Linear perturbations of the Kerr spacetime in quadratic gravity

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Outline

Motivation: Teukolsky-like approach

We would like to **follow this procedure** in quadratic gravity ...

PERTURBATIONS OF A ROTATING BLACK HOLE. I. FUNDAMENTAL EQUATIONS FOR GRAVITATIONAL, ELECTROMAGNETIC, AND NEUTRINO-FIELD PERTURBATIONS*

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How does one obtain linearized perturbation equations, say for gravitational perturbations? A straightforward way is to start with the Einstein equations for a metric $g_{\mu\nu}$, and to let $g_{\mu\nu} = g_{\mu\nu}{}^{A} + h_{\mu\nu}{}^{B}$, where the superscripts A and B denote background and perturbation quantities, respectively. The field equations are then expanded to first order in $h_{\mu\nu}{}^{B}$, yielding a set of linear equations for the perturbations.

Even in the Schwarzschild case, this procedure involves considerable algebraic complexity.

Fortunately, there is an alternative approach to the problem. This is provided by the Newman-Penrose (NP) formalism.

To do perturbation theory in this formalism, one specifies the perturbed geometry by $\boldsymbol{l} = \boldsymbol{l}^{A} + \boldsymbol{l}^{B}$, $\boldsymbol{n} = \boldsymbol{n}^{A} + \boldsymbol{n}^{B}$, etc. All the NP quantities can then be written in this form: $\psi_2 = \psi_2{}^{A} + \psi_2{}^{B}$, $D = D^{A} + D^{B}$, etc. The complete set of perturbation equations is obtained from the NP equations by keeping B terms only to first order.

The Schwarzschild and Kerr metrics are very similar from the NP point of view. This similarity allows us, in this paper, to derive decoupled Kerr-metric equations for $\psi_0{}^{B}$ and $\psi_4{}^{B}$. Moreover, we shall demonstrate the unexpected result that these equations, like those for Schwarzschild, are separable.

... let's start from the beginning ...

General relativity: Einstein's theory of gravity

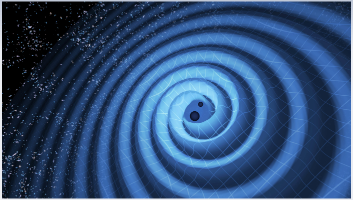
Gravity – **universal interaction** – inherent property of the '**arena**' (spacetime)

- main ingredient – **curvature** – Riemann tensor R_{abcd}
- WANTED: metric tensor g_{ab}

Einstein's field equations [*Einstein, 1916*]

$$R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

' *geometry* = *energy* and *momentum* '



(credit: ligo.org)

Non-linearity: *matter* → *space curvature* → *matter motion* → *curvature changed* → etc.

Solutions: exact spacetimes × perturbative models × numerical simulations

Successes: Mercury perihelion shift, light bending, cosmology, stellar evolution, GWs

Typical problems: matter in the universe, singularities quantum effects

Possible solution: **modifications of the theory**

Einstein–Hilbert action and its modifications

General relativity:

$$S = \int \left[\tfrac{1}{\mathsf{k}}(R - 2\Lambda) + L_M\right] \sqrt{-g}\, \mathrm{d}^4 \mathrm{x} \qquad \xrightarrow{\delta S = 0} \qquad R_{ab} - \tfrac{1}{2} R g_{ab} + \Lambda g_{ab} = \tfrac{\mathsf{k}}{2}\, T_{ab}$$

- R: the Ricci scalar and L_M: matter contribution
- Λ: the cosmological constant and k: theory parameter

Quadratic gravity: extension of the *geometric part* $\rightarrow$ *quadratic curvature* terms

[*Weyl, 1919; Bach, 1921; Stelle, 1977; 1978*]

$$S = \int \left[\tfrac{1}{\mathsf{k}}(R - 2\Lambda) - \mathfrak{a}\, C_{abcd}^2 + \mathfrak{b}\, R^2 + L_M\right] \sqrt{-g}\, \mathrm{d}^4 \mathrm{x}$$

- Λ, k, $\mathfrak{a}$, and $\mathfrak{b}$: theory constants
- C_{abcd}: Weyl tensor (i.e., the traceless part of the Riemann curvature tensor R_{abcd})
- field equations

$$\tfrac{1}{\mathsf{k}}\left(R_{ab} - \tfrac{1}{2} R g_{ab} + \Lambda g_{ab}\right) - 4\mathfrak{a}\, B_{ab} + 2\mathfrak{b}\left(R_{ab} - \tfrac{1}{4} R g_{ab} + g_{ab}\Box - \nabla_a \nabla_b\right) R = \tfrac{1}{2}\, T_{ab}$$

$$\text{with} \qquad B_{ab} = \left(\nabla^c \nabla^d + \tfrac{1}{2} R^{cd}\right) C_{acbd} \qquad \leftrightarrow \qquad \text{Bach tensor}$$

- Vacuum solutions to GR solve also QG $\rightarrow$ **backgrounds** for perturbations!

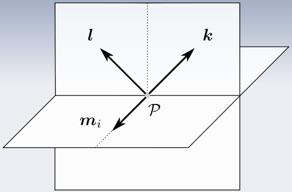
NP formalism: null frame and scalar quantities

Key idea: to suppress coordinate ambiguity and employ geometric quantities

Ingredients of the NP approach [*Newman and Penrose, 1962*]:

- null orthonormal frame:

$$\{\boldsymbol{k}\,,\boldsymbol{l}\,,\boldsymbol{m}\,,\bar{\boldsymbol{m}}\} \quad \text{with} \quad \boldsymbol{k}\cdot\boldsymbol{l} = -1\,, \qquad \boldsymbol{m}\cdot\bar{\boldsymbol{m}} = 1$$



Crucial quantities: projections onto the null frame

- the Weyl tensor projections

$$\Psi_0 = C_{abcd}k^a m^b k^c m^d \quad \text{and} \quad \Psi_1\,,\ \Psi_2\,,\ \Psi_3\,,\Psi_4$$

- the Ricci tensor projections

$$\Phi_{00} = \frac{1}{2}R_{ab}k^a k^b \quad \text{and} \quad \Phi_{01}\,,\ \Phi_{10}\,,\ \Phi_{11}\,,\Phi_{02}\,,\ \Phi_{20}\,,\ \Phi_{12}\,,\ \Phi_{21}\,,\ \Phi_{22}$$

- **covariant derivative** decomposition

$$\mathrm{D} = k^a\nabla_a \qquad \Delta = l^a\nabla_a \qquad \delta = m^a\nabla_a \qquad \bar{\delta} = \bar{m}^a\nabla_a$$

- components of the **frame vector derivatives**

$$\kappa = -k_{a;b}m^a k^b \quad \text{and} \quad \rho\,,\ \nu\,,\ \epsilon\,,\ \mu\,,\ \alpha\,,\ \beta\,,\ \gamma\,,\ \sigma\,,\ \lambda\,,\ \tau\,,\ \pi$$

NP formalism: geometric constraints

- the **commutation relations** (express the Lie bracket of all combinations of the frame vectors

$$\Delta \mathrm{D} - \mathrm{D}\Delta = (\gamma + \bar{\gamma})\,\mathrm{D} + (\epsilon + \bar{\epsilon})\,\Delta - (\bar{\tau} + \pi)\,\delta - (\tau + \bar{\pi})\,\bar{\delta}$$

  + 3 more ...

- the **Ricci identities** (correspond to the Riemann tensor nonzero components)

$$\mathrm{D}\sigma - \delta\kappa = \sigma\,(3\epsilon - \bar{\epsilon} + \rho + \bar{\rho}) + \kappa\,(\bar{\pi} - \tau - 3\beta - \bar{\alpha}) + \Psi_0$$

  + 17 more ...

- the **Bianchi identities** (the Riemann tensor derivative with cyclic indices exchange)

$$\begin{aligned} 0 = &- \bar{\delta}\Psi_0 + \mathrm{D}\Psi_1 + (4\alpha - \pi)\Psi_0 - 2(2\rho + \epsilon)\Psi_1 + 3\kappa\Psi_2 \\ &- \mathrm{D}\Phi_{01} + \delta\Phi_{00} + 2\,(\epsilon + \bar{\rho})\,\Phi_{01} + 2\sigma\Phi_{10} - 2\kappa\Phi_{11} - \bar{\kappa}\Phi_{02} \\ &+ (\bar{\pi} - 2\bar{\alpha} - 2\beta)\,\Phi_{00} \end{aligned}$$

  + 7 more ...

- the **contracted** Bianchi identities

$$\begin{aligned} &\bar{\delta}\Phi_{01} + \delta\Phi_{10} - D\left(\Phi_{11} + \tfrac{R}{8}\right) - \Delta\Phi_{00} \\ &= \bar{\kappa}\Phi_{12} + \kappa\Phi_{21} + (2\alpha + 2\bar{\tau} - \pi)\,\Phi_{01} + (2\bar{\alpha} + 2\tau - \bar{\pi})\,\Phi_{10} \\ &- 2\,(\rho + \bar{\rho})\,\Phi_{11} - \bar{\sigma}\Phi_{02} - \sigma\Phi_{20} + [\mu + \bar{\mu} - 2\,(\gamma + \bar{\gamma})]\,\Phi_{00} \end{aligned}$$

  + 2 more ...

NP formalism: gravity theories

General relativity: $R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{\kappa}{2} T_{ab}$

- projected field equations: **algebraic constraints on the Ricci tensor Φ_{AB}**

$$\tfrac{1}{2} T_{(0)(0)} = \tfrac{2}{\kappa} \Phi_{00} \qquad \tfrac{1}{2} T_{(0)(1)} = \tfrac{1}{\kappa} \left(2\Phi_{11} - \tfrac{R}{4}\right) \qquad \tfrac{1}{2} T_{(0)(2)} = \tfrac{2}{\kappa} \Phi_{01} \qquad \tfrac{1}{2} T_{(1)(1)} = \tfrac{2}{\kappa} \Phi_{22}$$
$$\tfrac{1}{2} T_{(1)(2)} = \tfrac{2}{\kappa} \Phi_{12} \qquad \tfrac{1}{2} T_{(2)(2)} = \tfrac{2}{\kappa} \Phi_{02} \qquad \tfrac{1}{2} T_{(2)(3)} = \tfrac{1}{\kappa} \left(2\Phi_{11} + \tfrac{R}{4}\right)$$

- Φ_{AB} has to be further combined with the **Ricci** and **Bianchi identities**
- the scalar curvature is $R = 4\Lambda - \frac{\kappa}{2} T$

Quadratic gravity: [*Švarc, Pravdova, Miškovský 2023*]

- rearranged field equations:

$$\left(\frac{1}{\kappa} + 2\mathfrak{b} R\right) R_{ab} - 2\mathfrak{a} R^{cd} C_{acbd} + Z_{ab} = \frac{1}{2} T_{ab}$$

with

$$Z_{ab} = -\frac{1}{\kappa}\left(\frac{1}{2} R g_{ab} - \Lambda g_{ab}\right) - 4\mathfrak{a}\, B^Z_{ab} - 2\mathfrak{b}\left(\frac{1}{4} R g_{ab} - g_{ab}\Box + \nabla_a \nabla_b\right) R$$
$$B^Z_{ab} = \nabla^c \nabla^d C_{acbd}$$

- it remains to calculate their **frame projections**

NP formalism: QG field equations

The **QG field equations** in terms of the null frame $\{\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{m}, \bar{\boldsymbol{m}}\}$:

$$\tfrac{1}{2}\,T_{(0)(0)} = -4\mathfrak{a}\left[\Phi_{20}\Psi_0 + \Phi_{02}\bar{\Psi}_0 - 2\Phi_{10}\Psi_1 - 2\Phi_{01}\bar{\Psi}_1 + \Phi_{00}(\Psi_2 + \bar{\Psi}_2)\right] + 2\left(\tfrac{1}{\mathsf{k}} + 2\mathfrak{b}R\right)\Phi_{00} + Z_{(0)(0)}$$

$$\tfrac{1}{2}\,T_{(0)(1)} = -4\mathfrak{a}\left[\Phi_{21}\Psi_1 + \Phi_{12}\bar{\Psi}_1 - 2\Phi_{11}(\Psi_2 + \bar{\Psi}_2) + \Phi_{01}\Psi_3 + \Phi_{10}\bar{\Psi}_3\right] + \left(\tfrac{1}{\mathsf{k}} + 2\mathfrak{b}R\right)\left(2\Phi_{11} - \tfrac{R}{4}\right) + Z_{(0)(1)}$$

$$\tfrac{1}{2}\,T_{(0)(2)} = -4\mathfrak{a}\left[\Phi_{21}\Psi_0 - 2\Phi_{11}\Psi_1 + \Phi_{02}\bar{\Psi}_1 + \Phi_{01}(\Psi_2 - 2\bar{\Psi}_2) + \Phi_{00}\bar{\Psi}_3\right] + 2\left(\tfrac{1}{\mathsf{k}} + 2\mathfrak{b}R\right)\Phi_{01} + Z_{(0)(2)}$$

$$\tfrac{1}{2}\,T_{(1)(1)} = -4\mathfrak{a}\left(\Phi_{22}(\Psi_2 + \bar{\Psi}_2) - 2\Phi_{12}\Psi_3 - 2\Phi_{21}\bar{\Psi}_3 + \Phi_{02}\Psi_4 + \Phi_{20}\bar{\Psi}_4\right) + 2\left(\tfrac{1}{\mathsf{k}} + 2\mathfrak{b}R\right)\Phi_{22} + Z_{(1)(1)}$$

$$\tfrac{1}{2}\,T_{(1)(2)} = -4\mathfrak{a}\left[\Phi_{22}\Psi_1 + \Phi_{12}(-2\Psi_2 + \bar{\Psi}_2) + \Phi_{02}\Psi_3 - 2\Phi_{11}\bar{\Psi}_3 + \Phi_{10}\bar{\Psi}_4\right] + 2\left(\tfrac{1}{\mathsf{k}} + 2\mathfrak{b}R\right)\Phi_{12} + Z_{(1)(2)}$$

$$\tfrac{1}{2}\,T_{(2)(2)} = -4\mathfrak{a}\left[\Phi_{22}\Psi_0 - 2\Phi_{12}\Psi_1 + \Phi_{02}(\Psi_2 + \bar{\Psi}_2) - 2\Phi_{01}\bar{\Psi}_3 + \Phi_{00}\bar{\Psi}_4\right] + 2\left(\tfrac{1}{\mathsf{k}} + 2\mathfrak{b}R\right)\Phi_{02} + Z_{(2)(2)}$$

$$\tfrac{1}{2}\,T_{(2)(3)} = -4\mathfrak{a}\left[\Phi_{21}\Psi_1 + \Phi_{12}\bar{\Psi}_1 - 2\Phi_{11}(\Psi_2 + \bar{\Psi}_2) + \Phi_{01}\Psi_3 + \Phi_{10}\bar{\Psi}_3\right] + \left(\tfrac{1}{\mathsf{k}} + 2\mathfrak{b}R\right)\left(2\Phi_{11} + \tfrac{R}{4}\right) + Z_{(2)(3)}$$

This system gives **algebraic constraints on the Ricci tensor** Φ_{AB} !

- $T_{(a)(b)}$ are components of the energy-momentum tensor T_{ab}
- Φ_{AB} has to be thus combined with the **Ricci** and **Bianchi identities**
- the GR case is given by $\mathfrak{a} = 0 = \mathfrak{b}$
- the scalar curvature is more complicated

$$R + \tfrac{1}{2}\,\mathsf{k}\,T = 6\mathfrak{b}\mathsf{k}\Box R + 4\Lambda$$

- $Z_{(a)(b)}$ are components of the tensor Z_{ab}, namely:

NP formalism: QG field equations

Relevant projections of the Z_{ab} tensor:

$$
\begin{aligned}
Z_{(0)(0)} &= -4\mathfrak{a}B^Z_{(0)(0)} + 2\mathfrak{b}\left[(\epsilon+\bar\epsilon)\mathrm{D}R - \mathrm{D}\mathrm{D}R - \bar\kappa\delta R - \kappa\bar\delta R\right]\\
Z_{(0)(1)} &= -4\mathfrak{a}B^Z_{(0)(1)} + \frac{1}{2\mathsf{k}}(R-2\Lambda) + 2\mathfrak{b}\Big[\frac{1}{4}R^2 - (\gamma+\bar\gamma-\mu-\bar\mu)\mathrm{D}R\\
&\quad -(\rho+\bar\rho)\Delta R + \Delta\mathrm{D}R + (\alpha-\bar\beta+\bar\tau)\delta R - \delta\bar\delta R + (\bar\alpha-\beta+\tau)\bar\delta R - \bar\delta\delta R\Big]\\
Z_{(0)(2)} &= -4\mathfrak{a}B^Z_{(0)(2)} + 2\mathfrak{b}\left[\bar\pi\mathrm{D}R - \mathrm{D}\delta R - \kappa\Delta R + (\epsilon-\bar\epsilon)\delta R\right]\\
Z_{(1)(1)} &= -4\mathfrak{a}B^Z_{(1)(1)} + 2\mathfrak{b}\left[-(\gamma+\bar\gamma)\Delta R - \Delta\Delta R + \nu\delta R + \bar\nu\bar\delta R\right]\\
Z_{(1)(2)} &= -4\mathfrak{a}B^Z_{(1)(2)} + 2\mathfrak{b}\left[\bar\nu\mathrm{D}R - \tau\Delta R - \Delta\delta R + (\gamma-\bar\gamma)\delta R\right]\\
Z_{(2)(2)} &= -4\mathfrak{a}B^Z_{(2)(2)} + 2\mathfrak{b}\left[\bar\lambda\mathrm{D}R - \sigma\Delta R + (-\bar\alpha+\beta)\delta R - \delta\delta R\right]\\
Z_{(2)(3)} &= -4\mathfrak{a}B^Z_{(2)(3)} - \frac{1}{2\mathsf{k}}(R-2\Lambda) + 2\mathfrak{b}\Big[-\frac{1}{4}R^2 + (\gamma+\bar\gamma-\bar\mu)\mathrm{D}R\\
&\quad -\mathrm{D}\Delta R + (\rho-\epsilon-\bar\epsilon)\Delta R - \Delta\mathrm{D}R + (-\alpha+\bar\beta+\pi-\bar\tau)\delta R + (\bar\pi-\tau)\bar\delta R + \bar\delta\delta R\Big]
\end{aligned}
$$

- $B^Z_{(c)(d)}$ represents projections of the Ricci-independent part of the Bach tensor

$$B^Z_{ab} = \nabla^c\nabla^d C_{acbd}$$

- it can be obtained after quite straightforward and very long calculation

$$
\begin{aligned}
B^Z_{(0)(0)} =&\bar{\delta}\bar{\delta}\Psi_0 - \mathrm{D}\bar{\delta}\Psi_1 - \bar{\delta}\mathrm{D}\Psi_1 + \mathrm{D}\mathrm{D}\Psi_2 + \lambda\mathrm{D}\Psi_0 + \bar{\sigma}\Delta\Psi_0 + (2\pi - 7\alpha - \bar{\beta})\bar{\delta}\Psi_0 \\
&+ (5\alpha + \bar{\beta} - 3\pi)\mathrm{D}\Psi_1 - \bar{\kappa}\Delta\Psi_1 - \bar{\sigma}\delta\Psi_1 + (3\epsilon + \bar{\epsilon} + 7\rho)\bar{\delta}\Psi_1 \\
&- (\epsilon + \bar{\epsilon} + 6\rho)\mathrm{D}\Psi_2 + \bar{\kappa}\delta\Psi_2 - 5\kappa\bar{\delta}\Psi_2 + 4\kappa\mathrm{D}\Psi_3 \\
&+ \Psi_0[\bar{\kappa}\nu + 4\alpha(3\alpha + \bar{\beta}) - (\epsilon + \bar{\epsilon} + 3\rho)\lambda + \pi(\pi - 7\alpha - \bar{\beta}) + \bar{\sigma}(\mu - 4\gamma) + \mathrm{D}\lambda - 4\bar{\delta}\alpha + \bar{\delta}\pi] \\
&+ 2\Psi_1[2\kappa\lambda + \bar{\kappa}(\gamma - \mu) + \rho(5\pi - 9\alpha - 2\bar{\beta}) + \bar{\sigma}(\beta + 2\tau) + \epsilon(2\pi - 4\alpha - \bar{\beta}) + \bar{\epsilon}(\pi - \alpha) \\
&\qquad + \mathrm{D}\alpha - \mathrm{D}\pi + \bar{\delta}\epsilon + 2\bar{\delta}\rho] \\
&+ 3\Psi_2[\kappa(3\alpha + \bar{\beta} - 3\pi) - \bar{\kappa}\tau + \rho(\epsilon + \bar{\epsilon} + 3\rho) - \sigma\bar{\sigma} - \mathrm{D}\rho - \bar{\delta}\kappa] \\
&+ 2\Psi_3[\kappa(\epsilon - \bar{\epsilon} - 5\rho) + \bar{\kappa}\sigma + \mathrm{D}\kappa] + 2\Psi_4\kappa^2 + c.c.
\end{aligned}
$$

$$
\begin{aligned}
B^Z_{(0)(1)} =&\bar{\delta}\Delta\Psi_1 - \mathrm{D}\Delta\Psi_2 - \bar{\delta}\delta\Psi_2 + \mathrm{D}\delta\Psi_3 - \lambda\Delta\Psi_0 - \nu\bar{\delta}\Psi_0 \\
&+ 2\nu\mathrm{D}\Psi_1 + (2\pi - \alpha + \bar{\beta})\Delta\Psi_1 + \lambda\delta\Psi_1 + (2\mu - \bar{\mu} - 2\gamma)\bar{\delta}\Psi_1 \\
&+ (\bar{\mu} - 3\mu)\mathrm{D}\Psi_2 + (2\rho - \epsilon - \bar{\epsilon})\Delta\Psi_2 + (\alpha - \bar{\beta} - 2\pi)\delta\Psi_2 + (\bar{\pi} + 3\tau)\bar{\delta}\Psi_2 \\
&+ (2\beta - \bar{\pi} - 2\tau)\mathrm{D}\Psi_3 - \kappa\Delta\Psi_3 + (\epsilon + \bar{\epsilon} - 2\rho)\delta\Psi_3 - 2\sigma\bar{\delta}\Psi_3 + \sigma\mathrm{D}\Psi_4 + \kappa\delta\Psi_4 \\
&+ \Psi_0[\lambda(4\gamma - \mu + \bar{\mu}) + \nu(\alpha - \bar{\beta} - 2\pi) - \bar{\delta}\nu] \\
&+ 2\Psi_1[\gamma(\alpha - \bar{\beta} - 2\pi) - \lambda(\beta + \bar{\pi} + 2\tau) + \mu(\bar{\beta} - \alpha + 2\pi) + \bar{\mu}(\alpha - \pi) + \nu(\epsilon + \bar{\epsilon} - 2\rho) \\
&\qquad + \mathrm{D}\nu - \bar{\delta}\gamma + \bar{\delta}\mu] \\
&+ 3\Psi_2[\kappa\nu + \mu(2\rho - \epsilon - \bar{\epsilon}) - \bar{\mu}\rho + \pi\bar{\pi} + \lambda\sigma + \tau(2\pi - \alpha + \bar{\beta}) - \mathrm{D}\mu + \bar{\delta}\tau] \\
&+ 2\Psi_3[\kappa(\bar{\mu} - 2\mu - \gamma) + \epsilon(\beta - \tau - \bar{\pi}) + \bar{\epsilon}(\beta - \tau) + \rho(\bar{\pi} - 2\beta + 2\tau) + \sigma(\alpha - \bar{\beta} - 2\pi) \\
&\qquad + \mathrm{D}\beta - \mathrm{D}\tau - \bar{\delta}\sigma] \\
&+ \Psi_4[\kappa(4\beta - \bar{\pi} - \tau) + \sigma(\epsilon + \bar{\epsilon} - 2\rho) + \mathrm{D}\sigma] + c.c.
\end{aligned}
$$

$$
\begin{aligned}
B^Z_{(0)(2)} =&\bar{\delta}\Delta\Psi_0 - \mathrm{D}\Delta\Psi_1 - \bar{\delta}\delta\Psi_1 + \mathrm{D}\delta\Psi_2 \\
&+ \nu\mathrm{D}\Psi_0 + (\pi - 3\alpha + \bar{\beta})\Delta\Psi_0 + (\mu - \bar{\mu} - 4\gamma)\bar{\delta}\Psi_0 \\
&+ (2\gamma - 2\mu + \bar{\mu})\mathrm{D}\Psi_1 + (\epsilon - \bar{\epsilon} + 3\rho)\Delta\Psi_1 + (3\alpha - \bar{\beta} - \pi)\delta\Psi_1 + (2\beta + \bar{\pi} + 4\tau)\bar{\delta}\Psi_1 \\
&- (\bar{\pi} + 3\tau)\mathrm{D}\Psi_2 - 2\kappa\Delta\Psi_2 - (\epsilon - \bar{\epsilon} + 3\rho)\delta\Psi_2 - 3\sigma\bar{\delta}\Psi_2 + 2\sigma\mathrm{D}\Psi_3 + 2\kappa\delta\Psi_3 \\
&+ \Psi_0[(4\gamma - \mu)(3\alpha - \bar{\beta} - \pi) + \bar{\mu}(4\alpha - \pi) + \nu(\bar{\epsilon} - \epsilon - 3\rho) - \lambda\bar{\pi} + \mathrm{D}\nu - 4\bar{\delta}\gamma + \bar{\delta}\mu] \\
&+ 2\Psi_1[2\kappa\nu + (\mu - \gamma)(\epsilon - \bar{\epsilon} + 3\rho) - \bar{\mu}(2\rho + \epsilon) + (\beta + 2\tau)(\pi - 3\alpha + \bar{\beta}) + \bar{\pi}(\pi - \alpha) \\
&\qquad + \mathrm{D}\gamma - \mathrm{D}\mu + \bar{\delta}\beta + 2\bar{\delta}\tau] \\
&+ 3\Psi_2[\kappa(\bar{\mu} - 2\mu) + \bar{\pi}\rho + \sigma(3\alpha - \bar{\beta} - \pi) + \tau(\epsilon - \bar{\epsilon} + 3\rho) - \mathrm{D}\tau - \bar{\delta}\sigma] \\
&+ 2\Psi_3[\kappa(2\beta - \bar{\pi} - 2\tau) + \sigma(\bar{\epsilon} - \epsilon - 3\rho) + \mathrm{D}\sigma] + 2\Psi_4\kappa\sigma \\
&+ \delta\delta\bar{\Psi}_1 - \delta\mathrm{D}\bar{\Psi}_2 - \mathrm{D}\delta\bar{\Psi}_2 + \mathrm{D}\mathrm{D}\bar{\Psi}_3 \\
&- 2\bar{\lambda}\delta\bar{\Psi}_0 + 3\bar{\lambda}\mathrm{D}\bar{\Psi}_1 + \sigma\Delta\bar{\Psi}_1 + (4\bar{\pi} - 3\bar{\alpha} - \beta)\delta\bar{\Psi}_1 \\
&+ (\bar{\alpha} + \beta - 5\bar{\pi})\mathrm{D}\bar{\Psi}_2 - \kappa\Delta\bar{\Psi}_2 + (\epsilon - \bar{\epsilon} + 5\bar{\rho})\delta\bar{\Psi}_2 - \sigma\bar{\delta}\bar{\Psi}_2 \\
&+ (3\bar{\epsilon} - \epsilon - 4\bar{\rho})\mathrm{D}\bar{\Psi}_3 - 3\bar{\kappa}\delta\bar{\Psi}_3 + \kappa\bar{\delta}\bar{\Psi}_3 + 2\bar{\kappa}\mathrm{D}\bar{\Psi}_4 \\
&+ \bar{\Psi}_0[\bar{\lambda}(5\bar{\alpha} + \beta - 3\bar{\pi}) - \bar{\nu}\sigma - \delta\bar{\lambda}] \\
&+ 2\bar{\Psi}_1[\kappa\bar{\nu} + \bar{\alpha}(\bar{\alpha} + \beta) + \bar{\pi}(2\bar{\pi} - 3\bar{\alpha} - \beta) - \bar{\lambda}(4\bar{\rho} + \epsilon) + \sigma(\bar{\mu} - \bar{\gamma}) + \mathrm{D}\bar{\lambda} - \delta\bar{\alpha} + \delta\bar{\pi}] \\
&+ 3\bar{\Psi}_2[2\bar{\kappa}\bar{\lambda} - \kappa\bar{\mu} + \bar{\pi}(\epsilon - \bar{\epsilon}) + \bar{\rho}(4\bar{\pi} - \bar{\alpha} - \beta) + \sigma\bar{\tau} - \mathrm{D}\bar{\pi} + \delta\bar{\rho}] \\
&+ 2\bar{\Psi}_3(\kappa(\bar{\beta} - \bar{\tau}) + \bar{\kappa}(\beta - 4\bar{\pi}) - \sigma\bar{\sigma} + (\bar{\rho} - \bar{\epsilon})(\epsilon - \bar{\epsilon} + 2\bar{\rho}) + \mathrm{D}\bar{\epsilon} - \mathrm{D}\bar{\rho} - \delta\bar{\kappa}) \\
&+ \bar{\Psi}_4[\bar{\kappa}(5\bar{\epsilon} - \epsilon - 3\bar{\rho}) + \kappa\bar{\sigma} + \mathrm{D}\bar{\kappa}]
\end{aligned}
$$

$$
\begin{aligned}
B^Z_{(1)(2)} =&\Delta\Delta\Psi_1 - \Delta\delta\Psi_2 - \delta\Delta\Psi_2 + \delta\delta\Psi_3 \\
&- 2\nu\Delta\Psi_0 + (4\mu - 3\gamma + \bar{\gamma})\Delta\Psi_1 + 3\nu\delta\Psi_1 - \bar{\nu}\bar{\delta}\Psi_1 \\
&+ \bar{\nu}\mathrm{D}\Psi_2 + (5\tau - \bar{\alpha} - \beta)\Delta\Psi_2 + (\gamma - \bar{\gamma} - 5\mu)\delta\Psi_2 + \bar{\lambda}\bar{\delta}\Psi_2 \\
&- \bar{\lambda}\mathrm{D}\Psi_3 - 3\sigma\Delta\Psi_3 + (\bar{\alpha} + 3\beta - 4\tau)\delta\Psi_3 + 2\sigma\delta\Psi_4 \\
&+ \Psi_0[\nu(5\gamma - \bar{\gamma} - 3\mu) + \lambda\bar{\nu} - \Delta\nu] \\
&+ 2\Psi_1[\nu(\bar{\alpha} - 4\tau) + \bar{\nu}(\alpha - \pi) - \lambda\bar{\lambda} + (\gamma - \mu)(\gamma - \bar{\gamma} - 2\mu) - \Delta\gamma + \Delta\mu + \delta\nu] \\
&+ 3\Psi_2[\mu(4\tau - \bar{\alpha} - \beta) + \bar{\lambda}\pi - \bar{\nu}\rho + 2\nu\sigma + \tau(\bar{\gamma} - \gamma) + \Delta\tau - \delta\mu] \\
&+ 2\Psi_3[\kappa\bar{\nu} - \sigma(\bar{\gamma} + 4\mu) + \tau(2\tau - \bar{\alpha} - 3\beta) + \beta(\bar{\alpha} + \beta) + \bar{\lambda}(\rho - \epsilon) - \Delta\sigma + \delta\beta - \delta\tau] \\
&+ \Psi_4[-\kappa\bar{\lambda} + \sigma(\bar{\alpha} + 5\beta - 3\tau) + \delta\sigma] \\
&- \Delta\mathrm{D}\bar{\Psi}_3 + \Delta\delta\bar{\Psi}_2 + \bar{\delta}\mathrm{D}\bar{\Psi}_4 - \bar{\delta}\delta\bar{\Psi}_3 \\
&- 2\bar{\lambda}\Delta\bar{\Psi}_1 - 2\bar{\nu}\delta\bar{\Psi}_1 + 2\bar{\nu}\mathrm{D}\bar{\Psi}_2 + (3\bar{\pi} + \tau)\Delta\bar{\Psi}_2 + (\bar{\gamma} - \gamma + 3\bar{\mu})\delta\bar{\Psi}_2 + 3\bar{\lambda}\bar{\delta}\bar{\Psi}_2 \\
&+ (\gamma - \bar{\gamma} - 3\bar{\mu})\mathrm{D}\bar{\Psi}_3 + (2\bar{\rho} - \rho - 2\bar{\epsilon})\Delta\bar{\Psi}_3 + (\alpha - 3\bar{\beta} + \bar{\tau})\delta\bar{\Psi}_3 - (2\bar{\alpha} + 4\bar{\pi} + \tau)\bar{\delta}\bar{\Psi}_3 \\
&+ (3\bar{\beta} - \alpha - \bar{\tau})\mathrm{D}\bar{\Psi}_4 - \bar{\kappa}\Delta\bar{\Psi}_4 + (4\bar{\epsilon} + \rho - \bar{\rho})\bar{\delta}\bar{\Psi}_4 \\
&+ 2\bar{\Psi}_0\bar{\lambda}\bar{\nu} + 2\bar{\Psi}_1[\bar{\lambda}(\gamma - \bar{\gamma} - 3\bar{\mu}) + \bar{\nu}(2\bar{\alpha} - 2\bar{\pi} - \tau) - \Delta\bar{\lambda}] \\
&+ 3\bar{\Psi}_2[\bar{\lambda}(3\bar{\beta} - \bar{\tau} - \alpha) + \bar{\pi}(3\bar{\mu} - \gamma + \bar{\gamma}) + \bar{\nu}(\rho - 2\bar{\rho}) + \bar{\mu}\tau + \Delta\bar{\pi} + \bar{\delta}\bar{\lambda}] \\
&+ 2\bar{\Psi}_3[2\bar{\kappa}\bar{\nu} + (\bar{\epsilon} - \bar{\rho})(\gamma - \bar{\gamma} - 3\bar{\mu}) - \rho(\bar{\gamma} + 2\bar{\mu}) + \tau(\bar{\tau} - \bar{\beta}) + (\bar{\alpha} + 2\bar{\pi})(\alpha - 3\bar{\beta} + \bar{\tau}) \\
&\qquad - \Delta\bar{\epsilon} + \Delta\bar{\rho} - \bar{\delta}\bar{\alpha} - 2\bar{\delta}\bar{\pi}] \\
&+ \bar{\Psi}_4[\bar{\kappa}(\gamma - \bar{\gamma} - 3\bar{\mu}) + \rho(4\bar{\beta} - \bar{\tau}) + \bar{\rho}(\alpha - 3\bar{\beta} + \bar{\tau}) + 4\bar{\epsilon}(3\bar{\beta} - \bar{\tau} - \alpha) - \bar{\sigma}\tau \\
&\qquad - \Delta\bar{\kappa} + 4\bar{\delta}\bar{\epsilon} - \bar{\delta}\bar{\rho}]
\end{aligned}
$$

$$
\begin{aligned}
B^Z_{(2)(2)} =&\Delta\Delta\Psi_0 - \Delta\delta\Psi_1 - \delta\Delta\Psi_1 + \delta\delta\Psi_2 \\
&+ (2\mu - 7\gamma + \bar{\gamma})\Delta\Psi_0 + \nu\delta\Psi_0 - \bar{\nu}\bar{\delta}\Psi_0 \\
&+ \bar{\nu}\mathrm{D}\Psi_1 + (7\tau - \bar{\alpha} + 3\beta)\Delta\Psi_1 + (5\gamma - \bar{\gamma} - 3\mu)\delta\Psi_1 + \bar{\lambda}\bar{\delta}\Psi_1 \\
&- \bar{\lambda}\mathrm{D}\Psi_2 - 5\sigma\Delta\Psi_2 + (\bar{\alpha} - \beta - 6\tau)\delta\Psi_2 + 4\sigma\delta\Psi_3 \\
&+ \Psi_0[\mu(\mu - 7\gamma + \bar{\gamma}) + \nu(\bar{\alpha} - \beta - 3\tau) + \bar{\nu}(4\alpha - \pi) + 4\gamma(3\gamma - \bar{\gamma}) - \lambda\bar{\lambda} \\
&\qquad - 4\Delta\gamma + \Delta\mu + \delta\nu] \\
&+ 2\Psi_1[2\nu\sigma - \bar{\nu}(\epsilon + 2\rho) + \bar{\lambda}(\pi - \alpha) + (\bar{\gamma} - 2\gamma)(\beta + 2\tau) + (\mu - \gamma)(5\tau - \bar{\alpha} + 2\beta) \\
&\qquad + \Delta\beta + 2\Delta\tau + \delta\gamma - \delta\mu] \\
&+ 3\Psi_2[\kappa\bar{\nu} + \bar{\lambda}\rho + \sigma(3\gamma - \bar{\gamma} - 3\mu) + \tau(3\tau - \bar{\alpha} + \beta) - \Delta\sigma - \delta\tau] \\
&+ 2\Psi_3[-\kappa\bar{\lambda} + \sigma(\bar{\alpha} + \beta - 5\tau) + \delta\sigma] + 2\Psi_4\sigma^2 \\
&+ \mathrm{D}\mathrm{D}\bar{\Psi}_4 - \mathrm{D}\delta\bar{\Psi}_3 - \delta\mathrm{D}\bar{\Psi}_3 + \delta\delta\bar{\Psi}_2 \\
&- 4\bar{\lambda}\delta\bar{\Psi}_1 + 5\bar{\lambda}\mathrm{D}\bar{\Psi}_2 + \sigma\Delta\bar{\Psi}_2 + (\bar{\alpha} - \beta + 6\bar{\pi})\delta\bar{\Psi}_2 \\
&+ (\beta - 3\bar{\alpha} - 7\bar{\pi})\mathrm{D}\bar{\Psi}_3 - \kappa\Delta\bar{\Psi}_3 + (\epsilon - 5\bar{\epsilon} + 3\bar{\rho})\delta\bar{\Psi}_3 - \sigma\bar{\delta}\bar{\Psi}_3 \\
&+ (7\bar{\epsilon} - \epsilon - 2\bar{\rho})\mathrm{D}\bar{\Psi}_4 - \bar{\kappa}\delta\bar{\Psi}_4 + \kappa\bar{\delta}\bar{\Psi}_4 \\
&+ 2\bar{\Psi}_0\bar{\lambda}^2 + 2\bar{\Psi}_1[\bar{\lambda}(\bar{\alpha} + \beta - 5\bar{\pi}) - \bar{\nu}\sigma - \delta\bar{\lambda}] \\
&+ 3\bar{\Psi}_2[\kappa\bar{\nu} + \bar{\lambda}(3\bar{\epsilon} - \epsilon - 3\bar{\rho}) + \bar{\mu}\sigma + \bar{\pi}(\bar{\alpha} - \beta + 3\bar{\pi}) + \mathrm{D}\bar{\lambda} + \delta\bar{\pi}] \\
&+ 2\bar{\Psi}_3[2\bar{\kappa}\bar{\lambda} - \kappa(2\bar{\mu} + \bar{\gamma}) + \sigma(\bar{\tau} - \bar{\beta}) + (\bar{\rho} - \bar{\epsilon})(2\bar{\alpha} - \beta + 5\bar{\pi}) + (\epsilon - 2\bar{\epsilon})(2\bar{\pi} + \bar{\alpha}) \\
&\qquad - \mathrm{D}\bar{\alpha} - 2\mathrm{D}\bar{\pi} - \delta\bar{\epsilon} + \delta\bar{\rho}] \\
&+ \bar{\Psi}_4[\kappa(4\bar{\beta} - \bar{\tau}) + \bar{\kappa}(\beta - \bar{\alpha} - 3\bar{\pi}) + (\bar{\rho} - 4\bar{\epsilon})(\epsilon - 3\bar{\epsilon} + \bar{\rho}) - \sigma\bar{\sigma} \\
&\qquad + 4\mathrm{D}\bar{\epsilon} - \mathrm{D}\bar{\rho} - \delta\bar{\kappa}]
\end{aligned}
$$

$$
\begin{aligned}
B^Z_{(1)(1)} =&\Delta\Delta\Psi_2 - \Delta\delta\Psi_3 - \delta\Delta\Psi_3 + \delta\delta\Psi_4 \\
&- 4\nu\Delta\Psi_1 + (\gamma + \bar{\gamma} + 6\mu)\Delta\Psi_2 + 5\nu\delta\Psi_2 - \bar{\nu}\bar{\delta}\Psi_2 \\
&+ \bar{\nu}\mathrm{D}\Psi_3 + (3\tau - \bar{\alpha} - 5\beta)\Delta\Psi_3 - (3\gamma + \bar{\gamma} + 7\mu)\delta\Psi_3 + \bar{\lambda}\bar{\delta}\Psi_3 \\
&- \bar{\lambda}\mathrm{D}\Psi_4 - \sigma\Delta\Psi_4 + (\bar{\alpha} + 7\beta - 2\tau)\delta\Psi_4 \\
&+ 2\Psi_0\nu^2 + 2\Psi_1[\nu(\gamma - \bar{\gamma} - 5\mu) + \lambda\bar{\nu} - \Delta\nu] \\
&+ 3\Psi_2[\mu(\gamma + \bar{\gamma} + 3\mu) + \nu(\bar{\alpha} + 3\beta - 3\tau) - \lambda\bar{\lambda} - \bar{\nu}\pi + \Delta\mu + \delta\nu] \\
&+ 2\Psi_3[\bar{\nu}(\epsilon - \rho) + \bar{\lambda}(\alpha + 2\pi) + \gamma(2\tau - \bar{\alpha} - 4\beta) + \bar{\gamma}(\tau - \beta) + \mu(5\tau - 2\bar{\alpha} - 9\beta) + 2\nu\sigma \\
&\qquad - \Delta\beta + \Delta\tau - \delta\gamma - 2\delta\mu] \\
&+ \Psi_4[\kappa\bar{\nu} + \bar{\lambda}(\rho - 4\epsilon) - \sigma(\gamma + \bar{\gamma} + 3\mu) + 4\beta(3\beta + \bar{\alpha}) + \tau(\tau - \bar{\alpha} - 7\beta) \\
&\qquad - \Delta\sigma + 4\delta\beta - \delta\tau] + c.c.
\end{aligned}
$$

Kerr geometry

Rotating black holes in GR [*Kerr 1963*] with properties:

- vacuum, stationary, axially symmetric, asymptotically flat ($\Lambda = 0$), Weyl type D

Spacetime metric: [*Boyer, Lindquist 1967*]

$$\mathrm{d}s^2 = -\frac{\triangle}{\Sigma}\left(\mathrm{d}t - a\sin^2\theta\,\mathrm{d}\phi\right)^2 + \frac{\Sigma}{\triangle}\mathrm{d}r^2 + \Sigma\mathrm{d}\theta^2 + \frac{\sin^2\theta}{\Sigma}\left((r^2+a^2)\mathrm{d}\phi - a\mathrm{d}t\right)^2$$

where

$$\triangle = r^2 - 2Mr + a^2 \qquad \Sigma = r^2 + a^2\cos^2\theta \qquad \varrho = r - ia\cos\theta$$

Kinnersley tetrad: adapted to the Weyl structure

$$\boldsymbol{k} = \tfrac{1}{\sqrt{2}\,\triangle}\left[\left(r^2+a^2\right)\partial_t + \triangle\,\partial_r + a\,\partial_\phi\right]$$

$$\boldsymbol{l} = \tfrac{1}{\sqrt{2}\,\Sigma}\left[\left(r^2+a^2\right)\partial_t - \triangle\,\partial_r + a\,\partial_\phi\right]$$

$$\boldsymbol{m} = \tfrac{1}{\sqrt{2}\,\bar{\varrho}}\left(ia\sin\theta\,\partial_t + \partial_\theta + i\csc\theta\,\partial_\phi\right)$$

- *vacuum*: $\Phi_{00} = \Phi_{01} = \Phi_{10} = \Phi_{11} = \Phi_{02} = \Phi_{20} = \Phi_{12} = \Phi_{21} = \Phi_{22} = 0 = R$
- *type D*: $\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 \qquad \Psi_2 = -M\varrho^{-3} \neq 0$
- *geometry*: $\kappa = \sigma = \nu = \lambda = \epsilon = 0 \qquad \rho\,, \mu\,, \alpha = \pi - \bar{\beta}\,, \beta\,, \gamma\,, \tau\,, \pi \neq 0$

This will be **background** for perturbations in QG.

Teukolsky-like approach: motivation

PERTURBATIONS OF A ROTATING BLACK HOLE. I. FUNDAMENTAL EQUATIONS FOR GRAVITATIONAL, ELECTROMAGNETIC, AND NEUTRINO-FIELD PERTURBATIONS*

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How does one obtain linearized perturbation equations, say for gravitational perturbations? A straightforward way is to start with the Einstein equations for a metric $g_{\mu\nu}$, and to let $g_{\mu\nu} = g_{\mu\nu}{}^{A} + h_{\mu\nu}{}^{B}$, where the superscripts A and B denote background and perturbation quantities, respectively. The field equations are then expanded to first order in $h_{\mu\nu}{}^{B}$, yielding a set of linear equations for the perturbations.

Even in the Schwarzschild case, this procedure involves considerable algebraic complexity.

Fortunately, there is an alternative approach to the problem. This is provided by the Newman-Penrose (NP) formalism.

To do perturbation theory in this formalism, one specifies the perturbed geometry by $\boldsymbol{l} = \boldsymbol{l}^{A} + \boldsymbol{l}^{B}$, $\boldsymbol{n} = \boldsymbol{n}^{A} + \boldsymbol{n}^{B}$, etc. All the NP quantities can then be written in this form: $\psi_2 = \psi_2{}^{A} + \psi_2{}^{B}$, $D = D^{A} + D^{B}$, etc. The complete set of perturbation equations is obtained from the NP equations by keeping B terms only to first order.

The Schwarzschild and Kerr metrics are very similar from the NP point of view. This similarity allows us, in this paper, to derive decoupled Kerr-metric equations for $\psi_0{}^{B}$ and $\psi_4{}^{B}$. Moreover, we shall demonstrate the unexpected result that these equations, like those for Schwarzschild, are separable.

... we follow this procedure in quadratic gravity ...

Teukolsky-like approach in quadratic gravity

Perturbed geometry: *all* quantities are written in the form

$$\text{complete quantity} \;=\; \text{background quantity} \;+\; \text{perturbation}$$
$$^{c}\zeta = \zeta + {}^{\varepsilon}\zeta$$

Linearity: the terms ${}^{\varepsilon}\zeta$ are kept only to the *first order*

Field equations:

$$\begin{aligned}
\tfrac{1}{2}\,{}^{\varepsilon}T_{(0)(0)} &= -\,4\mathfrak{a}\,{}^{\varepsilon}\Phi_{00}(\Psi_2+\bar{\Psi}_2)+\tfrac{2}{\mathsf{k}}\,{}^{\varepsilon}\Phi_{00}+{}^{\varepsilon}Z_{(0)(0)}\\
\tfrac{1}{2}\,{}^{\varepsilon}T_{(0)(1)} &= +\,8\mathfrak{a}\,{}^{\varepsilon}\Phi_{11}(\Psi_2+\bar{\Psi}_2)+\tfrac{1}{\mathsf{k}}\left(2\,{}^{\varepsilon}\Phi_{11}-\tfrac{{}^{\varepsilon}R}{4}\right)+{}^{\varepsilon}Z_{(0)(1)}\\
\tfrac{1}{2}\,{}^{\varepsilon}T_{(0)(2)} &= -\,4\mathfrak{a}\,{}^{\varepsilon}\Phi_{01}(\Psi_2-2\bar{\Psi}_2)+\tfrac{2}{\mathsf{k}}\,{}^{\varepsilon}\Phi_{01}+{}^{\varepsilon}Z_{(0)(2)}\\
\tfrac{1}{2}\,{}^{\varepsilon}T_{(1)(1)} &= -\,4\mathfrak{a}\,{}^{\varepsilon}\Phi_{22}(\Psi_2+\bar{\Psi}_2)+\tfrac{2}{\mathsf{k}}\,{}^{\varepsilon}\Phi_{22}+{}^{\varepsilon}Z_{(1)(1)}\\
\tfrac{1}{2}\,{}^{\varepsilon}T_{(1)(2)} &= -\,4\mathfrak{a}\,{}^{\varepsilon}\Phi_{12}(-2\Psi_2+\bar{\Psi}_2)+\tfrac{2}{\mathsf{k}}\,{}^{\varepsilon}\Phi_{12}+{}^{\varepsilon}Z_{(1)(2)}\\
\tfrac{1}{2}\,{}^{\varepsilon}T_{(2)(2)} &= -\,4\mathfrak{a}\,{}^{\varepsilon}\Phi_{02}(\Psi_2+\bar{\Psi}_2)+\tfrac{2}{\mathsf{k}}\,{}^{\varepsilon}\Phi_{02}+{}^{\varepsilon}Z_{(2)(2)}\\
\tfrac{1}{2}\,{}^{\varepsilon}T_{(2)(3)} &= +\,8\mathfrak{a}\,{}^{\varepsilon}\Phi_{11}(\Psi_2+\bar{\Psi}_2)+\tfrac{1}{\mathsf{k}}\left(2\,{}^{\varepsilon}\Phi_{11}+\tfrac{{}^{\varepsilon}R}{4}\right)+{}^{\varepsilon}Z_{(2)(3)}
\end{aligned}$$

- ${}^{\varepsilon}Z_{(a)(b)}$ and ${}^{\varepsilon}B^{Z}_{(a)(b)}$ have to be expressed explicitly
- the *Ricci* and *Bianchi identities* have to be employed

$\rightarrow$ coupled system for ${}^{\varepsilon}\Phi_{AB}$

Teukolsky-like approach in quadratic gravity

Expanding the QG vacuum constraints and separating linear contribution give:

Trace equation:

$$\tfrac{1}{2}\,\mathsf{k}\,{}^{\varepsilon}T = (6\mathfrak{b}\mathsf{k}\,\Box - 1)\,{}^{\varepsilon}R$$

Field equations: combined with the geometric identities

$$\begin{aligned}
\tfrac{\mathsf{k}}{4}\,{}^{\varepsilon}T_{(0)(0)} &= {}^{\varepsilon}\Phi_{00} - 2\mathfrak{a}\mathsf{k}\,{}^{\varepsilon}B_{(0)(0)} - \mathfrak{b}\mathsf{k}DD\,{}^{\varepsilon}R \\
\tfrac{\mathsf{k}}{4}\,{}^{\varepsilon}T_{(0)(1)} &= {}^{\varepsilon}\Phi_{11} - 2\mathfrak{a}\mathsf{k}\,{}^{\varepsilon}B_{(0)(1)} - \tfrac{1}{2}\mathfrak{b}\mathsf{k}\Big(D\Delta + \delta\bar{\delta} - \mu D + \bar{\rho}\Delta - \pi\delta - 2\bar{\alpha}\bar{\delta}\Big)\,{}^{\varepsilon}R \\
\tfrac{\mathsf{k}}{4}\,{}^{\varepsilon}T_{(0)(2)} &= {}^{\varepsilon}\Phi_{01} - 2\mathfrak{a}\mathsf{k}\,{}^{\varepsilon}B_{(0)(2)} - \mathfrak{b}\mathsf{k}\Big(D\delta - \bar{\pi}D\Big)\,{}^{\varepsilon}R \\
\tfrac{\mathsf{k}}{4}\,{}^{\varepsilon}T_{(1)(1)} &= {}^{\varepsilon}\Phi_{22} - 2\mathfrak{a}\mathsf{k}\,{}^{\varepsilon}B_{(1)(1)} - \mathfrak{b}\mathsf{k}\Big(\Delta\Delta + (\gamma + \bar{\gamma})\Delta\Big)\,{}^{\varepsilon}R \\
\tfrac{\mathsf{k}}{4}\,{}^{\varepsilon}T_{(1)(2)} &= {}^{\varepsilon}\Phi_{12} - 2\mathfrak{a}\mathsf{k}\,{}^{\varepsilon}B_{(1)(2)} - \mathfrak{b}\mathsf{k}\Big(\delta\Delta + \bar{\pi}\Delta - \mu\delta\Big)\,{}^{\varepsilon}R \\
\tfrac{\mathsf{k}}{4}\,{}^{\varepsilon}T_{(2)(2)} &= {}^{\varepsilon}\Phi_{02} - 2\mathfrak{a}\mathsf{k}\,{}^{\varepsilon}B_{(2)(2)} - \mathfrak{b}\mathsf{k}\Big(\delta\delta - (\beta - \bar{\alpha})\delta\Big)\,{}^{\varepsilon}R
\end{aligned}$$

... where ${}^{\varepsilon}B_{(a)(b)}$ encodes the linear contribution of the Bach tensor ...

Teukolsky-like approach in quadratic gravity

$$
\begin{aligned}
{}^{\varepsilon}B_{(0)(0)} &= \Box\,{}^{\varepsilon}\Phi_{00} + (\diamondsuit + \bar{\diamondsuit} - U_{00})\,{}^{\varepsilon}\Phi_{00} - \frac{1}{6} DD\,{}^{\varepsilon}R \\
&\quad - 4\left(\bar{\tau} D - \bar{\rho}\bar{\delta} + 2\bar{\rho}\alpha\right)^{\varepsilon}\Phi_{01} - 4\left(\tau D - \rho\delta + 2\rho\bar{\alpha}\right)^{\varepsilon}\bar{\Phi}_{01} + 8\rho\bar{\rho}\,{}^{\varepsilon}\Phi_{11} \\
{}^{\varepsilon}B_{(0)(1)} &= \Box\,{}^{\varepsilon}\Phi_{11} - U_{11}\,{}^{\varepsilon}\Phi_{11} - \frac{1}{12}\left(D\Delta + \delta\bar{\delta} - \mu D + \bar{\rho}\Delta - \pi\delta - 2\bar{\alpha}\bar{\delta}\right)^{\varepsilon} R \\
&\quad 2\mu\bar{\mu}\,{}^{\varepsilon}\Phi_{00} + 2\left(\pi\Delta - \mu\bar{\delta} + 2\alpha\mu - 2\gamma\pi\right)^{\varepsilon}\Phi_{01} + 2\pi\bar{\tau}\,{}^{\varepsilon}\Phi_{02} - 2\left(\bar{\tau}D - \bar{\rho}\bar{\delta} - 2\bar{\beta}\bar{\rho}\right)^{\varepsilon}\Phi_{12} + 2\,{}^{\varepsilon}\Phi_{22}\rho\bar{\rho} \\
&\quad + 2\left(\bar{\pi}\Delta - \bar{\mu}\delta + 2\bar{\alpha}\bar{\mu} - 2\bar{\gamma}\bar{\pi}\right)^{\varepsilon}\bar{\Phi}_{01} + 2\bar{\pi}\tau\,{}^{\varepsilon}\bar{\Phi}_{02} - 2\left(\tau D - \rho\delta + 2\rho\bar{\alpha} - 2\beta\rho\right)^{\varepsilon}\bar{\Phi}_{12} \\
{}^{\varepsilon}B_{(0)(2)} &= \Box\,{}^{\varepsilon}\Phi_{01} + (\diamondsuit - U_{01})\,{}^{\varepsilon}\Phi_{01} - \frac{1}{6}(D\delta - \bar{\pi}D)^{\varepsilon}R + 2\left(\bar{\pi}\Delta - \bar{\mu}\delta - 2\gamma\bar{\pi} - 2\bar{\gamma}\bar{\pi} + 2\bar{\mu}\bar{\pi}\right)^{\varepsilon}\Phi_{00} \\
&\quad + 4\tau\bar{\pi}\,{}^{\varepsilon}\bar{\Phi}_{01} - 4\left(\tau D - \rho\delta\right)^{\varepsilon}\Phi_{11} - 2\left(\bar{\tau}D - \bar{\rho}\bar{\delta} + 2(\alpha - \bar{\beta})\bar{\rho}\right)^{\varepsilon}\Phi_{02} + 4\,{}^{\varepsilon}\Phi_{12}\rho\bar{\rho} \\
{}^{\varepsilon}B_{(1)(1)} &= \Box\,{}^{\varepsilon}\Phi_{22} - (\diamondsuit + \bar{\diamondsuit} + U_{22})\,{}^{\varepsilon}\Phi_{22} - \frac{1}{6}\Big(\Delta\Delta + (\gamma + \bar{\gamma})\Delta\Big)^{\varepsilon}R \\
&\quad + 8\mu\bar{\mu}\,{}^{\varepsilon}\Phi_{11} + 4\left(\pi\Delta - \mu\bar{\delta} - 2\mu\bar{\beta} + 2\pi\bar{\gamma}\right)^{\varepsilon}\Phi_{12} + 4\left(\bar{\pi}\Delta - \bar{\mu}\delta - 2\beta\bar{\mu} + 2\bar{\pi}\gamma\right)^{\varepsilon}\bar{\Phi}_{12} \\
{}^{\varepsilon}B_{(1)(2)} &= \Box\,{}^{\varepsilon}\Phi_{12} - (\bar{\diamondsuit} + U_{12})\,{}^{\varepsilon}\Phi_{12} - \frac{1}{6}\Big(\Delta\delta + \tau\Delta - (\gamma - \bar{\gamma})\delta\Big)^{\varepsilon}R \\
&\quad + 4\mu\bar{\mu}\,{}^{\varepsilon}\Phi_{01} + 4\left(\bar{\pi}\Delta - \bar{\mu}\delta\right)^{\varepsilon}\Phi_{11} + 2\left(\pi\Delta - \mu\bar{\delta} - 2\pi(\gamma - \bar{\gamma}) + 2\mu(\alpha - \bar{\beta})\right)^{\varepsilon}\Phi_{02} \\
&\quad - 2\left(\tau D - \rho\delta - 2\rho\bar{\pi}\right)^{\varepsilon}\Phi_{22} + 4\tau\bar{\pi}\,{}^{\varepsilon}\bar{\Phi}_{12} \\
{}^{\varepsilon}B_{(2)(2)} &= \Box\,{}^{\varepsilon}\Phi_{02} + (\diamondsuit - \bar{\diamondsuit} - U_{02})^{\varepsilon}\Phi_{02} - \frac{1}{6}\Big(\delta\delta + (\bar{\alpha} - \beta)\delta\Big)^{\varepsilon}R \\
&\quad 4\left(\bar{\pi}\Delta - \bar{\mu}\delta + 2\beta\bar{\mu} - 2\gamma\bar{\pi}\right)^{\varepsilon}\Phi_{01} + 8\tau\bar{\pi}\,{}^{\varepsilon}\Phi_{11} - 4\left(\tau D - \rho\delta - 2\rho\bar{\alpha}\right)^{\varepsilon}\Phi_{12}
\end{aligned}
$$

... where $\diamondsuit = 4\gamma D - 4\alpha\delta - 4\beta\bar{\delta}$ and U_{AB} are functions of the background

System of equations for the linear perturbations ${}^{\varepsilon}\Phi_{AB}$ of the Kerr spacetime in QG!

What have we done?

Within the four-dimensional quadratic gravity:

- the NP form of the field equations was derived
- the suitable procedure how to combine them with the geometric identities was proposed (this is based on the linearity of the Ricci tensor in the field equations)
- it was applied to derive constraints on fully general linear perturbations of the Kerr geometry in terms of the Ricci tensor components ${}^{\varepsilon}\Phi_{AB}$
- for particular simple examples see poster by Šimon Knoška

We have a **nice toy** now, so let's play with it!

Let's unlock quadratic gravity through computation of perturbations!

This talk is based on:

- *Newman–Penrose formalism in quadratic gravity*
  Švarc R, Pravdová A, and Miškovský D, *Phys. Rev. D* **107** 024036 (2023)
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Thank you for your attention!