Tackling quantum gravity non-perturbatively

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The bare-bones story of Asymptotic Safety

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…or: ***Quantum Gravity as a Quantum Field Theory***

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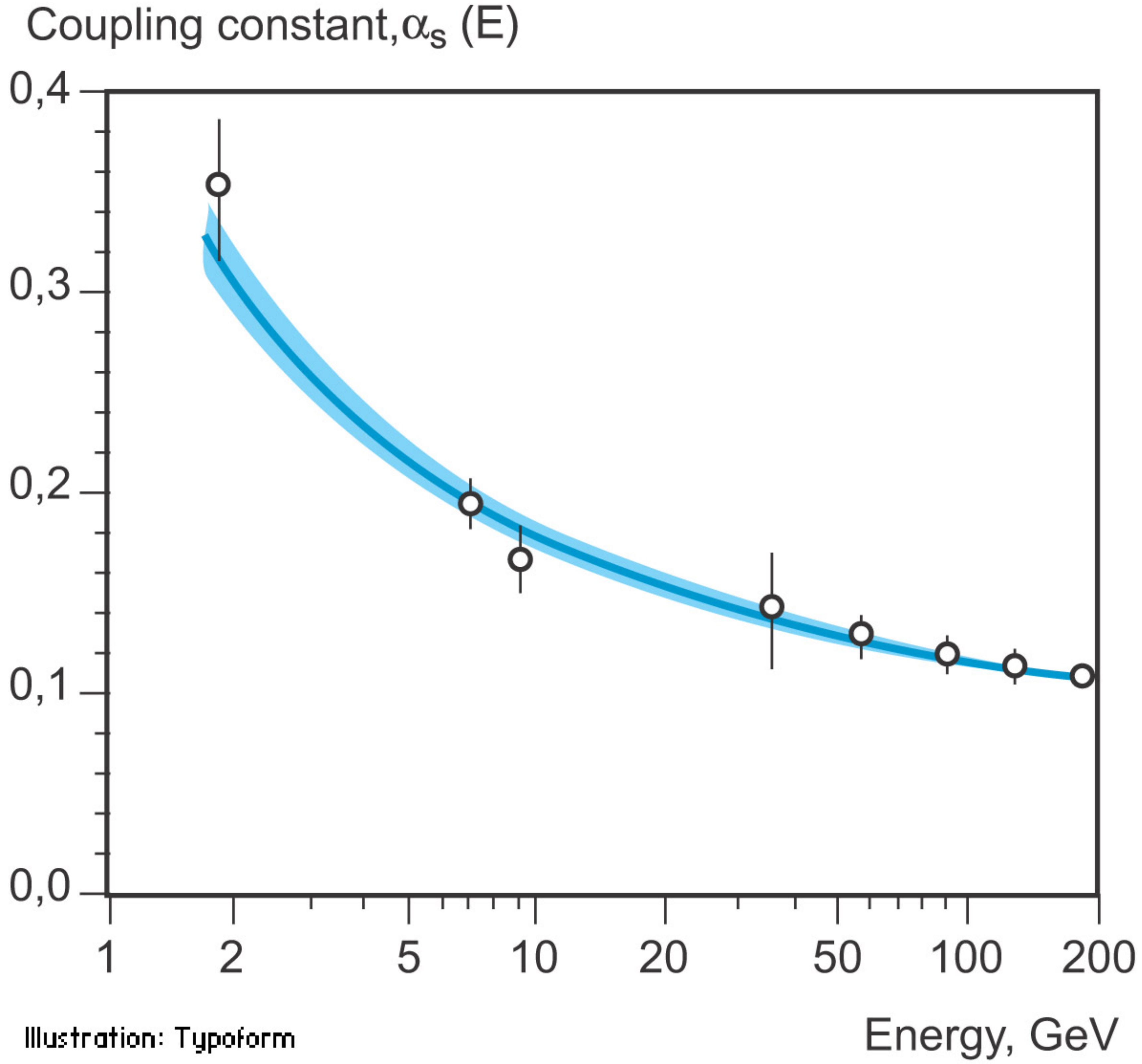

Nobel prize in Physics 2004
(Gross, Politzer, Wilczek)
“for the discovery of asymptotic freedom in the theory of the strong interaction”

nobelprize.org

Running coupling constants

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- measure scattering cross sections and compare them to theoretical predictions - coupling “constants” depend on energy scale dictated by their beta functions - **renormalisation group**

$$\beta_{\alpha_s} = -\left(11 - \frac{2}{3}N_f\right)\frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3)$$

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- Quo vadis, quantum gravity?

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- the actual problem: **predictivity**

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- two loops:

$$\Delta\Gamma^{\text{2-loops}}_{\text{div,OS}} \propto \frac{1}{\epsilon}\int \mathrm{d}^4x\,\sqrt{-g}\,\left[\tilde{a}\,C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\tau\omega}C_{\tau\omega}{}^{\mu\nu}\right]$$

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$$\tilde{a} \neq 0$$

Goroff, Sagnotti ’85, ’86
van de Ven ‘92

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Is GR non-perturbatively renormalisable?

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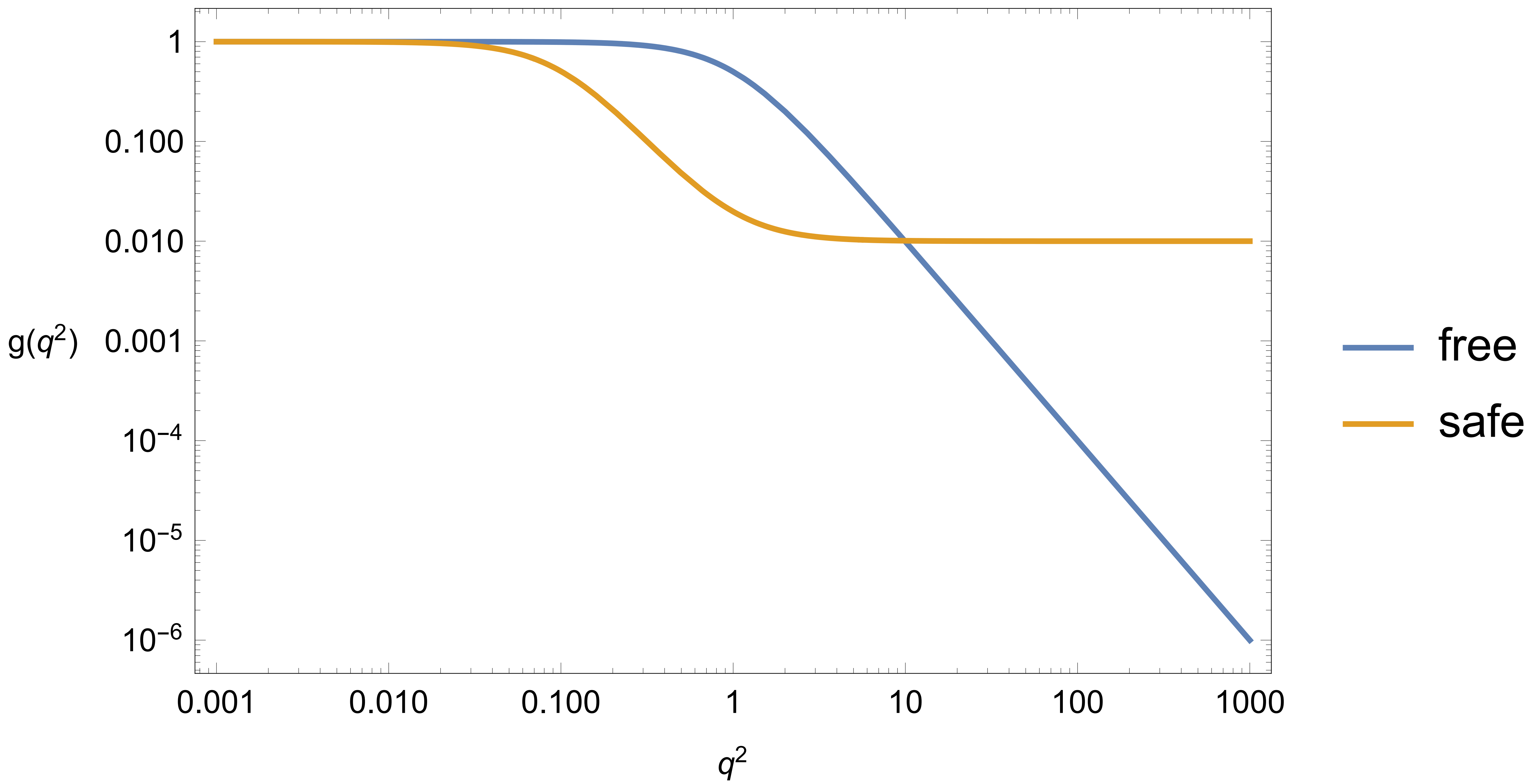

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  - all dimensionless versions of essential couplings approach a finite value at high energies = fixed point
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How to investigate:

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How to investigate: Functional Renormalisation Group

The functional RG

The functional RG

…or: *the working horse of Asymptotic Safety*

Asymptotic Safety via FRG

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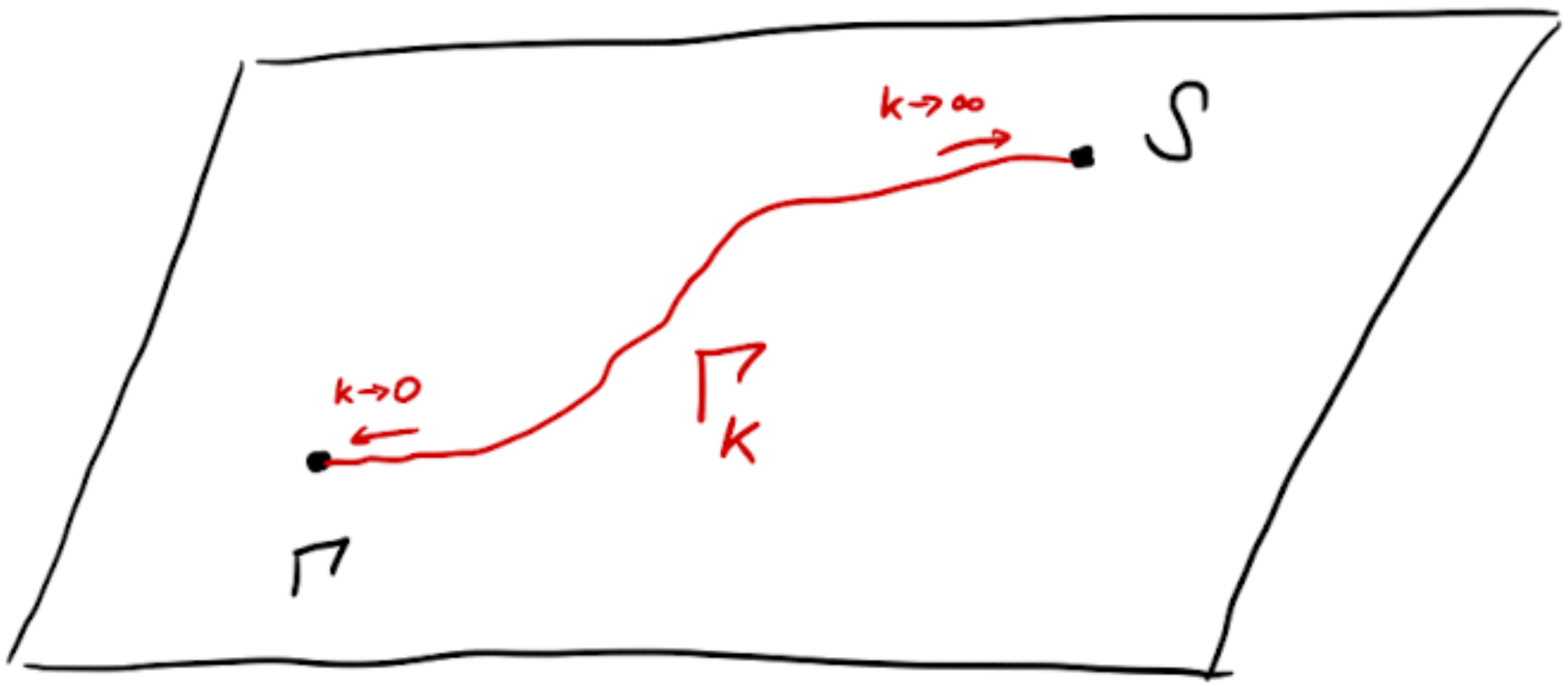

Asymptotic Safety via FRG

- Wilsonian idea of integrating out modes shell by shell
- governed by **exact non-perturbative** RG equation:

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathfrak{R}_k\right)^{-1} k\partial_k \mathfrak{R}_k\right]$$

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- no free dinner: standard implementation uses Euclidean signature

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Asymptotic Safety via FRG

the happy little challenges
(what keeps me up at night)

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The state of the art

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…or: *why I'm not doing string theory (yet)*

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finally published last week,
order now for the low price of 1278.97 EUR!

Two-loop counterterm

Q: What does AS do with the two-loop counterterm?

A. Baldazzi, K. Falls, Y. Kluth, BK 2312.03831

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- approximation:

$$\Gamma_k = \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{g} \left[2\Lambda_k - R + G_{C^3}\, C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\tau\omega} C_{\tau\omega}{}^{\mu\nu} \right]$$

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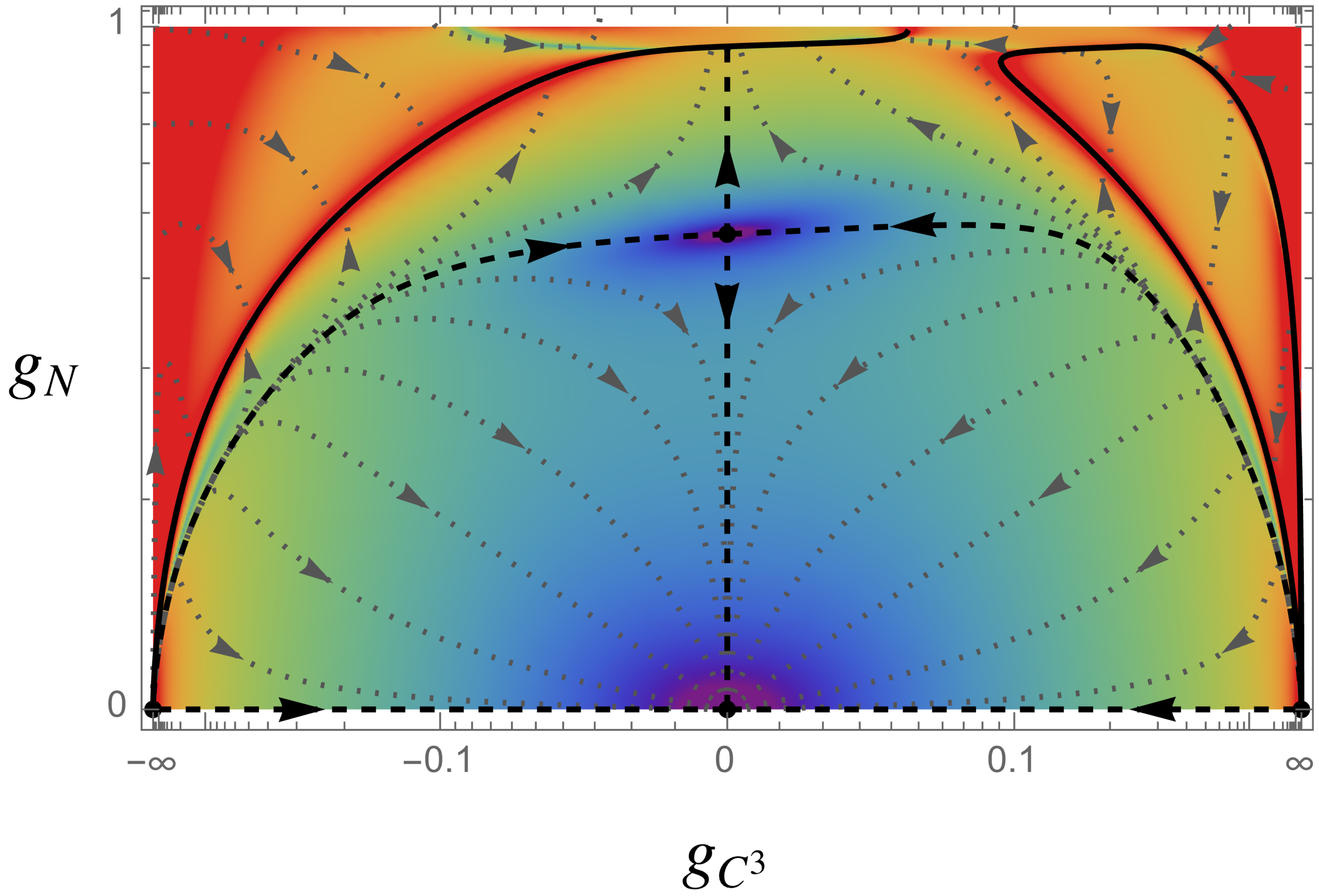

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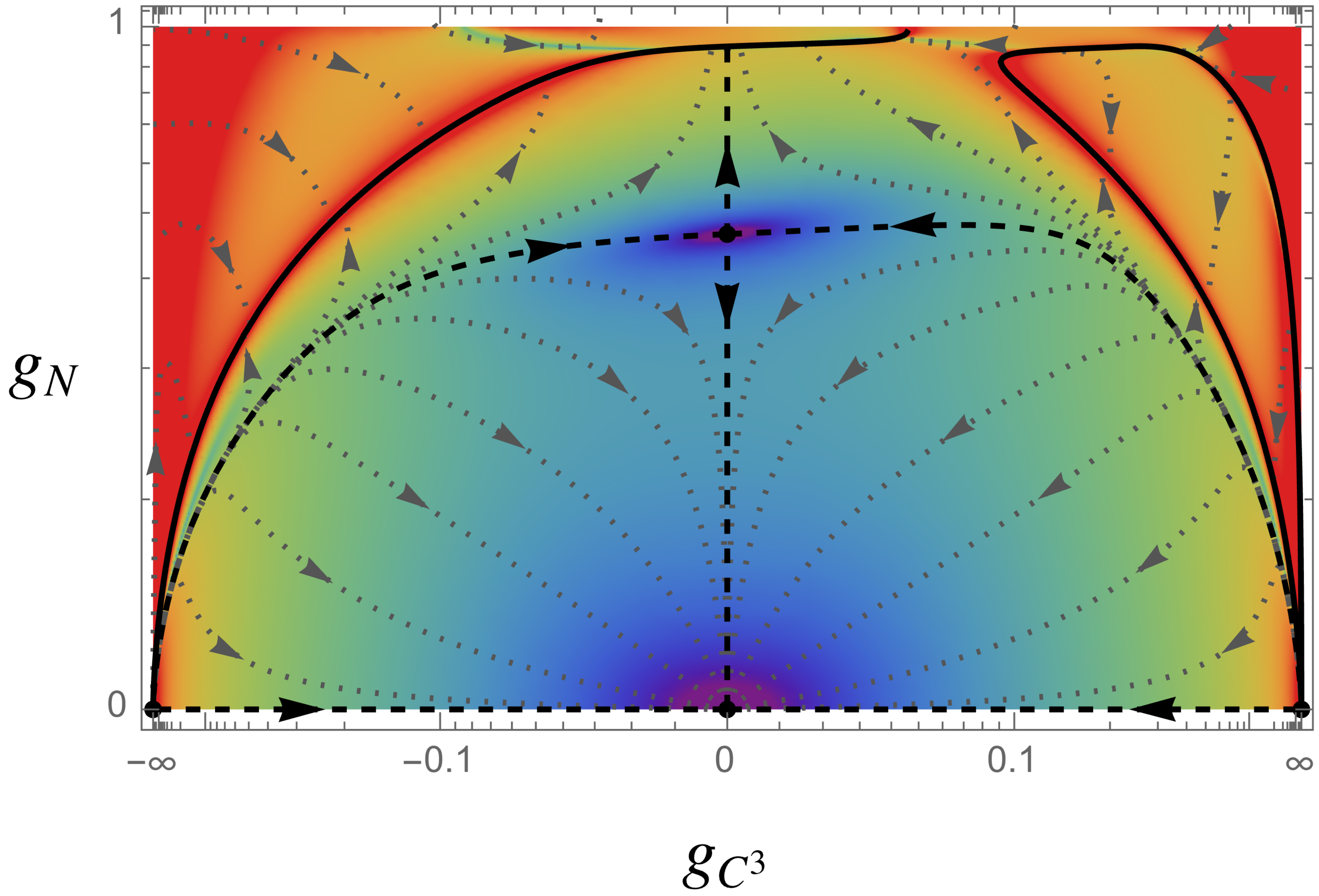

AS tames the two-loop counterterm!

see also H. Gies, BK, S. Lippoldt, F. Saueressig 1601.01800

A. Baldazzi, K. Falls, Y. Kluth, BK 2312.03831

Computational tools and where to apply them

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…or: *what I’m doing for a living*

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathfrak{R}_k\right)^{-1} k\partial_k \mathfrak{R}_k\right]$$

inversion

computing the two-point function

tensor contractions

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathfrak{R}_k\right)^{-1} k\partial_k\mathfrak{R}_k\right]$$

functional trace

Euclidean vs Lorentzian

beta functions: IDEs

computational tools

- computer tensor algebra

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key tools: xAct package (MMA), FORM

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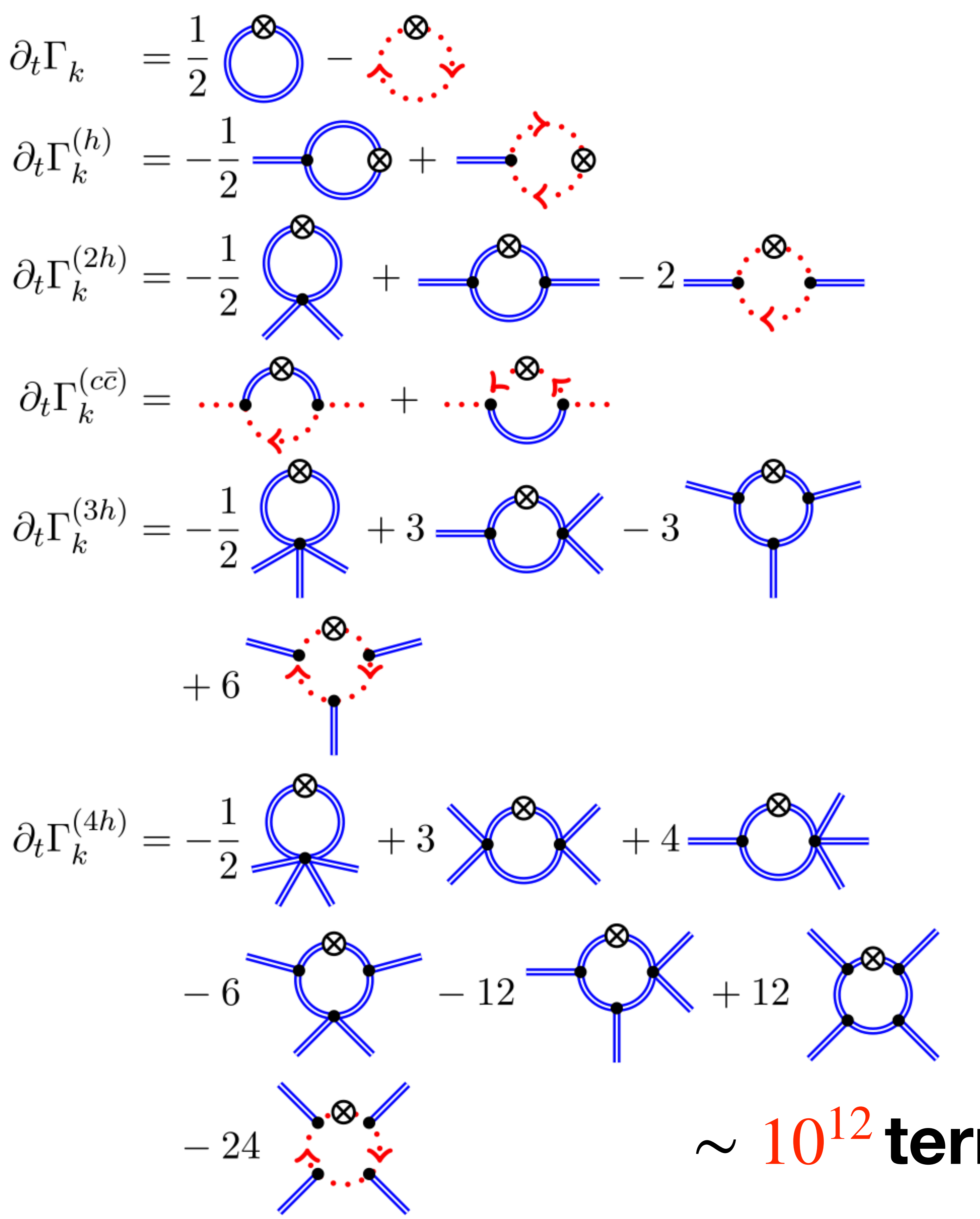

Denz, Pawlowski, Reichert
1612.07315

$\sim 10^{12}$ **terms**

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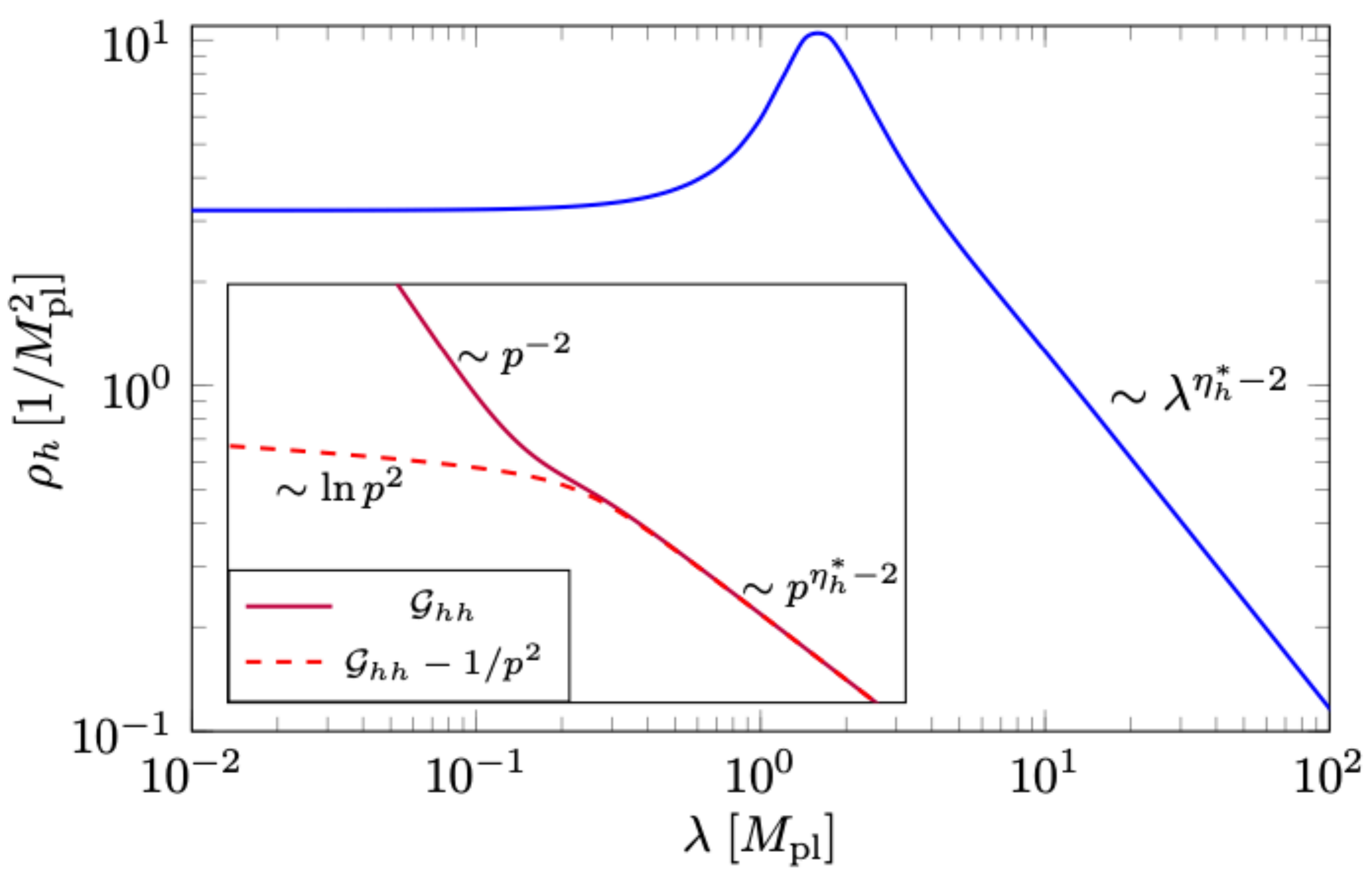

Bonanno, Denz, Pawlowski, Reichert 2102.02217
Fehre, Litim, Pawlowski, Reichert 2111.13232

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Borchardt, BK 1502.07511, 1603.06726

pseudo-spectral methods for high-precision results

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