Jakub Gizbert-Studnicki

in collaboration with

J. Ambjørn, A. Görlich and D. Németh

Causal Dynamical Triangulations From the infrared to the ultraviolet

5th EPS Conference on Gravitation

Prague, 9th December 2024

Is lattice quantum gravity asymptotically safe?
Making contact between causal dynamical triangulations and the functional renormalization group.
Phys. Rev. D 110, 126006 [arXiv:2408.07808]

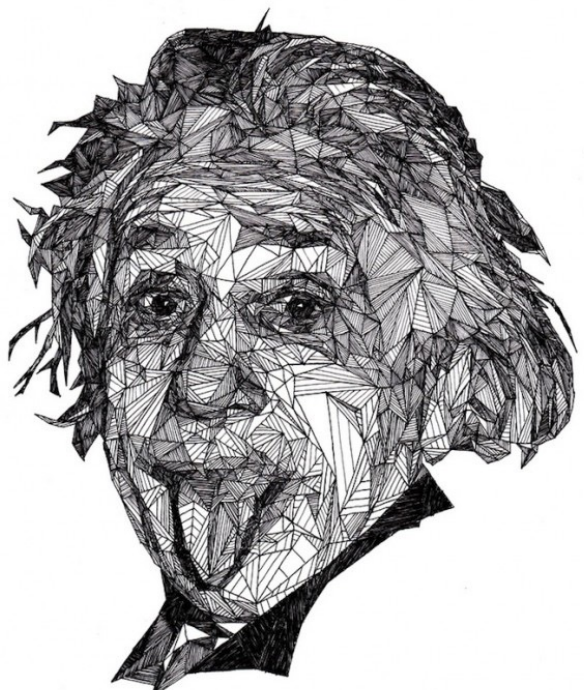
IR and UV limits of CDT and their relations to FRG
[arXiv:2411.02330]

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

UCZELNIA BADAWCZA
INICJATYWA DOSKONAŁOŚCI

Challenges of Quantum Gravity (QG)

- As early as in 1916 Einstein* pointed out that „quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation"
- After more than 100 years a complete, consistent quantum theory of gravity is still missing
- We have a number of interesting but incomplete research programs
  - string theory
  - loop quantum gravity
  - group field theory
  - causal set theory
  - noncommutative geometry
  - asymptotic safety (functional RG flow)
  - lattice QFT approaches (CDT, quantum Regge calc., …)
  - …

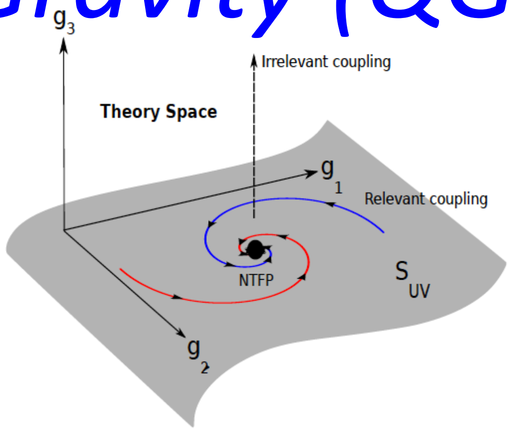


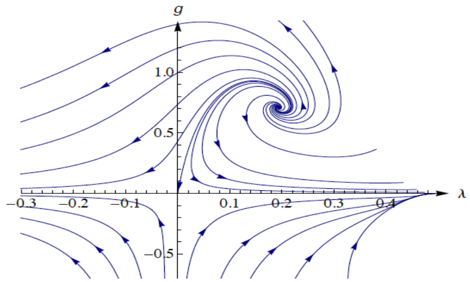
A. Einstein triangulation by J. Bryan

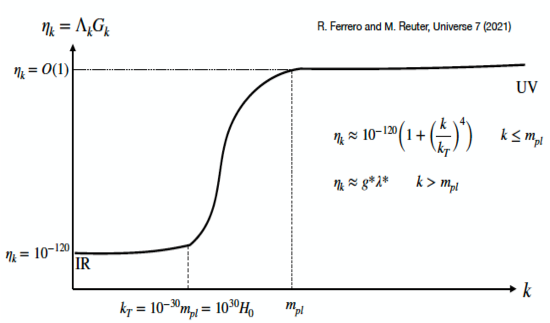
* Sitzungsber. Preuss. Akad. Wiss. Berlin (1916) 688

Challenges of Quantum Gravity (QG)

- ✧ *Lack of experimental guidance*
- ✧ *Conceptual issues: QFT based on Einstein's GR is perturbatively non-renormalizable in D > 2 dim.**
- ✧ *But it can be renormalizable in a non-perturbative regime: asymptotic safety conjecture* S. Weinberg, 1980
  - ✧ *renormalization group flow can lead to a non-Gaussian UV fixed point where QG becomes scale invariant (UV complete)*
- ✧ *Lattice formulation would allow to study a unitary, non-perturbative, background-independent and diffeomorphism-invariant quantum gravity*
  - ✧ *to encode geometry we need a dynamical lattice (DT)*
  - ✧ *UV fixed point should be associated with a 2nd order phase transition*
  - ✧ *one should be able to reproduce semi-classical gravity (IR limit)*
- ✧ *Imposing causal structure is important: CDT (J. Amjørn, J. Jurkiewicz, R. Loll)*

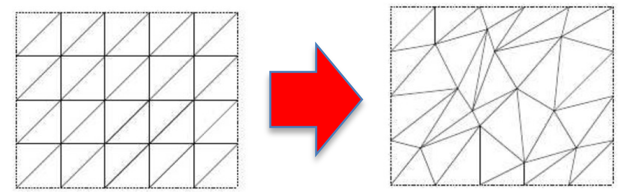




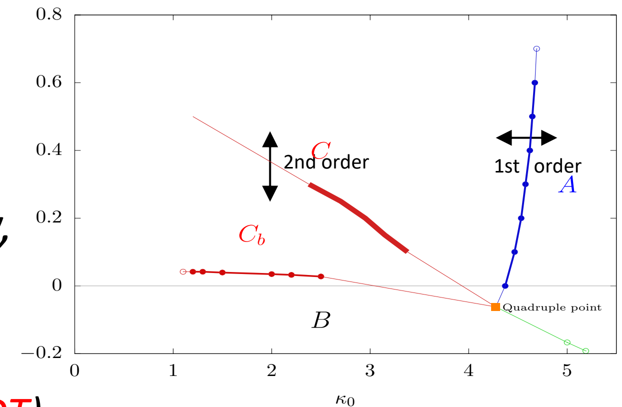


*Renormalizable extensions have problems with unitarity

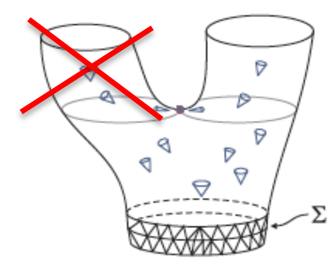
Challenges of Quantum Gravity (QG)



- *Lack of experimental guidance*
- *Conceptual issues: QFT based on Einstein's GR is perturbatively non-renormalizable in D > 2 dim.* *
- *But it can be renormalizable in a non-perturbative regime: asymptotic safety conjecture* S. Weinberg, 1980
  - *renormalization group flow can lead to a non-Gaussian UV fixed point where QG becomes scale invariant (UV complete)*



- *Lattice formulation would allow to study a unitary, non-perturbative, background-independent and diffeomorphism-invariant quantum gravity*
  - *to encode geometry we need a dynamical lattice (DT)*
  - *UV fixed point should be associated with a 2nd order phase transition*
  - *one should be able to reproduce semi-classical gravity (IR limit)*



- *Imposing causal structure is important: CDT (J. Amjørn, J. Jurkiewicz, R. Loll)*

2D: J. Ambjorn, R. Loll, Nucl.Phys. B 536 (1998) 407

3D: J. Ambjorn, J. Jurkiewicz, R. Loll, Phys.Rev.Lett. 85 (2000) 924

4D: J. Ambjorn, J. Jurkiewicz, R. Loll, Nucl.Phys. B610 (2001) 347

*Renormalizable extensions have problems with unitarity

Outline

- ✧ *Causal Dynamical Triangulations*
- ✧ *Phase structure*
- ✧ *Semi-classical phase*
- ✧ *Functional Renormalization Group*
- ✧ *RG flow on the lattice (ϕ^4 example)*
- ✧ *RG flow in CDT*
- ✧ *Conclusions*

Causal Dynamical Triangulations

- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*
- *One has to give a precise meaing to:*
  - *what class of geometries g should be included in the PI*
  - *what (classical) action $S_{\rm grav}$ should be used*
  - *which symmetries (GR diffeomorhisms ?) should be preserved and how to do that*
  - *how to compute the PI in practice*
  - *(how to include matter fields)*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\rm grav}[g])$$

← (Lorenzian) geometries

Causal Dynamical Triangulations

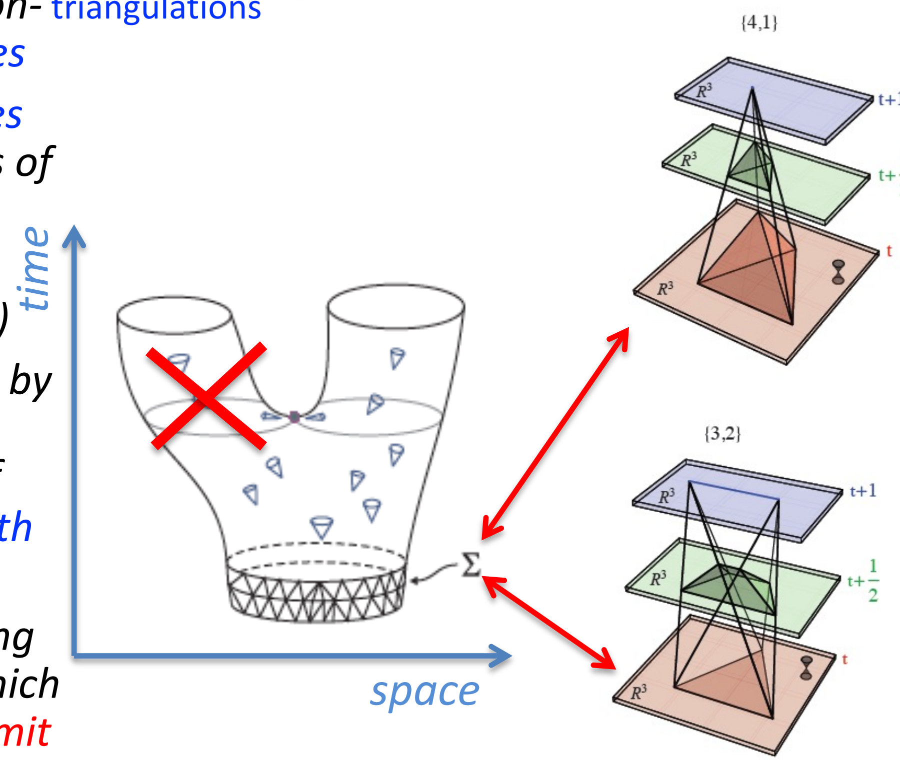
- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*
- *CDT (quantum) geometries:*
  - *In classical GR one deals with smooth (pseudo-)Riemannian manifolds*
  - *But in the PI one should also include non-smooth continuous (Lorenz.) geometries*
  - *Causality: globally hypebolic spacetimes which can be foliated into spacial slices of equal cosmological proper time*
  - *We fix the topology of the manifold (in here we will use the topology: S^3xS^1)*
  - *Geometries in the PI are approximated by piecewise linear simplicial manifolds (triangulations) built from two kinds of identical (internally flat) 4-simplices with fixed edge lengths (l_s)*
  - *As in ordinary lattice QFT: lattice spacing (l_s) plays a role of the UV cutoff l_s^{-1} which should be removed in the continuum limit ($l_s \to 0$, $N_4 \to \infty$)*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\text{grav}}[g])$$

← (Lorenzian) geometries

$$Z_{CDT}^{(L)} = \sum_T \frac{1}{C_T} \exp(i\, S_R[T])$$

causal → triangulations; ← # symmetries of T



Causal Dynamical Triangulations

- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\text{grav}}[g])$$

← (Lorenzian) geometries

- *CDT action & diffeom. symmetry:*
  - *We use the Einstein-Hilbert action*
  - *For a piesewise linear simplicial manifold (triangulation) it takes the form of the Regge action*

T. Regge, Nuovo Cim. A19 (1961) 558

$$Z_{CDT}^{(L)} = \sum_T \frac{1}{C_T} \exp(i\, S_R[T])$$

causal triangulations → ; ← # symmetries of T

$$S_{\text{grav}} = \frac{1}{16\pi G} \int_M d^4x \sqrt{-\det g}\,(R - 2\Lambda)$$

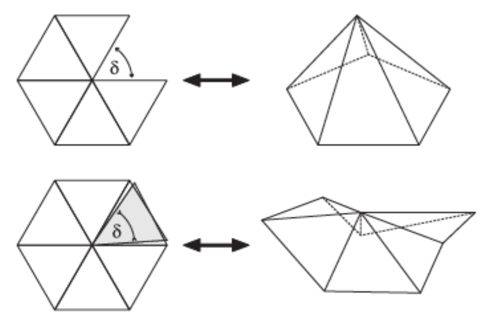
  - *Curvature is defined by deficit angles around D-2 dim. „hinges" (triangles in 4D)*
  - *Regge's formulation uses only geometric invariants (geodesic edge lengths and deficit angles) making it coordiante free and therefore manifestly diffeo-morphism invariant*

4-simplices; # {4,1} 4-simpl.; # vertices

$$S_R = -k_0 N_0 + K_4 N_4 + \Delta \left(N_4^{(4,1)} - 6 N_0 \right)$$

$1/G$; Λ ; α $(l_t^2 = -\alpha l_s^2)$

  - *In CDT one has only two types of bulding blocks with fixed edge lenghts thus the Regge action becomes very simple*



Causal Dynamical Triangulations

- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*
- *CDT computations:*
  - *In order to investigate the 4D PI one has to use Monte Carlo (MC) simulations*
  - *MC requires Euclidean formulation (Wick's rotation)*
  - *Due to the imposed time foliation each CDT Lorentzian geometry can be Wick-rotated to an Euclidean geom.*
  - *A the level of the Regge action Wick's rotation is achieved by an analytical continuation ($\alpha \to -\alpha$): general form of the action S_R is the same in (L) and (E)*
  - *MC algorithm performs a Markov chain in the space of triangulations by local moves* wchich change the geometry (in 4 dim CDT: 4 moves & 4 anti-moves)*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\text{grav}}[g])$$

(Lorenzian) geometries

$$Z_{CDT}^{(L)} = \sum_T \frac{1}{C_T} \exp(i\, S_R[T])$$

causal triangulations; # symmetries of T

$$S_{\text{grav}} = \frac{1}{16\pi G} \int_M d^4x \sqrt{-\det g}\,(R - 2\Lambda)$$

4-simplices; # {4,1} 4-simpl.; # vertices

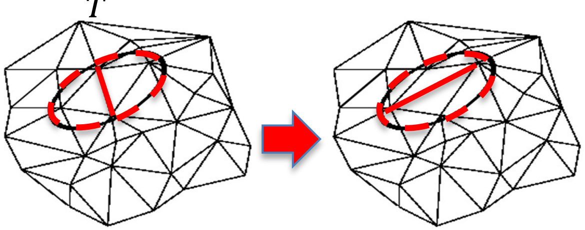
$$S_R = -k_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

$k_0 \leftrightarrow 1/G$, $K_4 \leftrightarrow \Lambda$, $\Delta \leftrightarrow \alpha$ $(l_t^2 = \alpha l_s^2)$

$\alpha \to -\alpha$

$$Z_{CDT}^{(E)} = \sum_T \frac{1}{C_T} \exp(-S_R[T])$$

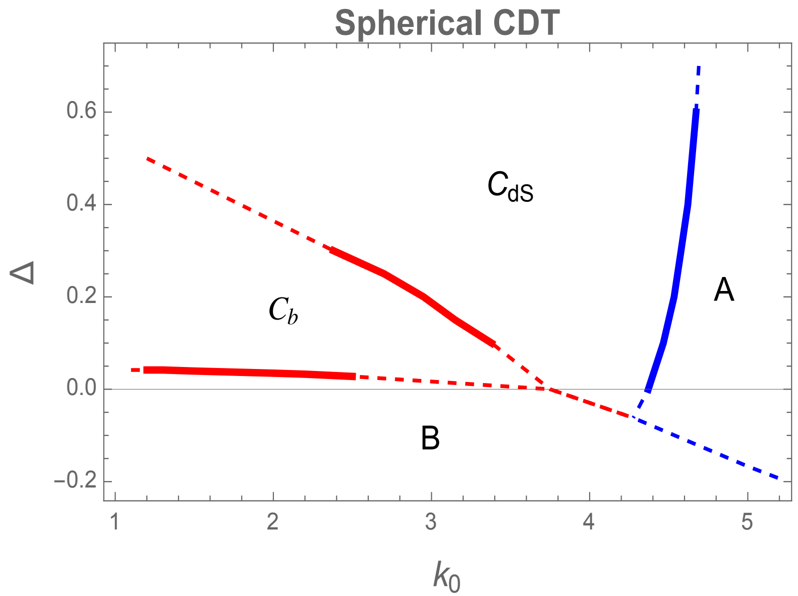
* Example of a („flip”) move in 2 dim. DT:



Phase structure

Phase structure

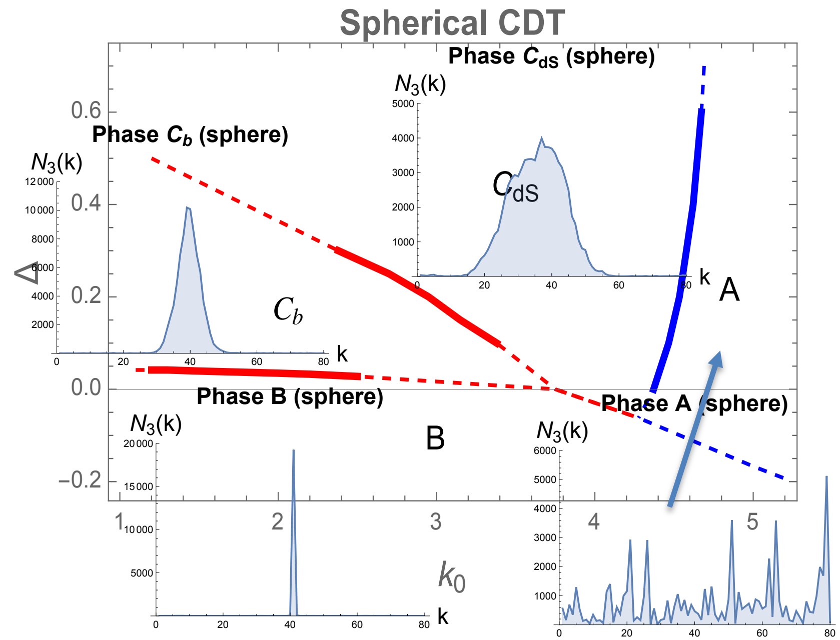
- ✧ *Phase structure:*
  - ✧ *We perform MC simulations with fixed N_4 The cosmological constant K_4 is tuned to N_4 and we effectively have two coupling constants: k_0 and Δ*
  - ✧ *Four phases (A, B, C_{dS}, C_b) of different generic geometries were discovered*
  - ✧ *The observable: physical 3-volume of spatial layers: $V_3(t_i) \propto N_3(i) \cdot l_s^3$*
  - ✧ *The difference between phases C_{dS} and C_b is captured by effective dimensions*
  - ✧ *One observes 1st order (blue lines) and 2nd order (red lines) phase transitions*



$$S_R = -k_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

Phase structure

- ✧ *Phase structure:*
  - ✧ *We perform MC simulations with fixed N_4. The cosmological constant K_4 is tuned to N_4 and we effectively have two coupling constants: k_0 and Δ*
  - ✧ *Four phases (A, B, C_{dS}, C_b) of different generic geometries were discovered*
  - ✧ *The observable: physical 3-volume of spatial layers:* $V_3(t_i) \propto N_3(i) \cdot l_s^3$
  - ✧ *The difference between phases C_{dS} and C_b is captured by effective dimensions*
  - ✧ *One observes 1st order (blue lines) and 2nd order (red lines) phase transitions*



lattice spacing in spatial directions

# of tetrahedra at lattice time i

physical proper time $t_i = i \cdot l_t$

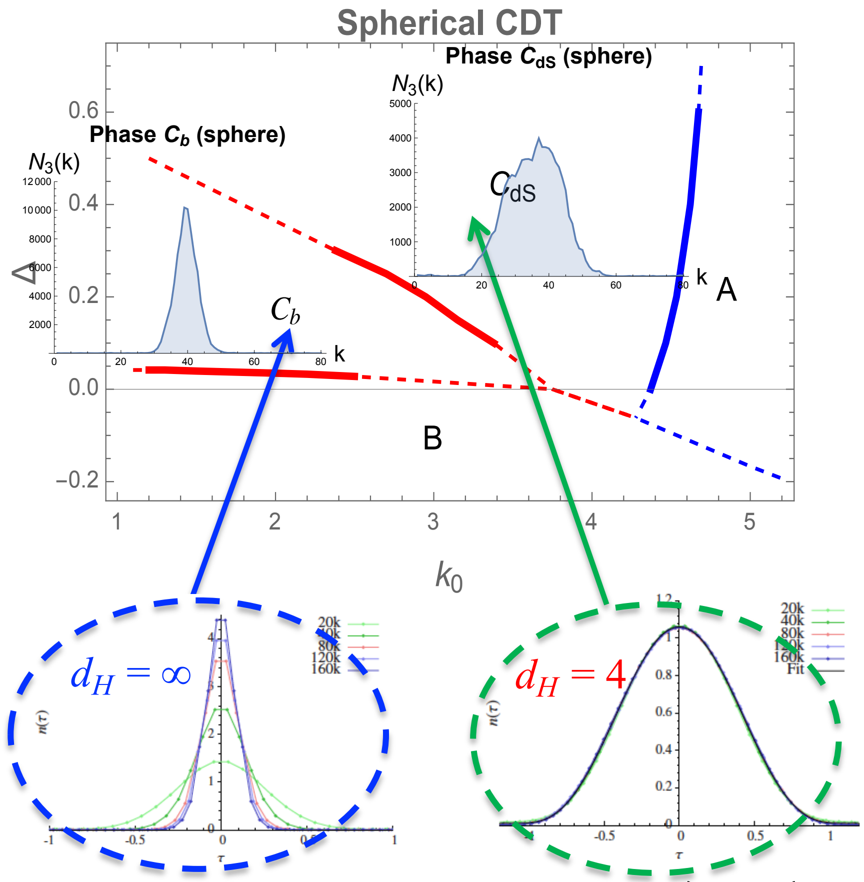
lattice time

lattice spacing in time direction

* *This is formally not a gauge invariant observable if we insist to keep full 4-d diffeo. symetry*

Phase structure

- ✧ *Phase structure:*
  - ✧ *We perform MC simulations with fixed N_4. The cosmological constant K_4 is tuned to N_4 and we effectively have two coupling constants: k_0 and Δ*
  - ✧ *Four phases (A, B, C_{dS}, C_b) of different generic geometries were discovered*
  - ✧ *The observable: physical 3-volume of spatial layers: $V_3(t_i) \propto N_3(i) \cdot l_s^3$*
  - ✧ *The difference between phases C_{ds} and C_b is captured by effective dimensions*
  - ✧ *One observes 1st order (blue lines) and 2nd order (red lines) phase transitions*



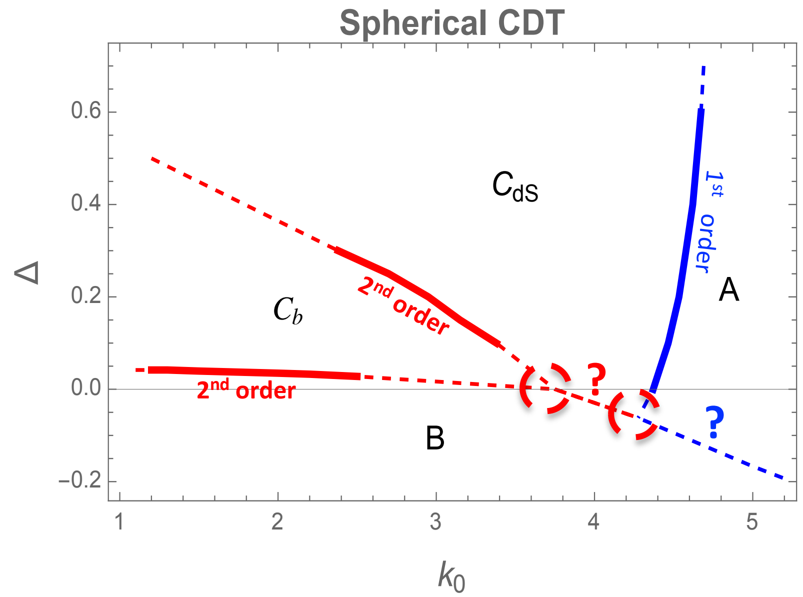
Hausdorff dimension: rescaled average volume profiles (scaling for $d_H = 4$)

$$\langle N_3(i) \rangle \rightarrow \frac{\langle N_3(i) \rangle}{N_4^{1-1/d_H}}$$

$$i \rightarrow \frac{i}{N_4^{1/d_H}}$$

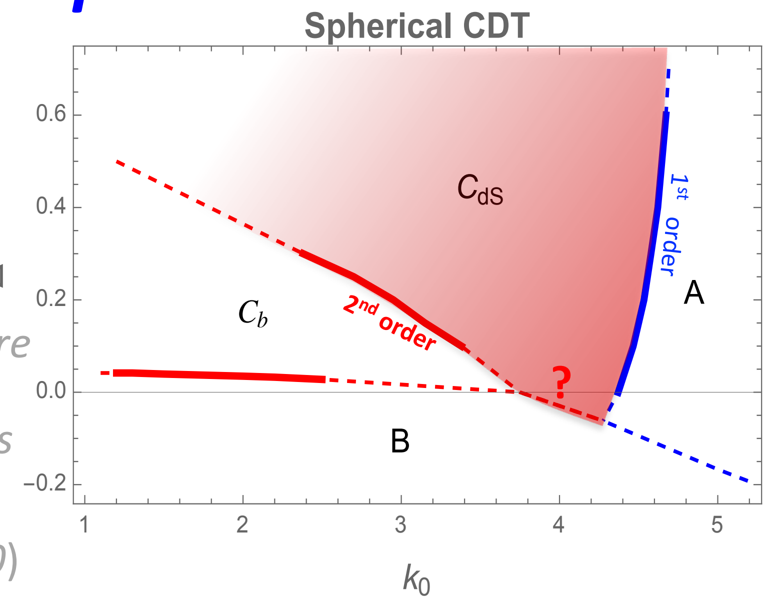
Phase structure

- *Phase structure:*
  - *We perform MC simulations with fixed N_4. The cosmological constant K_4 is tuned to N_4 and we effectively have two coupling constants: k_0 and Δ*
  - *Four phases (A, B, C_{dS}, C_b) of different generic geometries were discovered*
  - *The observable: physical 3-volume of spatial layers: $V_3(t_i) \propto N_3(i) \cdot l_s^3$*
  - *The difference between phases C_{dS} and C_b is captured by effective dimensions*
  - *One observes 1st order (blue lines) and 2nd order (red lines) phase transitions*

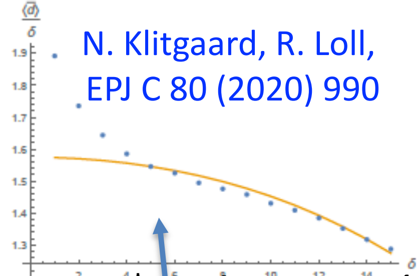


Semi-classical phase

Semi-classical phase



- *Phase* C_{ds} *(de Sitter phase) has good semi-classical properties !*
- *Effective dimensions consistent with d = 4*
- *Dynamicaly emerging background geom.* ◁
  - $\langle N_3(i)\rangle$ *profile of elongated* $(\widetilde{\omega} \neq \omega_0)$ *4-sphere*
  - *renormalizing* $l_t \rightarrow l_t = l_s \left(\frac{\omega_0}{\widetilde{\omega}}\right)^{4/3}$ *one obtains symmetric* S^4*, i.e., classicaly:* (Euclidean) de Sitter universe (max. sym. space with $\Lambda>0$)
  - local (average) curvature* consistent with S^4
  - ~homogenous and isotropic** on large scales
- *Minisuperspace behaviour of the scale factor*
  - *From quantum fluctuations of* $N_3(i)$ *one can recover the effective action of the scale factor*
  - *The effective action is consistent with the MS action (spatial homogeneity and isotropy)*
- *This was „derived" from first principles !*

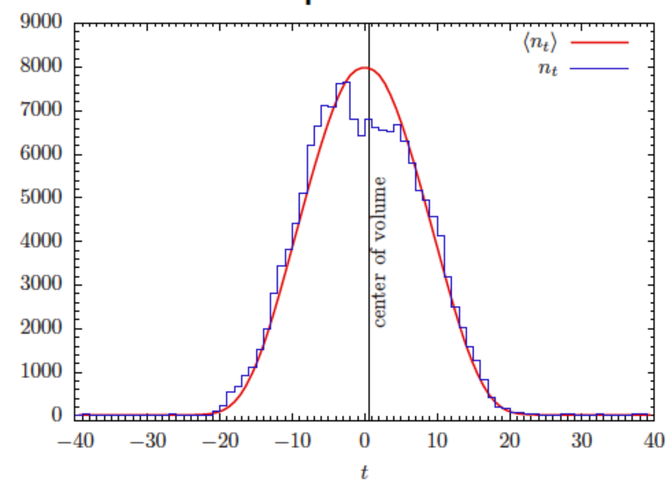


N. Klitgaard, R. Loll, EPJ C 80 (2020) 990

Semi-classical phase

- ✧ *Phase C_{dS} (de Sitter phase) has good semi-classical properties !*
- ✧ *Effective dimensions consistent with d = 4*
- ✧ *Dynamicaly emerging background geom.*
  - ✧ $\langle N_3(i)\rangle$ *profile of elongated* ($\widetilde{\omega} \neq \omega_0$) *4-sphere*
  - ✧ *renormalizing* $l_t \to l_t = l_s \left(\frac{\omega_0}{\widetilde{\omega}}\right)^{4/3}$ *one obtains symmetric S^4, i.e., classicaly:* (Euclidean) de Sitter universe (max. sym. space with *Λ>0*)
  - ✧ local (average) curvature* consistent with S^4
  - ✧ ~homogenous and isotropic** on large scales
- ✧ *Minisuperspace behaviour of the scale factor*
  - ✧ *From quantum fluctuations of $N_3(i)$ one can recover the effective action of the scale factor*
  - ✧ *The effective action is consistent with the MS action (spatial homogeneity and isotropy)*
- ✧ *This was „derived" from first principles !*

Spherical



$$\langle N_3(i)\rangle = \frac{3}{4} N_4 \frac{1}{\widetilde{\omega} N_4^{1/4}} \cos^3\left(\frac{i}{\widetilde{\omega} N_4^{1/4}}\right)$$

$V_3(t_i) = N_3(i) l_s^3$ $\qquad t = i\, l_t$

$$\langle V_3(t_i)\rangle = \frac{3}{4} V_4 \frac{1}{\omega_0 V_4^{1/4}} \cos^3\left(\frac{t_i}{\omega_0 V_4^{1/4}}\right)$$

$\omega_0 = \left(\frac{3}{8\pi^2}\right)^{1/4}$ $\qquad V_4 = l_s^4 \left(\frac{\omega_0}{\widetilde{\omega}}\right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$

Λ is fixing V_4

$$ds^2 = dt^2 + a^2(t) d\Omega_3^2$$

$$V_3(t) \propto a^3(t)$$

$$S_{HE} = -\frac{1}{16\pi G}\int d^4x \sqrt{\det g}\,(R - 2\Lambda)$$

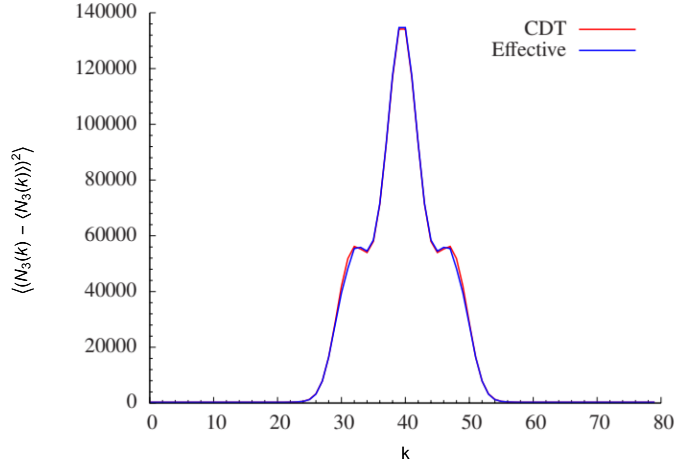
* Def. by Quantum Ricci Curvature: N. Klitgaard, R. Loll, PRD 97 (2018) 046008

** Homogeneity measures in CDT: R. Loll , A. Silva, PRD 107 (2023) 086013

Semi-classical phase

- *Phase C_{dS} (de Sitter phase) has good semi-classical properties !*
- *Effective dimensions consistent with d = 4*
- *Dynamicaly emerging background geom.*
  - $\langle N_3(i)\rangle$ *profile of elongated ($\tilde{\omega} \neq \omega_0$) 4-sphere*
  - *renormalizing* $l_t \to l_t = l_s \left(\frac{\omega_0}{\tilde{\omega}}\right)^{4/3}$ *one obtains symmetric S^4, i.e., classicaly:* (Euclidean) de Sitter universe (max. sym. space with *Λ>0*)
  - local (average) curvature* consistent with S^4
  - ~homogenous and isotropic** on large scales
- *Minisuperspace behaviour of the scale factor*
  - *From quantum fluctuations of* $N_3(i)$ *one can recover the effective action of the scale factor*
  - *The effective action is consistent with the MS action (spatial homogeneity and isotropy)*
- *This was „derived" from first principles !*

Spherical



$$S = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{\mu} N_3(i)^{1/3} \right)$$

Nucl. Phys. B 849 (2011) 144

agrees with Hartle–Hawking „noboundary" proposal

$\mu_0 = 9(2\pi^2)^{2/3}$

$$S_{MS} = \frac{1}{24\pi G} \int dt \left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu_0 V_3(t)^{1/3} \right)$$

$$24\pi G = \left(\frac{\tilde{\omega}}{\omega_0}\right)^{4/3} \tilde{\Gamma}\, l_s^2$$

$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}}\right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

$$ds^2 = dt^2 + a^2(t) d\Omega_3^2$$

$$V_3(t) \propto a^3(t)$$

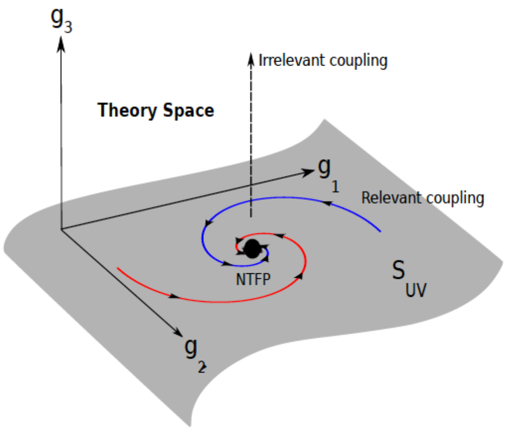
Λ is fixing V_4

$$S_{HE} = \frac{1}{16\pi G} \int d^4x \sqrt{\det g}\,(R - 2\Lambda)$$

Functional Renormalization Group

Functional Renormalization Group

- *As CDT the FRG is also based on non-petrurbative QFT framework to quantize gravity*
  - *Consider a (potenitally ∞ dim.) space of all effective actions* of QG (or in practice their truncations)*
  - Alternatively one has a space of scale-dependent dimensionless couplings related to operators appering in the effective actions
  - *Solve RG flow equations (based on β-functions) of the couplings with the cutoff scale k*
  - *Find RG trajectories linking IR ($k \to 0$) and UV ($k \to \infty$) fixed points ($\beta = 0$) of the RG flow*



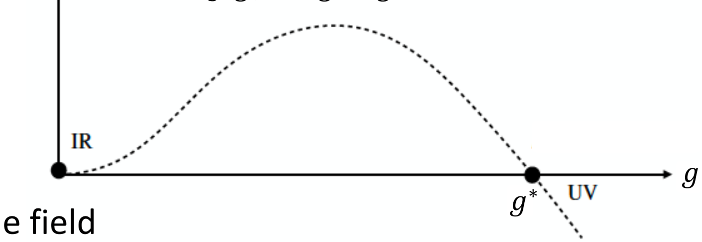
- *Asymptotic Safety conjecture (S. Weinberg)*
  - *Scale invariance of the UVFP imposes strong constraints on most operators (couplings)*
  - *On RG flow trajectories leading from IR to UV fixed points there is only a finite numer of relevant operators (finite dim. subspace of relevant couplings)*
  - *Even though the values of the couplings in the UV limit are not small one one can get a predictive theory of QG at all scales (nonperturbative renormalizability)*
  - *There is growing evidence from FRG in favour of AS*

Close to UVFP:

$$k_{\text{cutoff}} \sim 1/l_s$$

$$-l_s \frac{dg}{dl_s} = \beta(g) \approx \beta'(g^*)(g - g^*)$$

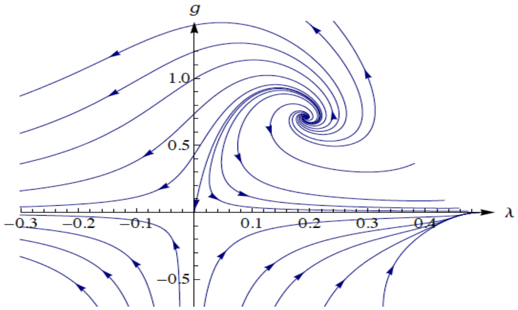
$$l_s(g) \propto |g - g^*|^{-1/\beta'(g^*)}$$



* Effective actions govern the expectation value and quantum fluctuations of the field

Functional Renormalization Group

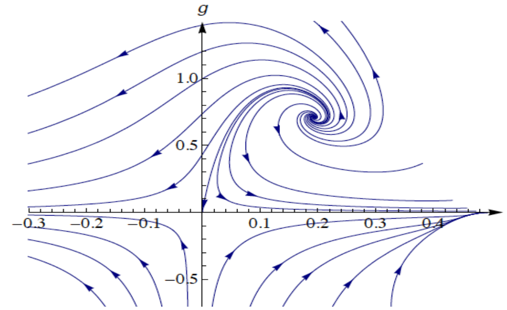
- Making contact between FRG and CDT: $S_k = \frac{1}{16\pi G_k}\int d^4x\sqrt{\det g}(R - 2\Lambda_k) + gauge + ghost$
  - In CDT one measures the (minisuperspace) Einstein-Hilbert effective action
  - Therefore in FRG we take the simplest Einstein-Hilbert truncation of the (Euclidean) effective actions with two scale-dependent couplings: G_k , Λ_k
  - An extremum of the E-H effective action is a de Sitter universe (the four-sphere S^4) with a 4-volume given by the cosmological constant $V_4 \propto \Lambda_k^{-2}$
  - As in CDT we measure only a behaviour of the scale factor $a(t)$ (or the 3-volume $V_3(t)$) we will also consider only minisuperspace fluctuations
  - The (relative) fluctuations are goverened by a dimensionless effective coupling $g_{ef}^2 \propto G_k\Lambda_k$



- In FRG one has both the IR and the UV fixed points
  - In the IR ($k \to 0$): $G_k\Lambda_k \to 0$ as $G_k \to G_0 \approx G_N, \Lambda_k \to 0$ so one recovers semiclassical universe with $V_4 \to \infty$
  - In the UV ($k \to \infty$): $G_k\Lambda_k \to g^*\lambda^* \sim 1$ as $G_k \to g^*k^{-2} \to 0$, $\Lambda_k \to \lambda^* k^2 \to \infty$ so $V_4 \to 0$

Functional Renormalization Group

- Making contact between FRG and CDT: $S_k = \frac{1}{16\pi G_k}\int d^4x\sqrt{\det g}(R - 2\Lambda_k) + gauge + ghost$
  - In CDT one measures the (minisuperspace) Einstein-Hilbert effective action
  - Therefore in FRG we take the simplest Einstein-Hilbert truncation of the (Euclidean) effective actions with two scale-dependent couplings: G_k , Λ_k
  - An extremum of the E-H effective action is a de Sitter universe (the four-sphere S^4) with a 4-volume given by the cosmological constant $V_4 \propto \Lambda_k^{-2}$
  - As in CDT we measure only a behaviour of the scale factor $a(t)$ (or the 3-volume $V_3(t)$) we will also consider only minisuperspace fluctuations
  - The (relative) fluctuations are goverened by a dimensionless effective coupling $g_{ef}^2 \propto G_k\Lambda_k$
- In FRG one has both the IR and the UV fixed points
  - In the IR ($k \to 0$): $G_k\Lambda_k \to 0$ as $G_k \to G_0 \approx G_N$, $\Lambda_k \to 0$ so one recovers semiclassical universe with $V_4 \to \infty$
  - In the UV ($k \to \infty$): $G_k\Lambda_k \to g^*\lambda^* \sim 1$ as $G_k \to g^* k^{-2} \to 0$, $\Lambda_k \to \lambda^* k^2 \to \infty$ so $V_4 \to 0$



$$V_3(t) = \frac{3}{4} V_4 \frac{1}{\omega_0 V_4^{1/4}} \cos^3\left(\frac{t}{\omega_0 V_4^{1/4}}\right)$$

$$S_{MS} = \frac{1}{24\pi G_k}\int dt\left(\frac{\dot V_3(t)^2}{V_3(t)} + \mu_0 V_3(t)^{1/3}\right)$$

$$\int dt\, V_3(t) = V_4 \propto \Lambda_k^{-2}$$

$$v_3 \equiv V_3/V_4^{3/4} \qquad s \equiv t/V_4^{1/4}$$

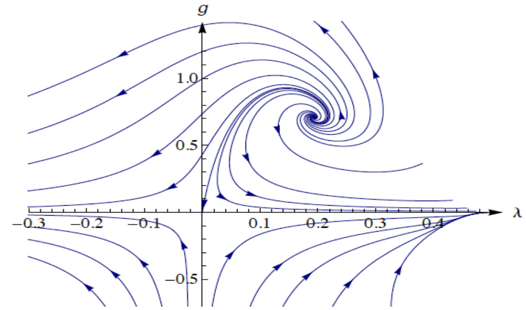
$$v_3(t) = \frac{3}{4}\frac{1}{\omega_0}\cos^3\left(\frac{t}{\omega_0}\right)$$

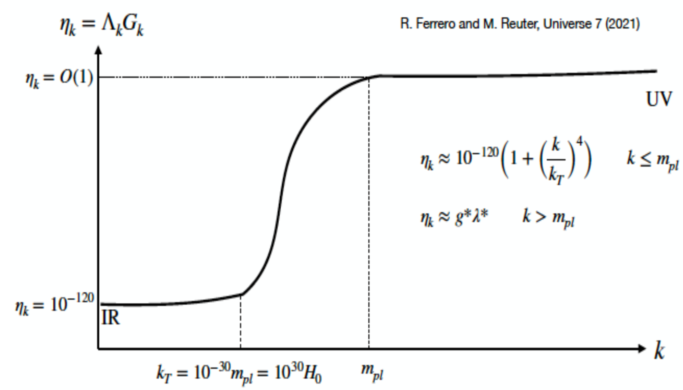
$$S_{MS} = \frac{\sqrt{6}}{4\, G_k\Lambda_k}\int ds\left(\frac{\dot v_3(t)^2}{v_3(t)} + \mu_0 v_3(t)^{1/3}\right)$$

$$\int ds\, v(s) = 1$$

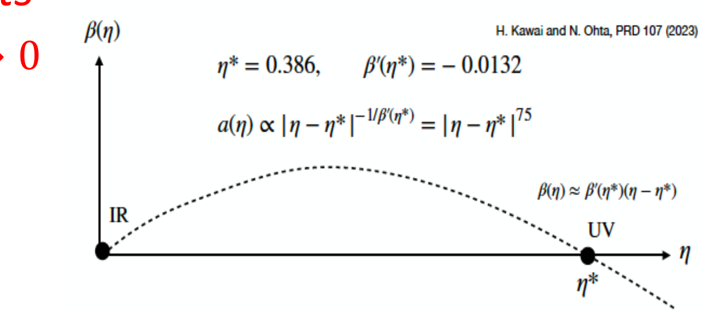
Functional Renormalization Group

- *Making contact between FRG and CDT:* $S_k = \frac{1}{16\pi G_k}\int d^4x\sqrt{\det g}(R - 2\Lambda_k) + gauge + ghost$
  - *In CDT one measures the (minisuperspace) Einstein-Hilbert effective action*
  - *Therefore in FRG we take the simplest Einstein-Hilbert truncation of the (Euclidean) effective actions with two scale-dependent couplings: G_k , Λ_k*
  - *An extremum of the E-H effective action is a de Sitter universe (the four-sphere S^4) with a 4-volume given by the cosmological constant $V_4 \propto \Lambda_k^{-2}$*
  - *As in CDT we measure only a behaviour of the scale factor $a(t)$ (or the 3-volume $V_3(t)$) we will also consider only minisuperspace fluctuations*
  - *The (relative) fluctuations are goverened by a dimensionless effective coupling $g_{ef}^2 \propto G_k\Lambda_k$*





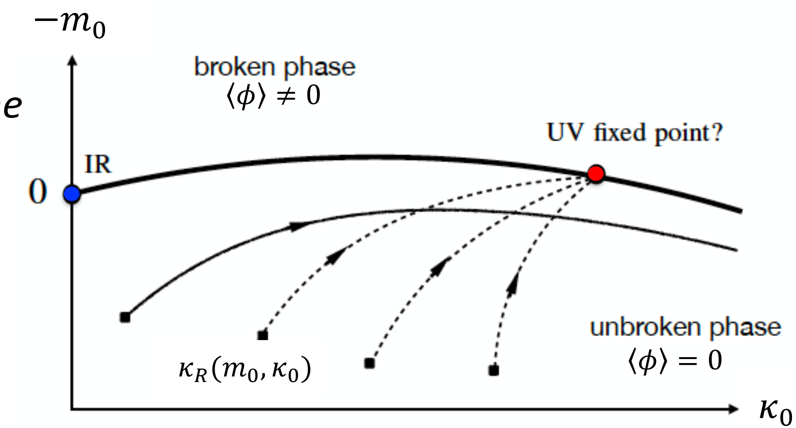
- *In FRG one has both the IR and the UV fixed points*
  - *In the IR ($k \to 0$): $G_k\Lambda_k \to 0$ as $G_k \to G_0 \approx G_N$, $\Lambda_k \to 0$ so one recovers semiclassical universe with $V_4 \to \infty$*
  - *In the UV ($k \to \infty$): $G_k\Lambda_k \to g^*\lambda^* \sim 1$ as $G_k \to g^*k^{-2} \to 0$, $\Lambda_k \to \lambda^*k^2 \to \infty$ so $V_4 \to 0$*



RG flow on the lattice (ϕ^4 example)

RG flow on the lattice (ϕ^4 example)

- *4D ϕ^4 (lattice) QFT example**
  - *2 dimensionless bare couplings: m_0, κ_0*
  - *for each choice of m_0, κ_0 one can compute the renormalized m_R, κ_R and the correl. length ξ*
  - *physical correl. length $\xi_{ph} = m_R^{-1} = \xi\, l_s$*
  - one can find RG flow where $\kappa_R, m_R = const.$
  - *there is a phase transition (where $\xi \to \infty$ so following the RG flow trajectory $l_s \to 0$)*
- *The UV limit*
  - *we approach the phase transition ($\xi \to \infty$) keeping the renormalized coupling κ_R fixed*
  - *in order to do that we have to tune the bare coupling κ_0*
- *The IR limit*
  - *we approach the phase transition ($\xi \to \infty$) keeping the bare coupling κ_0 fixed*
  - *we cross the $\kappa_R = const$ RG trajectories in the direction of $\kappa_R \to \kappa_R^{ir}$*



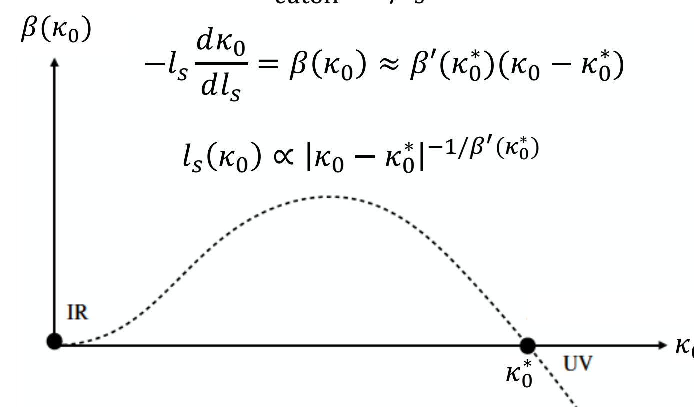
$$L = \left(\partial_\mu \phi\right)^2 + m_o \phi^2 + \kappa_0 \phi^4$$

$$\kappa_R \propto \Gamma_4(p_i = 0; m_0, \kappa_0)$$

$$k_{\text{cutoff}} \sim 1/l_s$$

$$-l_s \frac{d\kappa_0}{dl_s} = \beta(\kappa_0) \approx \beta'(\kappa_0^*)(\kappa_0 - \kappa_0^*)$$

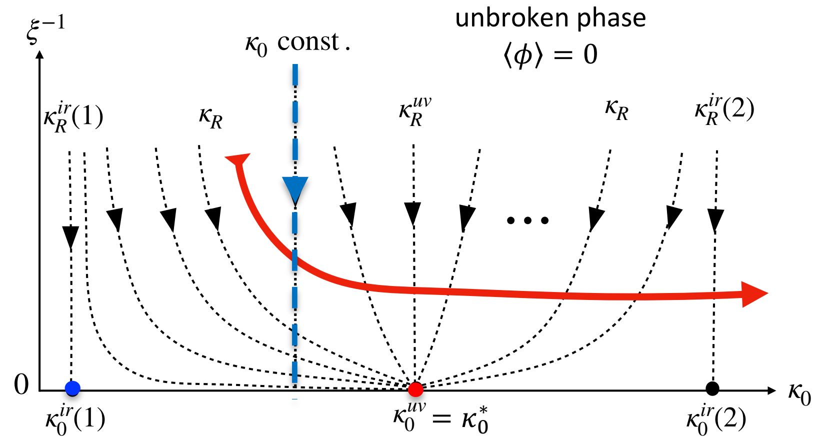
$$l_s(\kappa_0) \propto |\kappa_0 - \kappa_0^*|^{-1/\beta'(\kappa_0^*)}$$



**Unfortunately there is no UV fixed point in ϕ^4*

RG flow on the lattice (ϕ^4 example)

- *4D ϕ^4 (lattice) QFT example**
  - *2 dimensionless bare couplings: m_0, κ_0*
  - *for each choice of m_0, κ_0 one can compute the renormalized m_R, κ_R and the correl. length ξ*
  - *physical correl. length $\xi_{ph} = m_R^{-1} = \xi\, l_s$*
  - one can find RG flow where $\kappa_R, m_R = const.$
  - *there is a phase transition (where $\xi \to \infty$ so following the RG flow trajectory $l_s \to 0$)*
- *The UV limit*
  - *we approach the phase transition ($\xi \to \infty$)* keeping the renormalized coupling κ_R *fixed*
  - *in order to do that we have to tune the bare coupling κ_0*
- *The IR limit*
  - *we approach the phase transition ($\xi \to \infty$)* keeping the bare coupling κ_0 *fixed*
  - *we cross the $\kappa_R = const$ RG trajectories in the direction of $\kappa_R \to \kappa_R^{ir}$*



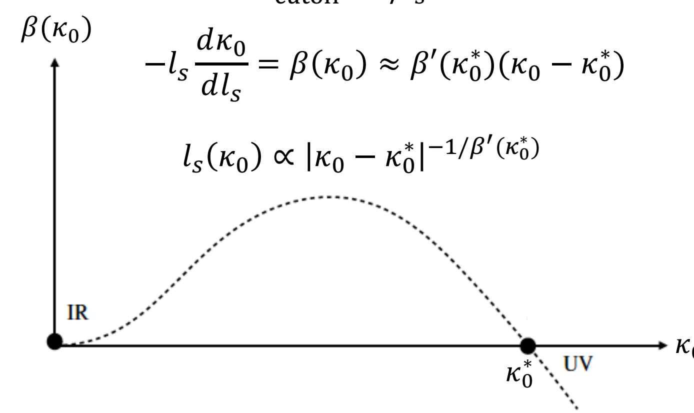
$$L = \left(\partial_\mu \phi\right)^2 + m_o \phi^2 + \kappa_0 \phi^4$$

$$\kappa_R \propto \Gamma_4(p_i = 0; m_0, \kappa_0)$$

$k_{\text{cutoff}} \sim 1/l_s$

$$-l_s \frac{d\kappa_0}{dl_s} = \beta(\kappa_0) \approx \beta'(\kappa_0^*)(\kappa_0 - \kappa_0^*)$$

$$l_s(\kappa_0) \propto |\kappa_0 - \kappa_0^*|^{-1/\beta'(\kappa_0^*)}$$

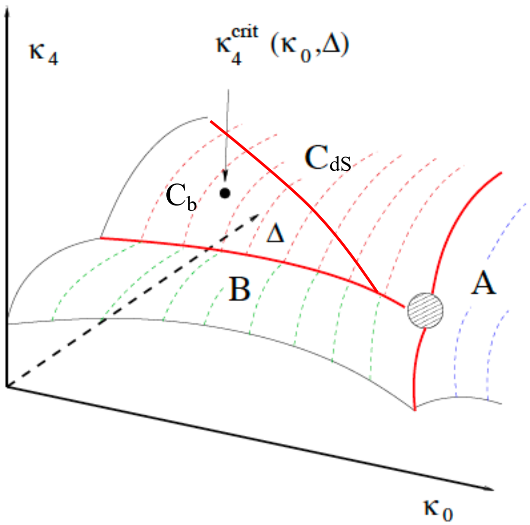


**Unfortunately there is no UV fixed point in ϕ^4*

RG flow in CDT

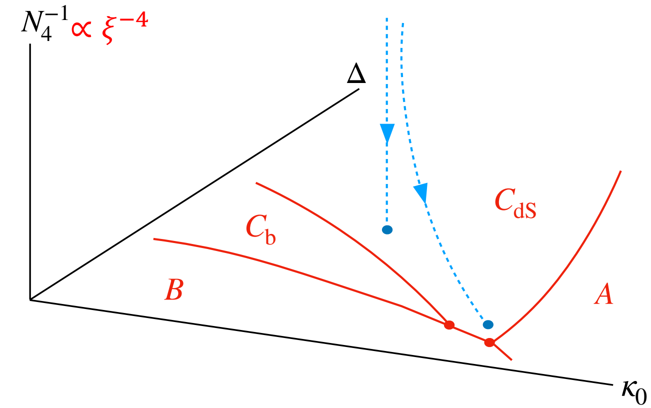
RG flow in CDT

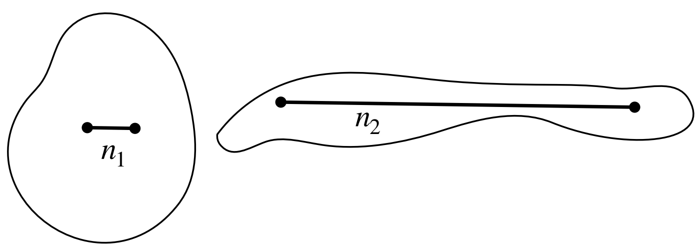
- ✧ *4D CDT (lattice) QFT*
  - ✧ *3 dimensionless bare couplings: K_4, k_0, Δ*
  - ✧ *The bare cosmol. const. K_4 is related to lattice volume N_4 : $K_4 \to K_4^{crit}(k_0, \Delta)$ when $N_4 \to \infty$*
  - ✧ *One can argue that inside phase C_{dS} the correl. length: $\xi \propto N_4^{1/4}$*
  - ✧ We assume that the CDT MS effective action is consistent with the E-H truncation in FRG
  - ✧ This implies relations between the effective couplings
- ✧ *The UV limit*
  - ✧ *we will approach the $K_4^*(k_0, \Delta)$* critical surface ($\xi \to \infty$) tuning the bare couplings k_0 *, Δ such that the effective coupling $G\Lambda$ stays fixed*
  - ✧ *we associate it with the UV limit of FRG*
- ✧ *The IR limit*
  - ✧ *we will approach the $K_4^*(k_0, \Delta)$* critical surface ($\xi \to \infty$) keeping the bare couplings k_0 *, Δ fixed*
  - ✧ *we associate it with the IR limit of FRG*



RG flow in CDT

- *4D CDT (lattice) QFT*
  - *3 dimensionless bare couplings: K_4, k_0, Δ*
  - *The bare cosmol. const. K_4 is related to lattice volume N_4 : $K_4 \to K_4^{crit}(k_0, \Delta)$ when $N_4 \to \infty$*
  - *One can argue that inside phase C_{dS} the correl. length: $\xi \propto N_4^{1/4}$*
  - We assume that the CDT MS effective action is consistent with the E-H truncation in FRG
  - This implies relations between the effective couplings
- *The UV limit*
  - *we will approach the $K_4^*(k_0, \Delta)$* critical surface ($\xi \to \infty$) tuning the bare couplings k_0, *Δ such that the effective coupling $G\Lambda$ stays fixed*
  - *we associate it with the UV limit of FRG*
- *The IR limit*
  - *we will approach the $K_4^*(k_0, \Delta)$* critical surface ($\xi \to \infty$) keeping the bare couplings k_0, *Δ fixed*
  - *we associate it with the IR limit of FRG*





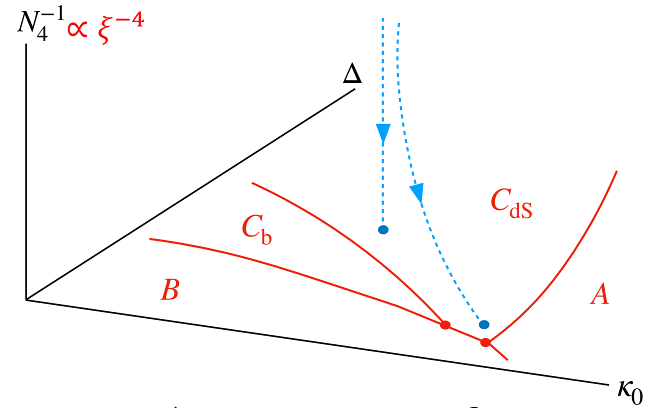
n is the geodesic distance between the two points in the case of DT

$$G_2(n) \propto \exp\left[-\frac{n}{c\,\langle N_4\rangle^{1/d_H}}\right], \qquad n \gg \langle N_4\rangle^{1/d_H}$$

ξ

RG flow in CDT

- *4D CDT (lattice) QFT*
  - *3 dimensionless bare couplings: K_4, k_0, Δ*
  - *The bare cosmol. const. K_4 is related to lattice volume N_4 : $K_4 \to K_4^{crit}(k_0, \Delta)$ when $N_4 \to \infty$*
  - *One can argue that inside phase C_{dS} the correl. length: $\xi \propto N_4^{1/4}$*
  - We assume that the CDT MS effective action is consistent with the E-H truncation in FRG
  - This implies relations between the effective couplings
- *The UV limit*
  - *we will approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$) tuning the bare couplings k_0 *, Δ such that the effective coupling $G\Lambda$ stays fixed*
  - *we associate it with the UV limit of FRG*
- *The IR limit*
  - *we will approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$) keeping the bare couplings k_0 *, Δ fixed*
  - *we associate it with the IR limit of FRG*



$$S = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{\mu} N_3(i)^{1/3} \right)$$

$$S_k = \frac{1}{16\pi G_k} \int d^4x \sqrt{\det g} (R - 2\Lambda_k)$$

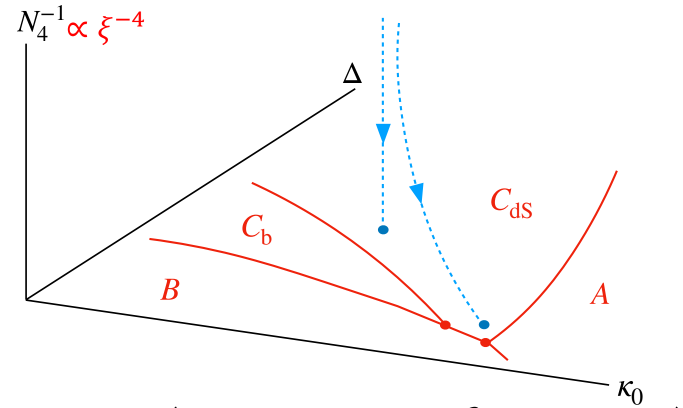
$$\frac{\tilde{\Gamma}}{\sqrt{N_4}} \left(\frac{\tilde{\omega}}{\omega_0} \right)^2 \propto G_k \Lambda_k$$

$$24\pi G = \left(\frac{\tilde{\omega}}{\omega_0} \right)^{4/3} \tilde{\Gamma}\, l_s^2$$

$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

RG flow in CDT

- 4D CDT (lattice) QFT
  - 3 dimensionless bare couplings: K_4, k_0, Δ
  - The bare cosmol. const. K_4 is related to lattice volume N_4 : $K_4 \to K_4^{crit}(k_0, \Delta)$ when $N_4 \to \infty$
  - One can argue that inside phase C_{dS} the correl. length: $\xi \propto N_4^{1/4}$
  - We assume that the CDT MS effective action is consistent with the E-H truncation in FRG
  - This implies relations between the effective couplings



$$S = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{\mu} N_3(i)^{1/3} \right)$$

$$S_k = \frac{1}{16\pi G_k} \int d^4x \sqrt{\det g}\,(R - 2\Lambda_k)$$

- The UV limit

$$k \to \infty : \; G_k \Lambda_k \to g^* \lambda^*$$
$$G_k \to g^* k^{-2}, \; \Lambda_k \to \lambda^* k^2$$

  - we will approach the $K_4^{crit}(k_0, \Delta)$ critical surface ($\xi \to \infty$) tuning the bare couplings k_0, Δ such that the effective coupling $G\Lambda$ stays fixed
  - we associate it with the UV limit of FRG

$$\frac{\tilde{\Gamma}}{\sqrt{N_4}} \left(\frac{\tilde{\omega}}{\omega_0} \right)^2 \propto G_k \Lambda_k$$

- The IR limit

$$k \to 0 : \; G_k \Lambda_k \to 0$$
$$G_k \to G_0 \approx G_N, \; \Lambda_k \to 0$$

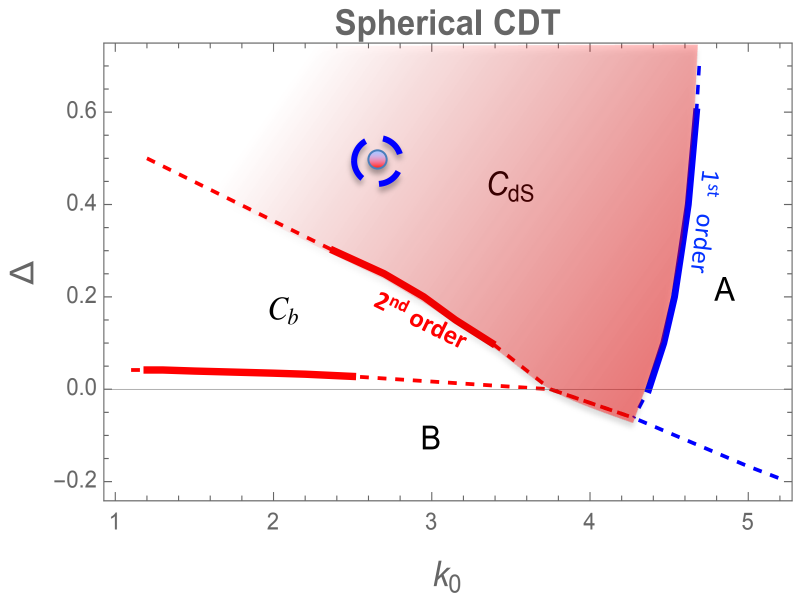
  - we will approach the $K_4^{crit}(k_0, \Delta)$ critical surface ($\xi \to \infty$) keeping the bare couplings k_0, Δ fixed
  - we associate it with the IR limit of FRG

$$24\pi G = \left(\frac{\tilde{\omega}}{\omega_0} \right)^{4/3} \tilde{\Gamma}\, l_s^2$$

$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

RG flow in CDT

- ✧ *The IR limit*
  - ✧ *we approach the* $K_4^{crit}(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) keeping the bare couplings k_0, *Δ fixed*
  - ✧ *for fixed* k_0, *Δ* we have $\tilde{\Gamma}$, $\tilde{\omega} = const. > 0$
  - ✧ *from FRG for* $k \to 0$: $G_k \to G_0 \approx G_N = \ell_{Pl}^2$
  - ✧ *therefore in CDT lattice spacing remains constant :* $l_s \sim \ell_{Pl}$
  - ✧ *as* $N_4 \to \infty$ *and* $l_s > 0$ *the volume of the CDT universe* $V_4 \to \infty$
  - ✧ *this is consistent with FRG as for* $k \to 0$: $\Lambda_k \to 0$ so $V_4 \propto \Lambda_k^{-2} \to \infty$
  - ✧ *CDT (relative) fluctuations vanish and one reproduces (semi) classical spacetime*
  - ✧ *this is also consistent with FRG where* $G_k \Lambda_k \to 0$



$$\langle N_3(i) \rangle = N_4^{3/4} \frac{3}{4\tilde{\omega}} \cos^3\left(\frac{i}{\tilde{\omega} N_4^{1/4}}\right)$$

$$\langle |\delta N_3(i)| \rangle = \tilde{\Gamma}^{1/2} N_4^{1/2} \; \mathrm{F}\left(\frac{i}{\tilde{\omega} N_4^{1/4}}\right)$$

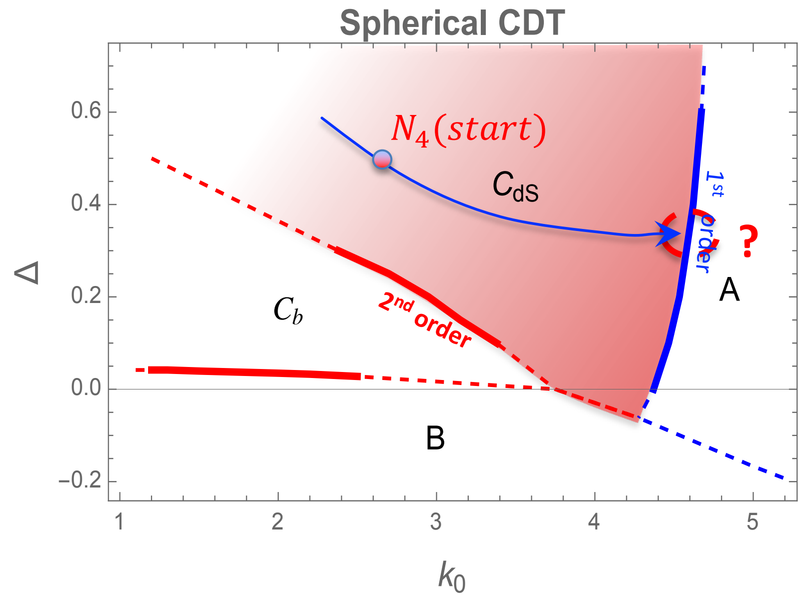
$$\frac{\langle |\delta V_3(t_i)| \rangle}{\langle V_3(t_i) \rangle} = \frac{\langle |\delta N_3(i)| \rangle}{\langle N_3(i) \rangle} \propto \frac{\sqrt{\tilde{\Gamma} \tilde{\omega}^2}}{N_4^{1/4}} \to 0$$

$$24\pi G = \left(\frac{\tilde{\omega}}{\omega_0}\right)^{4/3} \tilde{\Gamma} \, l_s^2$$

$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}}\right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

RG flow in CDT

- ✧ *The UV limit*
  - ✧ *we approach the* $K_4^{crit}(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) tuning the bare couplings k_0 *, Δ such that the effective* $G_k \Lambda_k = g^* \lambda^* = const$
  - ✧ *from FRG:* $G_k \to g^* k^{-2} \to 0$
  - ✧ *therefore in CDT :* $l_s \sim k^{-1} \to 0$
  - ✧ *(relative) fluctuations stay constant*
- ✧ *This requires finding RG flow trajectories* $(k_0(N_4), \Delta(N_4))$ *parametrized by* N_4
- ✧ *It is only possible by approaching the* $C_{dS} - A$ *phase transition line*
  - ✧ *we fix* Δ $(\Delta = 0)^*$ *and change only* k_0
  - ✧ *one can compute critical exponent:* $\gamma/4\nu_{uv}$ *related to scaling of* $\tilde{\Gamma}\,\tilde{\omega}^2$ *at the transition*
  - ✧ $\gamma/4\nu_{uv} = 0.54 \pm 0.04 \ \geq 1/2$ *so it may be possible to approach the UV limit*



$$\tilde{\Gamma}(k_0(N_4), \Delta(N_4))\tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4}$$

$$\frac{\tilde{\Gamma}}{\sqrt{N_4}}\left(\frac{\tilde{\omega}}{\omega_0}\right)^2 \propto G_k \Lambda_k = g^* \lambda^* = const$$

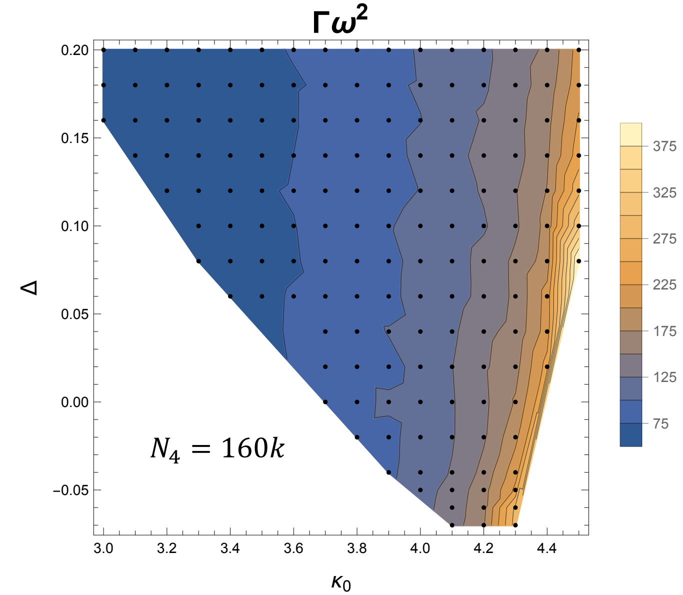
$$\frac{\langle|\delta V_3(t_i)|\rangle}{\langle V_3(t_i)\rangle} = \frac{\langle|\delta N_3(i)|\rangle}{\langle N_3(i)\rangle} \propto \frac{\sqrt{\tilde{\Gamma}\tilde{\omega}^2}}{N_4^{1/4}} \quad \to const \sim 1$$

$$24\pi G = \left(\frac{\tilde{\omega}}{\omega_0}\right)^{4/3} \tilde{\Gamma}\, l_s^2$$

$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}}\right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

RG flow in CDT

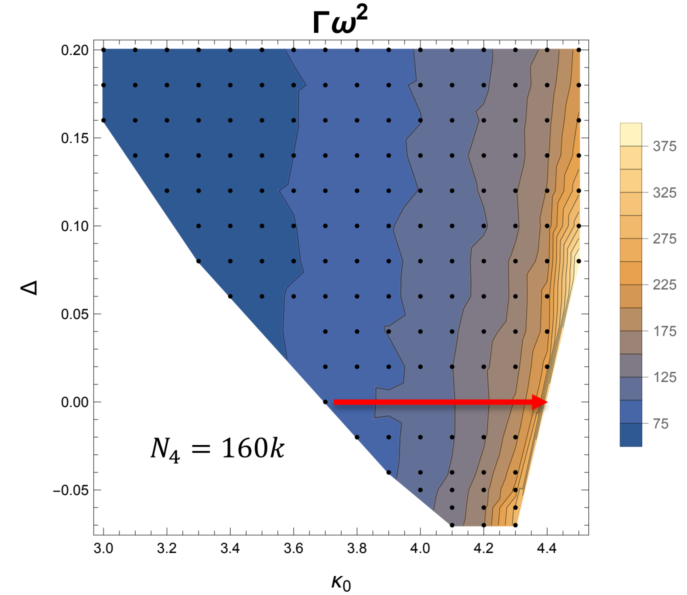
- *The UV limit*
  - *we approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) tuning the bare couplings k_0, Δ *such that the effective* $G_k \Lambda_k = g^* \lambda^* = const$
  - *from FRG:* $G_k \to g^* k^{-2} \to 0$
  - *therefore in CDT :* $l_s \sim k^{-1} \to 0$
  - *(relative) fluctuations stay constant*
- *This requires finding RG flow trajectories* $\left(k_0(N_4), \Delta(N_4)\right)$ *parametrized by* N_4
- *It is only possible by approaching the* $C_{dS} - A$ *phase transition* line
  - *we fix* Δ $(\Delta = 0)$ *and change only* k_0
  - *one can compute critical exponent:* $\gamma / 4\nu_{uv}$ *related to scaling of* $\tilde{\Gamma}\, \tilde{\omega}^2$ *at the transition*
  - $\gamma / 4\nu_{uv} = 0.54 \pm 0.04 \geq 1/2$ *so it may be possible to approach the UV limit*



$$\tilde{\Gamma}\left(k_0(N_4), \Delta(N_4)\right) \tilde{\omega}^2 \left(k_0(N_4), \Delta(N_4)\right) \propto \sqrt{N_4}$$

RG flow in CDT

- *The UV limit*
  - *we approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) tuning the bare couplings k$_0$, *Δ such that the effective* $G_k \Lambda_k = g^* \lambda^* = const$
  - *from FRG:* $G_k \to g^* k^{-2} \to 0$
  - *therefore in CDT :* $l_s \sim k^{-1} \to 0$
  - *(relative) fluctuations stay constant*
- *This requires finding RG flow trajectories* $\big(k_0(N_4), \Delta(N_4)\big)$ *parametrized by* N_4
- *It is only possible by approaching the* $C_{dS} - A$ *phase transition line*
  - *we fix* Δ $(\Delta = 0)$ *and change only* k_0
  - *one can compute critical exponent:* $\gamma/4\nu_{uv}$ *related to scaling of* $\tilde{\Gamma}\,\tilde{\omega}^2$ *at the transition*
  - $\gamma/4\nu_{uv} = 0.54 \pm 0.04 \;\geq 1/2$ *so it may be possible to approach the UV limit*

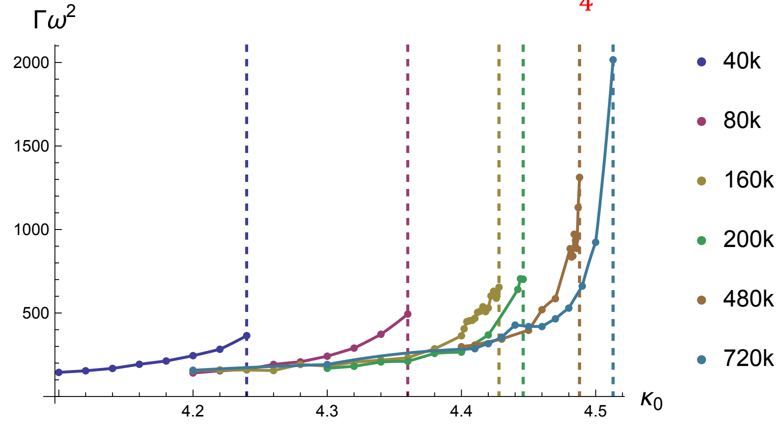


$$\tilde{\Gamma}\big(k_0(N_4), \Delta(N_4)\big)\tilde{\omega}^2\big(k_0(N_4), \Delta(N_4)\big) \propto \sqrt{N_4}$$

RG flow in CDT

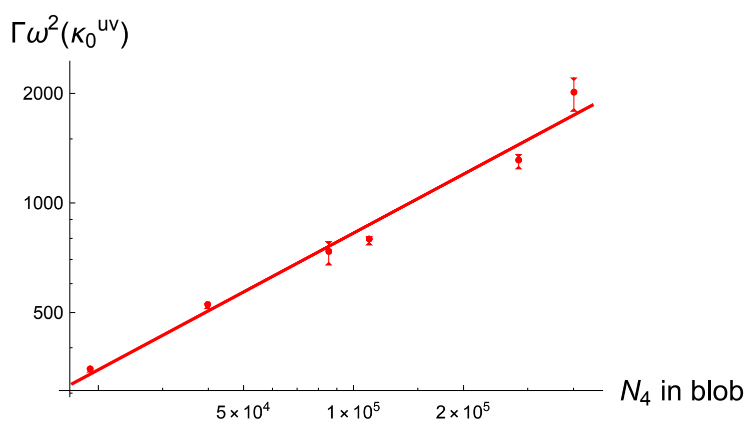
$$k_0^{uv}(N_4) = k_0^{uv} - \frac{c}{N_4^{1/4\nu_{uv}}}$$

- ✧ *The UV limit*
  - ✧ *we approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) tuning the bare couplings k_0 *, Δ such that the effective* $G_k \Lambda_k = g^* \lambda^* = const$
  - ✧ *from FRG:* $G_k \to g^* k^{-2} \to 0$
  - ✧ *therefore in CDT :* $l_s \sim k^{-1} \to 0$
  - ✧ *(relative) fluctuations stay constant*
- ✧ *This requires finding RG flow trajectories* $(k_0(N_4), \Delta(N_4))$ *parametrized by* N_4
- ✧ *It is only possible by approaching the* $C_{dS} - A$ *phase transition line*
  - ✧ *we fix* Δ ($\Delta = 0$) *and change only* k_0
  - ✧ *one can compute critical exponent:* $\gamma/4\nu_{uv}$ *related to scaling of* $\tilde{\Gamma}\,\tilde{\omega}^2$ *at the transition*
  - ✧ $\gamma/4\nu_{uv} = 0.54 \pm 0.04 \ \geq 1/2$ *so it may be possible to approach the UV limit*



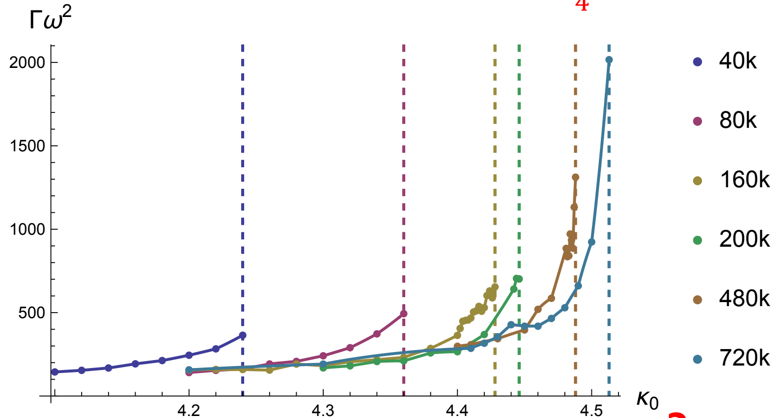
$$\tilde{\Gamma}\tilde{\omega}^2\left(k_0^{uv}(N_4)\right) \propto N_4^{\gamma/4\nu_{uv}}$$

$$\tilde{\Gamma}\left(k_0(N_4), \Delta(N_4)\right)\tilde{\omega}^2\left(k_0(N_4), \Delta(N_4)\right) \propto \sqrt{N_4}$$



RG flow in CDT

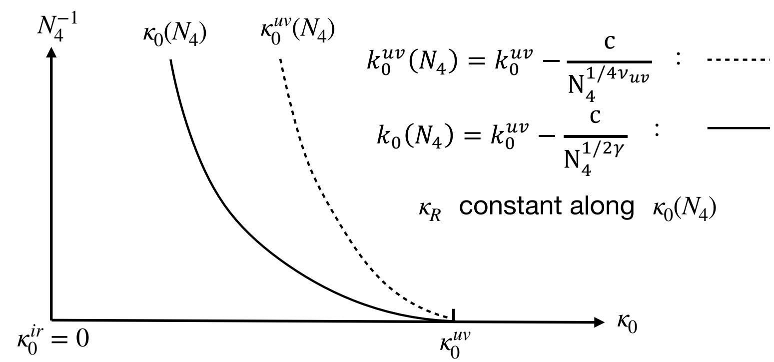
$$k_0^{uv}(N_4) = k_0^{uv} - \frac{c}{N_4^{1/4\nu_{uv}}}$$



$$\tilde{\Gamma}\tilde{\omega}^2\left(k_0^{uv}(N_4)\right) \propto N_4^{\gamma/4\nu_{uv}} \overset{?}{\geq} 1/2$$

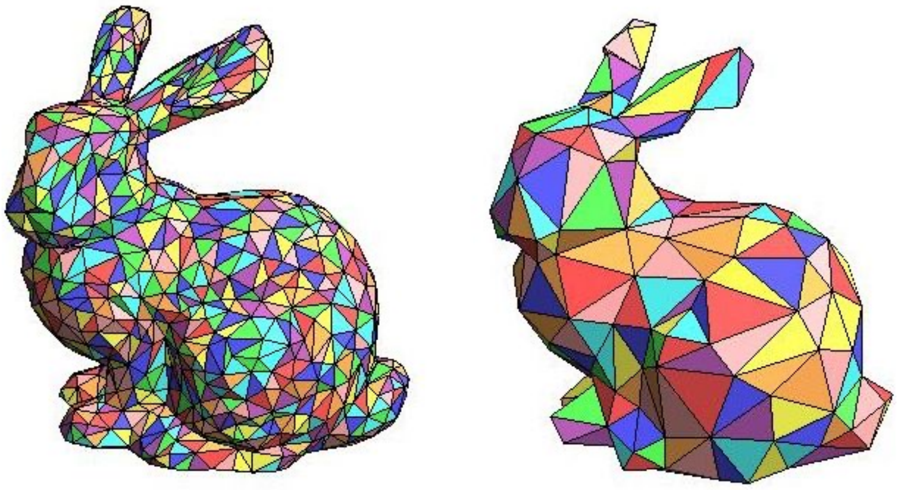
$$\tilde{\Gamma}\left(k_0(N_4), \Delta(N_4)\right)\tilde{\omega}^2\left(k_0(N_4), \Delta(N_4)\right) \propto \sqrt{N_4}$$

- *The UV limit*
  - *we approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) tuning the bare couplings k_0 , Δ *such that the effective* $G_k\Lambda_k = g^*\lambda^* = const$
  - *from FRG:* $G_k \to g^* k^{-2} \to 0$
  - *therefore in CDT :* $l_s \sim k^{-1} \to 0$
  - *(relative) fluctuations stay constant*
- *This requires finding RG flow trajectories* $\left(k_0(N_4), \Delta(N_4)\right)$ *parametrized by* N_4
- *It is only possible by approaching the* $C_{dS} - A$ *phase transition line*
  - *we fix* Δ ($\Delta = 0$) *and change only* k_0
  - *one can compute critical exponent:* $\gamma/4\nu_{uv}$ *related to scaling of* $\tilde{\Gamma}\,\tilde{\omega}^2$ *at the transition*
  - $\gamma/4\nu_{uv} = 0.54 \pm 0.04 \overset{?}{\geq} 1/2$ *so it may be possible to approach the UV limit*



Conclusions

- ✧ *CDT is a lattice QFT and a promising candidate for a UV complete theory of QG formulated in a fully non-perturbative and background independent way*
- ✧ *One can study dynamically emerging background geometry and quantum fluctuations*
- ✧ *CDT has a rich phase structure including the semi-classical phase C_{dS}*
  - ✧ *correct IR limit of the scale factor (spatial volume) consistent with (Eucl.) de Sitter space*
  - ✧ *quantum fluctuations of the scale factor are well described by the minisuperspace action*
- ✧ *CDT can provide independent tests of the asymptotic safety conjecture in a setting not dependent on FRG truncations*
- ✧ *One can make contact with FRG approach to QG by defining RG flow in CDT and searching for the IR and UV fixed points*
- ✧ *The results for the UV continuum limit seem promising but not conclusive*
- ✧ *Open problems and questions:*
  - ✧ *the (potential) UV limit of CDT is obtained at the 1st order phase transition (non-standard)*
  - ✧ *this is possible because we observe finite-size scaling and thus the correl. length $\xi \propto N_4^{1/4}$*
  - ✧ *this is actually a new (generic ?) feature of quantum gravity, where one can define correlations between fluctuating space-time points separated by a geodesic distance (not as in ordinary lattice QFTs, where space-time is fixed)*



Thank You !

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

UCZELNIA BADAWCZA
INICJATYWA DOSKONAŁOŚCI