Spin foams:
From spacetime quanta to quantum spacetime

Bianca Dittrich,
Perimeter Institute

5th EPS conference on gravitation

Spin foams

- Non-perturbative real time path integral over quantum geometries
- Available as path integral over Euclidean signature geometries and as path integral over Lorentzian signature geometries

$$Z_E = \int \mathscr{D}\text{geom} \;\; \exp(\imath S(\text{Eucl Geom})) \qquad\qquad Z_L = \int \mathscr{D}\text{geom} \;\; \exp(\imath S(\text{Lor Geom}))$$

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- Advantages:
  - Incorporates quantum geometry: discrete quantum geometric excitations/ atoms of quantum spacetime
  - Avoids conformal factor problem, which killed many Euclidean quantum gravity approaches
  - Can extract (effective) metric
  - Incorporates restriction to positive (semi-) definitive metrics

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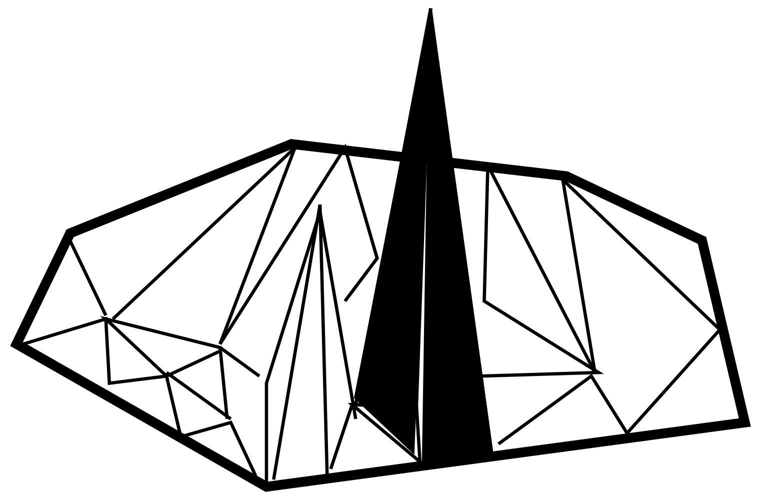
- Challenges:
  - Computational: extremely hard
  - Oscillating infinite sums (analytical continuation limited)
  - Many variables
  - Even with HPC: only very small triangulation treated so far - for a long time limited insight into dynamical properties

Avoidance of conformal factor problem



- 2D Euclidean Quantum Regge Calculus:
  Spikes render length expectation values ill-defined

[Ambjorn, Nielsen, Rolf, Savvidy 1997]

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  Spikes capture conformal factor: exponential enhancement of such configurations

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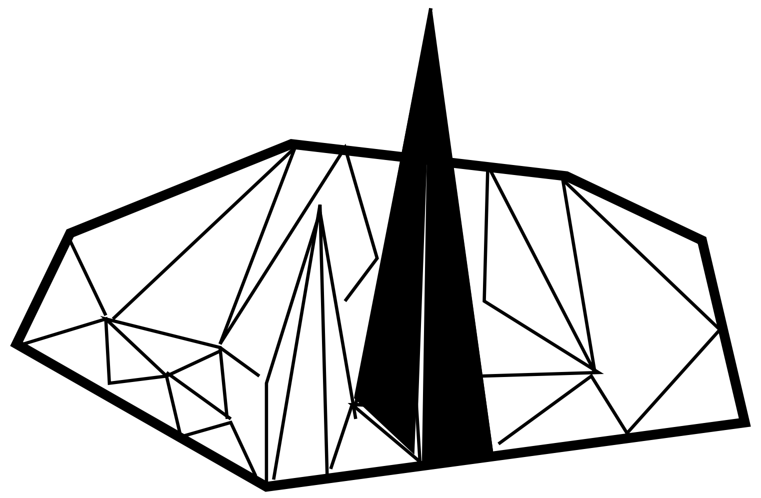

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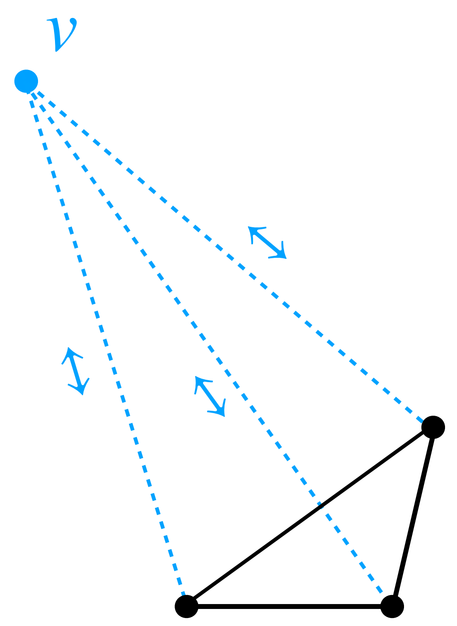
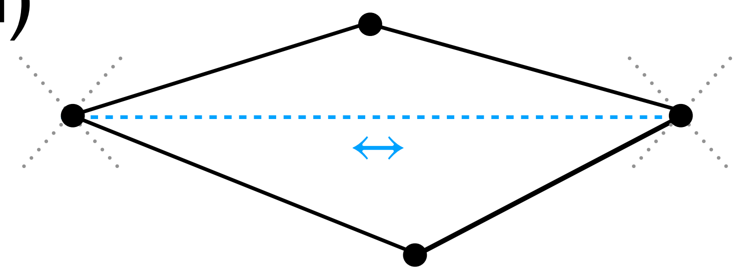


spike



(Lorentzian)
spine

• 3D and 4D Lorentzian Quantum Regge calculus with oscillating amplitudes:

Expectation values of arbitrary powers of lengths can be defined and are finite for spike and spine configurations.
Oscillatory amplitudes are essential - integrals do, in general, not converge absolutely.

[Borissova, BD, Qu, Schiffer 2024]

•Spin foams? - integral over length replaced by sum over area values

Discrete variable aligns with frequency of oscillation. Convergence may depend on measure.

[Borissova, BD, Qu, Schiffer 2024]

Spin foams as (decorated) TNW's

Computational: extremely hard — how to extract large scale limit (behaviour with many building blocks)?

Treat it as a (real time) lattice system.

- Tensor network renormalization algorithms: can deal with oscillating amplitudes
- But: variables with finite range; developed in lower dimensions, originally without gauge symmetries

- Developed and tested (decorated) tensor network algorithms for gauge theories, including first 3D algorithms for non-Abelian gauge theories
- Rich phase diagrams for spin foam analogue systems: potential for interesting continuum behaviour

[BD, Martin-Benito, Mizera, Steinhaus, … 2014 - 2020]

- Tensor network techniques also useful to compute 4D spin foam amplitudes more efficiently

[Asante, Steinhaus 2024]

- Challenges:
  - 4D models/ more realistic systems require significant increase in HPC capabilities
  - Better algorithms?

Dealing with oscillating sums

How to deal with unbounded oscillating sums? Acceleration techniques for series convergence. [Schmidt 41, Shanks 55, Wynn 56, …]
[BD, Padua-Arguelles 23]

A quite simple technique that allows to treat oscillating sums and integrals.
Contour deformation not necessary. But reproduces results for integrals treated via contour deformation.

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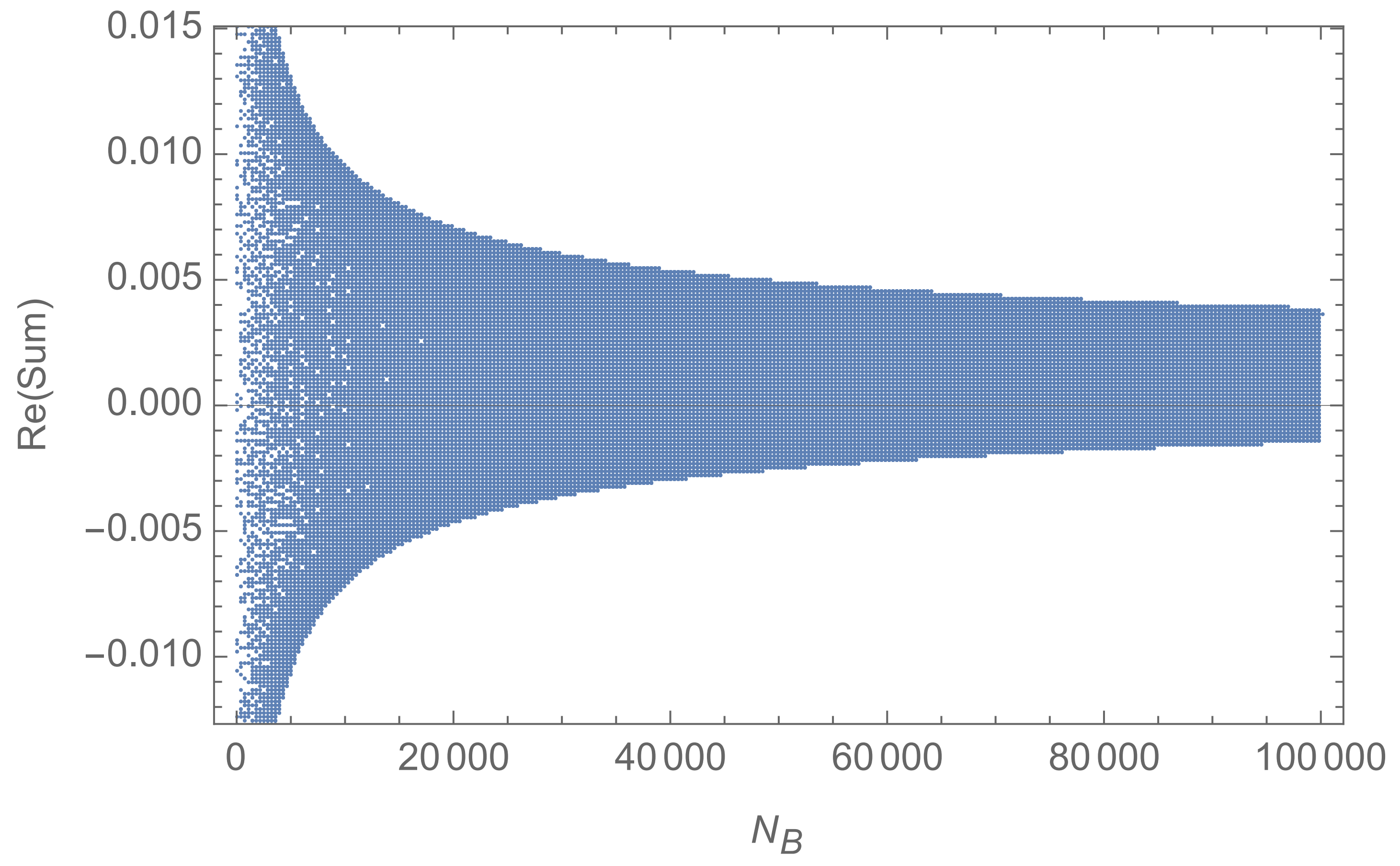

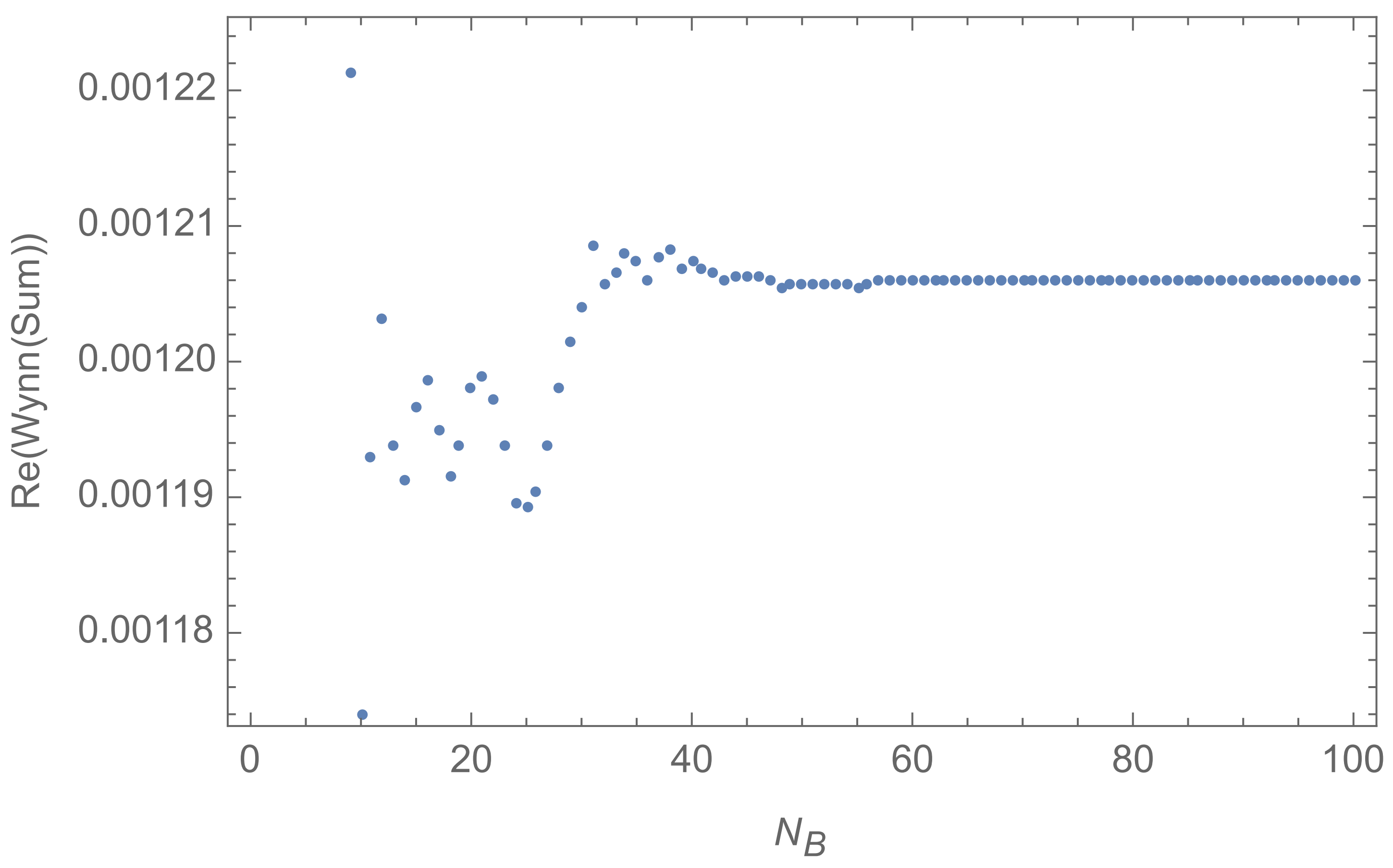

Rel. Error $\sim 10^{-11}$

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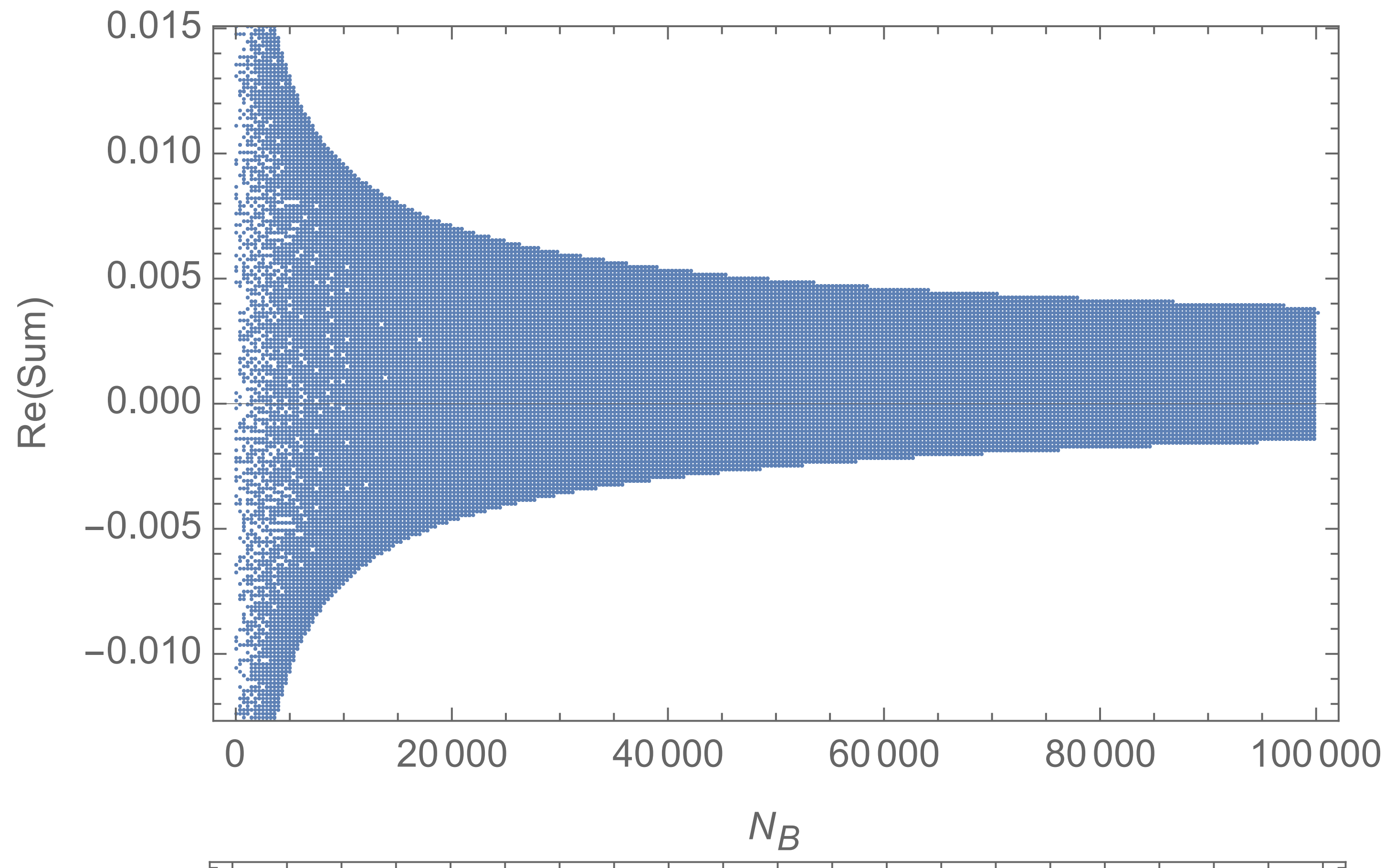
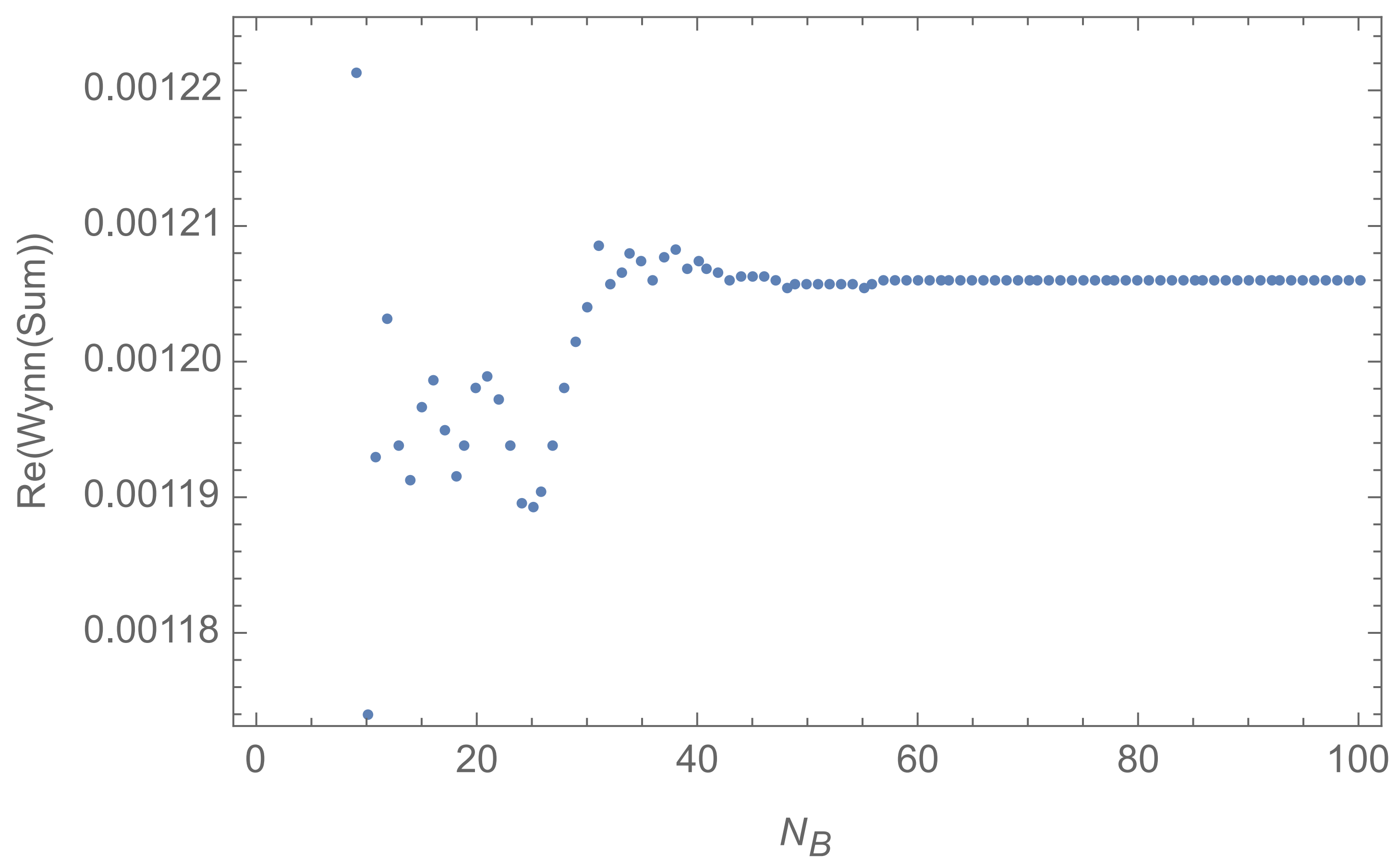
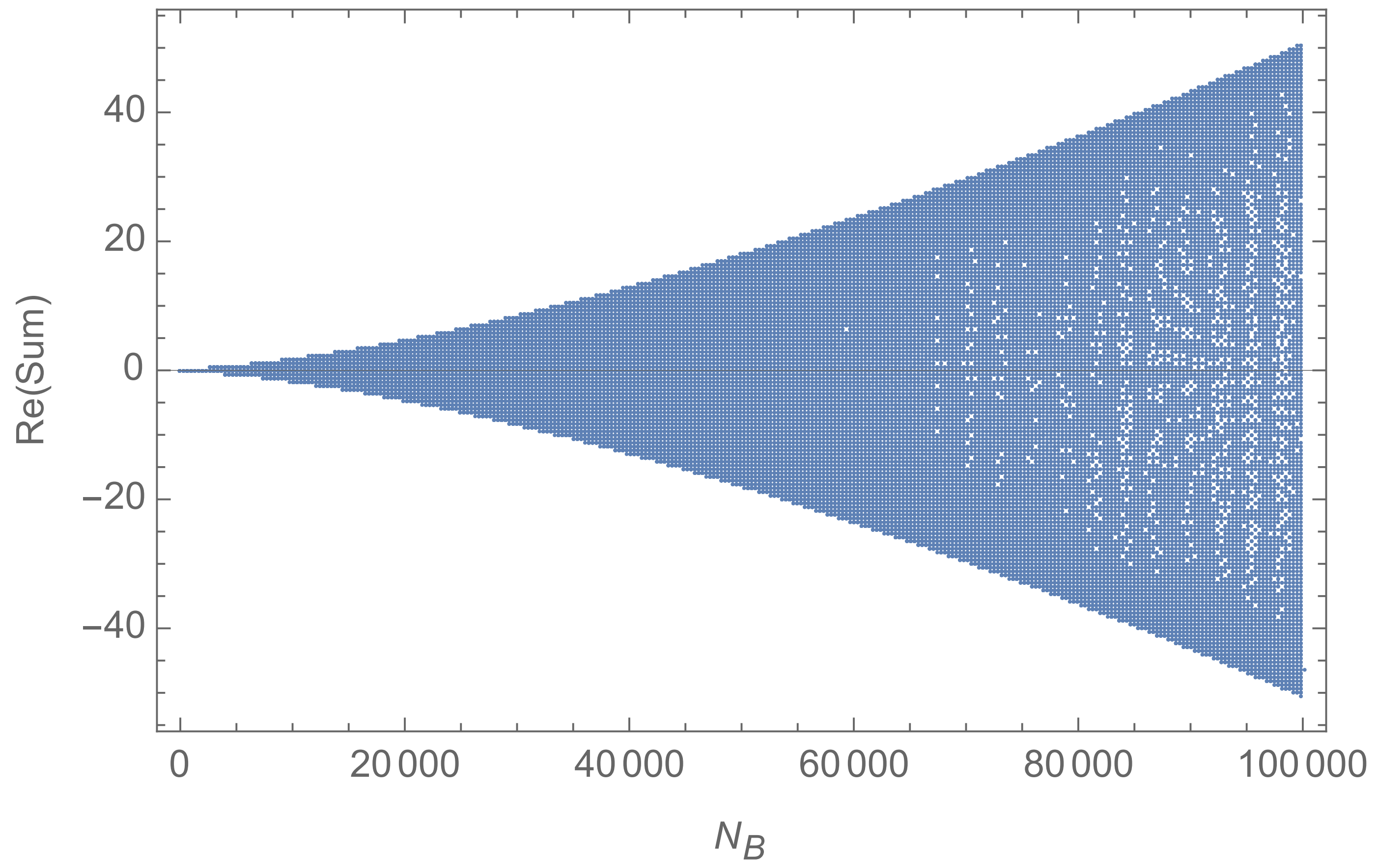
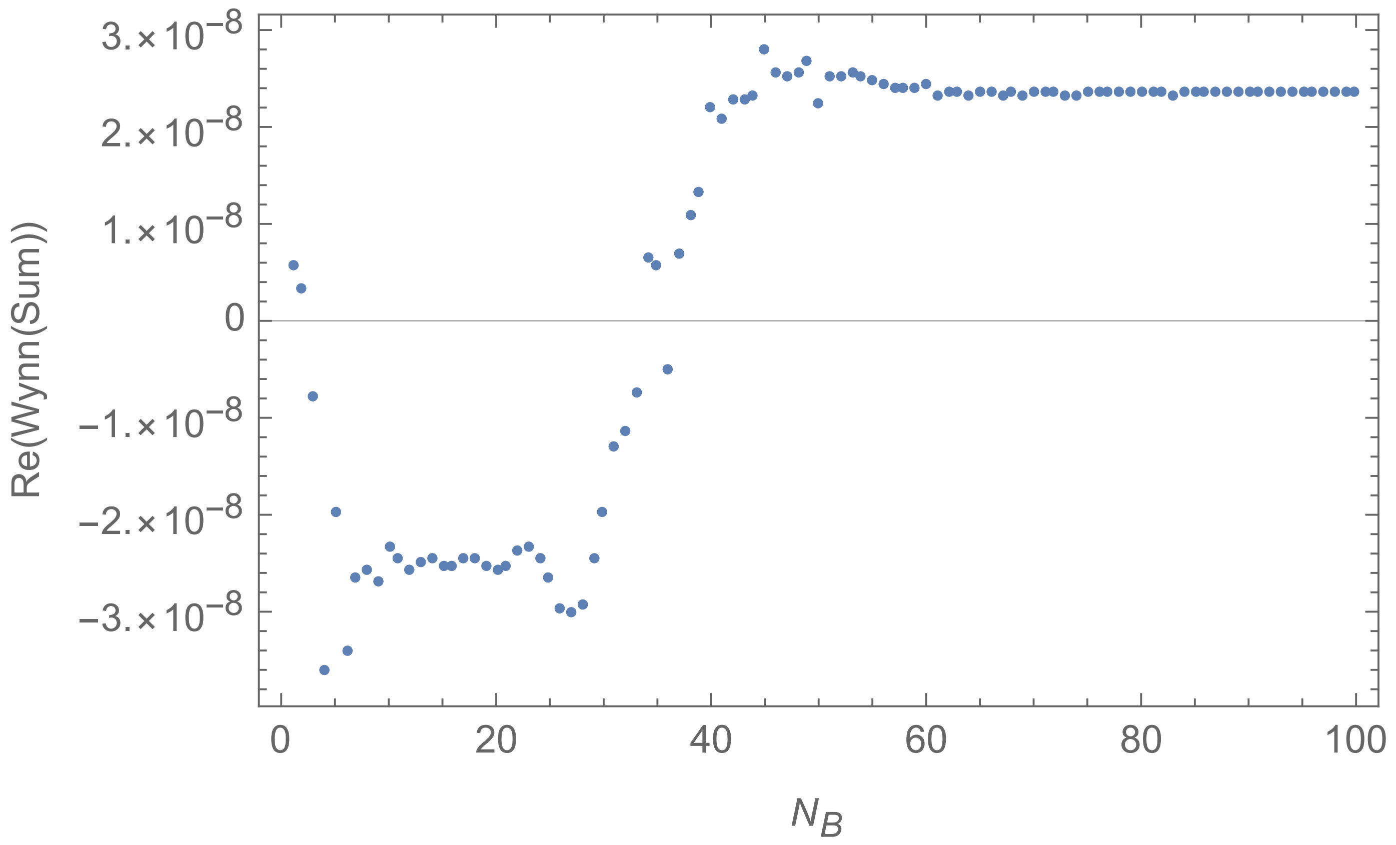




Rel. Error~10^{-11}

For computation of expectation value:





Rel. Error~10^{-8}

Works very well for sums with actions that are at most linear in the summation variable.
Consistent with quantum mechanics (Bohr correspondence principle).

Spin foams - Dynamics

$\Rightarrow$ Spin foam quantization leads to an extension of quantum configuration space: instead of length metrics we have truly an area metric space.

[BD, Ryan 2008-2012;]
[BD, Padua-Arguelles 23]

- Construction of spin foam amplitudes: to get equations of motion for GR, constraint implementation is essential.

- So do spin foams lead to GR?

Spin foams - Dynamics

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- Construction of spin foam amplitudes: to get equations of motion for GR, constraint implementation is essential.

- So do spin foams lead to GR?
  - There were indications that this was not the case: the flatness problem for spin foams (in semi-class. limit)
    [Bonzom 2009; Hellmann, Kaminski 2013, Han 2013, …, Engle, Kaminski, Oliveira 2020, Dona, Gozzini, Sarno 2020, Gozzini 2021]
  - Issue can be traced back to anomaly in constraint algebra and resulting weak implementation of simplicity constraints [Asante, BD, Haggard 2020 PRL]

- Existing spin foam models (up to 2020) lead to very involved amplitudes: explicit test of EOM even on small triangulation still not performed despite efforts to develop HPC tools for spin foams
  [Dona, Sarno, Gozzini, Frisoni, Steinhaus, Simao, Asante, Han, Liu, Qu, …]

Effective spin foams

[Asante, BD, Haggard PRL 2020]

[Asante, BD, Padua-Arguelles CQG 2021]

- Captures key ingredients of spin foams: discrete area spectrum and weak implementation of constraints.

- Much more transparent encoding of the dynamics, in particular with regard to simplicity constraints.
  Allowed resolution of the "flatness problem".

- Much much more amenable to numerical investigations: seconds on laptop compared to weeks on HPC

- Can be constructed from higher gauge theory, closely related to higher gauge topological field theory.

Allowed explicit test of EOM on small triangulation - by computing path integral and expectation values of areas without any truncation/ approximation:

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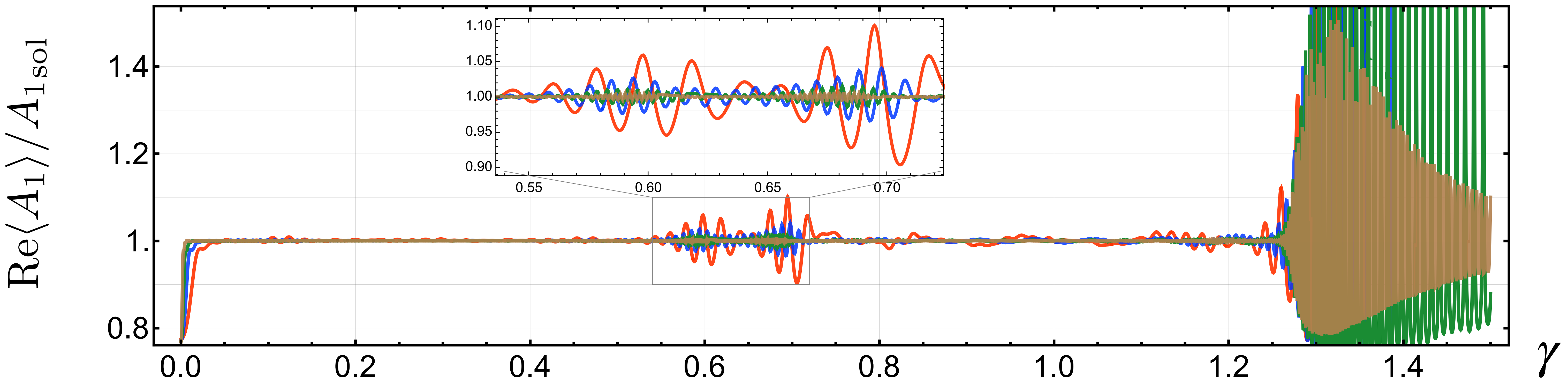

First explicit test of EOM in spin foams.

Results, show that spin foam do impose a discretized (Regge) gravitational dynamics.

Larger than expected range for γ is allowed, e.g. $\gamma \sim 0.1$ is possible.

Consistent with expectations from black hole entropy calculations.

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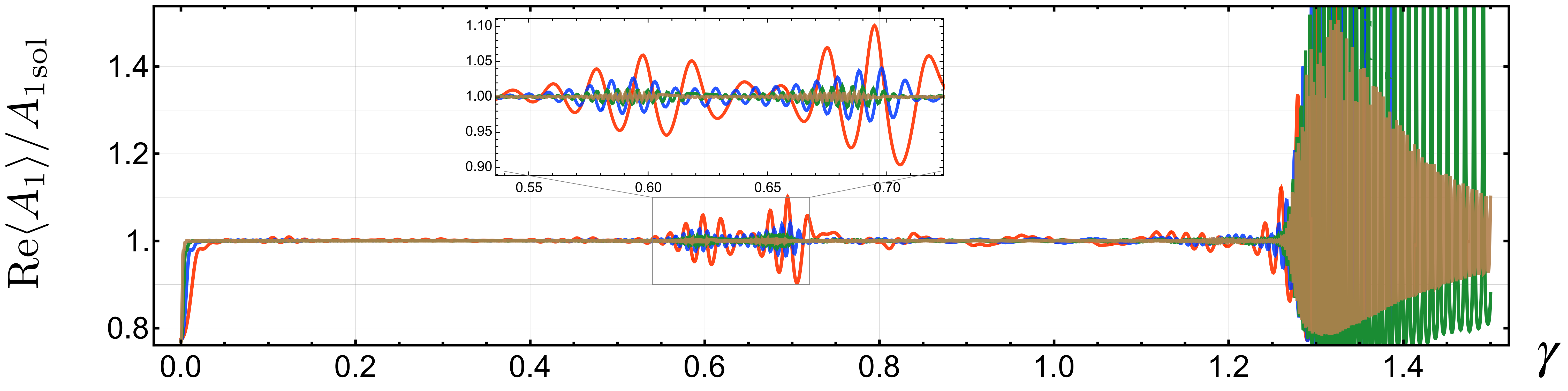

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What about triangulations with many building blocks? Do the additional dof's dominate in continuum limit?

Continuum limit for effective spin foams

- Fully non-perturbative calculation out of reach.
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- Consider continuum limit of Area Regge action on regular, infinite lattice

Area Regge action:

- discretization of Einstein-Hilbert Action
- but with length replaced by areas
- it was widely assumed that this action does not lead to general relativity since 90's
- But (perturbative) continuum limit was never established

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Length Regge action:
- discretization of Einstein-Hilbert Action, based on length variables attached to edges of a triangulation
- Continuum limit for linearized action performed in 80's
- Essential step:
  - -for standard triangulation of hypercubic lattice one finds 15 degrees of freedom per lattice site
  - -10 are massless (including 4 gauge), rest are massive and need to be decoupled [Rocek, Williams 84]
  - -this leads to linearized GR action
  - -finer triangulations of hypercubic lattices: have always 10 massless dof's, many many more massive dof [BD 2023]
  - -can be also done for Lorentzian signature [Asante, BD to appear]

Continuum limit of Area Regge action

[BD 2021,
BD, Kogios 2022,
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- Constructed linearized Area Regge action on a triangulation of the hyper-cubical lattice
- Depending on choice of lattice: 50 or 100 variables or more per lattice vertex

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Findings:

- Only graviton degrees of freedom are massless
- All other degrees of freedom are either gauge or (Planck) massive
- Lowest order in lattice lengths (second order in derivatives) reproduces (linearized) Einstein Hilbert action

$\Rightarrow$Spin foams do have additional degrees of freedom (to length metric), but these are very massive.

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- Next to leading order $\sim -\mathrm{Weyl}^2 + \mathcal{O}(\partial^6)$

- This correction can be explained by area metric (constructed from triangle areas on each hypercube)
- Has more dof's than length metric. Integrating these out leads to the correction term.
- Why Weyl? We integrate out traceless parts of Area metric - these couple to traceless parts of Riemann tensor.

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⇒Continuum limit for Area Regge action includes a Weyl squared correction. Induced from an area metric.

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- Even if we do: result does not change in an essential way.

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Universality: Spin foam models differ in how/ whether constraints are implemented.

But this does not seem to matter in the continuum limit.

Surprise: The Barret-Crane Spinfoam model could lead to GR.

Effective continuum dynamics for spin foams

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- Spin foams derived from quantization of Plebanski action
- Modified Plebanski actions: replace constraints by potential terms for constrained dof's [Kransov 08+]
- Chiral version: leads to ("deformed version of") GR [Kransov, Freidel]
- Non-chiral version (used in spin foams): leads to bi-metric theory of gravity [Speziale]

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[Borissova, BD 2022:]

- To mimic split of simplicity constraints into sharply and weakly imposed sets, replace only weakly imposed constraints by potential terms
- Solving for connection: leads to an action in terms of area metrics
- In linearized theory integrate out non-length metric degrees of freedom:

$$ {}^{(2)}\mathscr{L}(h) \quad = {}^{(2)}\mathscr{L}_{EH}(h) - {}^{(1)}C_{\mu\nu\rho\sigma}(h)\frac{1}{p^2+M^2}{}^{(1)}C^{\mu\nu\rho\sigma}(h) $$

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- Propagator: $$\text{Prop} = \left(\frac{1}{p^2} + \frac{1}{M^2}\right)$$

No additional poles!

[chiral Plebanski: Freidel '08, Krasnov '08, Area-metric: Borissova, BD, 2022]

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[chiral Plebanski: Freidel '08, Krasnov '08, Area-metric: Borissova, BD, 2022]

- Possible observational signature: mixing of Plus/Cross graviton modes (due to parity symmetry violating action).

[Borissova, BD, Krasnov 2023]

Area metric action captures enlargement of quantum configuration space in spin foams.

Summary

- Spin foams - promising Lorentzian path integral approach, but hard computational challenges
- Can avoid conformal factor problem which killed many Euclidean approaches
- Effective spin foams: led to huge computational simplifications, allowed for first explicit test of EOM
- Allowed for derivation of perturbative continuum limit: (linearized) GR plus Weyl squared correction
- Effective continuum action for spin foams based on area metrics: new avenues to phenomenology
- Possible observational signature: mixing of Plus/Cross graviton modes.

Outlook

- Phenomenology of area metric actions [wip w/ Borissova]
    - Area metric actions to higher order and area metric renormalization flow [wip w/ Borissova, Eichhorn, Schiffer]
    - Flow of Barbero-Immirzi parameter and investigation of special case with no additional pole

- Effective spin foams for (less and less) symmetry reduced sectors/ cosmology [BD, Padua-Arguelles 2023, wip]

- Continuum limit of spin foams - improve computational techniques

Thank you!

Area metrics from spin foams: three different ways

Micro-scopic:

[BD, Ryan ’08, Freidel, Speziale ’10]
4-simplex quantum geometry specified by 20 quantities
→
[BD, Padua-Arguelles ’23]
20 quantities define area metric for 4-simplex

Meso-scopic:

[Asante, BD, Haggard ’20, BD ’21]
Effective spin foams: allowing for lattice perturbation theory (50 variables per vertex)
→
[BD, Kogios ’22; Asante BD ’24]
Continuum limit: Leading order dynamics described by area metrics associated to hypercubes

Macro-scopic:

[Krasnov ’07]
Modified Plebanski framework
→
[Borissova, BD ’22]
Allows derivation of area metric action

} Actions are consistent.

(Naive) Semi-classical limit of spin foams

[Barrett-Williams, Barrrett-Dowdall-Fairbairn-Gomes-Hellmann, Conrady-Freidel, …]

Area Regge action

$$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

Equation of motion:

$$0 = \epsilon_t(A_t)$$

Seem to demand flatness

Length Regge action

$$S_{\mathrm{LR}}(L_e) = \sum_t A_t(L_e) \cdot \epsilon_t(L_e)$$

curvature angles

Equation of motion:

$$0 = \sum_t \frac{\partial A_t}{\partial L_e} \epsilon_t(L_e)$$

discretization of Einstein equations

(Naive) Semi-classical limit of spin foams

[Barrett-Williams, Barrrett-Dowdall-Fairbairn-Gomes-Hellmann, Conrady-Freidel, …]

Area Regge action $S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$

Equation of motion: $0 = \epsilon_t(A_t)$

Seem to demand flatness

Length Regge action $S_{\mathrm{LR}}(L_e) = \sum_t A_t(L_e) \cdot \epsilon_t(L_e)$

curvature angles

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discretization of Einstein equations

The continuum limit of the Area Regge dynamics was not understood until recently.
It was assumed that it does not lead to general relativity. [Barrett-Rocek-Williams 97]

This assumption lead to the “flatness problem” for spin foams.

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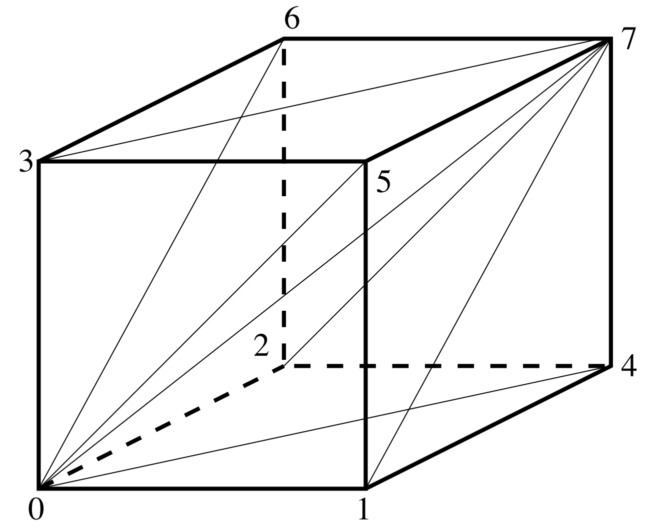
"Discrete" resolution: Together with $\hbar$ we have also to scale the anomaly parameter γ to be small. Explicit numerical proof for discrete dynamics.

[Asante, BD, Haggard '20, Asante, BD, Haggard '21, Asante, BD, Padua-Arguelles 21]

"Continuum" resolution: Surprise! The continuum limit of Area Regge calculus gives general relativity (+ corrections).

[BD 21, BD, Kogios 22]

Continuum limit of Area Regge action



Length Regge and Area Regge calculus are defined on general triangulations.
Dynamics appears non-transparent.

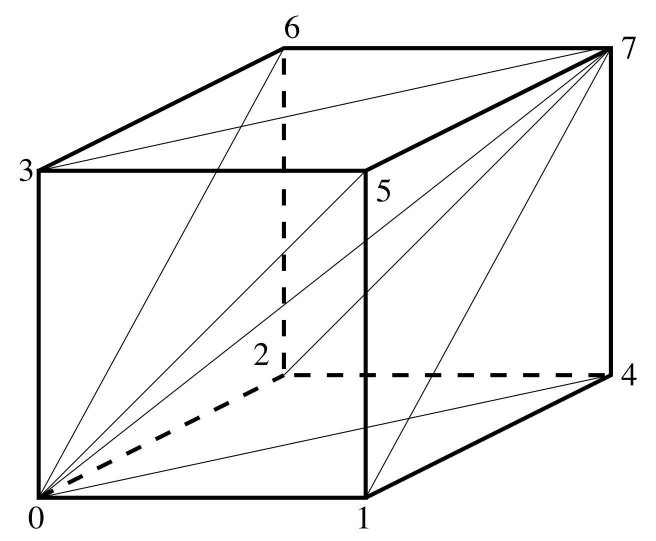
Perturbative expansion on triangulation of regular lattice (background describing flat space time).

Length Regge:

[Rocek, Williams '83]

Lattice action yields linearized Einstein-Hilbert action.
There are 15 degrees of freedom per lattice vertex,
but 5 are spurious (have a lattice constant dependent mass term).

Continuum limit of Area Regge action



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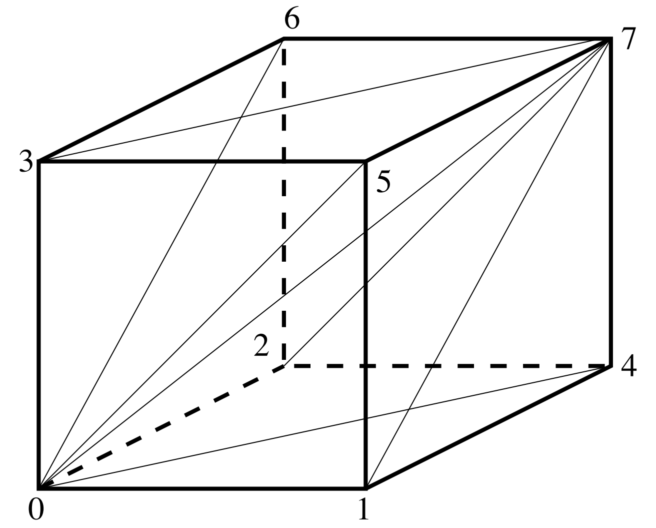
Zeroth order in lattice constant: Linearized Einstein-Hilbert action.
Lowest order correction: Arises from area metric dof's (additional to length dof's)

Continuum limit of Area Regge action



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linearized area metric $\rightarrow \; h, \chi$

$$S \approx S_{EH}(h) + \chi \cdot \mathrm{Weyl} - \chi \cdot (M^2 + \ldots) \cdot \chi + \ldots$$

$$S_{\mathrm{eff}} \approx S_{EH}(h) + \mathrm{Weyl} \cdot M^{-2} \cdot \mathrm{Weyl} + \ldots$$

higher derivative correction