Astronomický ústav AV ČR

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AN EFFECTIVE-ONE-BODY WAVEFORM MODEL FOR LARGE-MASS-RATIO BLACK HOLE BINARIES

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Based on:
PRD 110 (2024) 4, 044034, A. A., A. Nagar, J. Mathews, G. Lukes-Gerakopoulos
PRD 109 (2024) 4, 044022, A. A., R. Gamba, A. Nagar, S. Bernuzzi
PRD 106 (2022) 8, 084061 & 084062, A. A., A. Nagar, A. Pound, N. Warburton, B. Wardell, L. Durkan, J. Miller

5th EPS Conference on Gravitation – "Unlocking Gravity Through Computation"
Prague, 11th of December 2024

OUTLINE

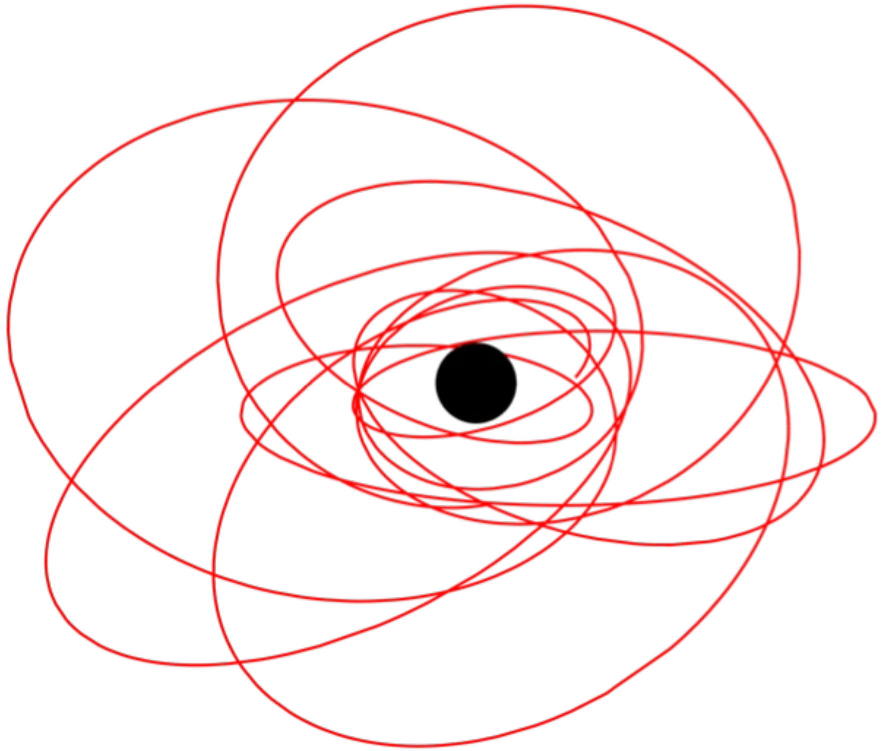
- Motivation
- Intro to the effective-one-body (EOB) approach and the model **TEOBResumS**
- Large-mass-ratio version benchmarked with gravitational self-force (GSF) results
- Current status: public code for inspiral waveforms with eccentricity and precession
- Computational challenges
- Conclusions & future work

LARGE-MASS-RATIO INSPIRALS

large-mass-ratio (LMR) inspirals

$$q \equiv \frac{m_1}{m_2} \sim 10^2 - 10^6$$

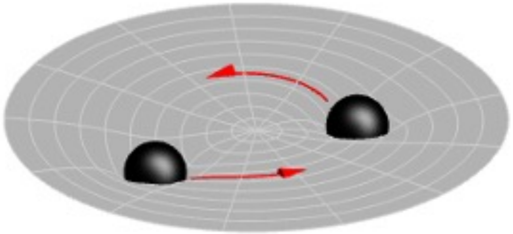
sources for the next generation of gravitational-wave detectors



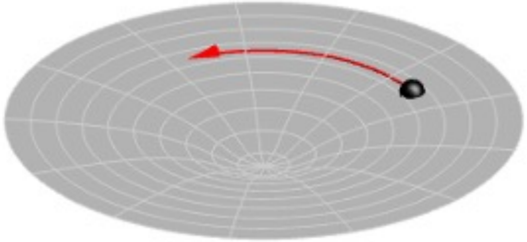
Created with the Black Hole Perturbation Toolkit

THE EFFECTIVE-ONE-BODY FORMALISM

A. Buonanno, T. Damour 1998



post-Newtonian (PN)
Hamiltonian
for the two-body problem



effective Hamiltonian
(motion in a deformed
Schwarzschild/Kerr metric)

$$\frac{H_{\rm EOB}}{\mu} = \frac{1}{\nu}\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

$$\mu = m_1 m_2/M, \quad \nu = \mu/M$$

TEOBRESUMS

EOB waveform model:

- Resumming PN results
- Two branches:
  GIOTTO – quasi-circular
  DALÍ – eccentric
- Aligned/precessing spins
- Comparable masses: informed & benchmarked with numerical relativity (NR)
- For large mass ratios: GSF takes the role of NR

DYNAMICS & WAVEFORM

G = c = 1

- Hamiltonian: function of the three EOB potentials (A, D and Q)
- The equations of motion are complemented by the radiation reaction:

$$
\begin{cases}
\dfrac{d\varphi}{dt} = \dfrac{\partial \hat{H}_{\text{EOB}}}{\partial p_\varphi} = \Omega \\
\dfrac{dr}{dt} = \left(\dfrac{A}{B}\right)^{1/2} \dfrac{\partial \hat{H}_{\text{EOB}}}{\partial p_{r_*}} \\
\dfrac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi \\
\dfrac{dp_{r_*}}{dt} = -\left(\dfrac{A}{B}\right)^{1/2} \dfrac{\partial \hat{H}_{\text{EOB}}}{\partial r} + \hat{\mathcal{F}}_r
\end{cases}
$$

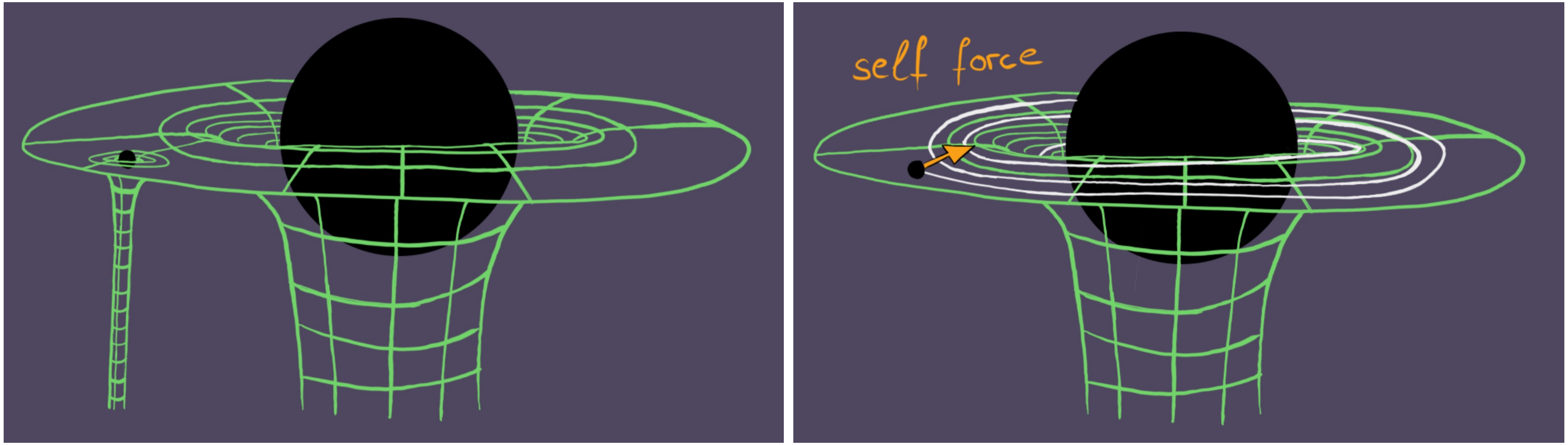
$$\mathcal{F}_{\ell m} \propto \left|\hat{h}_{\ell m}\right|^2$$

evaluated at every t

- Waveform: $$h_+ - ih_\times = \frac{1}{\mathcal{D}_L} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{\ell m}\, {}_{-2}Y_{\ell m}$$

we consider h_{22}

GRAVITATIONAL SELF-FORCE THEORY



- Small object's gravitational field → self-force

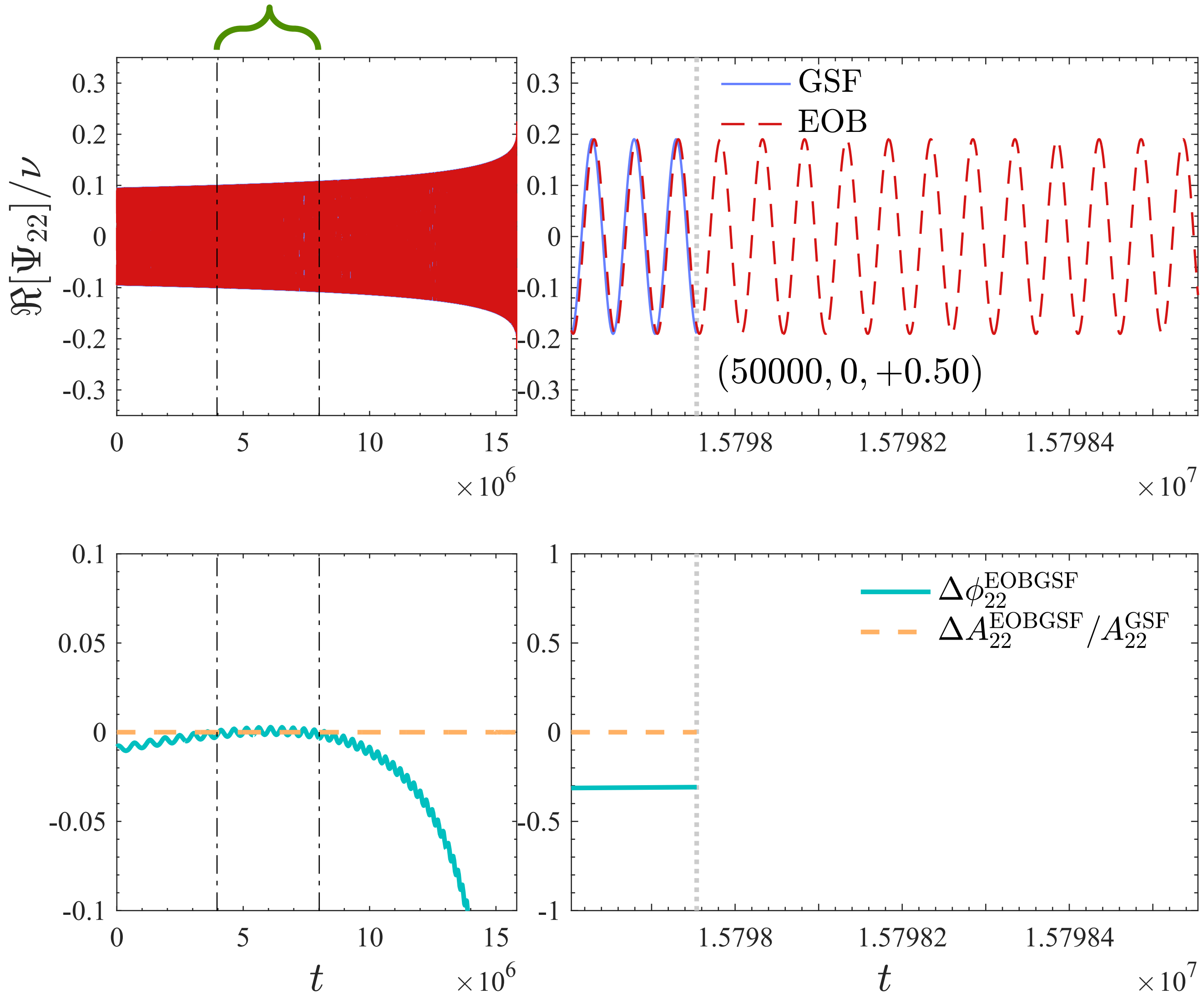
Images courtesy of Jakob Neef

COMPARISONS WITH GSF

- In [1 – 4] we compared TEOBResumS to:
  - GSF waveforms for **nonspinning** binaries (arXiv:2112.12265)
  - binaries with a **spinning secondary** based on arXiv:2112.13069 (not public yet)
- Changes in the model: **flux**, **potentials**, **spin-orbit** sector
- The outcome is a **new version of TEOBResumS** (with no NR information), that generates **large-mass-ratio** inspiral **waveforms**

[1] arXiv:2208.01049v2, [2] arXiv:2208.02055v2, [3] arXiv:2310.13578v2, [4] arXiv:2406:04108

PHASE DIFFERENCE WITH GSF
($q = 50\ 000, \chi_1 = 0, \chi_2 = 0.5$)



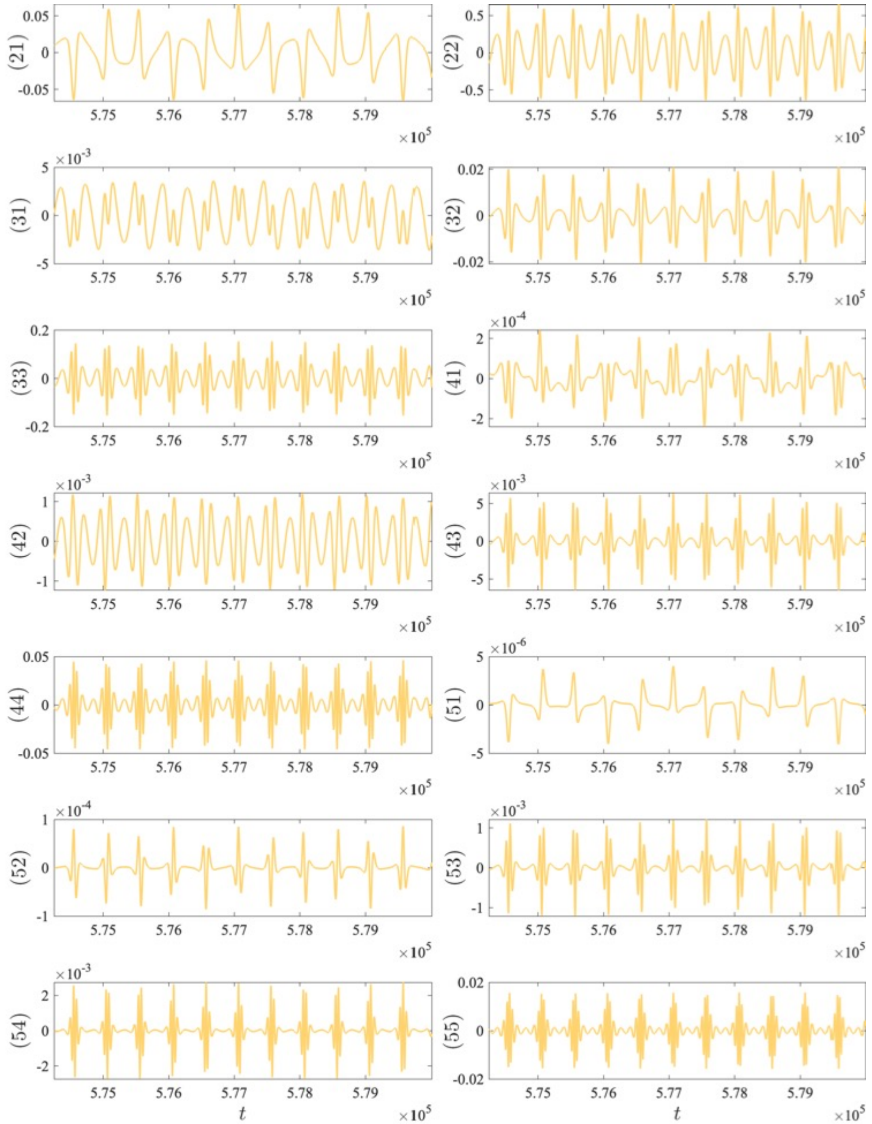
- Convention

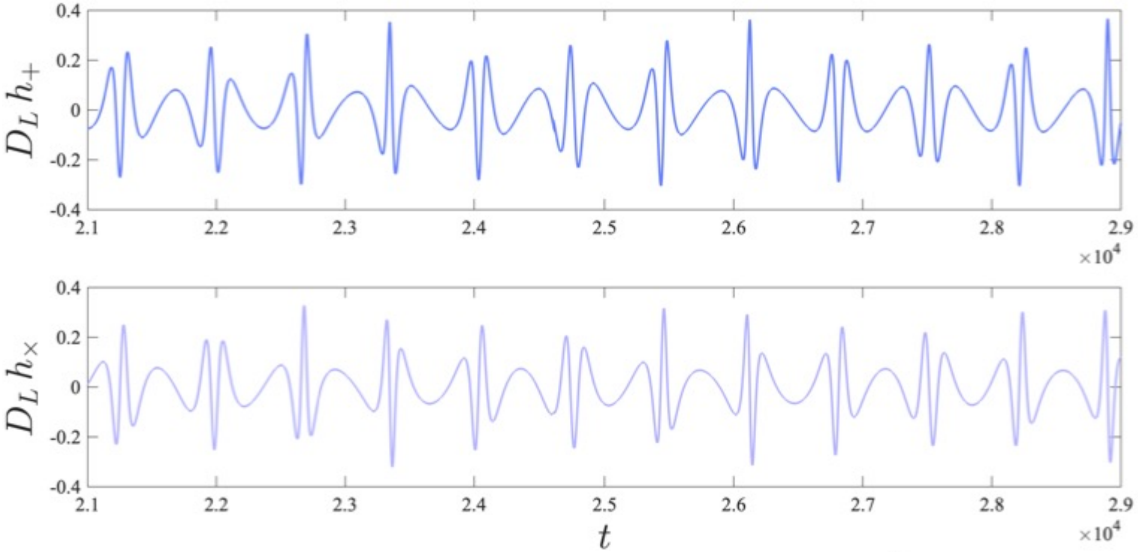
$$\Psi_{\ell m} \equiv \frac{h_{\ell m}}{\sqrt{(l+2)(l+1)l(l-1)}} = A_{\ell m} e^{-i\phi_{\ell m}}$$

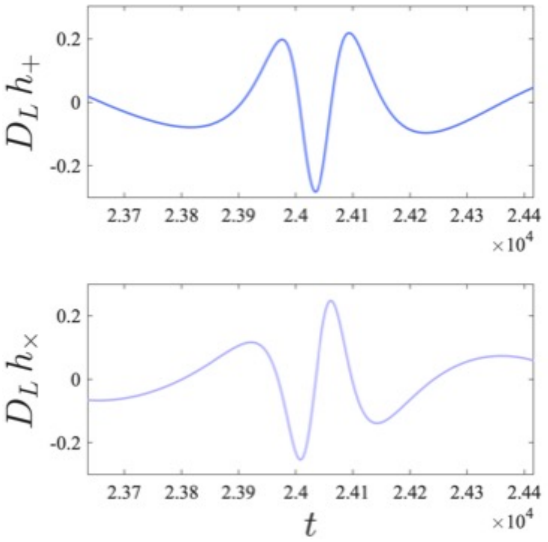
- Waveforms aligned by minimizing the root mean square of the phase difference on an interval
- Final dephasing:

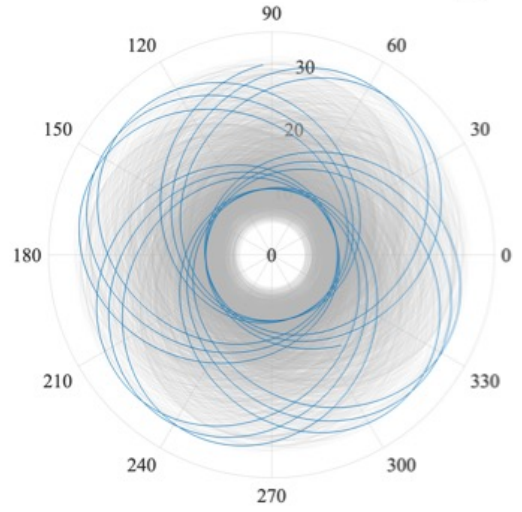
$\Delta\phi \simeq -0.3$ rad

$(q = 10^3, e = 0.5, \chi_1 = 0.3, \chi_2 = 0.1)$

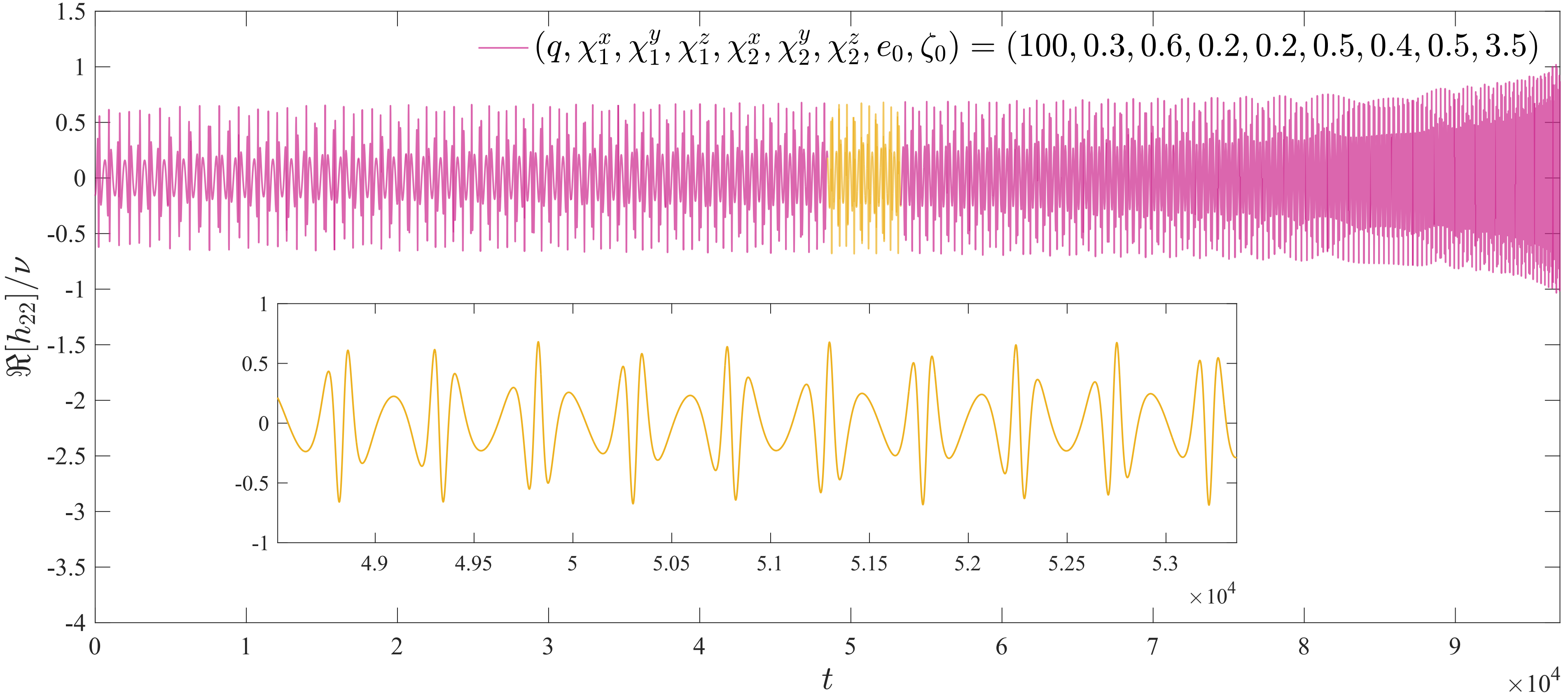






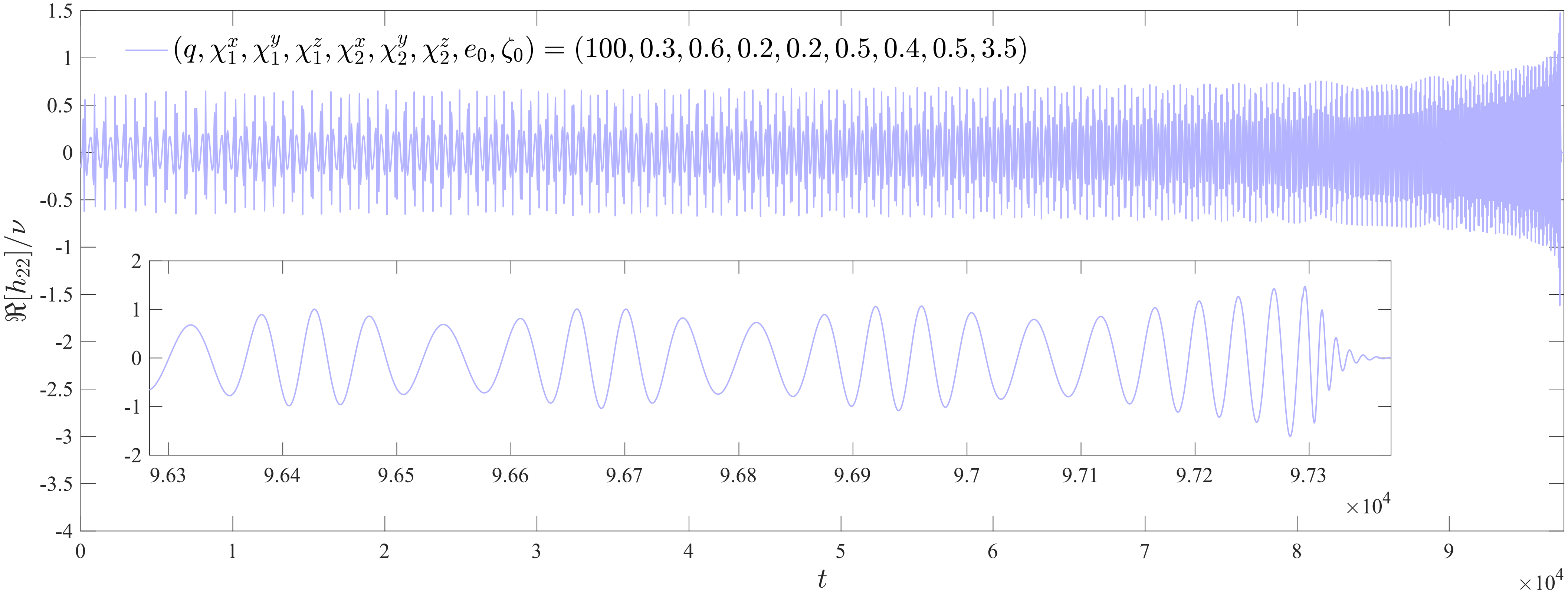


GENERIC (ECCENTRICITY + SPIN PRECESSION)



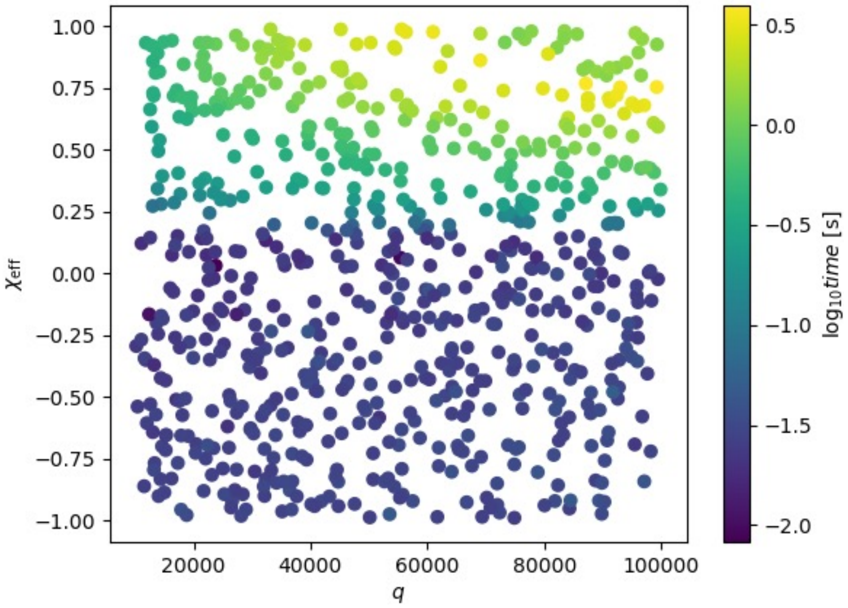
- LMR version of TEOBResumS + eccentricity & spin precession (arXiv:2404.15408)
- Precession is based on a PN twist of the waveform

INCORPORATING THE RINGDOWN (PRELIMINARY)

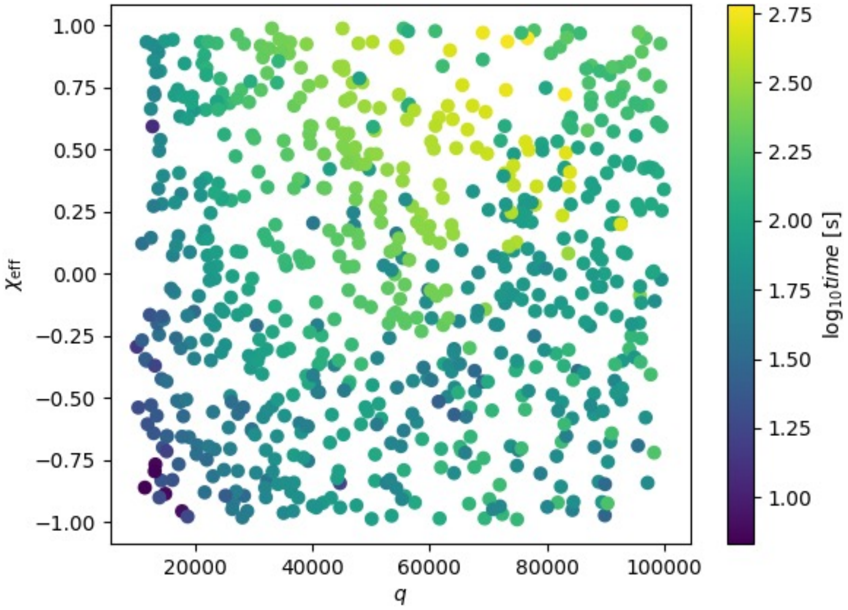


- Ringdown can be attached only with comparable-mass potentials right now

OUR CURRENT PACE



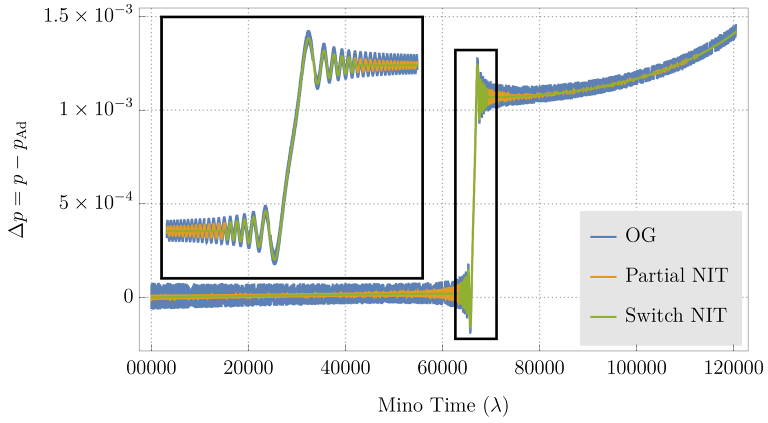
with post-adiabatic analytical solution to the equations of motion in the inspiral



without
(full solution of the ODEs)

COMPUTATIONAL CHALLENGES & POSSIBLE SOLUTIONS

- Evaluating the flux (up to $\ell = 10$) at every time step
  → could be precomputed on a grid & interpolated
- Solving the ODEs
  → developing the post-adiabatic approximation for generic binaries
  → parameter space of LMR inspirals too large for ML surrogate?
  → near-identity transformations:
  average out oscillations on the orbital timescale
  allow treatment of resonances



Lynch et al. 2024
2405.21072 [gr-qc]

CONCLUSIONS AND FUTURE WORK

- Subradian agreement with GSF data for quasi-circular binaries
- Finally fully generic!
- Public code: https://bitbucket.org/teobresums/teobresums/src/Dali/
- Next developments:
  - potentials/ringdown
  - improving the spin contribution to the flux
  - check precessing dynamics (resonances!)
  - **waveform acceleration**
- … and more comparisons with other models!

Thanks for your attention!
Questions? :)

Backup slides

DYNAMICAL BACKGROUND

$G = c = 1 \quad u = 1/r$

- Continuous deformation in ν of a Schwarzschild metric:

$$ds^2_{\text{eff}} = g^{\text{eff}}_{\mu\nu} dx^\mu_{\text{eff}} dx^\nu_{\text{eff}} = -A(r)dt^2 + B(r)dR^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- EOB Hamiltonian for nonspinning binaries:

$$\hat{H}_{\text{EOB}} \equiv \frac{H_{\text{EOB}}}{\mu} = \frac{1}{\nu}\sqrt{1 + 2\nu\left(\hat{H}_{\text{eff}} - 1\right)} \qquad p_{r_*} = (A/B)^{1/2} p_r$$

$$\hat{H}_{\text{eff}} = \sqrt{p^2_{r_*} + A(u)\left(1 + p^2_\varphi u^2 + Q(u, p_{r_*})\right)}$$

- $A(u)$, $D(u) \equiv A(u)B(u)$ and $Q(u, p_{r*})$ are the three EOB potentials

DYNAMICS & WAVEFORM

G = c = 1

- Hamiltonian: $$\hat{H}_{\rm EOB} \equiv \frac{H_{\rm EOB}}{\mu} = \frac{1}{\nu}\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)} \qquad p_{r_*} = (A/B)^{1/2} p_r$$

$$\hat{H}_{\rm eff} = \underbrace{\sqrt{p_{r_*}^2 + A\left(1 + \frac{p_\varphi^2}{r_c^2} + Q\right)}}_{\text{orbital}} + \underbrace{p_\varphi\left(G_S \hat{S} + G_{S_*}\hat{S}_*\right)}_{\text{spin-orbit}}$$

A, $D \equiv A\,B$ and Q are the three EOB potentials

- Hamiltonian equations of motion complemented by the radiation reaction:

$$\begin{cases} \dfrac{d\varphi}{dt} = \dfrac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} = \Omega \\ \dfrac{dr}{dt} = \left(\dfrac{A}{B}\right)^{1/2} \dfrac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}} \\ \dfrac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi \\ \dfrac{dp_{r_*}}{dt} = -\left(\dfrac{A}{B}\right)^{1/2} \dfrac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_r \end{cases}$$

- The multipoles are analytical functions of the phase space variables

$$h_+ - i h_\times = \frac{1}{\mathcal{D}_L} \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{\ell m}\, {}_{-2}Y_{\ell m}$$

- The flux expressions are also analytical (evaluated at every t in the solution of the ODEs)

$$\mathcal{F}_{\ell m} \propto \left|\hat{h}_{\ell m}\right|^2$$

RADIATION REACTION

$$\dot{J}_{\rm system} = \hat{\mathcal{F}}_\varphi = -\dot{J}_\infty - \dot{J}_{\rm H_1} - \dot{J}_{\rm H_2} \qquad \hat{\mathcal{F}}_\varphi = \hat{\mathcal{F}}^\infty_\varphi + \hat{\mathcal{F}}^{\rm H}_\varphi$$

asymptotic contribution ($\hat{\mathcal{F}}^\infty_\varphi$) horizon contribution ($\hat{\mathcal{F}}^{\rm H}_\varphi$)

$$\hat{\mathcal{F}}^\infty_\varphi = -\frac{32}{5}\nu\, r_\omega^4\,\Omega^5\,\hat{f}^\infty(v_\varphi^2;\nu)$$

$$1 = \Omega^2 r_\omega^3$$

Modified Kepler's law valid during the plunge

Reduced flux function: $\hat{f}^\infty = \dfrac{1}{\mathcal{F}^{\rm Newt}_{22}} \sum_{\ell m} \mathcal{F}_{\ell m}$

energy flux radiated at infinity ($\mathcal{F}_{\ell m}$)

$$\mathcal{F}_{\ell m} = \mathcal{F}^{\rm Newt}_{\ell m} \left|\hat{h}_{\ell m}\right|^2$$

correction from the waveform ($\left|\hat{h}_{\ell m}\right|^2$)

WAVEFORM: STRUCTURE AND CONVENTIONS

- Strain: $$h_+ - ih_\times = \frac{1}{\mathscr{D}_L}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} h_{\ell m}\,{}_{-2}Y_{\ell m}$$

Multipoles: computed with the phase space variables found by solving the Hamiltonian equations of motion

Newtonian prefactor

Resummed PN correction

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\,\hat{h}_{\ell m} = h_{\ell m}^{(N,\epsilon)}(x)\,\hat{S}_{\rm eff}^{\epsilon}(x)\,\hat{h}_{\ell m}^{\rm tail}(x)\,\left[\rho_{\ell m}(x)\right]^{\ell}$$

$$x = v_\varphi^2 \equiv (r_\omega \Omega)^2$$

- Regge-Wheeler-Zerilli normalized waveform:

$$\Psi_{\ell m} \equiv \frac{h_{\ell m}}{\sqrt{(l+2)(l+1)l(l-1)}} = A_{\ell m} e^{-i\phi_{\ell m}}$$

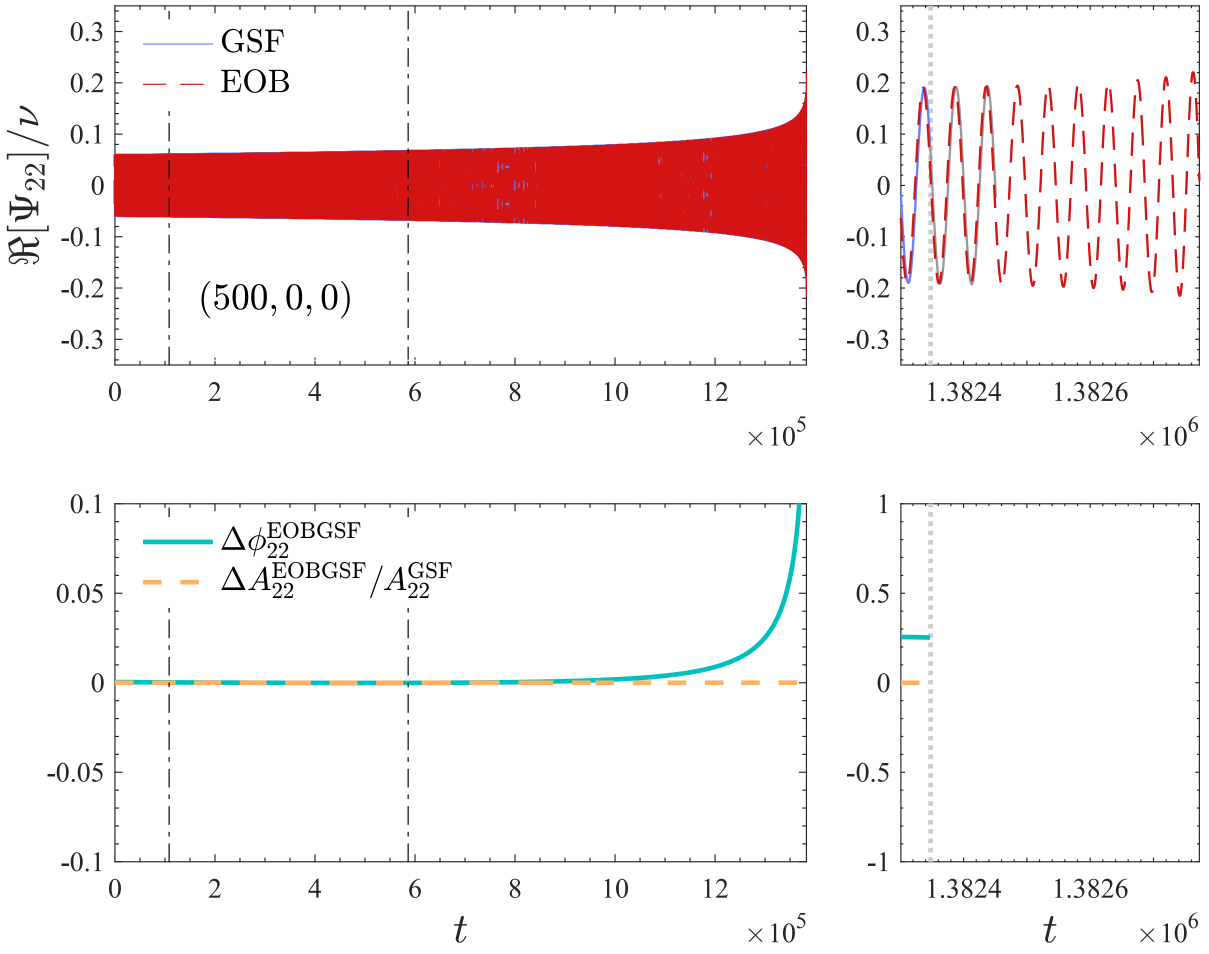
waveform frequency

$$\omega_{\ell m} = \dot{\phi}_{\ell m}$$

STANDARD PRACTICE: TIME-DOMAIN PHASING

GSF: $\Psi^1_{22} = A^1_{22}(t_1)e^{-i\phi_1(t_1)}$

EOB: $\Psi^2_{22} = A^2_{22}(t_2-\tau)e^{-i[\phi_2(t_2-\tau)-\alpha]}$



- We focus on the $\ell = m = 2$ strain multipole
- **Phasing**: finding the time and phase shift ($\boldsymbol{\tau}$, α) by minimizing the root-mean-square of the phase difference on a chosen interval

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\Delta\phi(t_i,\tau,\alpha))^2}$$

$$\Delta\phi(t_i,\tau,\alpha) = \left(\phi_2(t_i-\tau)-\alpha\right)-\phi_1(t_i)$$

GAUGE-INVARIANT ANALYSIS: Q_ω

- Another way of comparing waveforms is to consider the waveform frequency: $\omega \equiv \omega_{22} = \dot{\phi}_{22}$
- Adiabaticity parameter: $Q_\omega \equiv \dfrac{\omega^2}{\dot{\omega}}$ Q_ω >> 1 adiabatic motion
- Phase difference: $\Delta\phi_{(\omega_1,\omega_2)} = \int_{\omega_1}^{\omega_2} Q_\omega \, d\log\omega$
- Expanding in the symmetric mass ratio:

$$Q_\omega(\omega;\nu) = \underbrace{\frac{Q_\omega^{(0)}(\omega)}{\nu}}_{\text{0PA}} + \underbrace{Q_\omega^{(1)}(\omega)}_{\text{1PA}} + \underbrace{\nu Q_\omega^{(2)}(\omega)}_{\text{2PA}} + O(\nu^2)$$

0PA 1PA 2PA

0PA term: inversely proportional to ν, dominant for EMRIs!

GSF-INFORMED EOB POTENTIALS

$$A(u;\nu) = 1 - 2u + \nu a_{1\mathrm{SF}}(u) + O(\nu^2)$$

$$\bar{D}(u;\nu) = \frac{1}{AB} = 1 + \nu \bar{d}_{1\mathrm{SF}}(u) + O(\nu^2)$$

$$Q(u, p_{r_*};\nu) = \nu q_{1\mathrm{SF}}(u) p_{r_*}^4 + O(\nu^2)$$

New EOB orbital potentials
(the standard choice was 5PN for A and 3PN for D and Q, with all the available info in ν, Padé-resummed…)

Expressions for a_{1SF} , $\bar{d}_{1SF}$, q_{1SF} at 8.5PN order
+ suitable factorization & Padé-resummation
+ fit on the numerical GSF data of
Akcay & van de Meent, Phys. Rev. D 93, 064063 (2016)
(see arXiv:2207.14002v1)

Side note: these potentials are singular at the light ring!

RADIATION REACTION PT. 1 (FLUX AT ∞)

- Flux multipoles are factorized into different contributions, among which the residual amplitude corrections:

memo

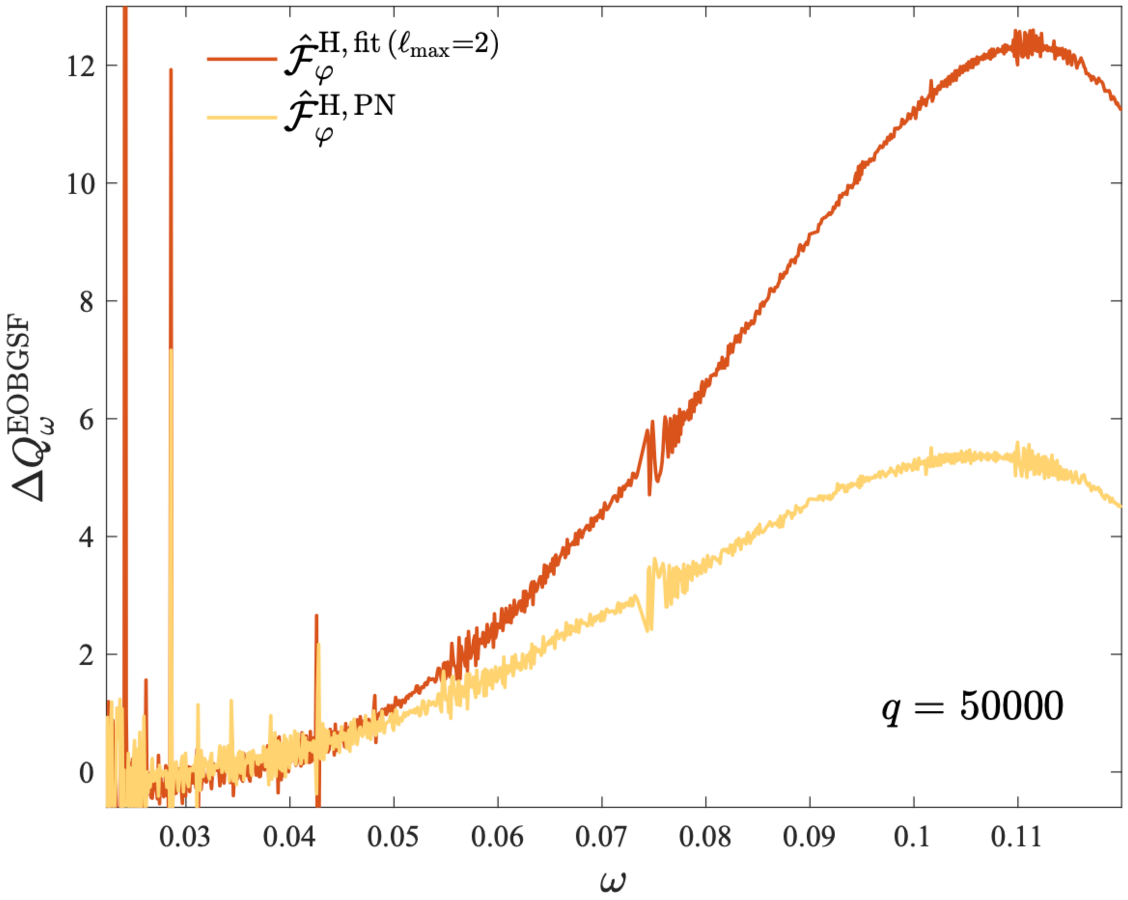
$$\mathcal{F}_{\ell m} = \mathcal{F}^{\rm Newt}_{\ell m} \left| \hat{h}_{\ell m} \right|^2$$

$$\hat{h}_{\ell m} = \hat{S}^{\epsilon}_{\rm eff}\, \hat{h}^{\rm tail}_{\ell m} \left(\rho_{\ell m}\right)^{\ell}$$

$$\rho_{\ell m} = \underbrace{1 + c_1 x + c_2 x^2 + \ldots}_{\text{PN series}}$$

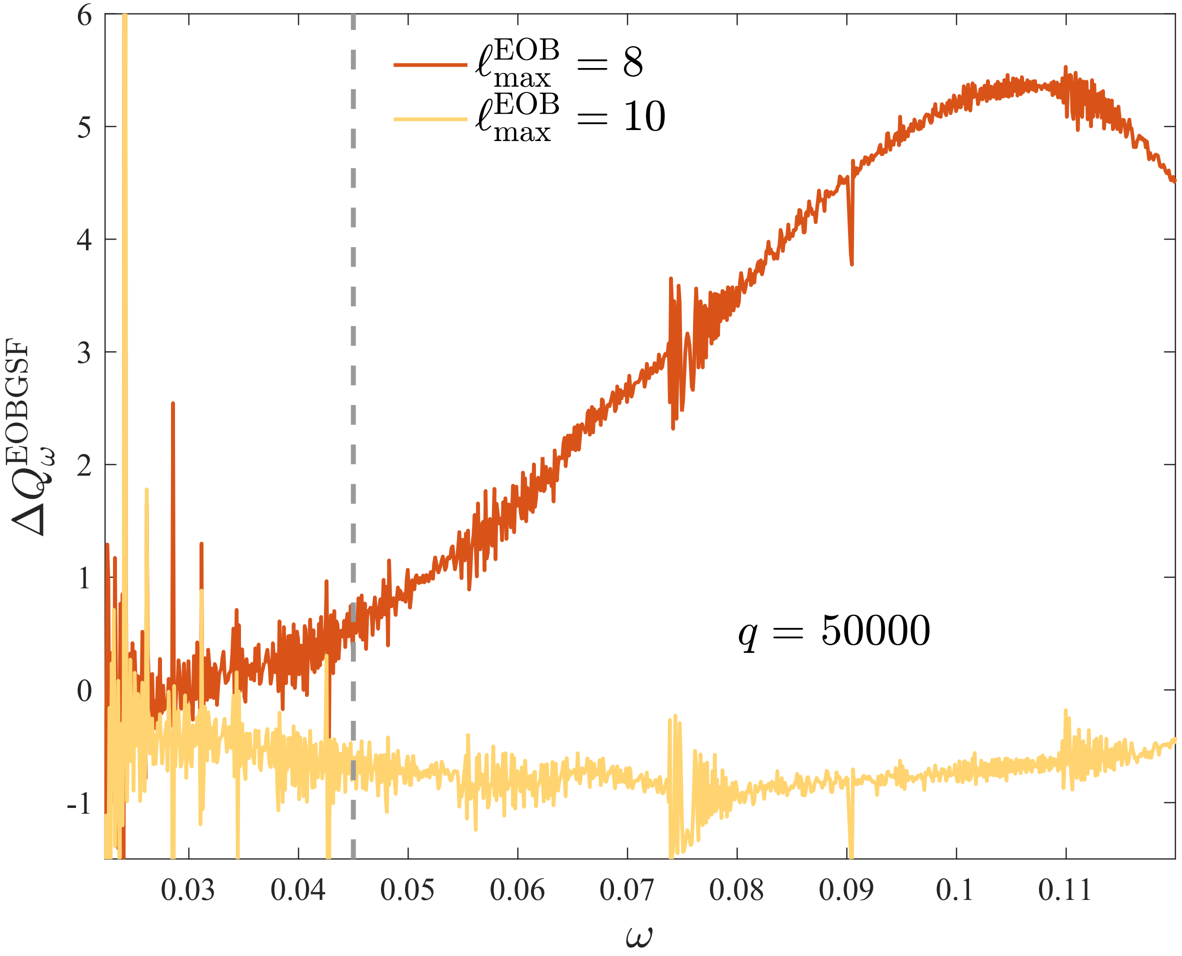
- The standard TEOBResumS had Padé-resummed 6PN expressions with the available info in ν up to 3PN, i.e. $c_1(\nu), c_2(\nu), c_3(\nu)$, hence dubbed 3^{+3}PN
- Now we use 22PN results (Fujita 2012), non resummed, with the same info in ν (hence 3^{+19}PN)

RADIATION REACTION PT. 2 (HORIZON FLUX)



- The standard horizon flux in TEOBResumS had only $\ell = 2$ multipoles
- We implemented a new 'hybrid' version with multipoles up to $\ell = 4$, where some of the multipoles are PN-expanded expressions, other ones are a fit to numerical data (arXiv:1207.0769v2)
- Halves the dephasing!

LAST UPDATE: ARXIV:2310.13578



- **Adding ℓ = 9, 10 to the infinity flux**
- Shorter frequency interval:

$\omega = [0.045, 0.12]$ $f = [0.003, 0.007]\ (Hz)$

if $m_2 = 10 M_\odot$

- Corresponding to ~1.2 years of EOB evolution, ~1.5×10^5 cycles
- Integrated phase differences:

Standard: $\Delta\phi \sim 2.99$
Improved: $\Delta\phi \sim -0.74$

NEXT COMPARISON: BINARIES WITH A SPINNING SECONDARY

- Talking of physical completeness:
  the EOB spin contributions both to the conservative sector and to the dissipative one could be improved
- Two considerations:
  - dissipative sector is really delicate
  (need a similar study to arXiv:1907.12233v2)
  - currently available results at 2GSF/1PA: arXiv:2112.13069v1
  (black hole binaries with a spinning secondary)
- Thus the next task is to **change to the EOB conservative spin-orbit sector** and compare the model to GSF results for binaries with a spinning secondary

QUICK INTRO TO SPINNING BINARIES IN EOB

Dimensionless spins $\chi_i = \frac{S_i}{M_i^2}, \quad i = 1,2$

$$X_1 = \frac{m_1}{M} = \frac{1}{2}\left(1 + \sqrt{1-4\nu}\right)$$

$$X_2 = \frac{m_2}{M} = 1 - X_1 \qquad \tilde{a}_i \equiv \chi_i X_i$$

Dimensionless effective Kerr parameter: $\tilde{a}_0 = \tilde{a}_1 + \tilde{a}_2 = \chi_1 X_1 + \chi_2 X_2$

The effective Hamiltonian has orbital + spin-orbit contribution:

$$\hat{H}_{\rm eff} = \hat{H}^{\rm orb}_{\rm eff} + p_\phi \left(G_S \hat{S} + G_{S_*} \hat{S}_* \right)$$

gyro-gravitomagnetic functions

$$S = S_1 + S_2 \qquad \hat{S} \equiv \frac{S}{M^2}$$

$$S_* = \frac{M_2}{M_1} S_1 + \frac{M_1}{M_2} S_2 \qquad \hat{S}_* \equiv \frac{S_*}{M^2}$$

The orbital Hamiltonian is a function of the centrifugal radius:

$$r_c^2 = r^2 + \tilde{a}_0^2 \left(1 + \frac{2}{r}\right) + \delta\hat{a}^2$$

spin-spin contribution

INSIGHT INTO THE SPIN-ORBIT SECTOR

- Standard version of the gyro-gravitomagnetic functions:

$$G_S = G_S^0 \hat{G}_S, \qquad G_S^0 = 2uu_c^2$$

$$G_{S_*} = G_{S_*}^0 \hat{G}_{S_*}, \qquad G_{S_*}^0 = (3/2)u_c^2$$

leading-order test-mass expressions are factorized out and the rest is (inverse) resummed

$$\hat{G}_S = \frac{1}{1 + c_{10}u_c + c_{20}u_c^2 + c_{30}u_c^3 + c_{02}p_{r*}^2 + c_{12}u_c p_{r*}^2 + c_{04}p_{r*}^4}$$

$$\hat{G}_{S_*} = \frac{1}{1 + c_{10}^* u_c + c_{20}^* u_c^2 + c_{30}^* u_c^3 + c_{40}^* u_c^4 + c_{02}^* p_{r*}^2 + c_{12}^* u_c p_{r*}^2 + c_{04}^* p_{r*}^4}$$

- All c_{ij}/c^*_{ij} coefficients depend on ν except from c30* and c40*, that are test-mass terms coming from the expansion of the exact G_{S*} of a spinning particle on Schwarzschild

(see arXiv:2003.11391v2)

TWO POSSIBILITIES FOR THE SPIN GAUGE

- The standard G_S and G_{S*} functions are expressed in the so-called "Damour-Jaranowski-Schäfer" (**DJS**) gauge:
  set requiring that G_S and G_{S*} have no dependence on p_φ
- In this gauge:
  - **G^0_{S*}** corresponds to **just the leading-order term in the PN expansion of the corresponding test-mass expression**
  - G^0_S coincides exactly with the test-mass expression of G_S.
- Different choice: **anti-DJS gauge**, that allows to choose as **G^0_{S*}** the EOB generalization of the **full G_{S*} of a spinning particle on a Kerr background**

ANTI-DJS GAUGE

- G_{S*} expression for a spinning particle on Kerr:

$$G_{S_*}^K = \frac{1}{(r_c^K)^2}\left\{ \frac{\sqrt{A^K}}{\sqrt{Q^K}}\left[1 - \frac{(r_c^K)'}{\sqrt{B^K}}\right] + \right.$$
$$\left. + \frac{r_c^K}{2\left(1+\sqrt{Q^K}\right)} \frac{(A^K)'}{\sqrt{A^K B^K}} \right\}, \qquad (15)$$

(see arXiv:1911.10818v2)

$$u_c^K = 1/r_c^K,$$
$$Q^K = 1 + p_\varphi^2 (u_c^K)^2 + \frac{p_r^2}{B^K}$$

- Our choice is to take G^0_{S*} as: formally equal to the complete spinning-particle expression BUT replacing the Kerr functions (r_c, A, B) with the EOB ν-dependent ones
- New results (arXiv:2401.12290) at next-to-next-to-next-to leading order (N3LO): we take the gyro-gravitomagnetic functions in this gauge (and the residual functions are again inversely resummed)

NAÏVE CHECK: TIME-DOMAIN PHASING

- We started by checking how the change in G_{S*} affected the dephasing… and the results seemed contradictory

q	χ_2	$\omega^{\mathrm{end}}_{\mathrm{GSF}}$	$\Delta\phi^{\mathrm{EOBGSF}}_{\mathrm{DJS}}$	$\Delta\phi^{\mathrm{EOBGSF}}_{\overline{\mathrm{DJS}}}$
5000	0.9	0.12451	0.24476	0.33948
	−0.9	0.12231	0.49008	0.39988

Positive spin: dephasing increases

Negative spin: dephasing decreases

- Note: positive dephasing in the time domain means the EOB evolution is shorter than the GSF one (whereas positive integrated Q_ω difference means more adiabatic, longer evolution)

Q_ω EXPANSION WITH SPIN

- Q_ω can be evaluated analytically for circular orbits (in EOB)
- Formal expansion of the flux:

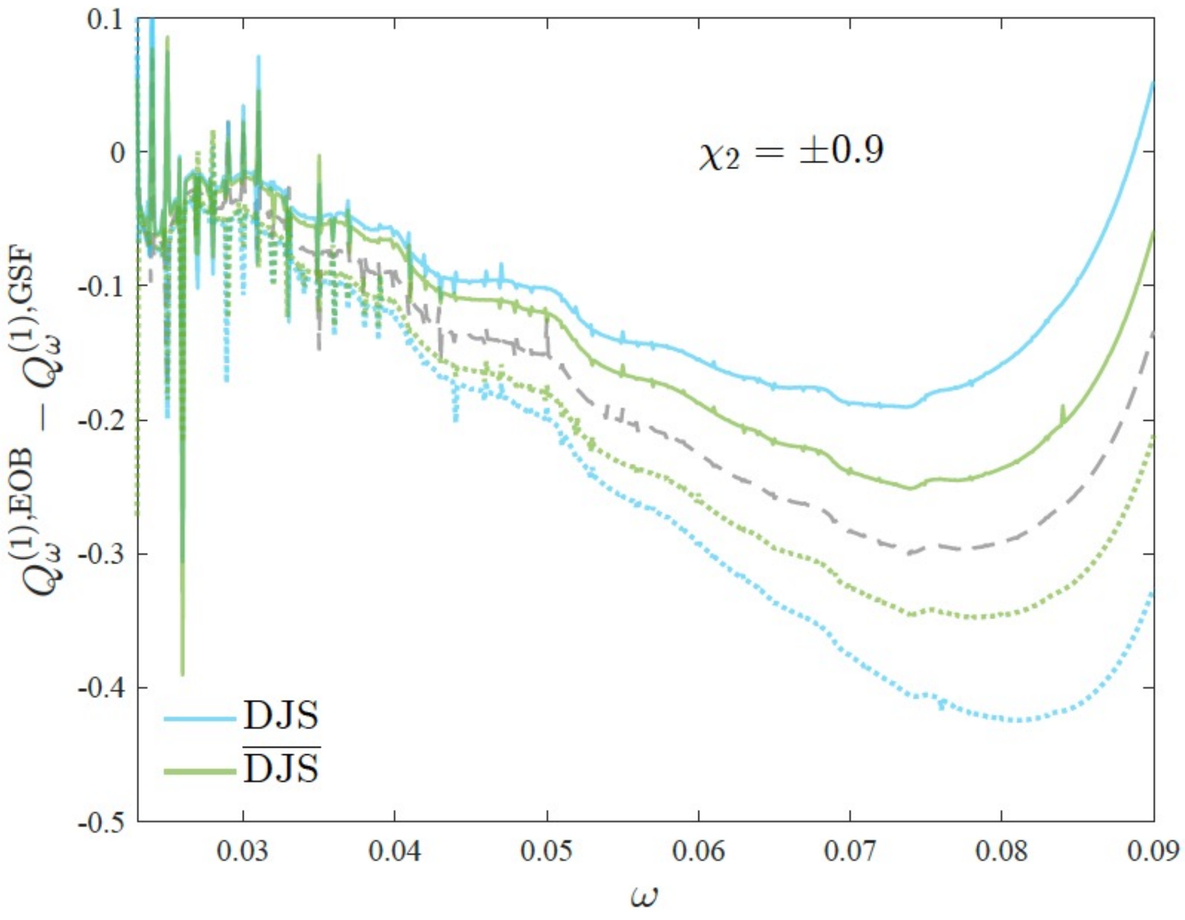
$$\mathcal{F} = \mathcal{F}_{1SF} + \nu\,\mathcal{F}_{2SF} + \nu^2\,\mathcal{F}_{3SF} + \chi_2\left(\nu\,\mathcal{F}^{\rm spin}_{2SF} + \nu^2\,\mathcal{F}^{\rm spin}_{3SF}\right)$$

- Then the coefficients have the following dependencies:

$$\begin{cases} Q_\omega^{(0)} = Q_\omega^{(0)}\left(\mathcal{F}_{1SF}\right) \\ Q_\omega^{(1)} = Q_\omega^{(1)}\left(a_1, \mathcal{F}_{1SF}, \mathcal{F}_{2SF}, \chi_2\cdot\mathcal{F}^{\rm spin}_{2SF}, \chi_2\right) \\ Q_\omega^{(2)} = Q_\omega^{(2)}\left(a_1, \mathcal{F}_{1SF}, \mathcal{F}_{2SF}, \chi_2\cdot\mathcal{F}^{\rm spin}_{2SF}, \mathcal{F}_{3SF}, \chi_2\cdot\mathcal{F}^{\rm spin}_{3SF}, \chi_2, \chi_2^2\right) \end{cases}$$

- Since the GSF model is not complete in its $Q_\omega{}^{(2)}$ contribution, **we focus on $Q_\omega{}^{(1)}$**

COMPARING $Q_\omega^{(1)}$ (EOB - GSF)



Anti-DJS:

the difference gets closer to the nonspin result (dashed grey)

➡ EOB gets less adiabatic for positive spins and more adiabatic for negative spins

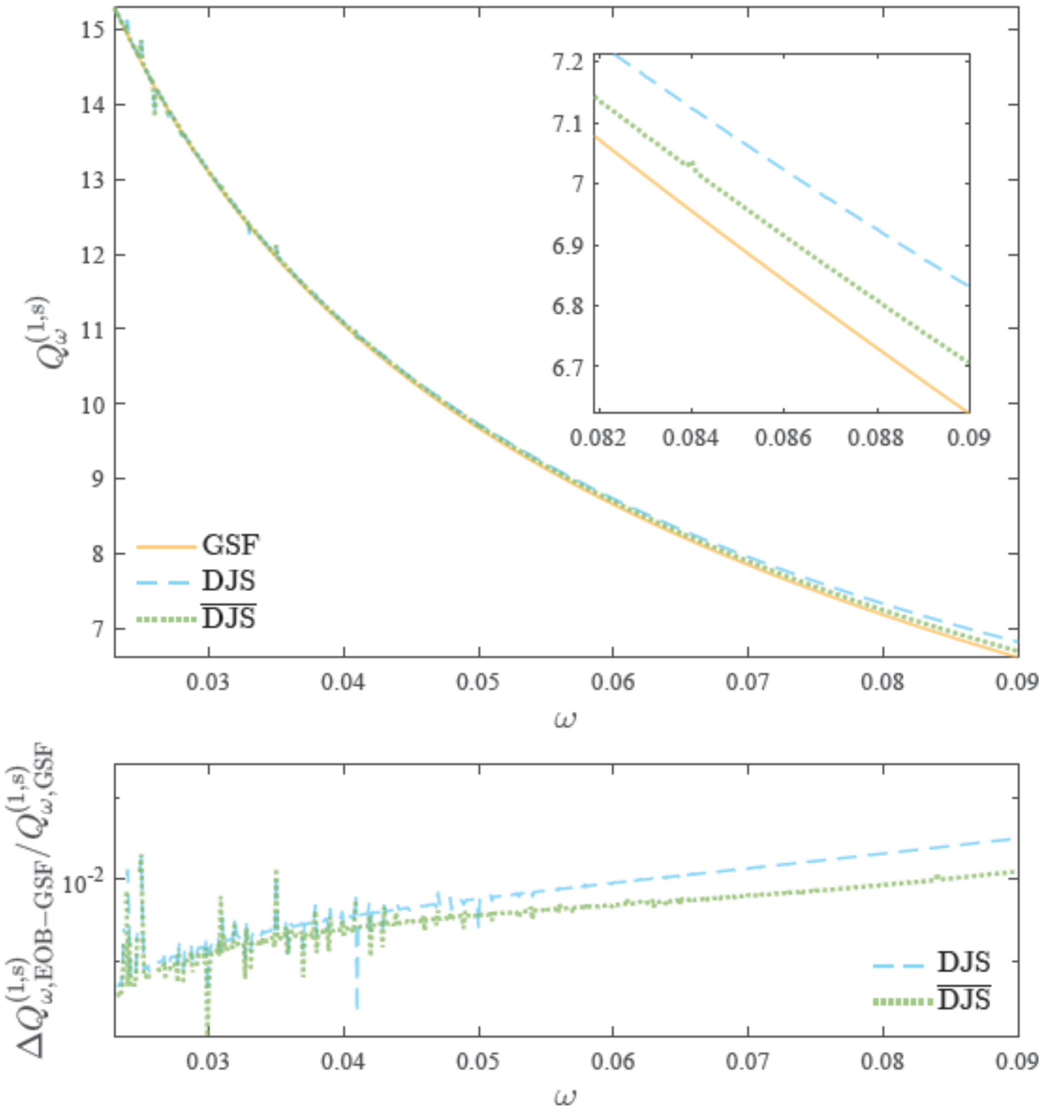
Translation in the time domain:

positive $\Delta\phi$ = EOB is shorter

So:

$\chi_2 > 0$, EOB gets less adiabatic = even shorter waveform, $\Delta\phi$ increases (viceversa for $\chi_2 < 0$)

COMPARING $Q_\omega^{(1)}$ (EOB - GSF)



$$Q_\omega^{(1)} = Q_\omega^{(1,\text{ns})} + \chi_2 Q_\omega^{(1,\text{s})}$$

$$Q^{(1)}_{\omega,\text{EOB}} - Q^{(1)}_{\omega,\text{GSF}} = Q^{(1,\text{ns})}_{\omega,\text{EOB}} - Q^{(1,\text{ns})}_{\omega,\text{GSF}} + \chi_2 \left(Q^{(1,\text{s})}_{\omega,\text{EOB}} - Q^{(1,\text{s})}_{\omega,\text{GSF}} \right)$$

Explaining further what we saw in the previous plot: the new choice for the spin-orbit sector brings the EOB $Q_\omega^{(1,s)}$ closer to the GSF one, so that overall the difference in $Q_\omega^{(1)}$ is closer to the nonspin result

MEANINGFULNESS OF OUR CHOICE

- We can check the impact of the new choice on the dephasing:

$$\Delta\phi^{\rm EOBGSF,ns}_{(\omega_1,\omega_2)} = \int_{\omega_1}^{\omega_2} \left(Q^{(1,s)}_{\omega,\rm EOB} - Q^{(1,s)}_{\omega,\rm GSF}\right) d\log\omega$$

- Can impact the overall 1PA contribution to $\Delta\phi$ from 1% to 10%, depending on χ_2. But the complete $\Delta\phi$ is also more dominated by the 0PA contribution as the mass ratio gets more extreme (smaller ν):

$$\Delta\phi_0 \equiv \frac{1}{\nu}\int_{\omega_1}^{\omega_2} \log(\omega)\ \left(Q_\omega^{0,\rm EOB} - Q_\omega^{0,\rm GSF}\right),$$

$$\Delta\phi_1 \equiv \int_{\omega_1}^{\omega_2} \log(\omega)\ \left(Q_\omega^{1,\rm EOB} - Q_\omega^{1,\rm GSF}\right),$$

$$\Delta\phi_2 \equiv \nu\int_{\omega_1}^{\omega_2} \log(\omega)\ \left(Q_\omega^{2,\rm EOB} - Q_\omega^{2,\rm GSF}\right),$$

$$\Delta\phi^{\rm EOBGSF}_{(\omega_1,\omega_2)} = \Delta\phi_0 + \Delta\phi_1 + \Delta\phi_2.$$

the impact of the anti-DJS choice depends on mass ratio and spin, but we still keep it since it represents an upgrade to some parts of the parameter space