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Measuring the spacetime of SMBH with pulsar timing

5th EPS Conference on Gravitation

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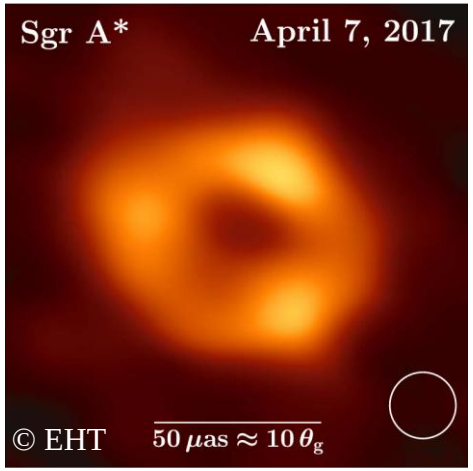
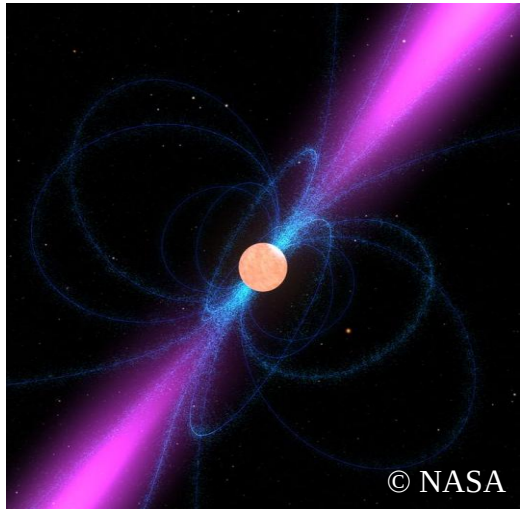
Background: black hole and pulsar

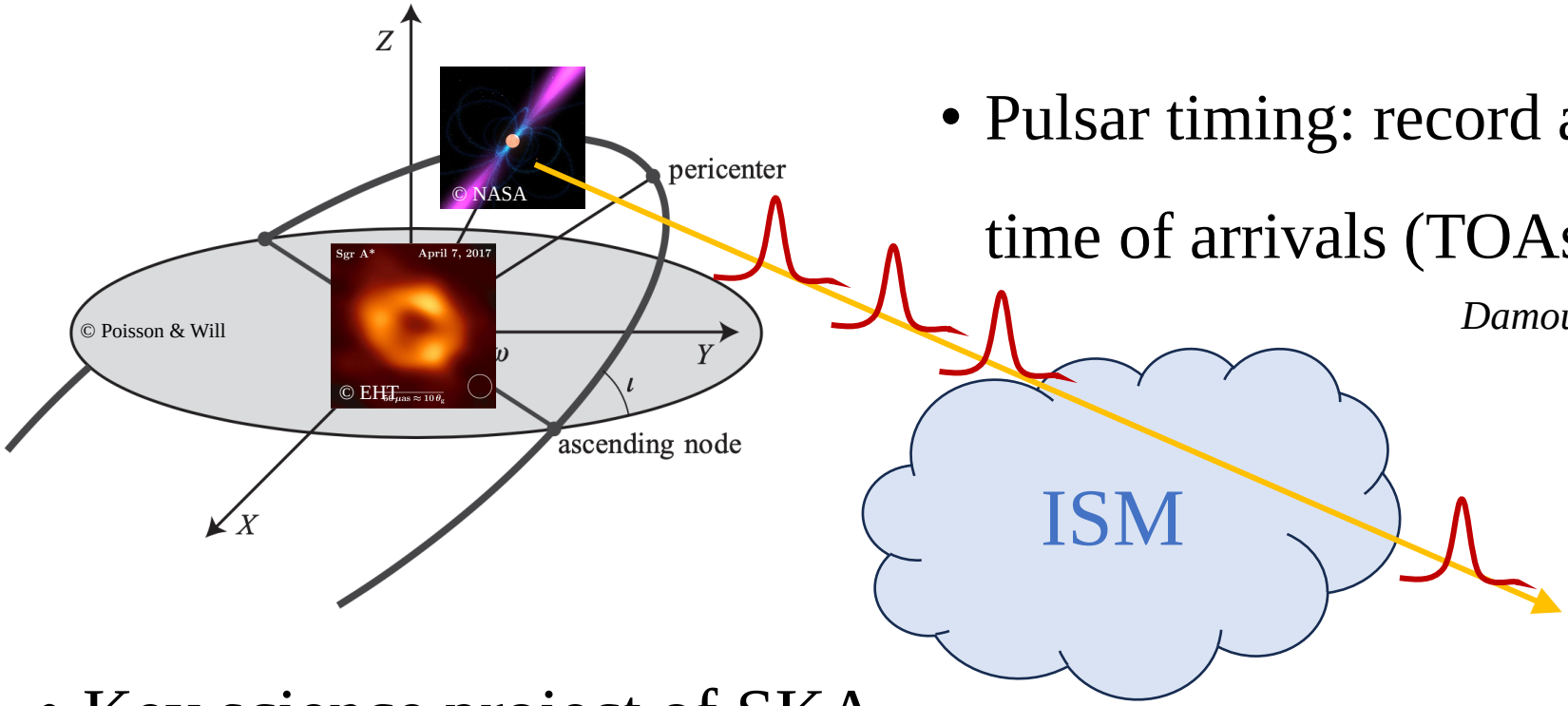


- Black holes (BHs) are the most compact objects in the universe
- Hard to observe directly

- Pulsars are **stably** rotating neutron stars
- High density & strong gravitational/magnetic field
- Rich observational effects

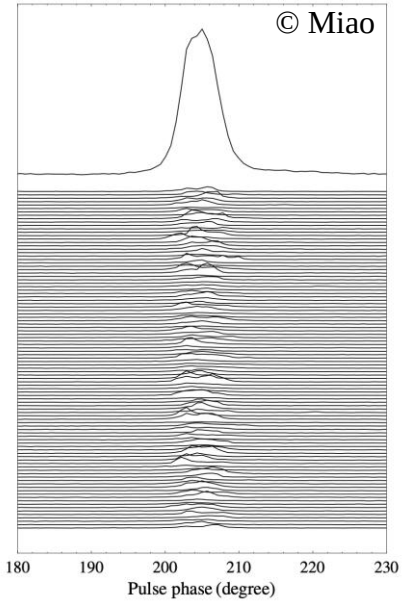


Background: pulsar timing

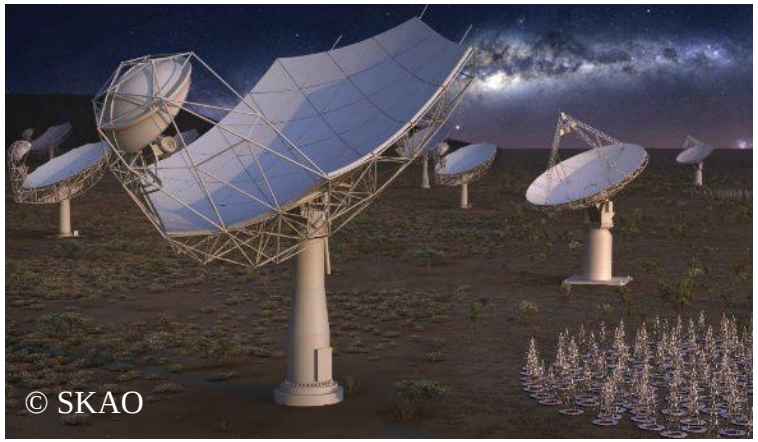


- Pulsar timing: record and predict time of arrivals (TOAs) of pulses

Damour & Taylor, 1992



- Key science project of SKA
- Pulsar-SMBH system in our Galactic Center (GC)



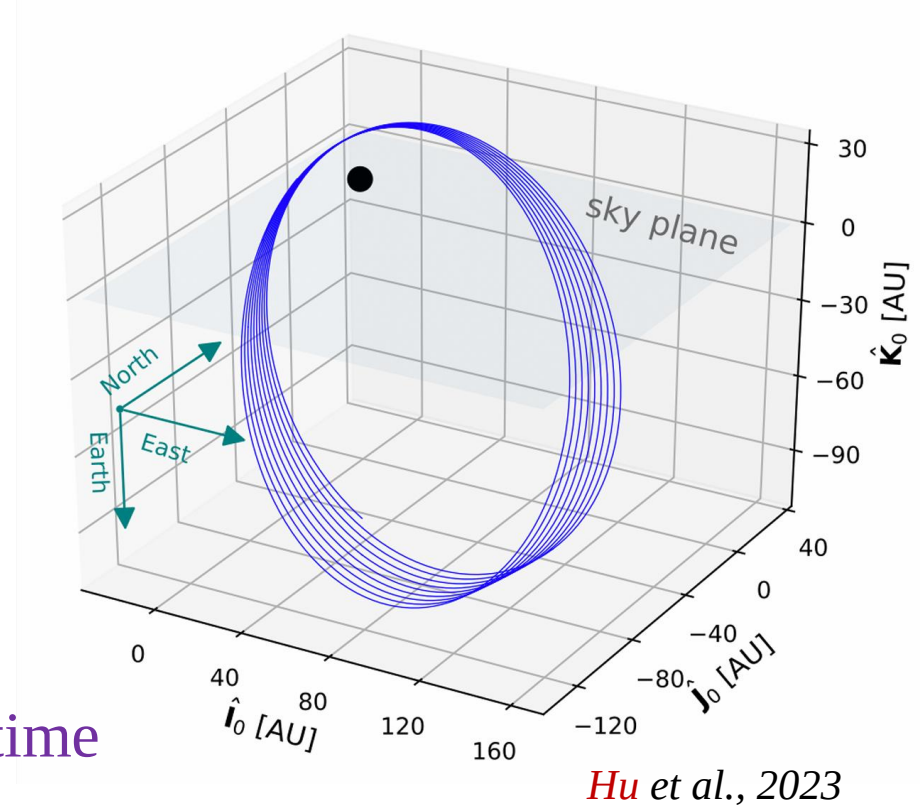
Timing model

- Pulsar timing model:
- $N(T) = N_0 + \nu T + \frac{1}{2}\dot{\nu}T^2 + \cdots$
- $t^{\mathrm{TOA}} = T + \Delta_{\mathrm{E}} + \Delta_{\mathrm{R}} + \Delta_{\mathrm{S}}$

Shapiro delay: curved spacetime

Römer delay: z/c

Einstein delay: coordinate transformation



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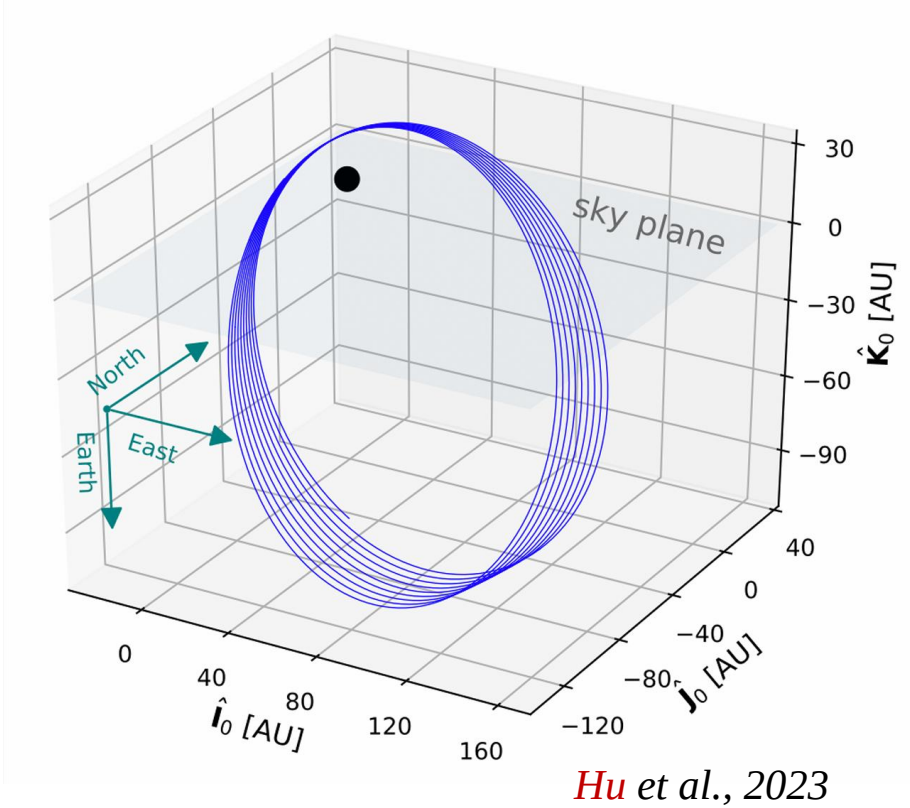
orbital motion + light propagation

Damour & Deruelle, 1986

Timing model: orbital motion

- Analytic model: *Damour & Deruelle, 1986; Wex & Kopeikin, 1999;Liu et al., 2012; Psaltis et al., 2016*
- Fully GR model: *Zhang & Saha, 2017*
- Numerical model based on post-Newtonian (PN) equation of motion:
- $\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{r}}_{\mathrm{N}} + \ddot{\boldsymbol{r}}_{\mathrm{1PN}} + \ddot{\boldsymbol{r}}_{\mathrm{SO}} + \ddot{\boldsymbol{r}}_{\mathrm{Q}} + \cdots$



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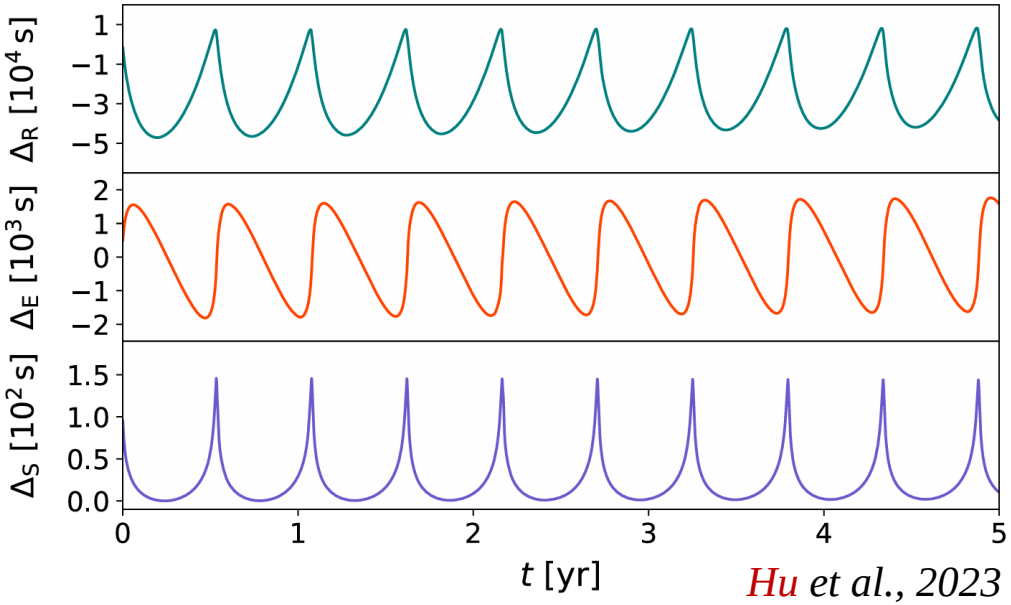
Timing model: light propagation and inverse

- Emitter-observer problem
- $\Delta_{\mathrm{R}} = z/c$
- $\Delta_{\mathrm{S}} = -\frac{2GM}{c^3}\ln(r - z) + \cdots$
- 2PN correction, frame dragging effect…
- Inverse timing model: $t^{\mathrm{TOA}} \rightarrow N\left(\Theta; t^{\mathrm{TOA}}\right)$
- Efficient approach: $\mathrm{d}t^{\mathrm{TOA}}/\mathrm{d}t$



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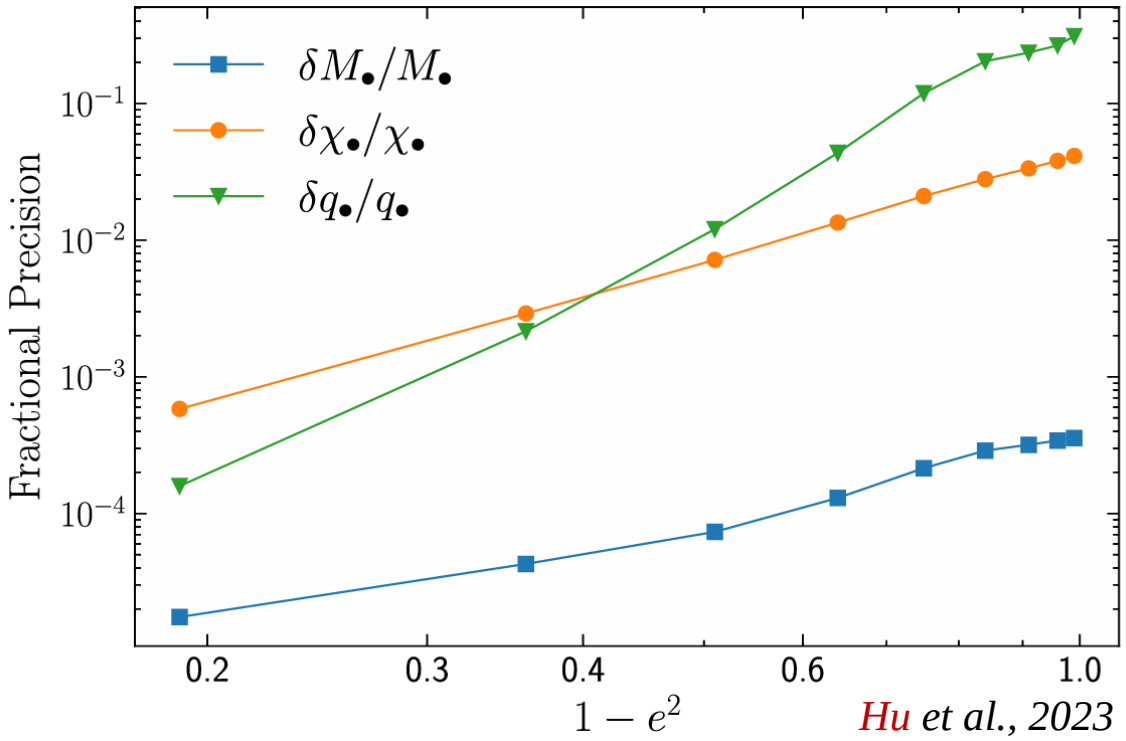
Measuring the BH properties

- Time span: $T_{obs} = 5$ yr
- Weekly observation: $N_{\mathrm{TOA}} = 260$
- Timing precision: $\sigma_{\mathrm{TOA}} = 1$ ms
- Orbital period: $P_b = 0.5$ yr
- $\mathcal{L} = -\frac{1}{2\nu^2}\sum_{i=1}^{N_{\mathrm{TOA}}} \frac{[N_i(\Theta) - N_i(\bar{\Theta})]^2}{\sigma_{\mathrm{TOA}}^2}$



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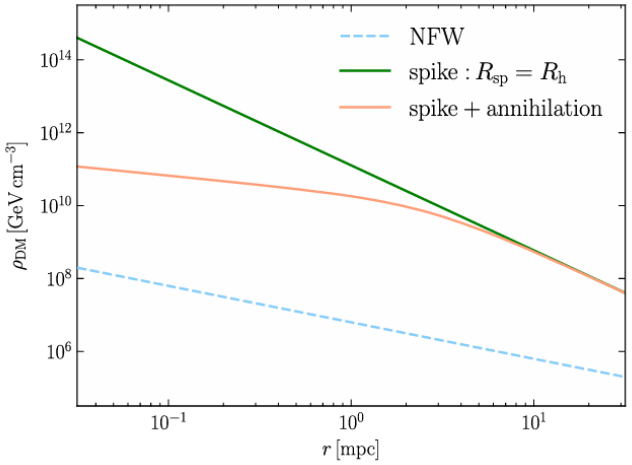
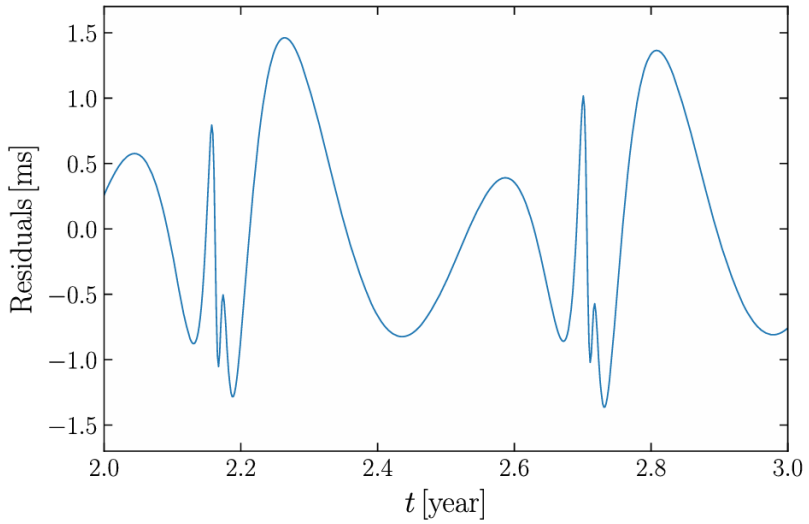
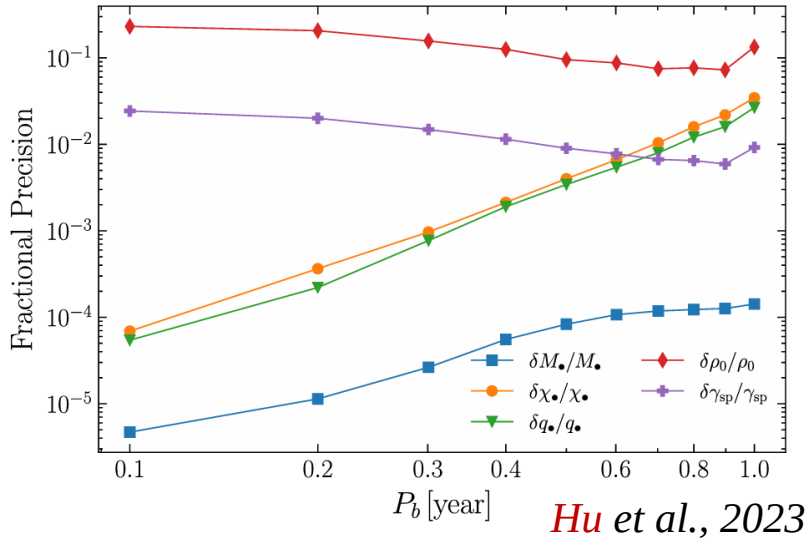
Tests of cosmic censorship conjecture and no hair theorem!

Measuring the small-scale dark matter distribution

- Dark matter distribution around SMBH
- Spike vs core?
- $\ddot{\boldsymbol{r}} = \ddot{\boldsymbol{r}}_{\mathrm{N}} + \ddot{\boldsymbol{r}}_{1\mathrm{PN}} + \ddot{\boldsymbol{r}}_{\mathrm{SO}} + \ddot{\boldsymbol{r}}_{\mathrm{Q}} + \ddot{\boldsymbol{r}}_{\mathrm{DM}}$







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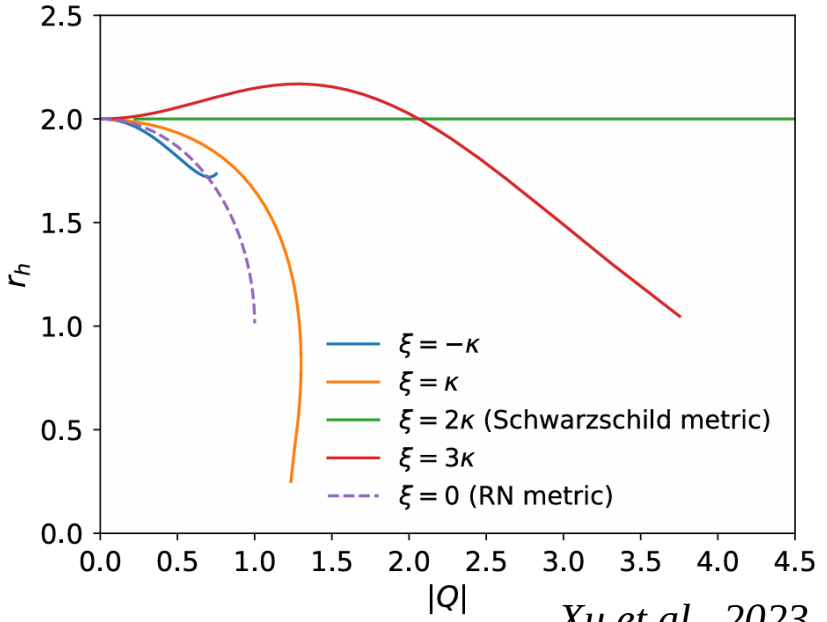
Beyond GR: Bumblebee gravity

- The bumblebee gravity model
- $S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1}{2\kappa} R + \frac{\xi}{2\kappa} B^\mu B^\nu R_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V \right) + S_m$
- $B_{\mu\nu} = D_\mu B_\nu - D_\nu B_\mu$
- $\left. \frac{\mathrm{d}V}{\mathrm{d}(B^\lambda B_\lambda)} \right|_{B_\mu = b_\mu}$: numerical static spherical solution



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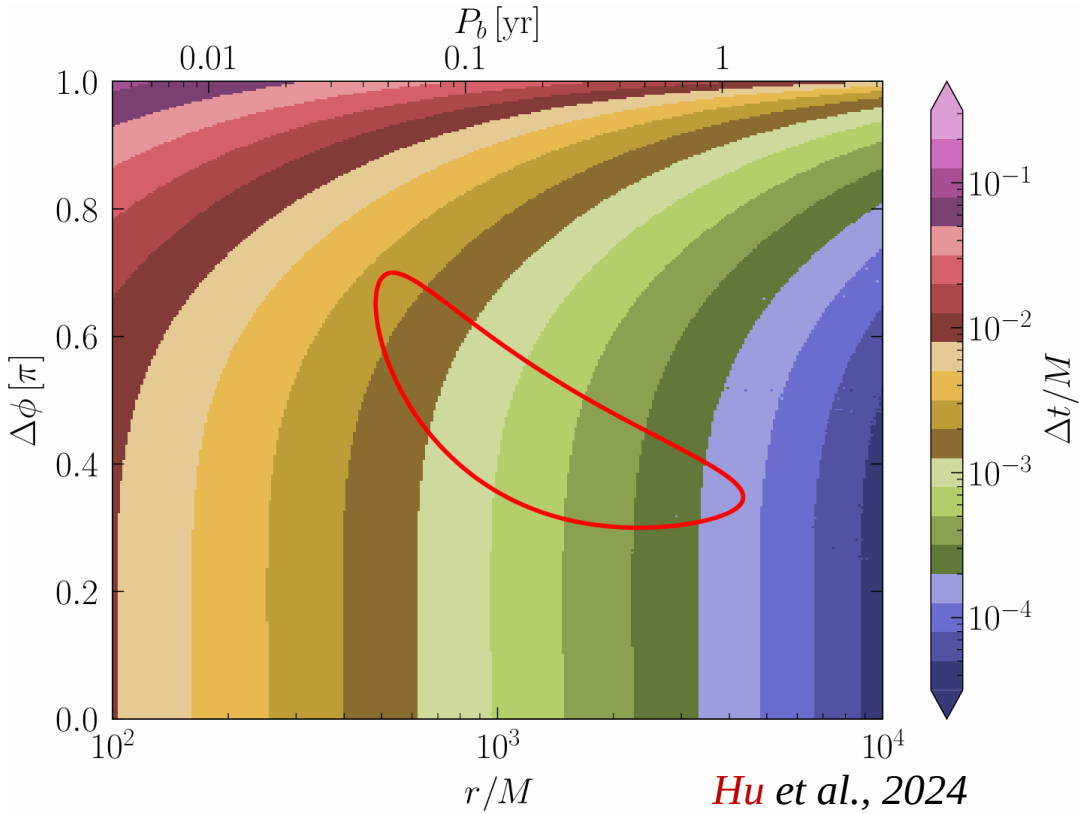
Beyond GR: Bumblebee gravity

- $\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = e^{2\nu-2\mu}\left(1-\frac{l^2}{\epsilon^2}\frac{e^{2\nu}}{r^2}-\frac{e^{2\nu}}{\epsilon^2}\right)$
- $\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{l}{\epsilon}\frac{e^{2\nu}}{r^2}$, $\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{\epsilon}e^{2\nu}$
- Full emitter-observer problem
- Leading order model in Q/M



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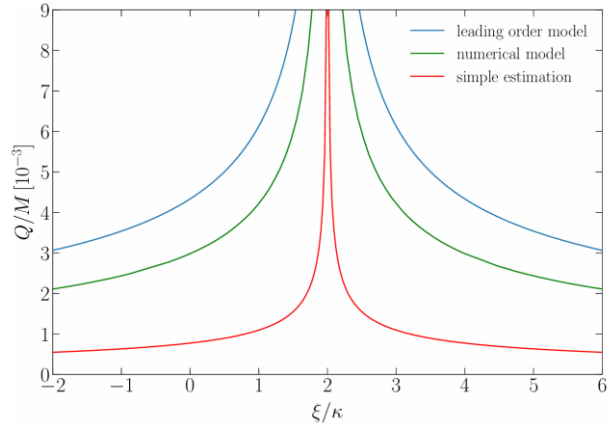
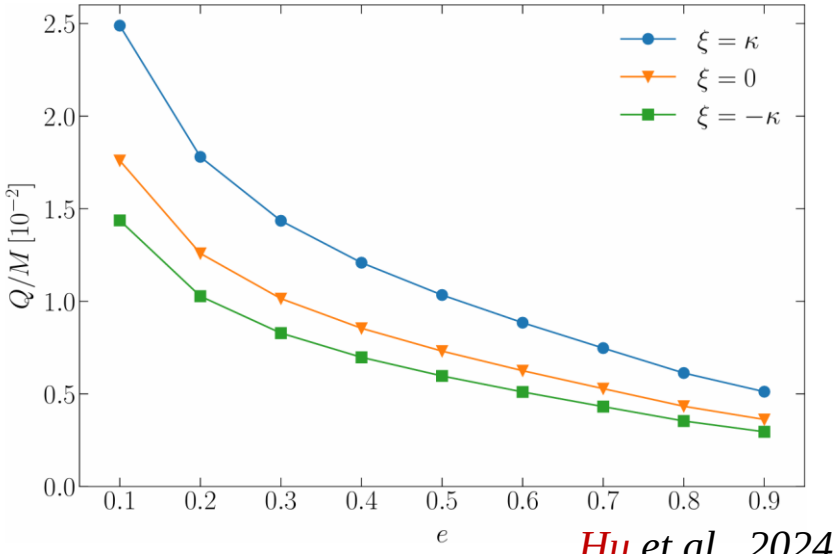
Beyond GR: Bumblebee gravity

- For $\xi/\kappa \sim 1$
- Current EHT observation: $|Q/M| \lesssim 1$
- Future pulsar-SMBH observation: $|Q/M| \lesssim 10^{-2}$
- Future EMRI observation: $|Q/M| \lesssim 10^{-3}$





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Conclusion

- Pulsars orbiting around SMBH are powerful tool for testing gravity
- We build a **extensible numerical timing model** for pulsar-SMBH system
- Future discovery and timing observation of GC pulsars orbiting around Sgr A* can provide various **high-precision** tests and measurements of gravity theory and GC environment

Thank you!