# Open issues in the construction of non-singular black holes

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Mainly based on:

R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. *Phys.Rev.Lett.* 133 (2024) 18, 181402.

F. Di Filippo. *Phys.Rev.D* 110 (2024) 8, 084026.

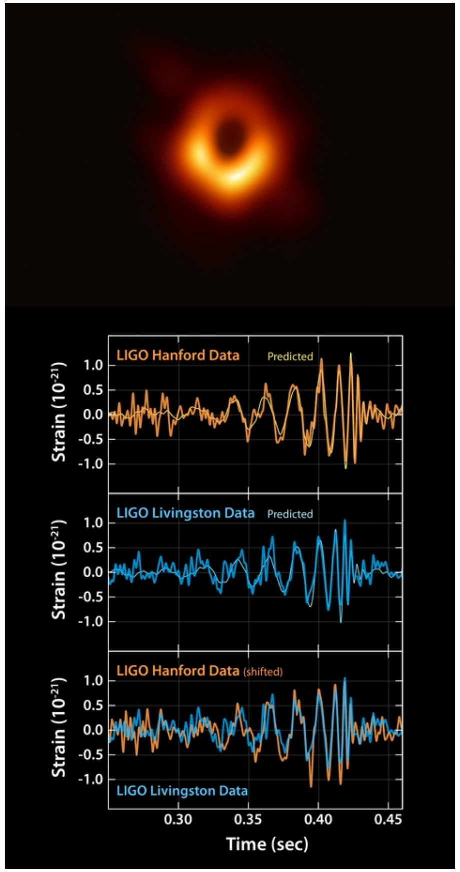
# Introduction

General Relativity is an extremely elegant and successful theory.

Observations coming from the LIGO/Virgo/KAGRA, the EHT, and the GRAVITY collaborations are in agreement with the prediction of general relativity.

However, there are also reasons to extend GR. In particular, the theory predicts its own breakdown due to the formation of singularities

Weak Cosmic Censorship Conjecture:
*Singularities are hidden from an observer at infinity by the event horizon.*

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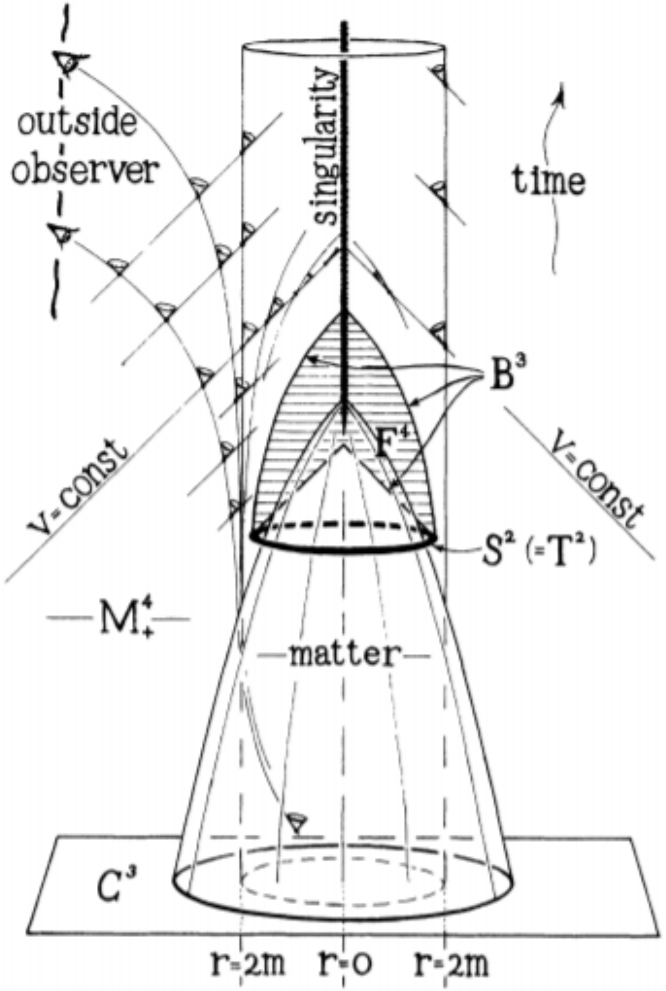


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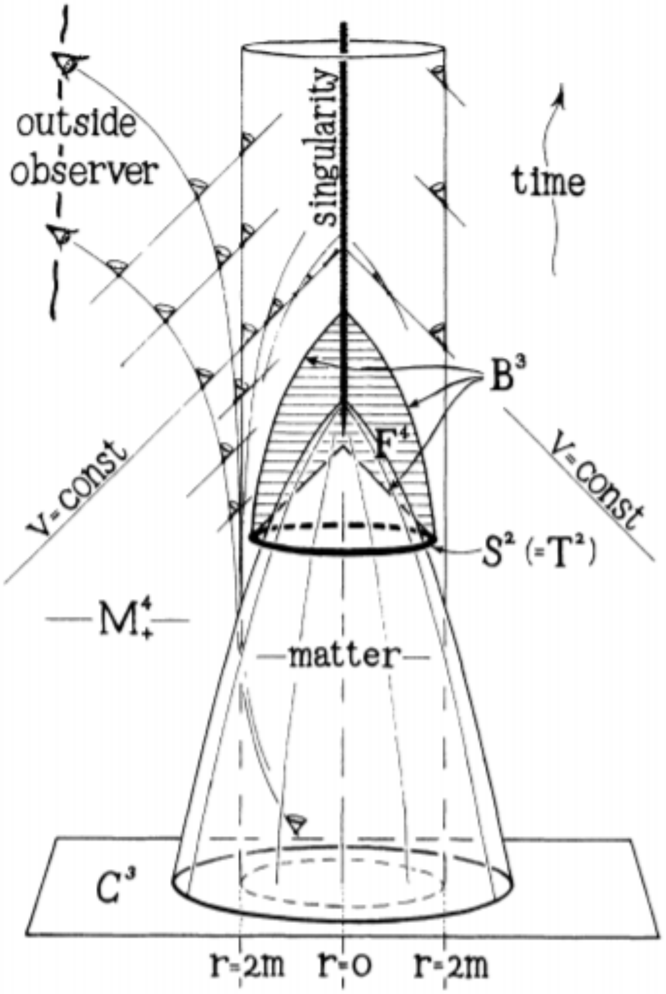
# Singularity avoidance: possibilities

As a framework, we use standard tools of Pseudo-Riemannian geometry. By studying the possible ways out of Penrose theorem we can classify non-singular geometries [R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. Phys. Rev. D 101 (2020), 084047, arXiv:1911.11200.]




- **Regular black holes.** Both outer and inner horizon;
- **Wormholes.** Local or global minimum radius surface, with or without outer horizon;
- **Asymptotic regular black holes or wormholes.** Inner horizon or minimum radius are pushed at infinite affine distance;
- **Ultracompact horizonless objects.** Surface close to the would be horizon.

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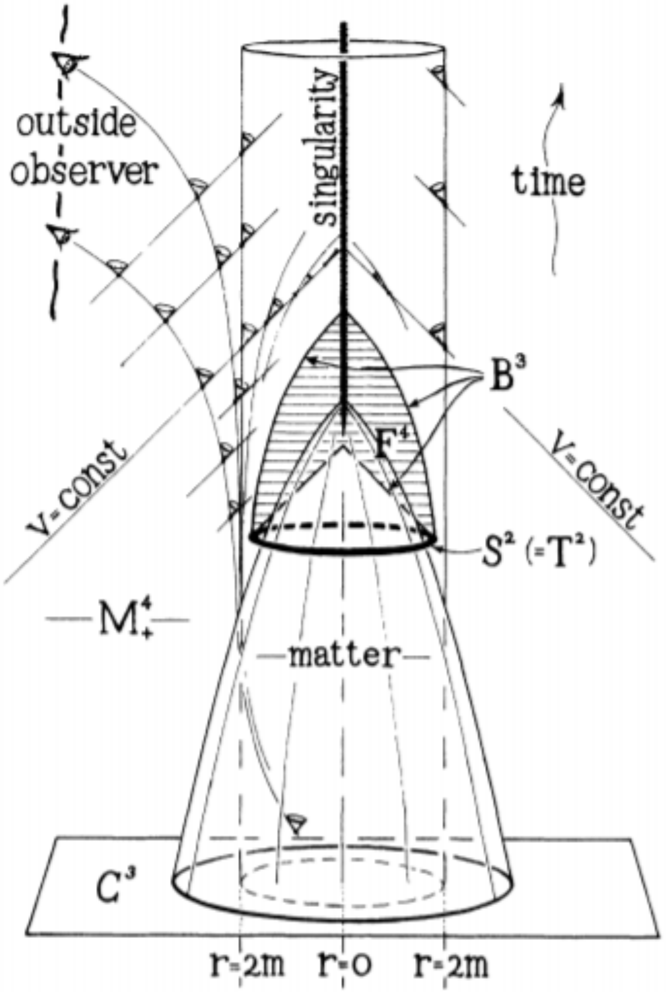


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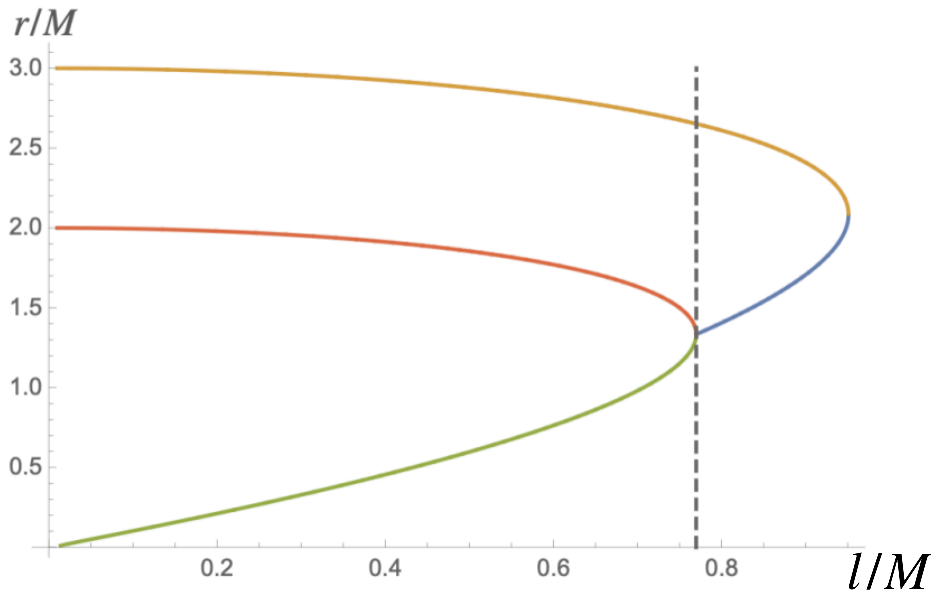
# Regular BH and Ultra-compact horizonless objects

We will focus on regular black holes and horizonless Ultra-compact objects.

As an example

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$




R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. JHEP 08 (2023) 046, arXiv:2211.05817.

# Viability and self-consistency

## Regular black holes instability

R. Carballo Rubio, F. Di Filippo, S. Liberati, C. Pacilio, M. Visser. *On the viability of regular black holes.*
JHEP 07 (2018), 023, arXiv:1805.02675.

R. Carballo Rubio, F. Di Filippo, S. Liberati, C. Pacilio, M. Visser. *Inner horizon instability and the unstable cores of regular black holes.*
*JHEP* 05 (2021) 132. arXiv:2101.05006.

R. Carballo Rubio, F. Di Filippo, S. Liberati, C. Pacilio, M. Visser. *Regular black holes without mass inflation instability.*
*JHEP* 09 (2022) 118. arXiv:2205.13556.

R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. *Mass inflation without Cauchy horizons.*
*Phys.Rev.Lett.* 133 (2024) 18, 181402.

# Regular black holes

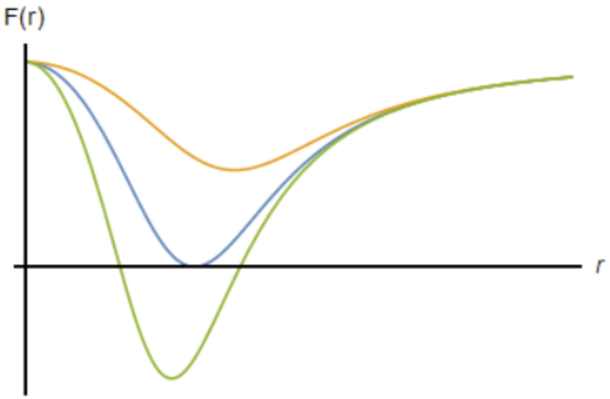
Let us start studying a static configuration, the analysis will trivially extend to the dynamical case.

$$ds^2 = -\, e^{-2\phi(r)}F(r)dv^2 + 2e^{-\phi(r)}dvdr + r^2 d\Omega^2$$

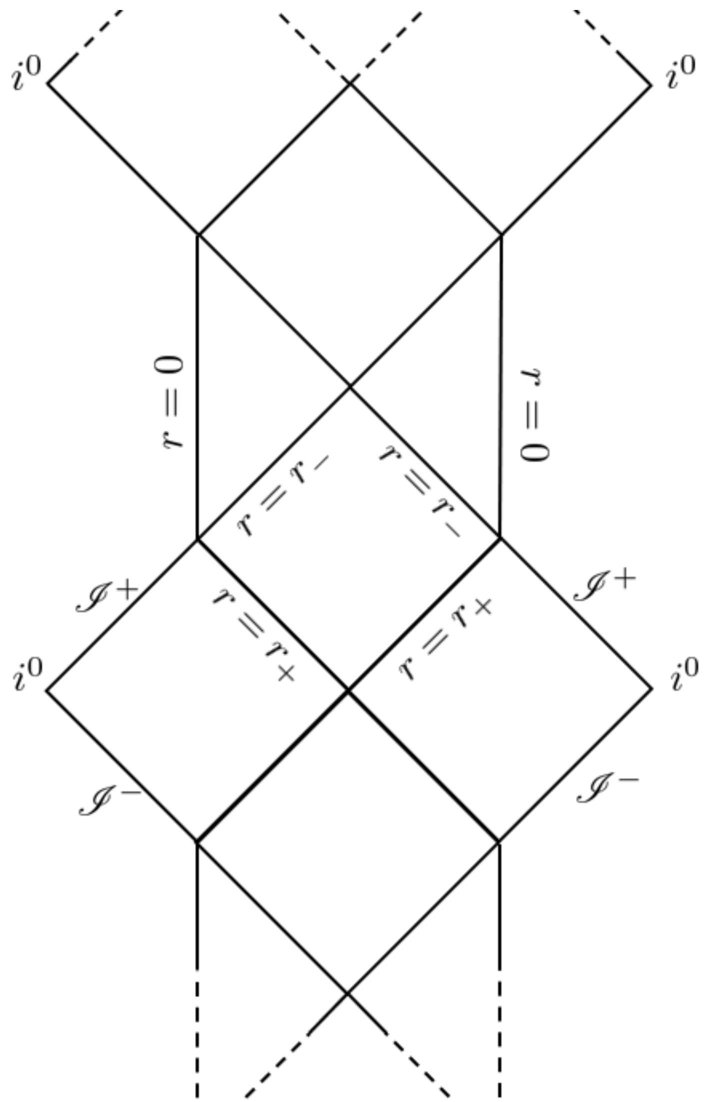
The horizon condition is $F(r) = 0$ .

- $\lim_{r->\infty} F(r) = 1$
- $\lim_{r->0} F(r) = 1$

There is an even number of horizons
The surface gravity:




$$\kappa_{\pm} = \frac{1}{2} e^{-\phi(r_{\pm})} \frac{dF}{dr}\bigg|_{r=r_{\pm}} \implies \kappa_{-} < 0\,, \kappa_{+} > 0\,.$$



# Mass Inflation

Inner horizons for GR black holes are unstable.
We want to study the stability of the inner horizon for regular black holes.

## Problem:

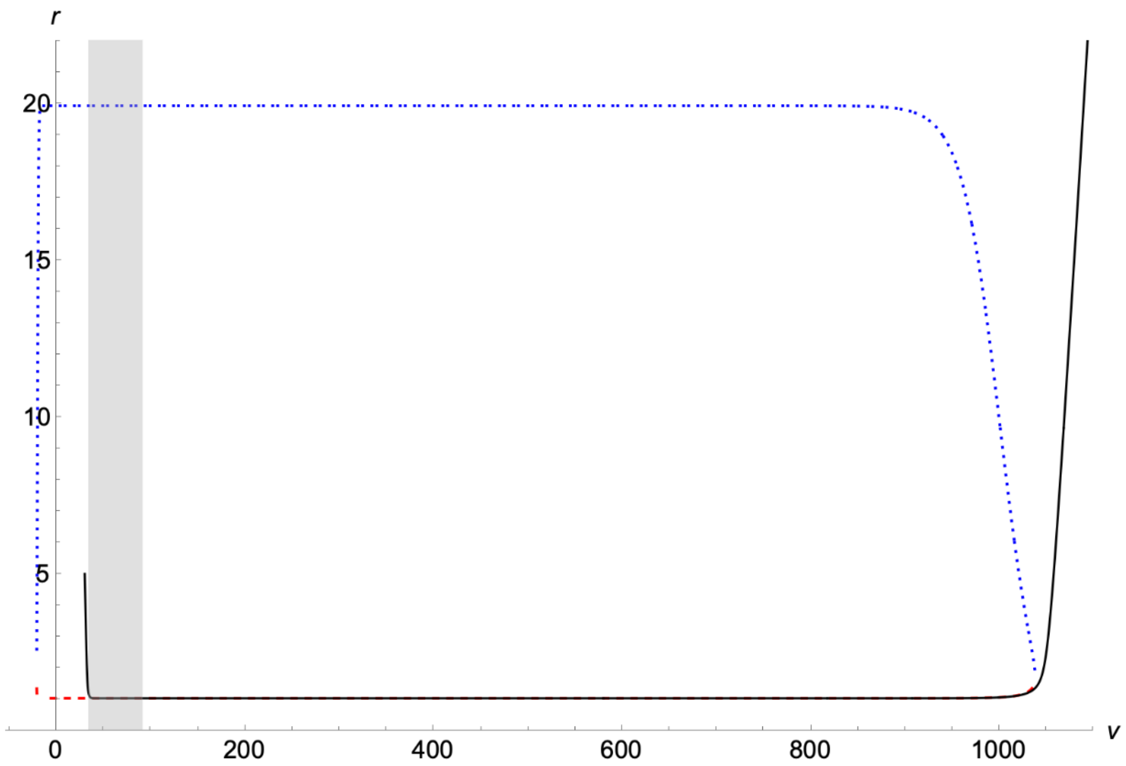
We do not know the field equations.

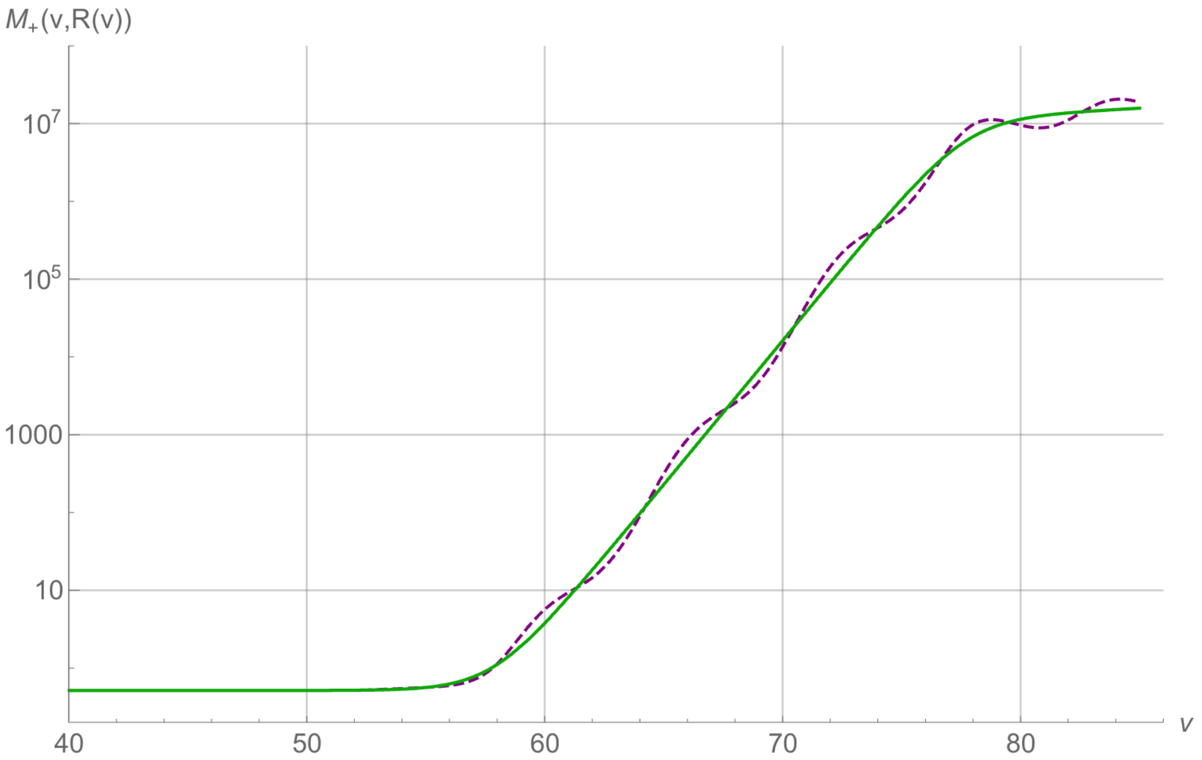
## Solution:

Consider a geometrical approach with few assumptions

What are these assumptions? What are the limitations of the approach?

# Dynamical regular black holes



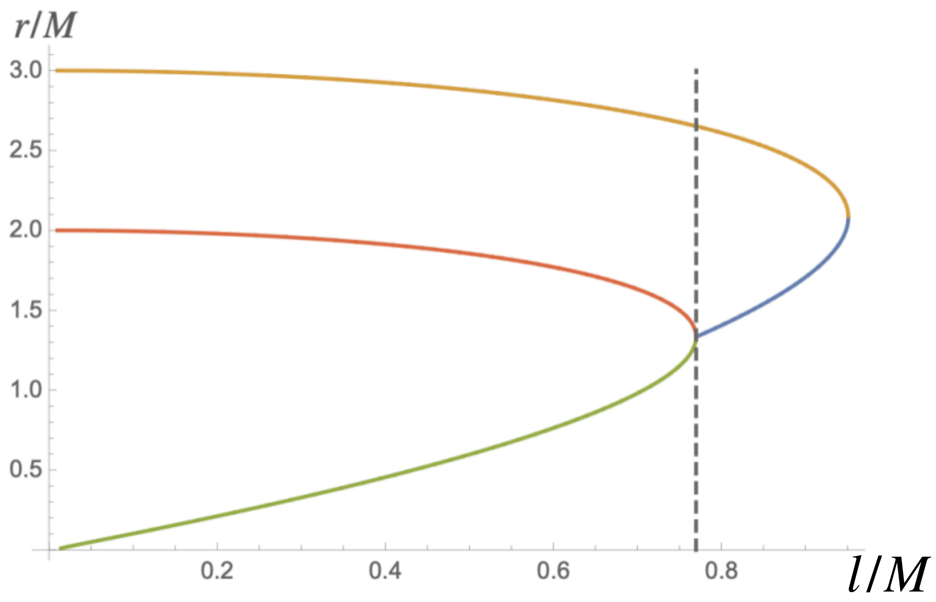

A small perturbation has a huge backreaction on the geometry.

R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. *Phys.Rev.Lett.* 133 (2024) 18, 181402.

# Regular BH and Ultra-compact horizonless objects

Let us go back to the plot we saw before

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right)dv^2 + 2dvdr + r^2 d\Omega^2$$

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# Viability and self-consistency
## Horizonless ultracompact objects instability

F.Di Filippo *On the nature of inner light-rings PRD* 110 (2024) 8, 084026.

# Light-rings

Light-rings (LR) correspond to the location of circular orbits.

For axisymmetric spacetimes ($(t, r, \theta, \phi)$, $\partial_t$ and $\partial_\phi$ are Killing vectors), at the LRs are region null geodesics have only $t$ and $\phi$ directions.

Light-rings correspond to the stationary points of the effective potential [Cunha et al. 2022];

$$H_\pm = \pm \frac{-g_{t\phi} \pm \sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}}{g_{\phi\phi}}$$

Why are we interested in light-rings?

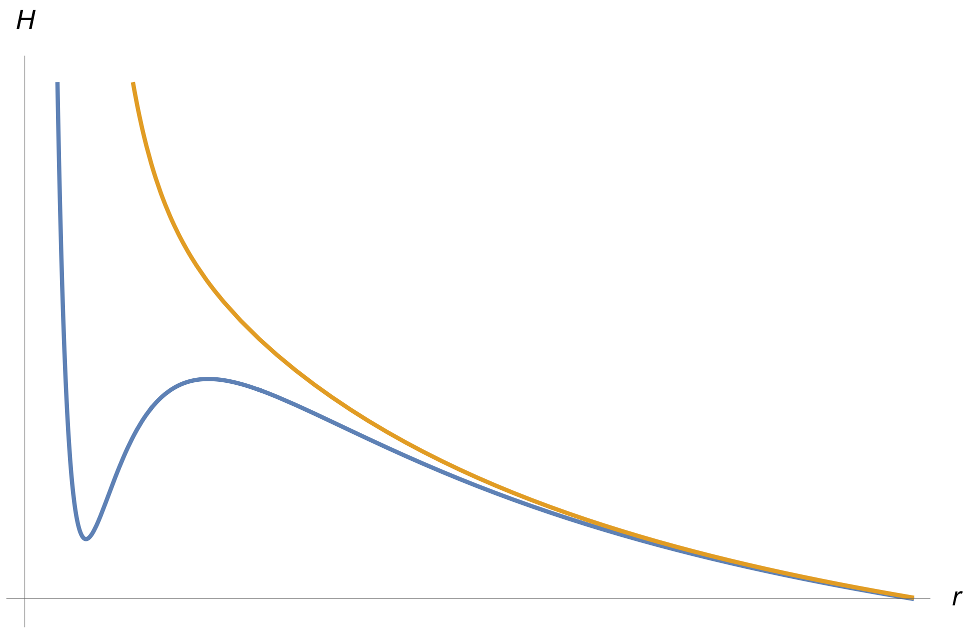
- The standard LR in Kerr plays a crucial role in the phenomenology of astrophysical compact objects.
- Stable LRs, corresponding to a minimum of the potential, can lead to an unstable spacetime (but there a lot of open questions)

# Number of light-rings

The light-rings in Kerr are saddle points of the effective potential.

Horizonless objects must have an even number of light-rings.

There must be an equal number of saddle points and extrema.



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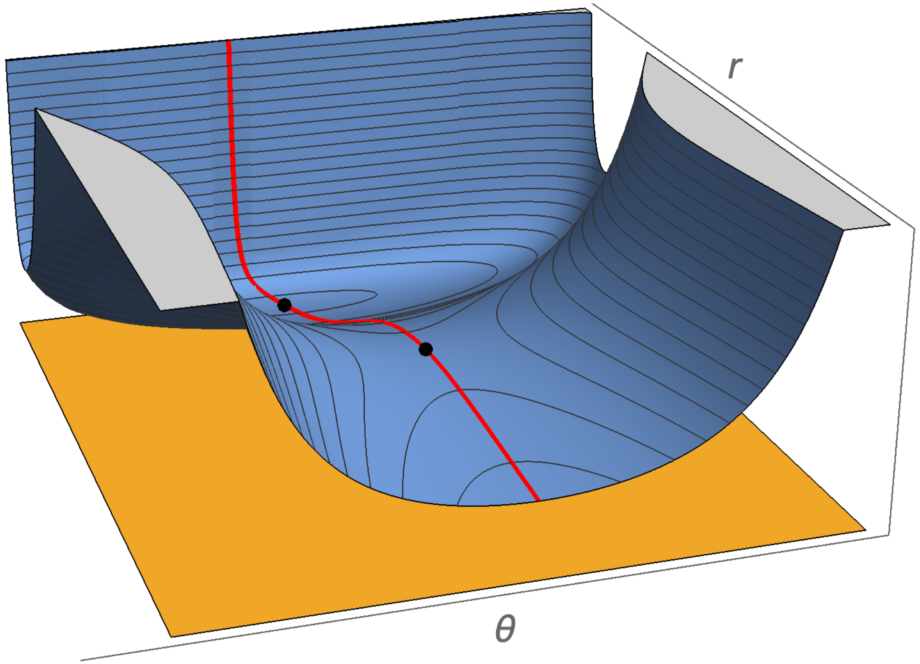


# Nature of light-rings

Using the Einstein equations and the null energy conditions, it was proved that the extremum of the effective potential is a minimum [Cunha et al. 2022]

***Theorem***: Any axisymmetric UCO invariant under simultaneous reflections $(t, \varphi) \rightarrow (-t, -\varphi)$ that has an outer light-ring that mimics the light-ring of a Kerr black hole must possess at least one stable inner light-ring for each rotation sense.
[F.Di Filippo *PRD* 110 (2024) 8, 084026.]

The statement "…outer light-ring that mimics the light-ring…" can be made precise.

Crucially, we do not need any assumption at the scale of the extra LR.

Violation of this result would lead to observational signatures!

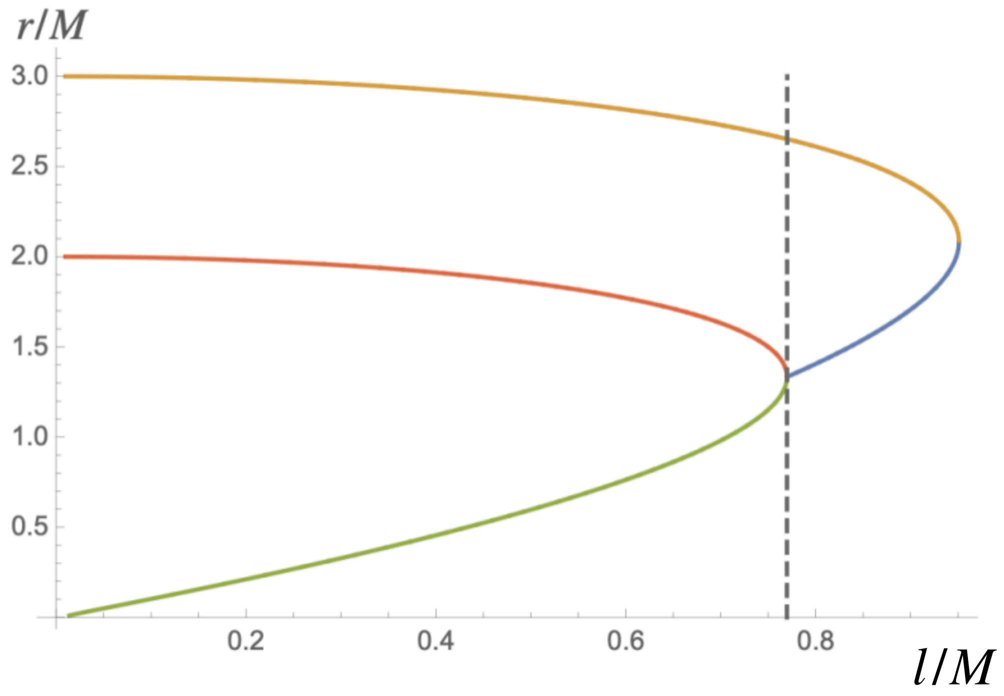
# Inner light-ring instability

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right)dv^2 + 2dvdr + r^2 d\Omega^2$$

For some boson stars models this leads to a non-linear instability [Cunha et al. 2022];

As soon as we get rid of the inner horizon instability, we get another source of instability.

Does this mean that there is no stable alternative to BH?



# Final remarks

- Black holes hide theoretical evidence of the failure of general relativity.

- Regular black holes and ultracompact objects seem to be unstable.

- Stable regular black hole geometries exist. Can we form them?
  [R. Carballo Rubio, F. D. F., S. Liberati, C. Pacilio, M. Visser. arXiv:2205.13556; F. D. F., I. Kolar, D. Kubiznak, arXiv:2404.07058]

- Alternatively, will we have deviations at scales larger than the Planck scale?

- The instability of horizonless objects is less understood.

- Can we construct a stable ultracompact object?

- Working with agnostic models has its limitations. We also need to study specific models

Still work to do to obtain a viable alternative to black holes (at least for the classes discussed in this talk).
This might not be a negative feature!

# Thank You