Unlocking the Spin Foam Path Integral

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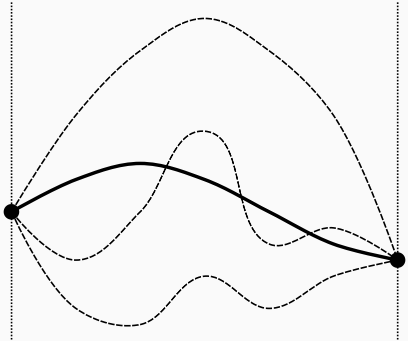
Path integral for 1d particle

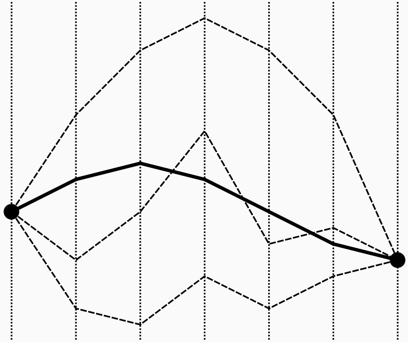


$$Z = \int_{q_i, t_i}^{q_f, t_f} \mathcal{D}q \; e^{\frac{i}{\hbar} S(q, \dot{q}, t)}$$

Sum over **all path** weighted by the exponentiated action.

We must **regularize**, e.g. by **discretization**.

Path integral for 1d particle

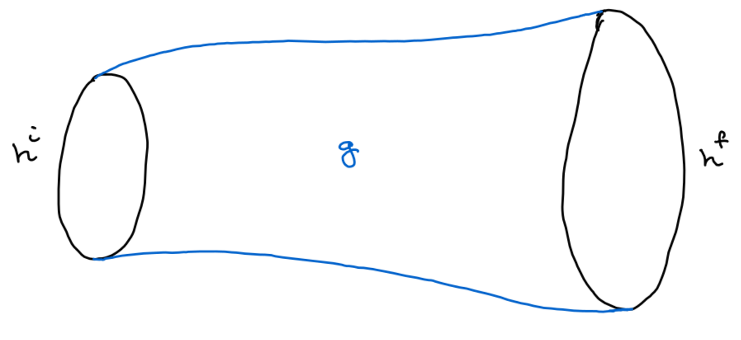


$$Z_{\text{disc}} = \int_{q_i,t_i}^{q_f,t_f} \prod_{i=1}^{N} dq_i \; e^{\frac{i}{\hbar} S_{\text{disc}}(q_i,t_i)}$$

Discretize position and time: **ambiguities**, e.g. in S_{disc}!

Must define / take **continuum limit** to remove regulator.

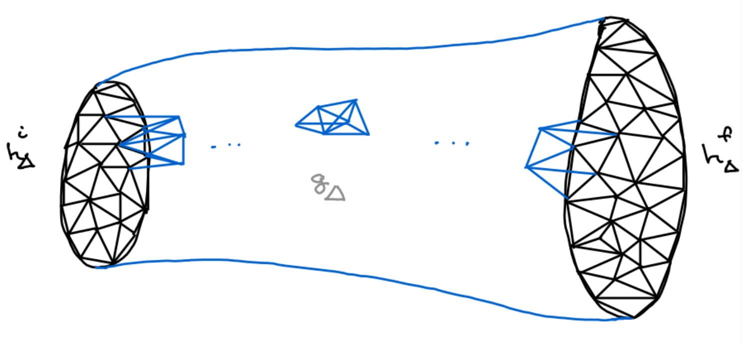
Discretize gravitational path integral



$$Z = \int_{h_i}^{h_f} \mathcal{D}g \; e^{\frac{i}{\hbar} S_{\mathrm{EH}}[g]}$$

Formal definition: integral over all **geometries**!

Discretize gravitational path integral

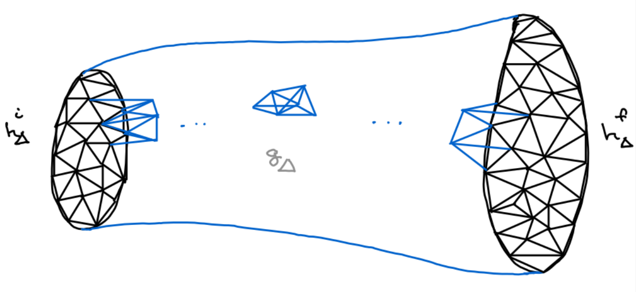


$$Z(\Delta) = \int_{h_i^\Delta}^{h_f^\Delta} \prod_e dl_e \; e^{\frac{i}{\hbar} S_{\text{Regge}}[l_e]}$$

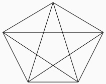
Regularize by replacing manifold by a **triangulation**.

Replace metric by **edge lengths** in (quantum) **Regge calculus** [Regge, Hamber, Williams, Rocek,...].

Discretize gravitational path integral



$$Z(\psi_i, \psi_f, \Delta) = \sum_{\rho_t, \iota_\tau} \prod_{t \in \Delta} \mathcal{A}_f(\rho_t) \prod_{\tau \in \Delta} \mathcal{A}_\tau(\iota_\tau) \prod_{\sigma \in \Delta}$$



$$\psi_f^* \, \psi_i$$

Spin foam quantum gravity [Rovelli, Reisenberger, Barrett, Crane, Freidel, Livine, Krasnov, Perez, Speziale, Engle, Pereira, Kaminski...]

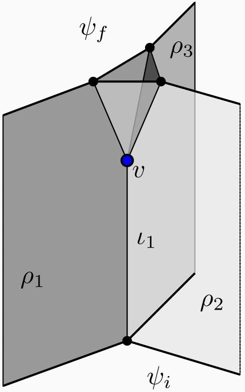
Quantum geometric building blocks with **quantum amplitudes**.

Group theoretic data: **area and shape variables**

Derived from **general relativity** as a **constrained topological field theory**

Numerical Challenges

Spin foam Challenges



- **Non-perturbative → Numerical methods indispensable**
- Numerical **challenges**
- **Complexity**: Number of variables
  - Summing over areas and angles
  - Discrete (possibly infinite) set of labels
- **Quantum amplitudes**
  - **Sign problem**: obstacle to Monte Carlo?
  - Tensor network contractions
  - Acceleration operators [Shanks '55, Wynn '56]
- **Setup** of calculations

Goals for spin foam quantum gravity

Explore a **continuum limit**

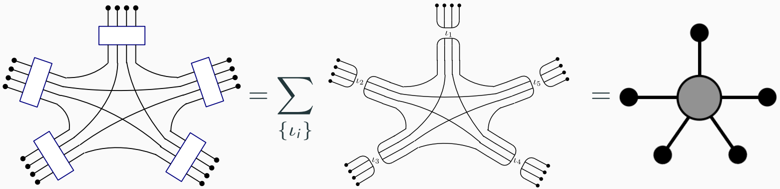
Extract **semi-classical physics**

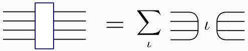
Couple **matter** and define **observables**

Prototype: one 4-simplex

Coherent SU(2) 4-simplex amplitude [Dona, Fanizza, Sarno, Speziale '17, Doná, Fanizza, Sarno, Speziale '19, Gozzini '21]

- **Single 4-simplex**
  - Coherent boundary states → peaked on classical simplicial geometry



- 12-dim. **integration** of **oscillatory** functions → **inefficient**
  - Stationary phase analysis: Oscillate with **Regge action** [Regge '61] of 4-simplex [Conrady, Hnybida '08, Barrett, Dowdall, Fairbairn, Gomes, Hellmann '09,...]
- Perform **group integrations analytically**: 

  - $\{15j\}$-symbol efficient to compute
- Write as **tensor network contraction** [Gozzini '21]

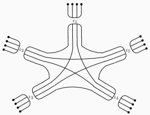
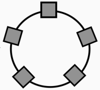
Responsible for **numerical advances** in recent years!

Costs grow **exponentially**: computing and storing 5-valent tensor.

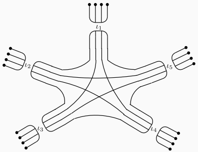
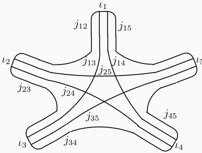
Optimizations

Smaller tensors and Monte Carlo

- Write amplitude as **matrix trace** [Asante, S.St. '24]
  - Avoid storing whole $\{15j\}$ tensor

$\sum_{\{\iota_i\}}$  $= (-1)^{\chi} \sum_{x} d_x$ 

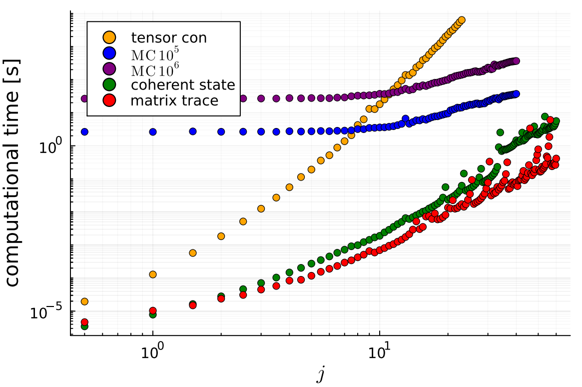
- Monte Carlo **importance sampling** for intertwiners [S.St. '24]
  - Probability distribution from coherent states
  - **Sign problem** present, but (mostly) **under control**

$\sum_{\{\iota_i\}}$  $\approx \sum_{\iota_i \text{ samples}} \left(\prod_i n_{\iota_i} \frac{c_{\iota_i}}{|c_{\iota_i}|} \right)$ 

Drastic optimization: calculations possible on **consumer level hardware**

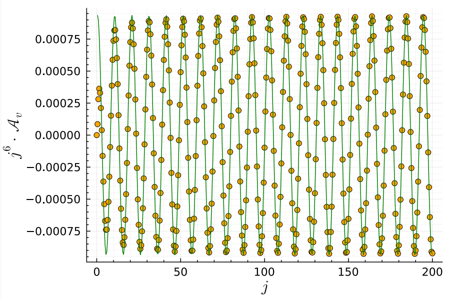
Numerical costs: equilateral case

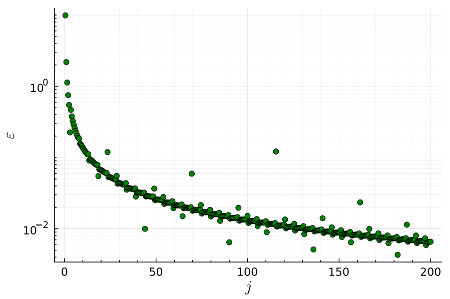
- **Less computational time, less memory usage**
  - $\{15j\}$-tensor-contraction $\sim j^6$
  - Matrix trace $\sim j^4$
  - Monte Carlo $\sim$ number of samples



Results: Equilateral vertex [Asante, S.St.'24]

- Comparison to **asymptotic results**
  - ϵ relative error: below 1% around $j \sim 200$



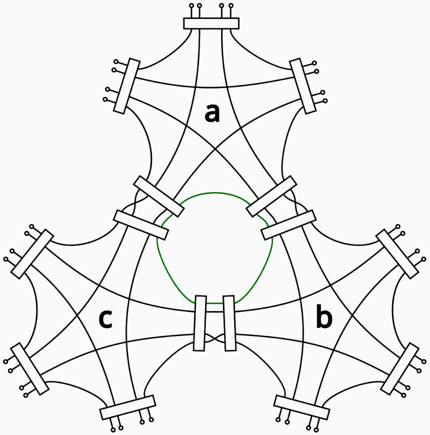


Numerically reach to asymptotic / **semi-classical** regime!

What about **larger triangulations**?

Setting up simulations

Spin foam vertex amplitudes for larger triangulations

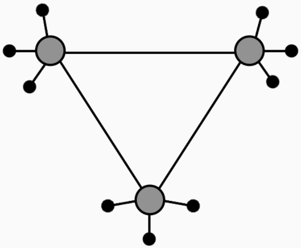


Spin foam amplitude for **3 glued 4-simplices** sharing one **dynamical triangle**

Constructive algorithm for amplitudes [Asante, Siebert, Simão, S.St. w.i.p.]

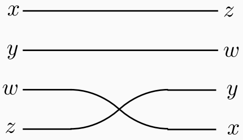
JFoams: Constructing (simplicial) 2-complexes [Asante, Siebert, Simão, S.St. w.i.p.]

- **Build** the 2-complex in a **few easy steps**
- Written in **Julia**



Algorithm to build foam

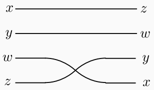
- Create **number of 4-simplices**
  - Fixed internal wiring
- Fix **wiring of faces** in edges
- Specify **gluing** of 4-simplices
  - Different wiring per edge



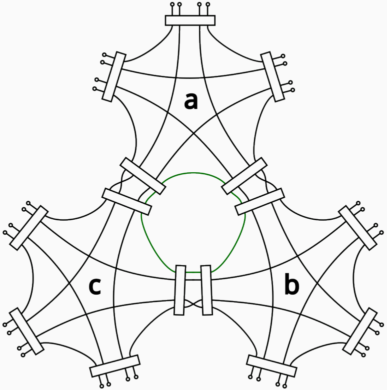
- Code **identifies** bulk / boundary faces
- Provide **coherent data** for boundary tetrahedra
- Compatible with **Lorentzian theory**
  - Incorporate package **sl2cfoam-next** [Dona, Fanizza, Sarno, Speziale '19, Gozzini '21]

Example: Δ_3 calculation

```
wiring = [[["x", "z"], ["y", "w"], ["w", "x"], ["z", "y"]] for i in 1:3]
d3 = foam_complex(["a", "b", "c"],[["a1","b1"],["b2","c3"],["c4","a5"]], wiring=wiring);
```



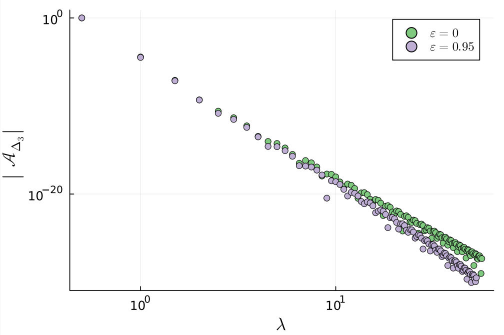

```
ntt = [[0.0, 0.0, 1.0], [0.0, 0.9428090415820634, -0.33333333333333334],
      [0.816496580927726, -0.4714045207910318, -0.3333333333333332],
      [-0.816496580927726, -0.47140452079103157, -0.3333333333333332]];
bd_data=Dict("a2,x" => ntt[1], "a2,y" => ntt[2], "a2,w" => ntt[3], "a2,z" => ntt[4],
"a3,x" => ntt[1], "a3,y" => ntt[2], "a3,w" => ntt[3], "a3,z" => ntt[4],
"a4,x" => ntt[1], "a4,y" => ntt[2], "a4,w" => ntt[3], "a4,z" => ntt[4],
"b3,x" => ntt[1], "b3,y" => ntt[2], "b3,w" => ntt[3], "b3,z" => ntt[4],
"b4,x" => ntt[1], "b4,y" => ntt[2], "b4,w" => ntt[3], "b4,z" => ntt[4],
"b5,x" => ntt[1], "b5,y" => ntt[2], "b5,w" => ntt[3], "b5,z" => ntt[4],
"c1,x" => ntt[1], "c1,y" => ntt[2], "c1,w" => ntt[3], "c1,z" => ntt[4],
"c2,x" => ntt[1], "c2,y" => ntt[2], "c2,w" => ntt[3], "c2,z" => ntt[4],
"c5,x" => ntt[1], "c5,y" => ntt[2], "c5,w" => ntt[3], "c5,z" => ntt[4]);

d3._bd_data = bd_data;
```

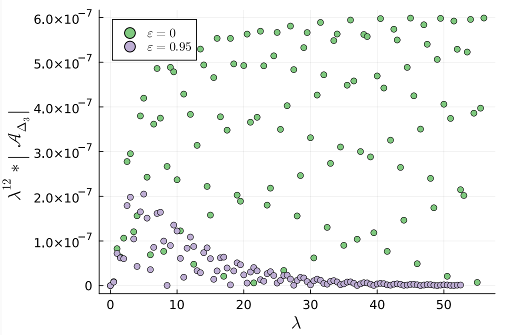


Make it **easy to set up simulations**!

Some Δ_3 results [Dona, Siebert, S.St. w.i.p.]

- **Classically**, geometry **completely fixed** by boundary edge lengths
  - Investigate different **deficit angles** ϵ

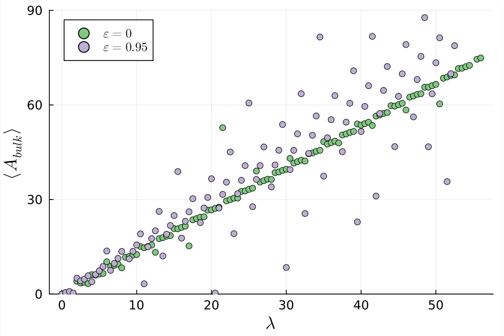


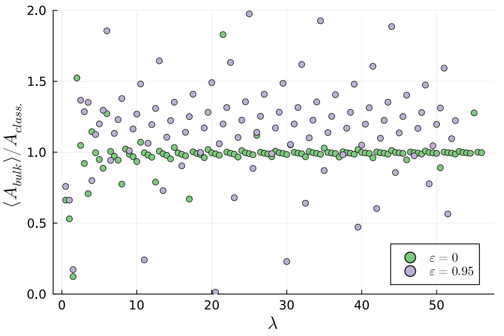


Exponential suppression for non-flat ($\epsilon \neq 0$) geometries [Dona, Gozzini, Sarno '20]

Some Δ_3 results [Dona, Siebert, S.St. w.i.p.]

- Study **expectation value** of **bulk triangle** $\langle A_{\text{bulk}} \rangle$
    - Comparison to boundary geometry





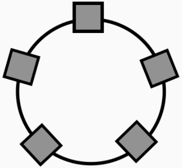
Classical triangle area $\sim$ recovered for **flat** geometries

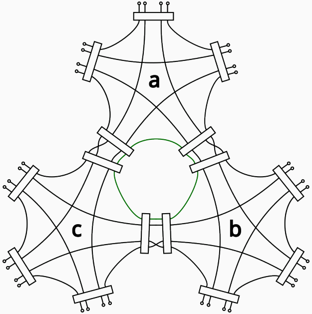
Proof of principle on **consumer level hardware**.

Summary & Outlook

Summary

- **Numerical challenges** in spin foams
  - **Complexity**: number of variables
  - Quantum amplitudes → sign problem
  - **Simulation setup**
- **Substantial optimization**
  - **Efficient tensor network methods** [Asante, S.St. '24]
  - Monte Carlo methods [S.St. '24]
  - **Constructive algorithm** [Asante, Siebert, Simão, S.St. w.i.p.]





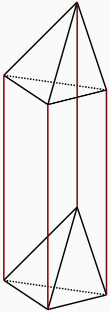
Strategy for full theory

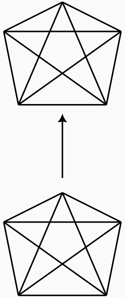
- **Reliable**, yet **costly** methods
  - Tensor Network contractions
  - Acceleration operators
- **Benchmarks** for Monte Carlo
  - Random sampling [Dona, Frisoni '23]
  - Importance sampling in bulk? [Siebert, S.St. w.i.p.]

Further **approximations** and **simplifications** necessary!

Outlook

- **Lorentzian theory**
- Make contact to **effective / asymptotic methods**
  - Effective spin foams [Asante, Dittrich, Haggard '20, '21, Asante, Dittrich, Padua-Argüelles '21]
  - Complex critical points [Han, Huang, Liu, Qu '21, Han, Liu, Qu '23]
- Testbed: **Cosmological models** [Hartle '85, da Silva, Williams '99, Liu, Williams '16, Dittrich, Gielen, Schander '21, Dittrich, Padua-Argüelles '23, Jercher, S.St. '23, Jercher, Simão, S.St. '24, Han, Liu, Qu, Vidotto, Zhang '24]
  - Symmetry reduction: less variables
  - Time-reparametrization invariance
  - Matter fields as relational clocks
  - Contact to (discrete) classical solutions
  - Explore continuum limit
  - Lorentzian signature: irregular causal structure





Thank you for your attention!