{\nu_{\ell m}(\omega)}_{\text{Dissipative effects}} \,.$$

Static Love Numbers for Kerr BHs

Kerr BH static tidal deformation coefficients defined by

$$\hat{R}_s(r) \xrightarrow[r\to\infty]{} \tilde{c_1}\, r^\ell \left(1 + \left(\frac{r}{r_s}\right)^{-(1+2\ell)} k_{\ell m}\right),$$

$$k_{\ell m}(\omega=0) = \kappa_{\ell m}(\omega=0) + i\,\nu_{\ell m}(\omega=0)\,,$$

where

Static Love Numbers $\kappa_{\ell m}(\omega=0) = 0\,,$

Static Dissipation Coeff. $\nu_{\ell m}(\omega=0) = (-1)^{s+1} m\gamma \frac{(\ell+s)!(\ell-s)!}{(2\ell+1)!(2\ell)!}\left(\prod_{n=1}^{\ell}(n^2+4m^2\gamma^2)\right)\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}$

Love Number vanishes, dissipation does not.
Is this a realization of something more fundamental?
Yes: symmetries. In that case we could observe it in the GW data.

Hidden symmerties for vanishing Love numbers for Kerr BHs

SL(2,R) x U(1)

arXiv:2209.02091 [hep-th]

→ Love Symmetry

SO(4,2)

arXiv:2203.08832 [hep-th]

→ Starobisnky Symmetry

Dynamical Tidal Coefficients from Starobinsky symmetry

$$k_{\ell m}^{Eff} = \frac{\Gamma(-2\ell-1)\Gamma(1+\ell-s)\Gamma(1+\ell+2i\bar{Q})}{\Gamma(2\ell+1)\,\Gamma(-\ell-s)\,\Gamma(-\ell+2i\bar{Q})}\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}$$

$$\bar{Q} = Q - M\omega$$

Low frequency solutions

Mano and E. Takasugi arXiv:gr-qc/9603020 [gr-qc]

Dubowsky et al arXiv:2209.02091 [hep-th].

$$k_{\ell m}(\omega) = \frac{\Gamma(-2\nu-1)\,\Gamma(1+\nu-s-2iM\omega)\,\Gamma\,(1+\nu+2iQ)}{\Gamma(2\nu+1)\,\Gamma(-\nu-s-2iM\omega)\,\Gamma\,(-\nu+2iQ)}\left[1-2\Delta\ell\,\log\left(\frac{r_+-r_-}{r}\right)\right] \times (1+A_{\ell m}\,\omega)\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2l)} + \mathcal{O}(\omega^2)$$

where

$$Q = -\frac{2M}{r_+-r_-}(M\omega - r_+ m\Omega)\,,$$

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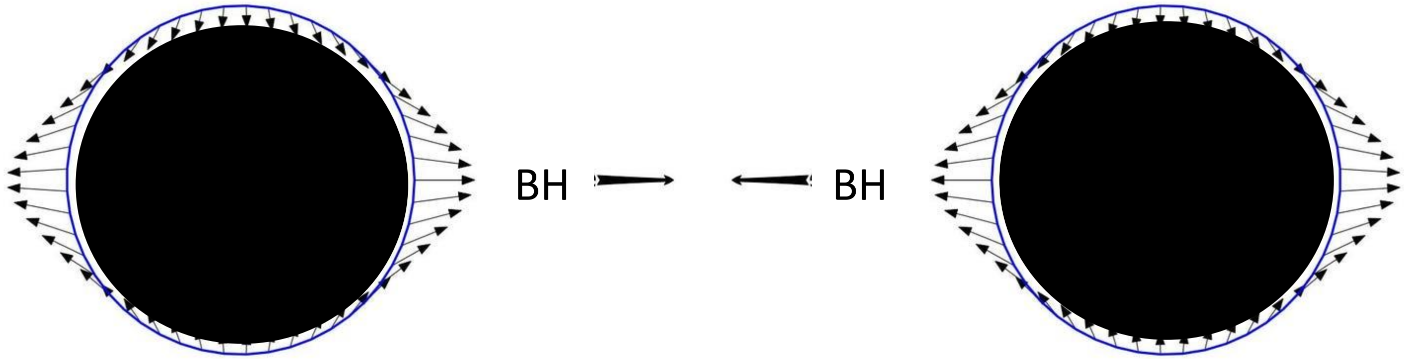
The vanishing of black hole Love numbers became however a controversial topic.

Potential uncertainties in the tidal coefficient arising from this Newtonian/GR matching could be effectively addressed by utilizing analytic continuations of the GR solutions into higher dimensions.

A different strategy to address these uncertainties involves utilizing the framework defined within the point-particle effective field theory (EFT) for binary inspirals.

Dynamical Tidal deformations of BHs



One possibility to compute the Love numbers for Kerr is to work in a regime where

$$\omega M \ll 1 \,, \qquad \omega r \ll 1 \,.$$

Such that the scalar/ Teukolsky's equation becomes

SL(2,R) x SL(2,R)
Hidden Symmetry

$$\left[\partial_r \Delta \partial_r + \frac{\left(2M\omega r_+ - \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r-r_+)(r_+-r_-)} - \frac{\left(2M\omega r_- + \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r-r_-)(r_+-r_-)} - \hat{K}_{\ell,s}\right]\hat{R}_s = 0 \quad (4.2)$$

$$\left[\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) - \frac{(m+s\cos\theta)^2}{\sin^2\theta} + K_{\ell,s}\right] S_s(\theta) = 0 \ .$$

Spheroidal eigenvalues

$$K_{\ell,s} = (\ell - s)(\ell + s + 1) + s.$$

Teukolsky's radial equation

SL(2,R) x SL(2,R)
Hidden Symmetry

$$\left[\partial_r \Delta \partial_r + \frac{\left(2M\omega r_+ - \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r - r_+)(r_+ - r_-)} - \frac{\left(2M\omega r_- + \frac{i}{2}s(r_+ - r_-) - am\right)^2}{(r - r_-)(r_+ - r_-)} - \hat{K}_{\ell,s}\right]\hat{R}_s = 0 \; (4.2)$$

1) Coordinate transformation and field redefintion

$$z = \frac{r - r_+}{r - r_-}\,, \qquad \hat{R}_s(r) = (r - r_-)^p\,(r - r_+)^q\,w(r)\,,$$

$$p = \frac{2iMr_+(\omega - m\Omega)}{r_+ - r_-} - (1 + \ell)\,, \qquad q = -\frac{2iMr_+(\omega - m\Omega)}{r_+ - r_-} = -i\,\alpha_+\,,$$

2) Identifying

$$\mathfrak{a} = 1 + \ell - i\frac{4M}{r_+ - r_-}(M\omega - r_+ m\Omega)\,, \qquad \mathfrak{b} = 1 + \ell - 2iM\omega - s\,,$$

$$\mathfrak{c} = 1 - i\frac{4M\,r_+}{r_+ - r_-}(\omega - m\Omega) - s\,.$$

$$z\,(1 - z)\,\frac{d^2 w}{dz^2} + \left[\mathfrak{c} - (\mathfrak{a} + \mathfrak{b} + 1)\,z\right]\frac{dw}{dz} - ab\;w = 0$$

Teukolsky's radial equatio

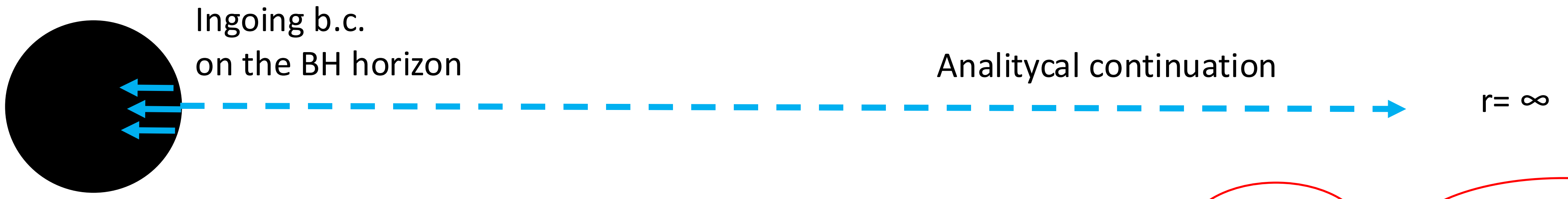
$$z\,(1-z)\,\frac{d^2w}{dz^2}+\left[\mathfrak{c}-(\mathfrak{a}+\mathfrak{b}+1)\,z\right]\frac{dw}{dz}-ab\;w=0$$

SL(2,R) x SL(2,R)
Hidden Symmetry

The solution takes the form

$$\hat{R}_s(z) \;=\; (1-z)^p\,z^q\left(c_1 F[\mathfrak{a},\mathfrak{b},\mathfrak{c};z]+c_2\,z^{1-\mathfrak{c}}\,F[\mathfrak{a}-\mathfrak{c}+1,\mathfrak{b}-\mathfrak{c}+1,2-\mathfrak{c};z]\right),$$

Boundary Conditions



$$\hat{R} \;=\; \left(\frac{r-r_+}{r-r_-}\right)^{-i\frac{2Mr_+}{r_+-r_-}(\omega-m\Omega)-s/2}(r-r_-)^{-1-\ell} \qquad (4.10)$$
$$F\left(1+\ell-i\frac{4M(M\omega-r_+m\Omega),}{r_+-r_-},1+\ell-2iM\omega-s,1-i\frac{4Mr_+}{r_+-r_-}(\omega-m\Omega)-s;\frac{r-r_+}{r-r_-}\right).$$

$$R\xrightarrow[r\to\infty]{}\tilde{c}_1\left[\frac{\Gamma(k)\Gamma(\mathfrak{a}+\mathfrak{b}-k)}{\Gamma(\mathfrak{a})\Gamma(\mathfrak{b})}r^\ell+\frac{\Gamma(\mathfrak{a}+\mathfrak{b}-k)}{k!\,\Gamma(\mathfrak{a}-k)\Gamma(\mathfrak{b}-k)}\log\left(\frac{r_+-r_-}{r}\right)r^{-\ell-1}\right]$$

$$\mathfrak{a}=1+\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\,,\qquad \mathfrak{b}=1+\ell-2iM\omega-s\,,$$
$$\mathfrak{c}=1-i\frac{4M\,r_+}{r_+-r_-}(\omega-m\Omega)-s\,.$$

Generic dimensionless tidal coefficient

$$k_{\ell m}(\omega)=\frac{\Gamma(\mathfrak{a})\,\Gamma(\mathfrak{b})}{\Gamma(k+1)\,\Gamma(\mathfrak{a}-k)\Gamma(\mathfrak{b}-k)}\log\left(\frac{r_+-r_-}{r}\right).$$

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Tidal coefficients for dynamical external gravitational sources

$$
\begin{aligned}
k_{\ell m}(\omega) &= \frac{\Gamma\left(1+\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\right)\,\Gamma(1+\ell-2iM\omega-s)}{(2\ell+1)!\,\Gamma(2\ell+1)\,\Gamma\left(-\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\right)\Gamma(-\ell-2iM\omega-s)} \\
&\quad\times\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}\log\left(\frac{r_+-r_-}{r}\right) \qquad (8
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4Mi(M\omega-r_+m\Omega)(2iM\omega+s)}{(2\ell+1)!\,\Gamma(2\ell+1)\,(r_+-r_-)}\left[\prod_{m=1}^{\ell}\left(m^2+\frac{16M^2(M\omega-r_+m\Omega)^2}{(r_+-r_-)^2}\right)\right] \\
&\quad\times\left[\prod_{n=1}^{\ell}(n^2+(2M\omega-is)^2)\right]\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}\log\left(\frac{r}{r_+-r_-}\right)\,. \qquad (
\end{aligned}
$$

$$\gamma = a/(r_+-r_-)\,.$$

Kerr Dynamical Tidal Coefficients

$$k_{lm}(\omega) = \sum_{n=1}^{\infty} k_{lm}^{(n)} \, \omega^n = \sum_{n=1}^{\infty} (\kappa^{(n)} + i\,\nu^{(n)}) \left(\frac{r_+ - r_-}{r_+ + r_-}\right)^{(1+2l)} \omega^n \,,$$

where the first few orders yield

$$\begin{aligned}
\kappa^{(1)} &= \frac{(-1)^{l+s+1}(l+s)!\,\Gamma(1+l-s)\,\Gamma(1+l+2mi\gamma)}{(2l+1)!\,\Gamma(2l+1)\,\Gamma(-l+2mi\gamma)} \log\left(\frac{r_+ - r_-}{r}\right) 2Mi \\
&= \frac{(-1)^s(l+s)!\,(l-s)!}{(2l+1)!\,(2l)!} 4M\,m\,\gamma \log\left(\frac{r_+ - r_-}{r}\right) \prod_{n=1}^{l}(n^2 + 4m^2\gamma^2) \\
&= \nu^{(0)}\, 4M \log\left(\frac{r_+ - r_-}{r}\right) \\
\kappa^{(2)} &= \kappa^{(1)} \frac{4iM^2}{(r_+ - r_-)} (\psi(-l+2im\gamma) - \psi(1+l+2im\gamma))\,, \\
&= -\kappa^{(1)} \frac{4M^2}{(r_+ - r_-)} \left(\frac{1}{2m\gamma} + 2\sum_{n=1}^{l} \frac{2m\gamma}{(2m\gamma)^2 + (n^2)}\right) \\
\nu^{(2)} &= \kappa^{(1)}\, 2M(\psi(1+l+s) - \psi(1+l-s)) \\
&= \kappa^{(1)}\, 2M \sum_{n=0}^{2s-1} \frac{1}{n+\ell+1-s}\,.
\end{aligned}$$

** Dynamical Love numbers for Kerr are generically not zero at all orders in the frequency ω and exhibit logarithmic running,*

**No frequency-dependent dissipation in Kerr by scalar perturbations (s = 0)*

** Kerr black holes do not universally behave like rigidly rotating dissipative spheres*

** Agreement with low frenquency results*

Tidal Coefficients for mor general D=4 BHs

$$k_{\ell m}(\omega) = \frac{\Gamma\left(1+\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\right)\Gamma(1+\ell-2iM\omega-s)}{(2\ell+1)!\,\Gamma(2\ell+1)\,\Gamma\left(-\ell-i\frac{4M}{r_+-r_-}(M\omega-r_+m\Omega)\right)\Gamma(-\ell-2iM\omega-s)} \times\left(\frac{r_+-r_-}{r_++r_-}\right)^{(1+2\ell)}\log\left(\frac{r_+-r_-}{r}\right)$$

(8

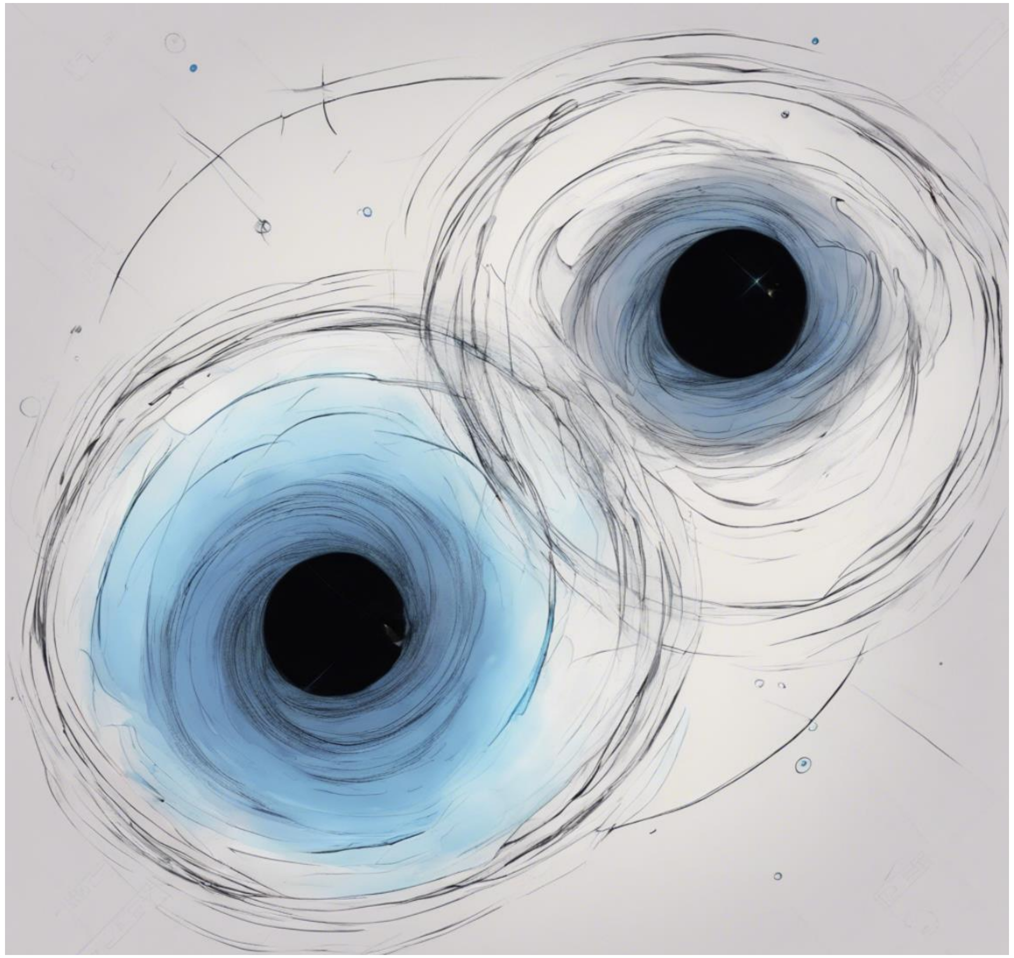
Kerr-NUT black holes

$$r_\pm \to r_\pm^{KTN} = M \pm \sqrt{M^2+N^2-a^2}\,,$$

Kerr-MOG black hole of the Scalar Tensor Vector Gravity (STVG),

$$r_\pm \to r_\pm^{MOG} = r(1+\alpha) \pm \sqrt{M^2(1+\alpha)-a^2}\,,$$

Tidal Coefficients for Extremal Kerr Bhs



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Tidal Coefficients for Extremal Kerr Bhs

2 Static Love numbers for Extremal Kerr

We aim to find the static Love numbers in extremal Kerr. For the polar angle dependence

$$\left[\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) - \frac{(m+s\cos\theta)^2}{\sin^2\theta} + K_{\ell,s}\right] S_s(\theta) = 0\ . \tag{2.1}$$

The corresponding eigenvalues

$$K_{\ell,s} = (\ell - s)(\ell + s + 1) + s \tag{2.2}$$

The radial equation is now

$$\left[\partial_r\left((r-M)^2\partial_r\right) + \frac{m\,M\,(m\,M + 2i(r-M)\,s)}{(r-M)^2} - s^2 - K_{\ell,s}\right]\hat{R}_s(r) = 0\ .$$

Tidal Coefficients for Extremal Kerr Bhs

For s=0 we can reduce the radial equation to a spheroidal Bessel equation

Special case $s = 0$

One can make the following coordinate transformation

$$z = \frac{mM}{(r-M)}\,, \qquad \partial_r = -\frac{mM}{(r-M)^2}\partial_z$$

to bring the equation to a Bessel spherical differential equation

$$z^2\,\frac{d^2\hat{R}_{s=0}}{dz^2} + (z^2 - \ell(\ell+1))\hat{R}_{s=0} = 0$$

Solutions are of the form

$$\hat{R}_{s=0} = \, z\sqrt{\frac{2}{\pi}}\left(c_1\, h_\ell^{(1)}(z) + c_2\, h_\ell^{(2)}(z)\right)\,.$$

$$h_\ell^{(1)}(z) \;=\; \sqrt{\frac{\pi}{2z}}\left(j_{\frac{1}{2}+\ell}(z) + i\, y_{\frac{1}{2}+\ell}(z)\right)$$

$$h_\ell^{(2)}(z) \;=\; \sqrt{\frac{\pi}{2\,z}}\left(j_{\frac{1}{2}+\ell}(z) - i\, y_{\frac{1}{2}+\ell}(z)\right)\,.$$

Plugging the expressions of the solution (2.11) in R we can determine the behavior of the solutions on the black hole horizon $r \to M$

$$\hat{R}_{s=0} \to \sqrt{\frac{2}{\pi}}\left[c_1\, e^{\frac{imM}{(r-M)}}\,(1+...) + c_2\, e^{-\frac{imM}{(r-M)}}\,(1+...)\right] \tag{2.12}$$

in the asymptotic region $r \to \infty$. The behavior of the solutions yields

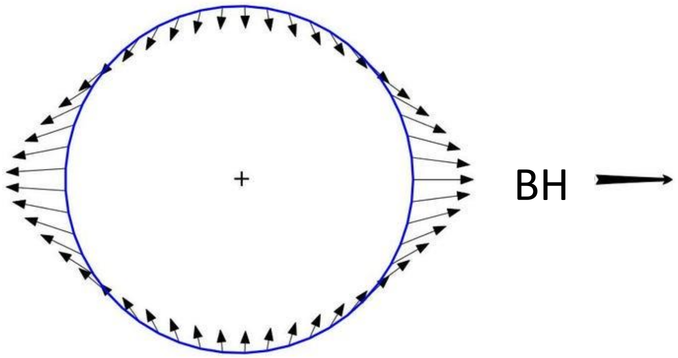
$$\hat{R}_{s=0} \to \tilde{c}_2 r^\ell \left(1 + k_{\ell m}\, r^{-2\ell-1} + ...\right)\,.$$

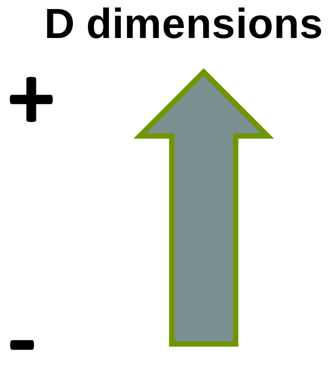
We can define the tidal coefficient

$$k_{\ell m} = -\frac{i\pi\,(mM)^{2\ell+1}}{2^{(2\ell+1)}\Gamma\left(\frac{1}{2}+\ell\right)\Gamma\left(\frac{3}{2}+\ell\right)}\,.$$

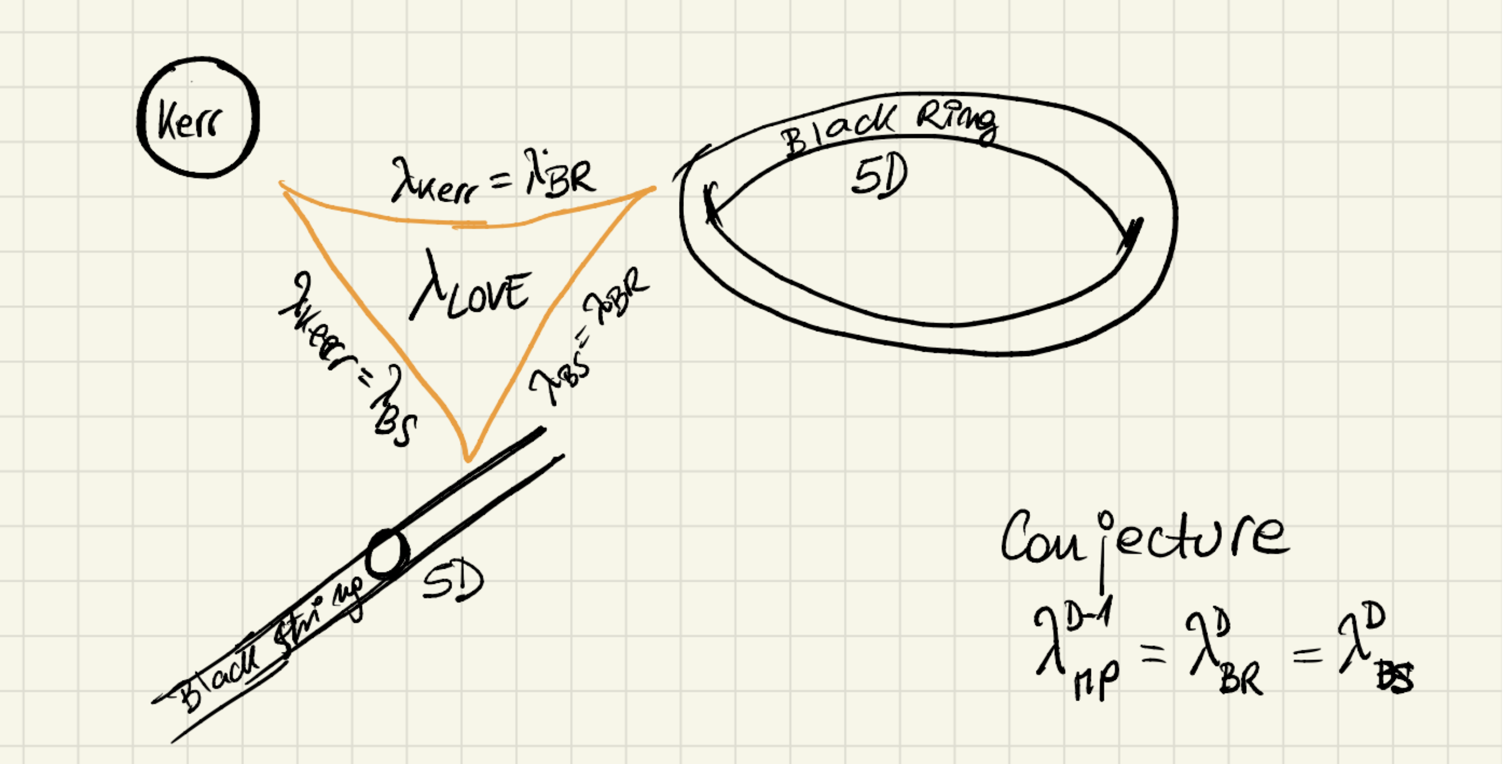
Tidal Coefficients for Higher Dimensional Bhs

Black Hole

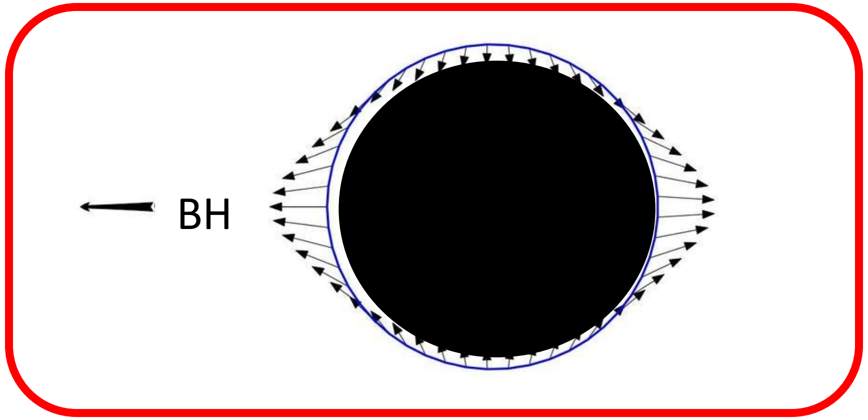




Static Tidal Coefficients for D=5 BHs, Black Rings and Black Strings



Contributions



– Reviewed how tidal deformations for BHs are defined in General Relativity

– Offered steps toward a better understanding of the computation of static Love numbers, and discussed the vanishing controversies for BHs

– Determined the dynamical tidal coefficients for Kerr through the study of the tidal deformations of Kerr BHs in dynamical external fields

– Argued that the Love numbers for Kerr have an approximate SL(2,R) x SL(2,R) hidden symmetry and match both, the low frequency regimes and Post-Newtonian computations.

Tidal squeezing in the farm.

Myers-Perry D=5 BH Love Numbers are non-vanishing

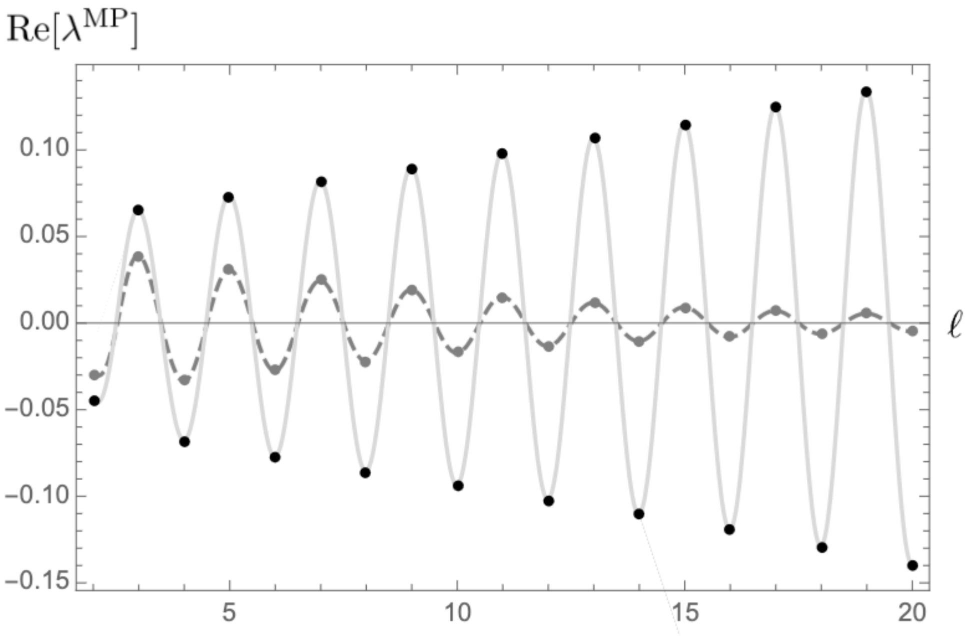


Figure 2: Visualization of the response coefficients $\lambda_{\ell m}^{\rm MP}$ for the single spinning $5D$ Myers–Perry black holes (3.35) as a function of the multipole moments ℓ. The imaginary part of the coefficients vanish, leading to vanishing dissipative response coefficients. The Love numbers, defined as the real part of the $\lambda_{\ell m}^{\rm MP}$, are represented in the plot for fixed mass $M = 1$ and angular momenta $J/M^{3/2} = 0.26, 0.29$ (from *gray* to *black* curves), respectively below and above the critical value $(J/M^{3/2})_{\rm crit} \sim 0.286$. As the multipole moments increase the Myers–Perry black holes with $J/M^{3/2} > (J/M^{3/2})_{\rm crit}$ exhibit increasing values of the Love numbers. This behavior reverts for slowly rotating Myers–Perry black holes where the tidal distortion tends to zero as $\ell \to \infty$.

Myers-Perry BHs (= Kerr in D>4 space-time dimensions) ?

Schwarzschild Dynamical Tidal Coefficients

$$\Omega \to 0,\ r_+ \to 2M \text{ and } r_- \to 0$$

$$\begin{aligned} k_\ell^{Schw}(\omega) &= \frac{\Gamma(1+\ell-s-2iM\omega)\Gamma(1+\ell-2iM\omega)}{(2\ell+1)!\,\Gamma(2\ell+1)\,\Gamma(-\ell-s-2iM\omega)\,\Gamma(-\ell-2iM\omega)} \log\left(\frac{2M}{r}\right) \\ &= \frac{(2iM\omega\ s-4M^2\omega^2)}{(2\ell+1)!\,\Gamma(2\ell+1)} \left[\prod_{j=1}^{\ell}(j^2+4M^2\omega^2)\right]\left[\prod_{n=1}^{\ell}(n^2+(2M\omega-is)^2)\right] \log\left(\frac{r}{2M}\right). \end{aligned} \tag{4.27}$$

$$k_{s=2}^{Schw}(\omega) = -\frac{(\ell-2)!(\ell-1)!\,\ell!(\ell+2)!}{2\,(1+2\ell)!(2\ell-1)!} 4M^2\omega^2 \log\left(\frac{2M}{r}\right) + \mathcal{O}(\omega^3)\,.$$

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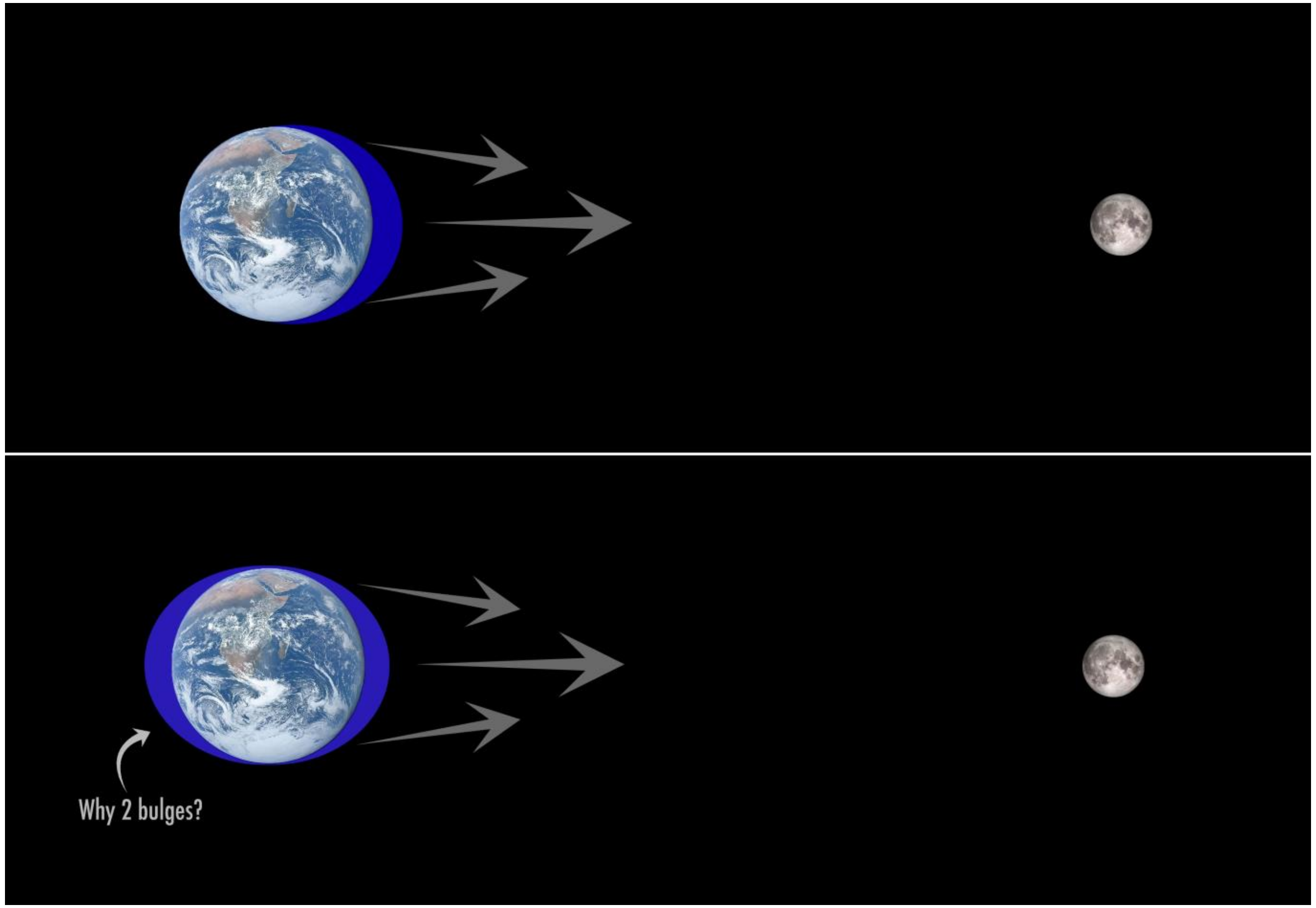
Why 2 bulges?