

# Do we live inside a Hayward black hole?

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based on:

MB *A non-singular universe out of Hayward black hole*, arXiv:2404.12243.

# A link between BHs and cosmology

## Classical Oppenheimer-Snyder Collapse (1939)

The collapse of a dust ball within General Relativity.

- Interior metric: dust spatially flat Friedmann-Robertson-Walker metric

$$ds_{\text{int}}^2 = -dT^2 + a(T)^2 dr^2 + r^2 a(T)^2 d\Omega^2,$$

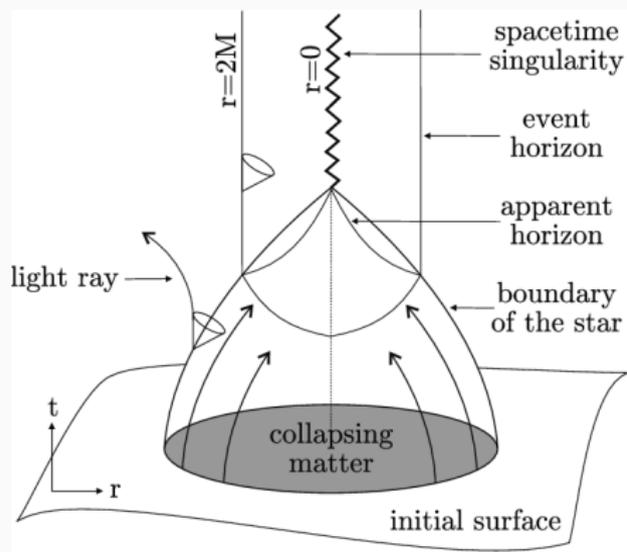
$$a(T) \sim T^{2/3}$$

- Exterior metric: Schwarzschild solution (simplest black hole)

## My speculation

People did not know that the exterior metric could be derived from the junction conditions at the boundary without referring to Einstein Field Equations. And vice versa for the interior metric.

This utility can be exploited beyond Einstein Field Equations.



Dust ball collapse in General Relativity

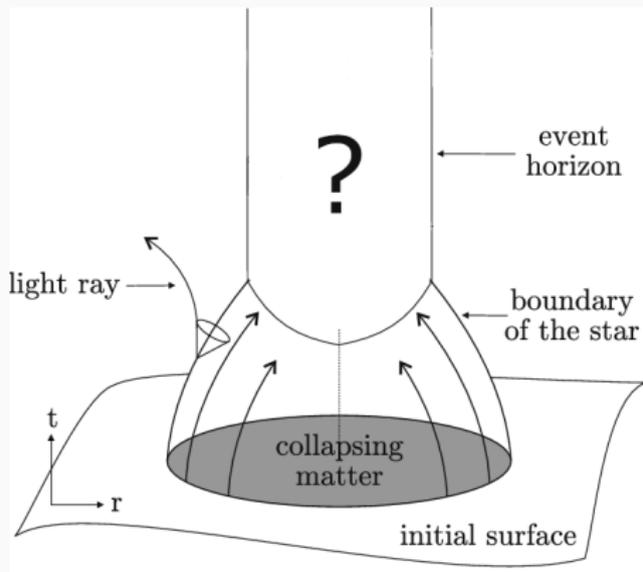
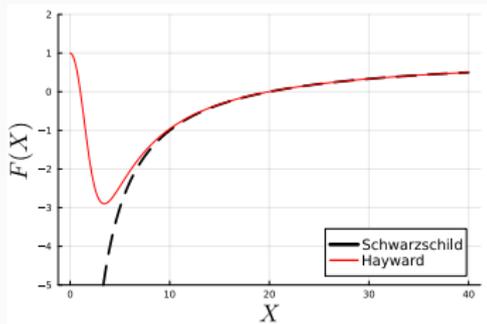
## This talk

Suppose in the exterior we have a regular (non-singular) geometry – a celebrated Hayward black hole – what is the dynamics of the corresponding dust universe in Oppenheimer-Snyder scenario?

Quantum-corrected Schwarzschild metric – Hayward black hole (Hayward, 2005):

$$ds^2 = -F(X)dt^2 + F(X)^{-1}dX^2 + X^2 d\Omega^2 \quad (1)$$

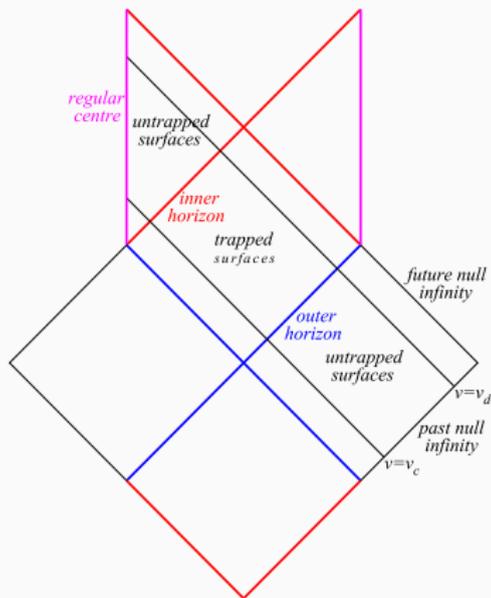
$$F(X) = 1 - \frac{2MX^2}{X^3 + 2l^2 M}$$



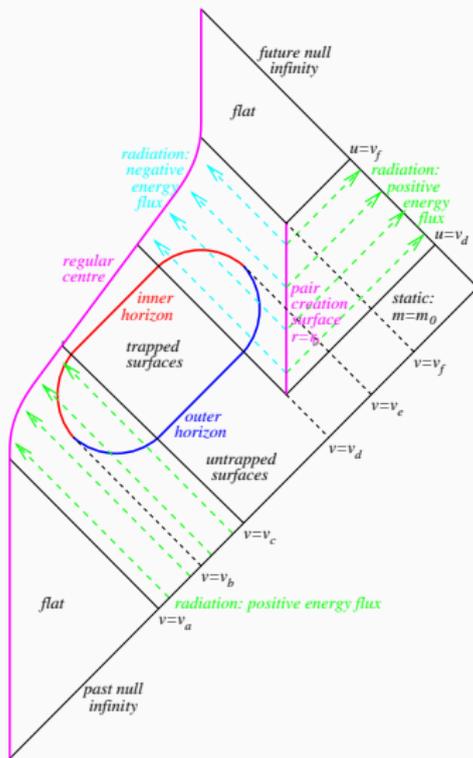
What is the FRW geometry?

$$ds_{\text{int}}^2 = -dT^2 + a(T)^2 dr^2 + r^2 a(T)^2 d\Omega^2. \quad (2)$$

Remarkably, for vacuum Hayward geometry the information paradox is (probably) non-existent. This is why it got so much attention in the literature.



Static Hayward black hole



Evaporating Hayward black hole. Consistent with Ashtekar-Bojowald paradigm (2005)

Very recently Hayward black hole was derived as a unique spherically symmetric vacuum solution within

- $D \geq 5$  Quasi-topological gravity – Einstein-Hilbert action supplemented with infinite tower of higher-curvature corrections (Bueno et al., 2024)

$$S_{\text{QT}} = \frac{1}{16\pi G} \int d^D x \sqrt{|g|} \left[ R + \sum_{n=2}^{n_{\text{max}}} \alpha_n \mathcal{Z}_n \right]$$

$\alpha_n = l^{n-1}$ ,  $n_{\text{max}} \rightarrow \infty$ ,  $\mathcal{Z}_n$  are contractions of Riemann tensor (def. property: eq. of motion of the second order in spherical symmetry).  $\mathcal{Z}_2$  corresponds to Gauss-Bonnet density. Different, specific choices of  $\alpha_n$  can deliver other regular black holes e.g. Bardeen, Dymnikova ones. Such an infinite tower is what one could expect from String Theory as UV completion of GR.

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- Loop-Quantum-Gravity-inspired Lemaitre-Tolman-Bondi (LTB) models (Giesel, et al., 2024). Contrary to standard LQG models, the 'polymerization' function of gravitational Hamiltonian is unbounded.

# A universe out of Hayward black hole

Consider a modified Oppenheimer-Snyder scenario with assumptions

- the exterior is given by Hayward geometry
- interior FRW metric is smoothly joined with the exterior one. Its exact form will be determined by the junction conditions.
- the conservation of EM tensor holds. In particular  $\rho = M / \left(\frac{4}{3}\pi r_b^3 a^3\right)$

These assumptions are compatible with Quasi-Topological gravity and LQG-inspired LTB model.

The exterior is given by

$$ds_{\text{ext}}^2 = -F(X)dt^2 + F(X)^{-1}dX^2 + X^2d\Omega^2 \quad (3)$$

$$F(X) = 1 - \frac{2MX^2}{X^3 + 2l^2M}$$

The interior geometry is described by

$$ds_{\text{int}}^2 = -dT^2 + a(T)^2 dr^2 + r^2 a(T)^2 d\Omega^2. \quad (4)$$

with yet-to-be-determined (via junction conditions) scale factor  $a$ .

**Junction conditions:** continuity of the induced metrics and extrinsic curvatures (first derivatives of induced metrics) at the junction surface – surface of the collapsing dust ball

# Cosmological dynamics from junction conditions

The derived inverse of the scale factor

$$T(a) = \frac{1}{3} \sqrt{\frac{2a_0^3 r_b^3}{M} + 4l^2} - \frac{1}{3} \sqrt{\frac{2a^3 r_b^3}{M} + 4l^2} - \frac{1}{3} \log \left( \frac{\left( \sqrt{\frac{2a_0^3 r_b^3}{M} + 4l^2} + 2l \right) \left( \sqrt{\frac{2a^3 r_b^3}{M} + 4l^2} - 2l \right)}{\left( \sqrt{\frac{2a_0^3 r_b^3}{M} + 4l^2} - 2l \right) \left( \sqrt{\frac{2a^3 r_b^3}{M} + 4l^2} + 2l \right)} \right). \quad (5)$$

allows me to extract **quantum-corrected** Friedmann equations

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{(T'(a) a)^2} = \frac{8\pi\rho}{3 + 8\pi l^2 \rho}, \quad \dot{H} + H^2 = \frac{4\pi\rho(-3 + 16\pi l^2 \rho)}{(3 + 8\pi l^2 \rho)^2}, \quad (6)$$

where  $H = \dot{a}/a$ . The expansion around  $\rho = 0$  yields

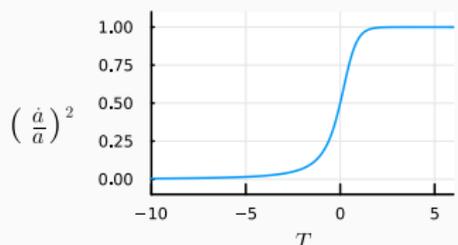
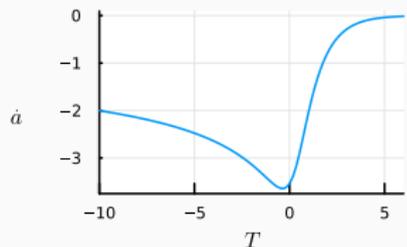
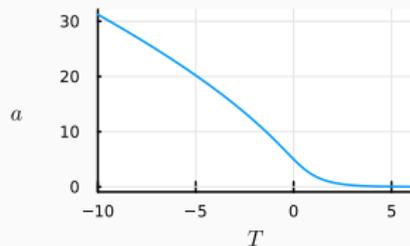
$$\frac{8\pi\rho}{3 + 8\pi l^2 \rho} = \frac{8\pi\rho}{3} - \frac{64}{9} (\pi^2 l^2) \rho^2 + \frac{512}{27} \pi^3 l^4 \rho^3 - \frac{4096}{81} (\pi^4 l^6) \rho^4 + \dots \quad (7)$$

This looks like infinite tower of **quantum corrections**. **Bouncing universe when the series is truncated at the second order!** General Relativity is reproduced both in low energy limit ( $\rho$  small) and  $l \rightarrow 0$ .

# The features of the derived universe

- No curvature singularities, the Kretschmann scalar (full contraction of Riemann tensor) is bounded everywhere. In particular,  $\lim_{a \rightarrow 0} K = \ell^2/24$ .
- Universe is timelike geodesically complete. Free-falling observers "fall" for infinite proper time  $T \in (-\infty, \infty)$
- Smooth transition from power-law contraction (expansion) to exponential one. Graceful exit from inflation.
- The energy density  $\rho = M / (\frac{4}{3}\pi r_b^3 a^3)$  diverges in the final collapsing point  $T \rightarrow \infty$  (equivalently  $a \rightarrow 0$ ). However, neither timelike observer nor light ray reaches this point in finite affine time (I will discuss it in the next slide)

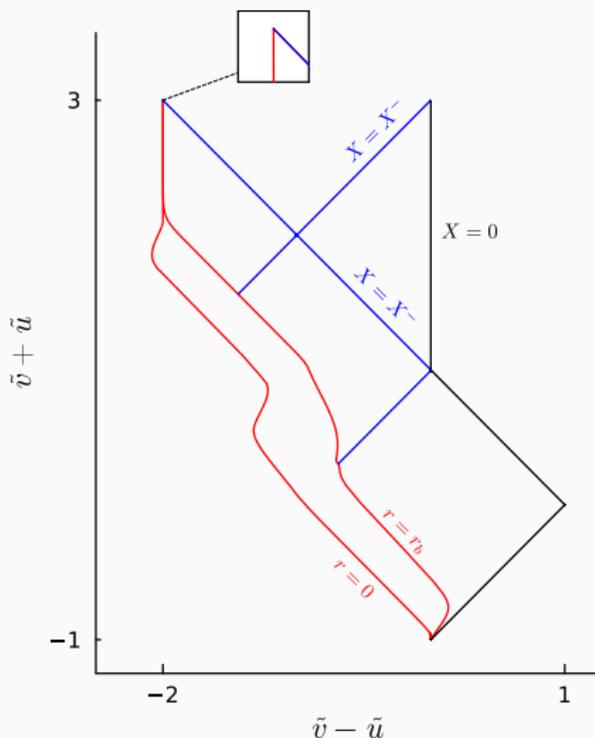
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3 + 8\pi\ell^2\rho}$$



# Numerically computed conformal diagram

- The matter collapses forever unlike in GR where the collapsing dust ball hits singularity in finite proper time.
- The dust ball crosses a pair of horizons: outer and inner horizon. The latter is unstable in general, however, the situation may be cured by Hawking radiation (Bonanno et al. 2022)
- There is only one radial light (null) ray reaching the final collapsing point. It arrives there in infinite affine time.
- In more realistic scenario accounting for the backreaction with the Hawking quanta, the infinite density  $\rho \rightarrow \infty$  will not be reached since the black hole should evaporate in finite time, that is, before  $T \rightarrow \infty$ .

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3 + 8\pi\ell^2\rho}$$



# Discussion

- Two descriptions identify a Hayward black hole as a unique vacuum solution: gravity with higher-curvature corrections (Quasi-topological gravity) and LQG-inspired LTB models. Within a modified Oppenheimer-Snyder collapse scenario, I derived non-singular cosmological dynamics from assumptions compatible with these descriptions. The derived model accounts for Planck scale corrections  $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3+8\pi l_p^2\rho}$

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- The resulting dust collapse model resembles a recently obtained one within the model based on Asymptotically Safe Gravity.  $S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} [R + 2\chi(\epsilon)\mathcal{L}]$ ,  $\chi(\epsilon = 0) = 8\pi G_N$ . (Bonanno et al. 2023)

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Thank you for your attention!