Post-Minkowskian self-force in the low-velocity limit: scalar field scattering

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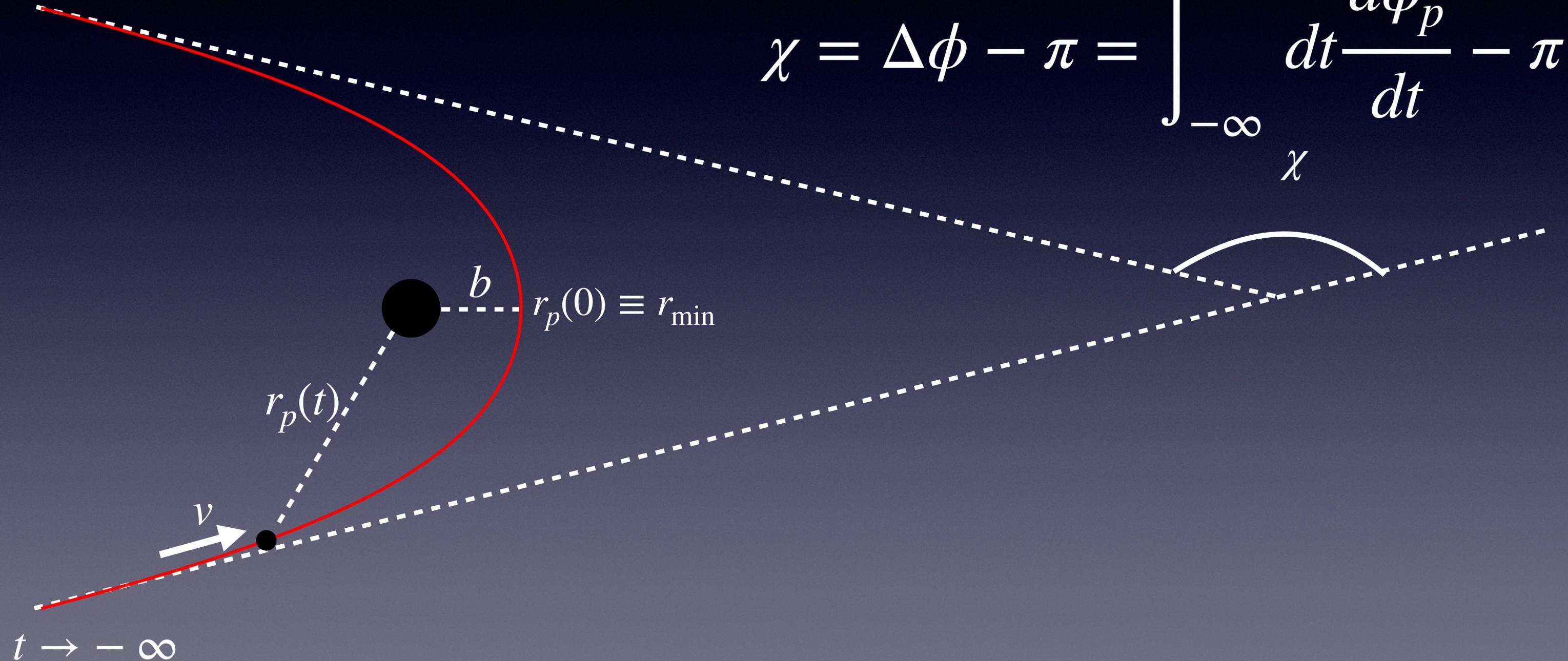9th - 11th December 2024
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Unlocking Gravity through Computation

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Work done in collaboration with D. Bini, A. Geralico, C. Kavanagh and A. Pound
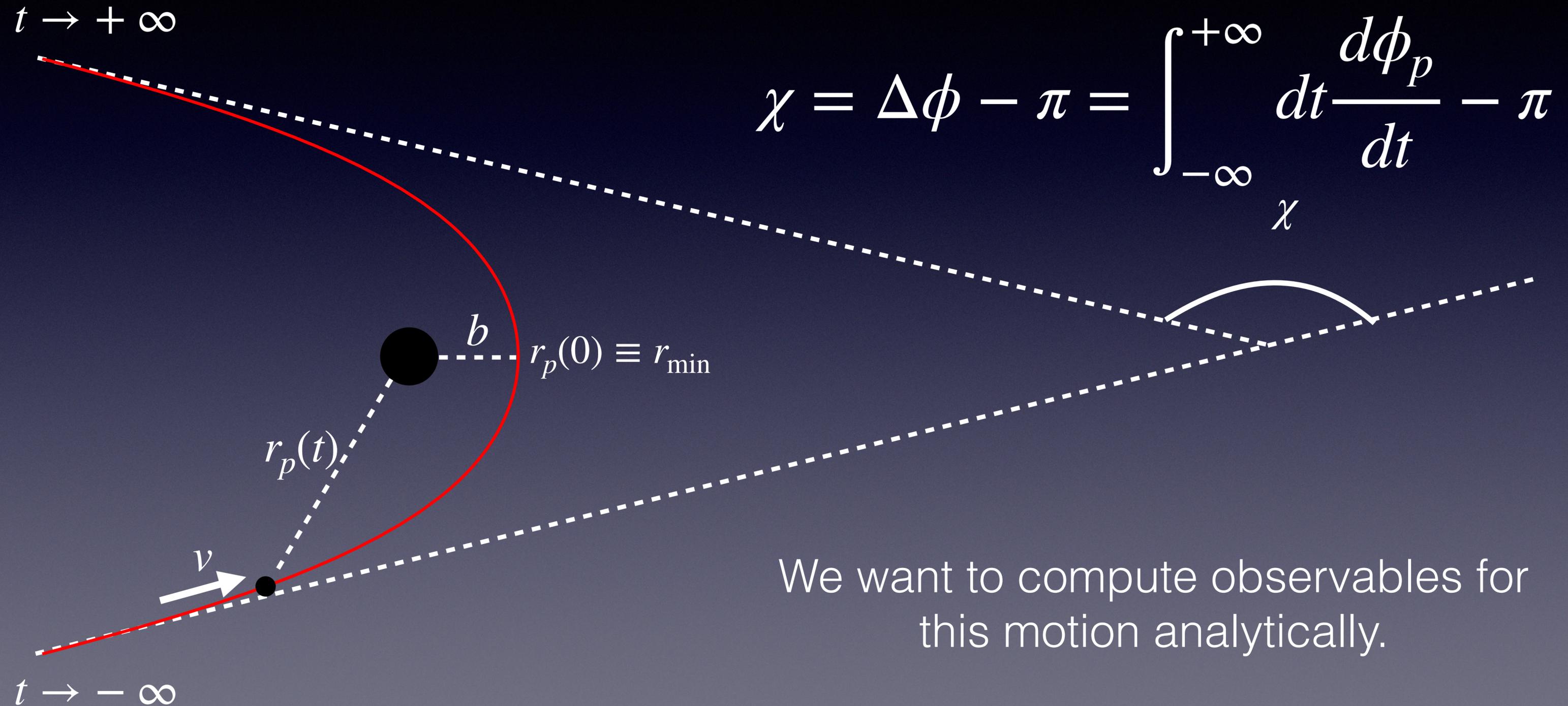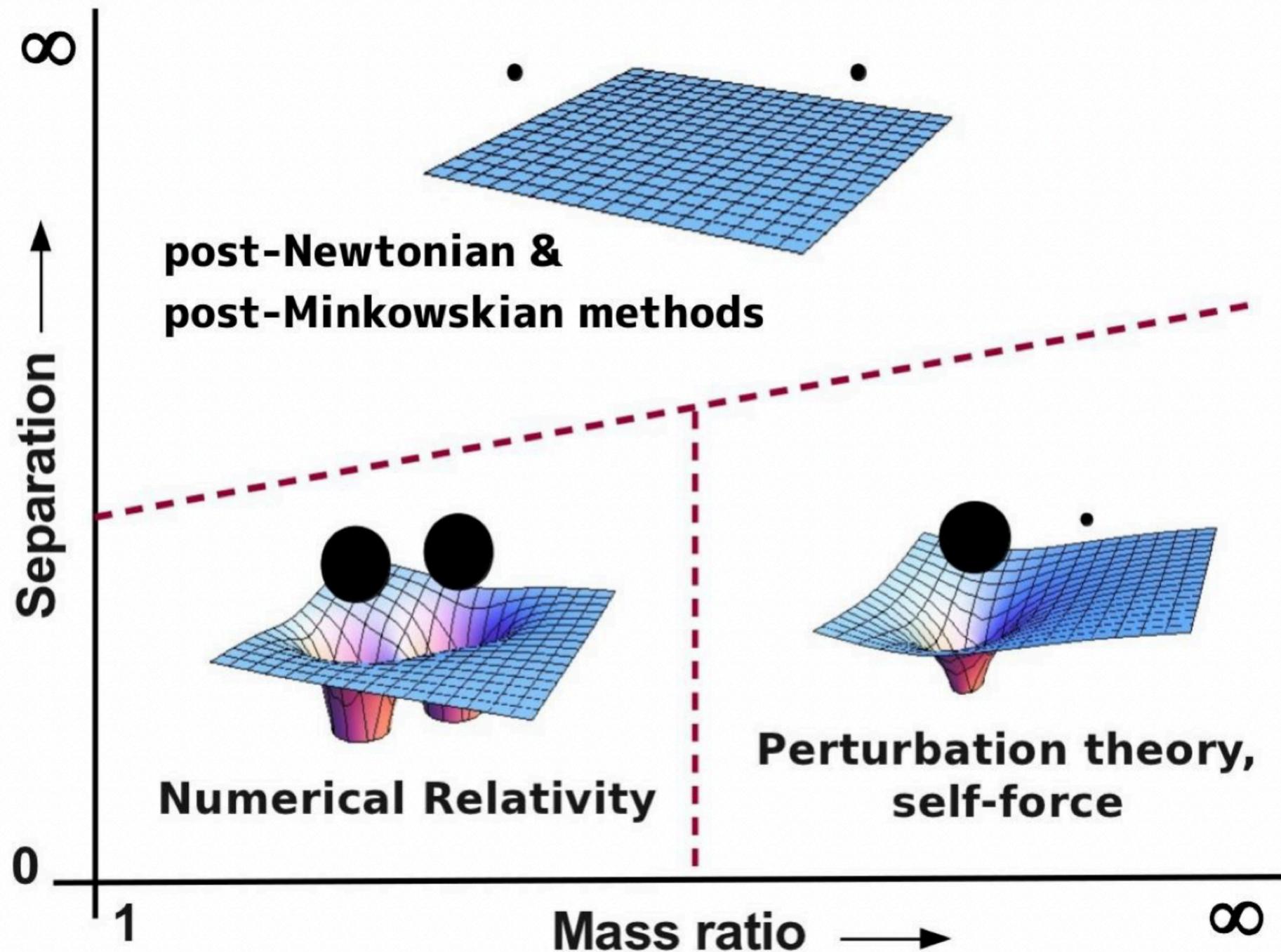
Scattering Orbit

$t \to +\infty$

$t \to -\infty$

v

b

$r_p(t)$

$r_p(0) \equiv r_{\min}$

χ

$$\chi = \Delta\phi - \pi = \int_{-\infty}^{+\infty} dt \frac{d\phi_p}{dt} - \pi$$

Scattering Orbit

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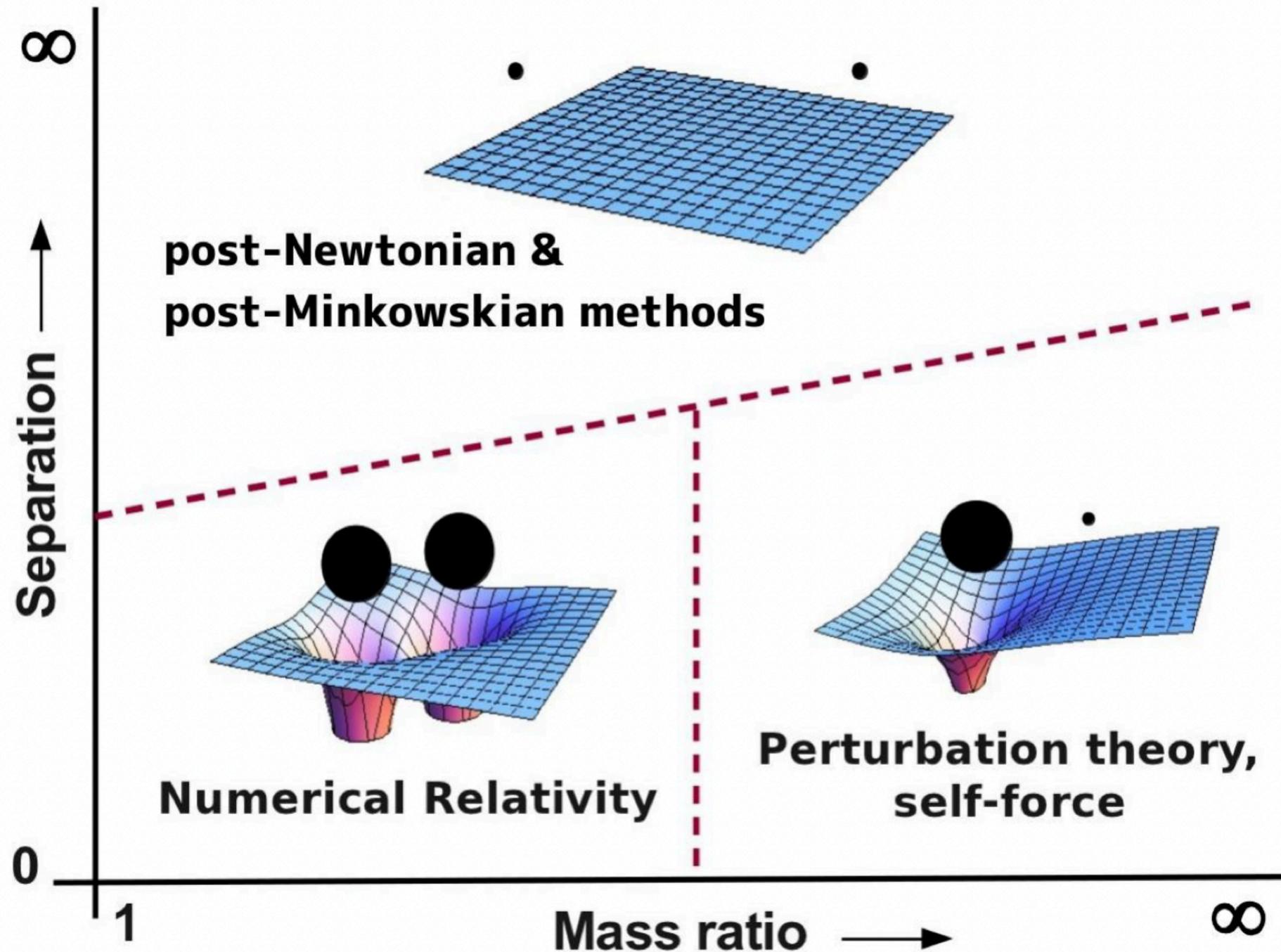
We want to compute observables for this motion analytically.

Self Force

Self Force

Self Force means expansion in the mass ratio.

Analytical Self Force

We have to solve analytically the Teukolsky equation for generic spin

$$_s\mathcal{O}\,_s\psi = \,_sT$$

and the procedure is very well known.

1) Solve the mode decomposed homogeneous solutions in the Fourier domain;
2) Convolve the homogeneous solutions with the source;
3) Return to the time domain;
4) Sum over the modes and regularize.

Analytical Self Force

We have to solve analytically the Teukolsky equation for generic spin

$$_s\mathscr{O}\,_s\psi = \,_sT$$

and the procedure is very well known.

The analytical solutions always relies on some kind of additional expansions: Post-Minkowskian (PM) and/or Post-Newtonian (PN).

PM means large impact parameter b
PN means small v/c

(Analytical) Scalar Self Force

We have to solve analytically the Teukolsky equation

$$_s\mathcal{O}\,_s\psi = \,_sT$$

$s = 0$ scalar field

$|s| = 2$ gravitational perturbation

(Analytical) Scalar Self Force

We have to solve analytically the Teukolsky equation

$$_0\mathcal{O}_0\psi = {}_0T$$

$$\Box\psi = 4\pi\rho$$

(Analytical) Scalar Self Force

We have to solve analytically the Teukolsky equation

$$_0\mathcal{O}\,_0\psi = {}_0T$$

Information about the background geometry

Information about the orbit

$$\square\,\psi = 4\pi\rho$$

Forces, Fluxes and Scattering angle

We define the components of the force as

$$F_\alpha = \frac{q}{\mu} P_\alpha{}^\beta \nabla_{\beta 0} \psi$$

The presence of a force modifies the energy, the angular momentum of the scattered body. Because of this, it also changes the scattering angle

$$E(\tau) = E_- + \int_{-\infty}^{\tau} d\tau\, F_t = E_- + \delta E_0 + \delta E(\tau) \qquad L(\tau) = L_- + \int_{-\infty}^{\tau} d\tau\, F_\phi = L_- + \delta L_0 + \delta L(\tau)$$

$$\chi = \hat{\chi} + \delta\hat{\chi} + \delta\tilde{\chi} = \hat{\chi} + \frac{\partial\hat{\chi}}{\partial E_-}\delta E_0 + \frac{\partial\hat{\chi}}{\partial L_-}\delta L_0 + \sum_{\pm} \int_{\hat{r}_{\min}}^{\infty} dr \left(\frac{d\delta\phi}{dr}\right)^{\pm}$$

Forces, Fluxes and Scattering angle

We define the components of the force as

$$F_\alpha = \frac{q}{\mu} P_\alpha{}^\beta \boxed{\nabla_{\beta 0} \psi} \longrightarrow$$

We need Teukolsky Eq for s=0!

The presence of a force modifies the energy, the angular momentum of the scattered body. Because of this, it also changes the scattering angle

$$E(\tau) = E_- + \int_{-\infty}^{\tau} d\tau\, F_t = E_- + \delta E_0 + \delta E(\tau) \qquad L(\tau) = L_- + \int_{-\infty}^{\tau} d\tau\, F_\phi = L_- + \delta L_0 + \delta L(\tau)$$

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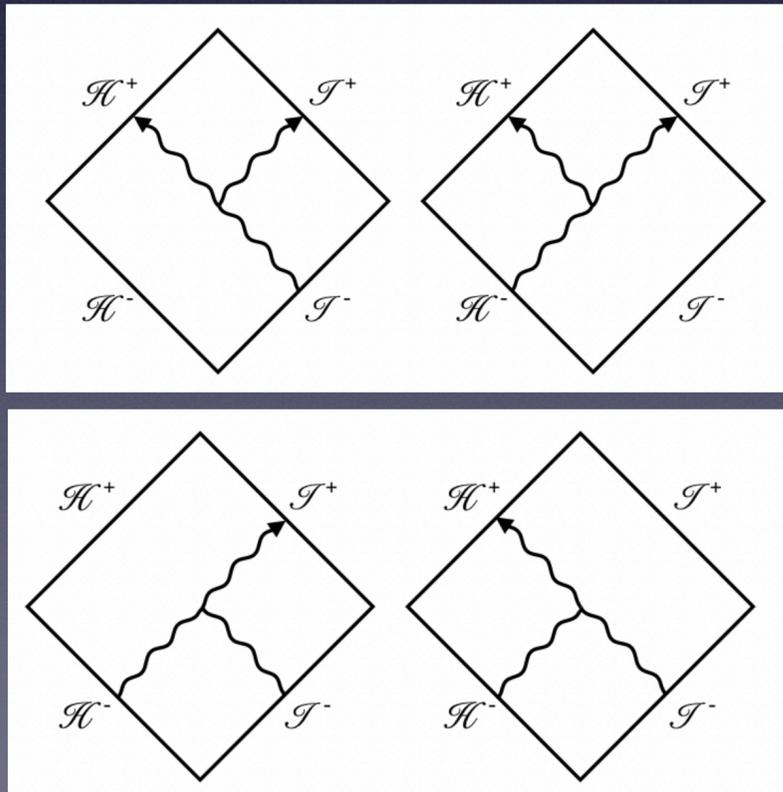
Solving Teukolsky Equation

1) Solve the mode decomposed homogeneous solutions in the Fourier domain;

2) Convolve the homogeneous solutions with the source;

3) Return to the time domain;

4) Sum over the modes and regularize.

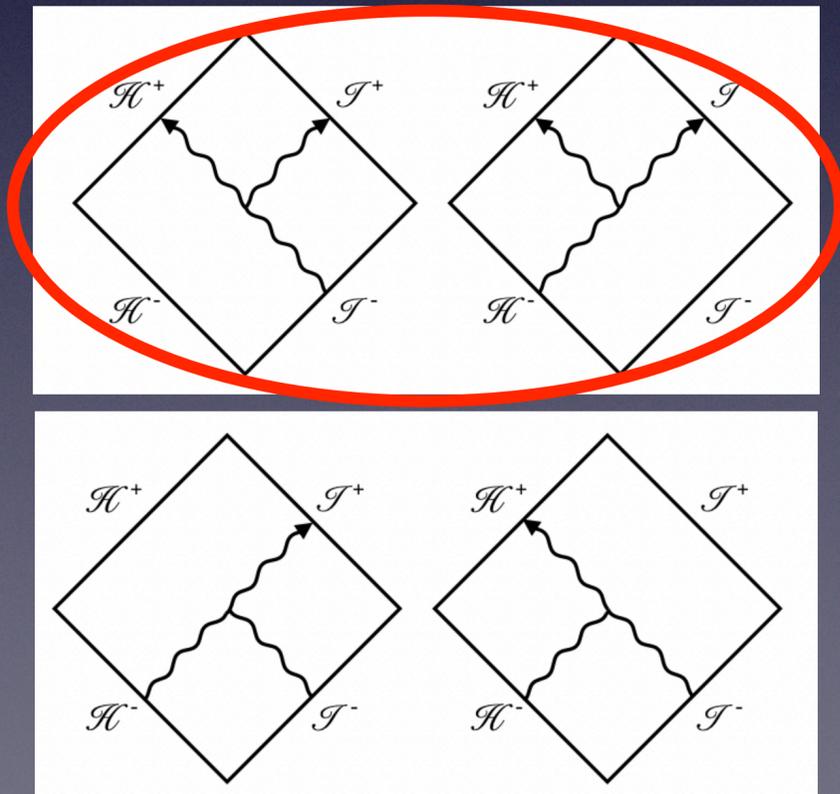# 1) Homogeneous Solutions

$$_0\mathcal{O}\,_0\psi = \,_0T$$

We have four different (physical) boundary condition that we could impose

$$
_sR^{\text{in}}_{\ell m \omega}(r) \sim \begin{cases} 0 & + \; _sR^{\text{in,trans}}_{\ell m \omega}\Delta^{-s}e^{-ikr_*} & r \to r_+ \\ _sR^{\text{in,ref}}_{\ell m \omega}r^{-1-2s}e^{+i\omega r_*} & + \; _sR^{\text{in,inc}}_{\ell m \omega}r^{-1}e^{-i\omega r_*} & r \to \infty \end{cases}
$$

$$
_sR^{\text{up}}_{\ell m \omega}(r) \sim \begin{cases} _sR^{\text{up,inc}}_{\ell m \omega}e^{+ikr_*} & + \; _sR^{\text{up,ref}}_{\ell m \omega}\Delta^{-s}e^{-ikr_*} & r \to r_+ \\ _sR^{\text{up,trans}}_{\ell m \omega}r^{-1-2s}e^{+i\omega r_*} & + \quad 0 & r \to \infty \end{cases}
$$

$$
_sR^{\text{out}}_{\ell m \omega}(r) \sim \begin{cases} _sR^{\text{out,trans}}_{\ell m \omega}e^{+ikr_*} & + \quad 0 & r \to r_+ \\ _sR^{\text{out,inc}}_{\ell m \omega}r^{-1-2s}e^{+i\omega r_*} & + \; _sR^{\text{out,ref}}_{\ell m \omega}r^{-1}e^{-i\omega r_*} & r \to \infty \end{cases}
$$

$$
_sR^{\text{down}}_{\ell m \omega}(r) \sim \begin{cases} _sR^{\text{down,ref}}_{\ell m \omega}e^{+ikr_*} & + \; _sR^{\text{down,inc}}_{\ell m \omega}\Delta^{-s}e^{-ikr_*} & r \to r_+ \\ 0 & + \; _sR^{\text{down,trans}}_{\ell m \omega}r^{-1}e^{-i\omega r_*} & r \to \infty \end{cases}
$$

1) Homogeneous Solutions

$R^{\text{in}}_{lm\omega}(r)$ has no outcoming wave from past horizon, while $R^{\text{up}}_{lm\omega}(r)$ has no incoming wave at past infinity.

$$
{}_sR^{\text{in}}_{\ell m\omega}(r) \sim \begin{cases} 0 & + \; {}_sR^{\text{in,trans}}_{\ell m\omega}\Delta^{-s}e^{-ikr_*} & r \to r_+ \\ {}_sR^{\text{in,ref}}_{\ell m\omega}r^{-1-2s}e^{+i\omega r_*} & + \; {}_sR^{\text{in,inc}}_{\ell m\omega}r^{-1}e^{-i\omega r_*} & r \to \infty \end{cases}
$$

$$
{}_sR^{\text{up}}_{\ell m\omega}(r) \sim \begin{cases} {}_sR^{\text{up,inc}}_{\ell m\omega}e^{+ikr_*} & + \; {}_sR^{\text{up,ref}}_{\ell m\omega}\Delta^{-s}e^{-ikr_*} & r \to r_+ \\ {}_sR^{\text{up,trans}}_{\ell m\omega}r^{-1-2s}e^{+i\omega r_*} & + \; 0 & r \to \infty \end{cases}
$$

$$
{}_sR^{\text{out}}_{\ell m\omega}(r) \sim \begin{cases} {}_sR^{\text{out,trans}}_{\ell m\omega}e^{+ikr_*} & + \; 0 & r \to r_+ \\ {}_sR^{\text{out,inc}}_{\ell m\omega}r^{-1-2s}e^{+i\omega r_*} & + \; {}_sR^{\text{out,ref}}_{\ell m\omega}r^{-1}e^{-i\omega r_*} & r \to \infty \end{cases}
$$

$$
{}_sR^{\text{down}}_{\ell m\omega}(r) \sim \begin{cases} {}_sR^{\text{down,ref}}_{\ell m\omega}e^{+ikr_*} & + \; {}_sR^{\text{down,inc}}_{\ell m\omega}\Delta^{-s}e^{-ikr_*} & r \to r_+ \\ 0 & + \; {}_sR^{\text{down,trans}}_{\ell m\omega}r^{-1}e^{-i\omega r_*} & r \to \infty \end{cases}
$$

1) Homogeneous Solutions

The solution obtained with MST approach takes the form

$$R_{in} = R_0^{\nu} + R_0^{-1-\nu}$$

$$R_0^{\nu} = e^{i\epsilon\kappa x}(-x)^{-s-(i/2)(\epsilon+\tau)}(1-x)^{(i/2)(\epsilon+\tau)+\nu} \sum_{n=-\infty}^{\infty} f_n^{\nu} \frac{\Gamma(1-s-i\epsilon-i\tau)\Gamma(2n+2\nu+1)}{\Gamma(n+\nu+1-i\tau)\Gamma(n+\nu+1-s-i\epsilon)}$$

$$\times (1-x)^n F\left(-n-\nu-i\tau, -n-\nu-s-i\epsilon; -2n-2\nu; \frac{1}{1-x}\right)$$

$$R_{up} = 2^{\nu} e^{-\pi\epsilon} e^{-i\pi(\nu+1+s)} e^{i\hat{z}} \hat{z}^{\nu+i\epsilon_+}(\hat{z}-\epsilon\kappa)^{-s-i\epsilon_+} \sum_{n=-\infty}^{\infty} i^n \frac{(\nu+1+s-i\epsilon)_n}{(\nu+1-s+i\epsilon)_n} f_n^{\nu}(2\hat{z})^n$$

$$\times \Psi(n+\nu+1+s-i\epsilon, 2n+2\nu+2; -2i\hat{z})$$

1) Homogeneous Solutions

However, we don't need the full expressions and we can use the PN expanded solutions

$R_{in}^{l=2}(r) =$

$$\frac{r^4}{16} + \frac{1}{24}\, \mathbb{i}\, r^5\, \eta\, \omega + \eta^2\left(-\frac{r^3}{4} - \frac{11\, r^6\, \omega^2}{672}\right) + \eta^3\left(-\frac{7}{96}\, \mathbb{i}\, r^4\, \omega - \frac{1}{224}\, \mathbb{i}\, r^7\, \omega^3\right) +$$

$$\eta^4\left(\frac{r^2}{4} - \frac{17\, r^5\, \omega^2}{504} + \frac{23\, r^8\, \omega^4}{24\,192}\right) + \eta^5\left(-\frac{1}{12}\, \mathbb{i}\, r^3\, \omega - \frac{113\, \mathbb{i}\, r^6\, \omega^3}{4032} + \frac{\mathbb{i}\, r^9\, \omega^5}{6048}\right) +$$

$$\eta^6\left(\frac{127\, r^7\, \omega^4}{12\,096} - \frac{13\, r^{10}\, \omega^6}{532\,224} + \frac{r^4\, \omega^2\left(24\,253 - 1960\, \pi^2 + 5992\, \text{Log}[2] - 5992\, \text{Log}[r]\right)}{47\,040}\right)$$

$R_{up}^{l=2}(r) =$

$$-3\,\eta - \frac{3\,\mathbb{i}}{r\,\omega} + \eta^2\left(-\frac{3\,\mathbb{i}}{r^2\,\omega} + \frac{3\,\mathbb{i}\,r\,\omega}{2}\right) + \frac{\eta^3\left(4 - 12\,\text{EulerGamma} + 12\,\mathbb{i}\,\pi + r^3\,\omega^2\right)}{2\,r} +$$

$$\eta^4\left(-\frac{24\,\mathbb{i}}{7\,r^3\,\omega} + \left(-\frac{13\,\mathbb{i}}{2} + 6\,\mathbb{i}\,\text{EulerGamma} + 6\,\pi\right)\omega - \frac{1}{8}\,\mathbb{i}\,r^3\,\omega^3\right) +$$

$$\eta^5\left(-\frac{6\,(2 + 7\,\text{EulerGamma} - 7\,\mathbb{i}\,\pi)}{7\,r^2} + 3\,(-2 + \text{EulerGamma} - \mathbb{i}\,\pi)\,r\,\omega^2 + \frac{31\,r^4\,\omega^4}{40}\right) +$$

$$\eta^6\left(-\frac{30\,\mathbb{i}}{7\,r^4\,\omega} + \frac{1265\,\mathbb{i}\,\omega}{49\,r} - \frac{354\,\mathbb{i}\,\text{EulerGamma}\,\omega}{35\,r} + \frac{6\,\mathbb{i}\,\text{EulerGamma}^2\,\omega}{r} + \frac{74\,\pi\,\omega}{35\,r} + \frac{12\,\text{EulerGamma}\,\pi\,\omega}{r} - \frac{7\,\mathbb{i}\,\pi^2\,\omega}{r} + \frac{31}{8}\,\mathbb{i}\,r^2 \right.$$

$$\left. \omega^3 - \mathbb{i}\,\text{EulerGamma}\,r^2\,\omega^3 - \pi\,r^2\,\omega^3 + \frac{43}{80}\,\mathbb{i}\,r^5\,\omega^5 - \frac{214\,\mathbb{i}\,\omega\,\text{Log}[2]}{35\,r} - \frac{428\,\mathbb{i}\,\omega\,\text{Log}[-\mathbb{i}\,r\,\eta\,\omega]}{35\,r} + \frac{214\,\mathbb{i}\,\omega\,\text{Log}[r\,\eta\,\omega]}{35\,r}\right)$$

2) Inhomogeneous Solution

We construct the Green function

$$_sG_{lm\omega}(r,r') = \frac{1}{r^2 f(r)W_{lm\omega}}\left\{{}_sR_{\text{in}}^{lm\omega}(r)\,{}_sR_{\text{up}}^{lm\omega}(r')H(r'-r) + {}_sR_{\text{in}}^{lm\omega}(r')\,{}_sR_{\text{up}}^{lm\omega}(r)H(r-r')\right\}$$

Then the inhomogeneous equation can be formally written as

$$_s\psi_{\ell m}(r) = \int dr' \int_{-\infty}^{\infty} d\omega \, {}_sG_{\ell m\omega}(r,r') \int_{-\infty}^{\infty} dt'\, e^{i\omega(t'-t)}\,{}_sT_{\ell m}(t',r'),$$

3) Time domain solution

$$_s\psi_{\ell m}(r) = \int dr' \int_{-\infty}^{\infty} d\omega \, _sG_{\ell m\omega}(r, r') \int_{-\infty}^{\infty} dt' e^{i\omega(t'-t)} \, _sT_{\ell m}(t', r'),$$

This means performing these three integrals!

3) Time domain solution

In the bound case we have specific frequencies, hence the integration is trivial. In the unbound we have to perform the integrations explicitly!

We rewrite the PN-expanded Green function as

3) Time domain solution

In the bound case we have specific frequencies, hence the integration is trivial. In the unbound we have to perform the integrations explicitly!

We rewrite the PN-expanded Green function as

$$G_{lm\omega}(r,r') = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} G_{lm}^{(j,k)}(r,r')\omega^j \log^k(-i\omega)$$

$$= \sum_{j=0}^{\infty}\left\{ G_{lm}^{(j,0)}(r,r')\omega^j + \sum_{k=1}^{\infty} G_{lm}^{(j,k)}(r,r')\omega^j \log^k(-i\omega) \right\}$$

$$= G_{lm}^{\mathrm{loc}}(r,r') + G_{lm}^{\mathrm{non-loc}}(r,r')$$

3) Time domain solution: local terms

The local terms are straightforward

$$_s\psi_{\ell m}^{\mathrm{L}} = \frac{q}{b} \sum_j i^j v^j \frac{\partial^j}{\partial \bar{t}^j} \int d\bar{r}' \, _sG_{\ell m}^{(j,0)}(\bar{r}, \bar{r}') \, _sT_{\ell m}(\bar{t}, \bar{t}')$$

3) Time domain solution: non-local terms

The non-local terms required great care and we will discuss only
$$k = 1$$

We define $y = \bar{t}' - \bar{t}$ have

$$_s\psi_{\ell m}^{\mathrm{NL}} = q \sum_j \frac{v^j}{2\pi} \int d\bar{r}' \, G_{\ell m}^{(j,1)}(\bar{r}, \bar{r}') \int_{-\infty}^{\infty} dy \, {}_sT_{\ell m}(\bar{t} + y, \bar{r}') \int_{-\infty}^{\infty} d\bar{\omega} \, \bar{\omega}^j \log(-i\bar{\omega}) e^{i\bar{\omega}y}$$

There are multiple ways for actually doing the $\omega-$integral, but the infrared divergence for $\omega \to 0$ must be suitably regularized.[1]

1. This divergence was already identified in the paper by C. Whittall and L. Barack via numerical methods.

3) Time domain solution: non-local terms

By using dimensional regularisation we are able to regularize
the integrals and we get

$$
{}_s\psi_{lm}^{\rm NL} = -\frac{q}{b}\lim_{\epsilon\to 0}\sum_{j\geq 0}i^j v^j\int d\bar{r}'\Bigg\{\left(\gamma_E + \log\epsilon\right)\frac{d^j}{d\bar{t}^j}\left[G_{lm}^{(j,1)}(\bar{r},\bar{r}')\,{}_sT_{lm}(\bar{t},\bar{r}')\right]
$$

$$
+\int_{\epsilon}^{+\infty}\frac{dy}{y}\frac{d^j}{d\bar{t}'^j}\left[G_{lm}^{(j,1)}(\bar{r},\bar{r}')\,{}_sT_{lm}(\bar{t}',\bar{r}')\right]\Bigg|_{\bar{t}'=\bar{t}-y}\Bigg\}
$$

PolyLog structure comes from here!

4) Summation over (l, m) and regularisation at particle position

This step uses standard procedure for summation of spherical harmonics.

We used $l = 0...4$ from MST and $l > 4$ by using a PN ansatz.

$$\psi_{\text{reg}}(t) = \sum_l \left\{ \sum_m \left(\psi_{lm}^{\text{L}}(t, r) + \psi_{lm}^{\text{NL}}(t, r) \right) Y_{lm}(\theta, \phi) \Big|_{(r,\theta,\phi) \rightarrow (r(t), \pi/2, \phi(t))} - B(t) \right\}$$

Results

We firstly need the forces, that can be split in *conservative* and *dissipative*

$$F_\alpha^{\mathrm{cons}}(r) = \frac{1}{2}[F_\alpha(r) - F_\alpha(r)] \qquad F_\alpha^{\mathrm{diss}}(r) = \frac{1}{2}[F_\alpha(r) + F_\alpha(r)]$$

where we just need $\alpha = t, \phi$.

We solved the problem up to 5PM-4.5PN.

Dissipative Scattering Angle

$$\varepsilon = q^2/\mu M$$

$$\delta\chi^{\mathrm{diss}}_{\mathrm{1PM}} = \delta\chi^{\mathrm{diss}}_{\mathrm{2PM}} = 0,$$

$$\delta\chi^{\mathrm{diss}}_{\mathrm{3PM}} = \frac{\varepsilon M^3}{b^3 v^3}\left[\frac{2}{3} + \frac{5v^2}{3} + \frac{19v^4}{12} + \frac{25v^6}{24} + O(v^7)\right]$$

$$\delta\chi^{\mathrm{diss}}_{\mathrm{4PM}} = \frac{\pi\varepsilon M^4}{b^4 v^5}\left[\frac{1}{2} + \frac{10v^2}{3} + \frac{89v^4}{48} + \frac{53v^6}{80} + O(v^7)\right],$$

$$\delta\chi^{\mathrm{diss}}_{\mathrm{5PM}} = \frac{4\varepsilon M^5}{3b^5 v^7}\left[1 + \left(\frac{91}{6} + \frac{3\pi^2}{4}\right)v^2 - 4v^3 + \left(\frac{1357}{120} + \frac{39\pi^2}{32}\right)v^4 - \frac{14v^5}{15} + \left(\frac{16103}{560} - \frac{45\pi^2}{16}\right)v^6 + O(v^7)\right]$$

Conservative Scattering Angle

$$\varepsilon = q^2/\mu M$$

$$\delta\chi_{2PM}^{cons} = -\frac{\pi\varepsilon M^2}{4b^2}$$

$$\delta\chi_{3PM}^{cons} = -\frac{\varepsilon M^3}{b^3 v^2}\left[4 + \frac{2v^2}{3} + \frac{5v^4}{6} + O(v^5)\right]$$

$$\delta\chi_{4PM}^{cons} = -\frac{\pi\varepsilon M^4}{b^4 v^4}\left\{\frac{9}{4} + v^2\left[\frac{91}{24} + \frac{21\pi^2}{128} + \log(v/2)\right]\right.$$

$$\left. + v^4\left[\frac{493}{480} + \frac{4335\pi^2}{8192} - \frac{3\log(b/M)}{2} + 2\log\left(\frac{v}{2}\right)\right] + O(v^5)\right\}$$

$$\delta\chi_{5PM}^{cons} = -\frac{16\varepsilon M^5}{3b^5 v^6}\left\{1 + v^2\left[\frac{47}{6} + \frac{21\pi^2}{64} + 2\log(2v)\right]\right.$$

$$\left. - v^4\left[\frac{797}{72} - \frac{205\pi^2}{128} - \frac{43\log(2)}{3} + 4\log\left(\frac{b}{M}\right) - \frac{19\log(v)}{3}\right] + \frac{19v^5}{15} + O(v^6)\right\}$$

Conclusion

- Our results are the first attempts to systematically address analytically the scattering problem in Self Force;

- We are now computing fluxes in the gravitational scattering, but pushing the PN expansion means performing the integrals of $\log^k(-i\omega)$ with $k > 1$;

- The *local* gravitational flux at infinity reproduces results from PN literature;

- 5 and higher PM are not complete because we lack the 2SF contributions;

(Very Reduced) Bibliography

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Thanks for the attention!

Questions?

If you are interested, more details can be found in our paper!

Backup

Results

$$c_1 = \frac{1}{6} \qquad\qquad c_2(\overline{\mu}) = -\frac{11}{6} - 4\gamma - 2\log(2M^2\overline{\mu}^2)$$

c_1 agrees with numerical fits but $c_2(\overline{\mu})$ is rather different, but the fit is quite unstable.

Thanks to Oliver Long we were able to test our results against the numerics.

Analytics vs Numerics

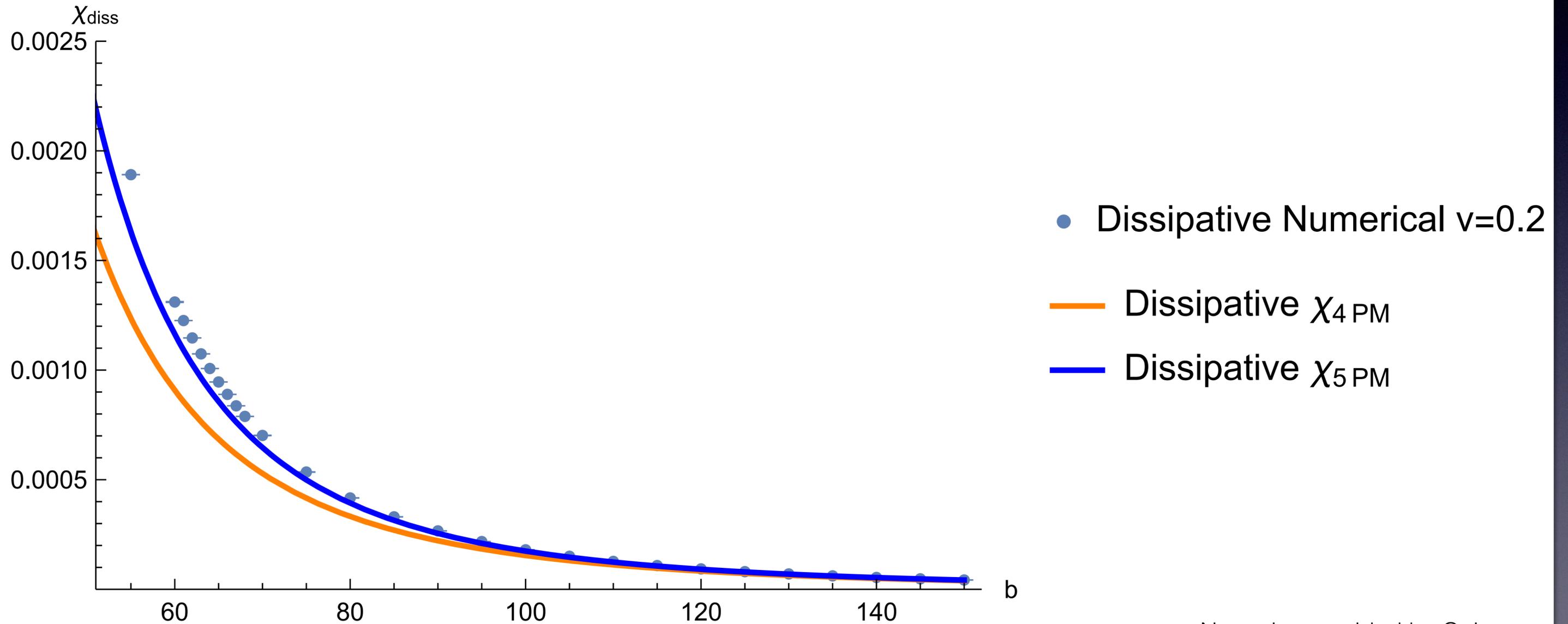We compared his numerical results for $\nu = 0.2$ and our analytical expressions.

We compared both the dissipative and conservative scattering angle, and then we compared the residue, defined as

$$\chi_{\text{res}} = |\chi^{\text{analytical}} - \chi^{\text{numerical}}|$$
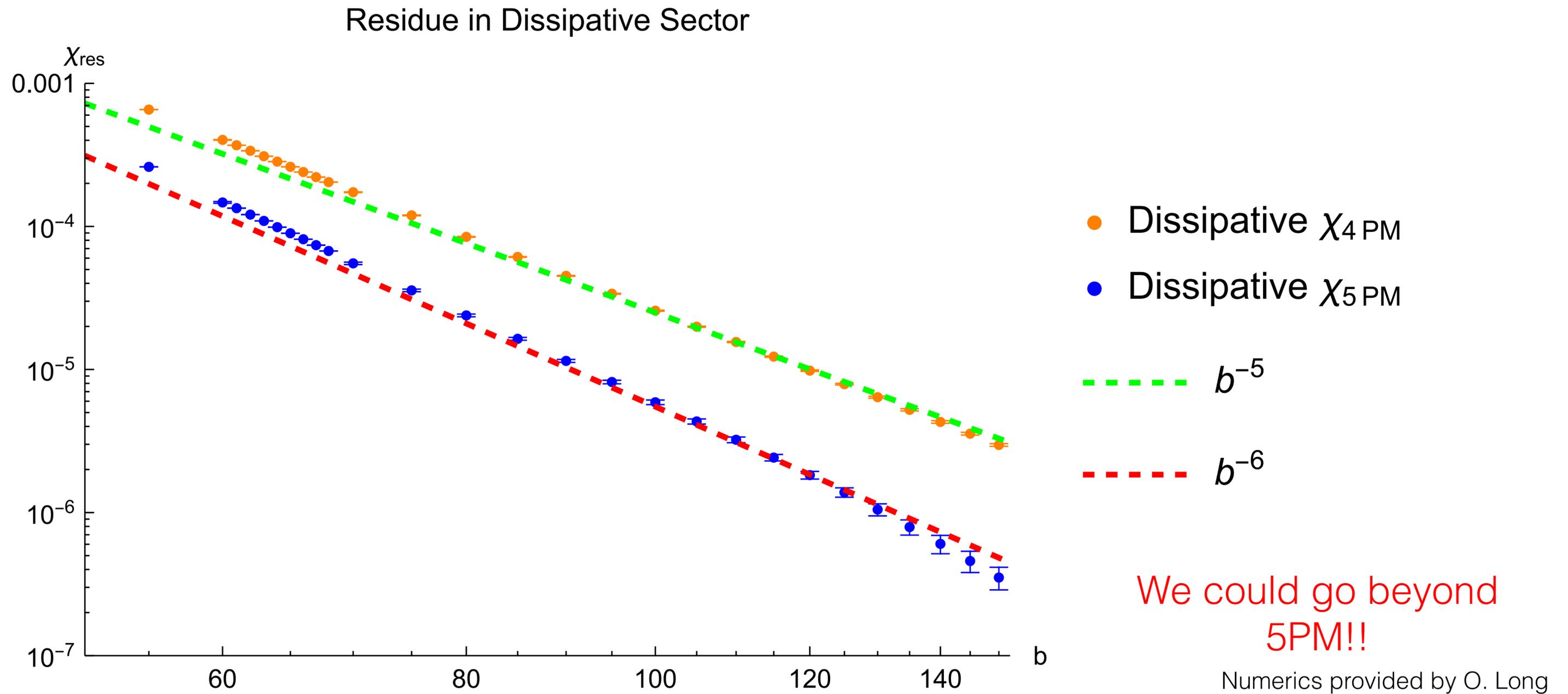
at both 4PM and 5PM!

Dissipative Sector

Comparison Dissipative NUM vs ANALYTICAL
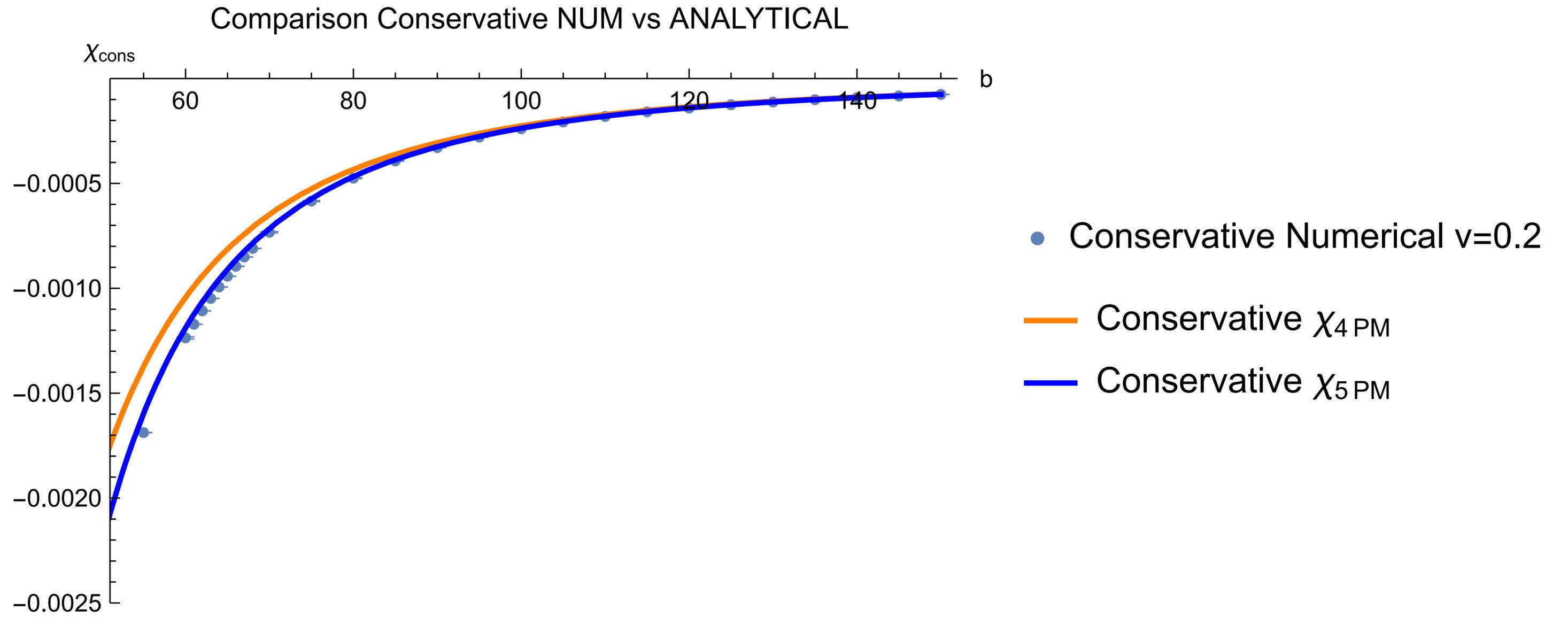
χ_{diss}

- Dissipative Numerical v=0.2
- Dissipative $\chi_{4\,\text{PM}}$
- Dissipative $\chi_{5\,\text{PM}}$

b

Numerics provided by O. Long

Dissipative Sector

Residue in Dissipative Sector

Numerics provided by O. Long

Conservative Sector

Comparison Conservative NUM vs ANALYTICAL

- Conservative Numerical v=0.2
- Conservative $\chi_{4\,\mathrm{PM}}$
- Conservative $\chi_{5\,\mathrm{PM}}$

Numerics provided by O. Long

Conservative Sector

Residue in Conservative Sector

Conservative $\chi_{4\,\text{PM}}$

Conservative $\chi_{5\,\text{PM}}$

b^{-5}

b^{-6}

5PM is already hitting the numerical precision!
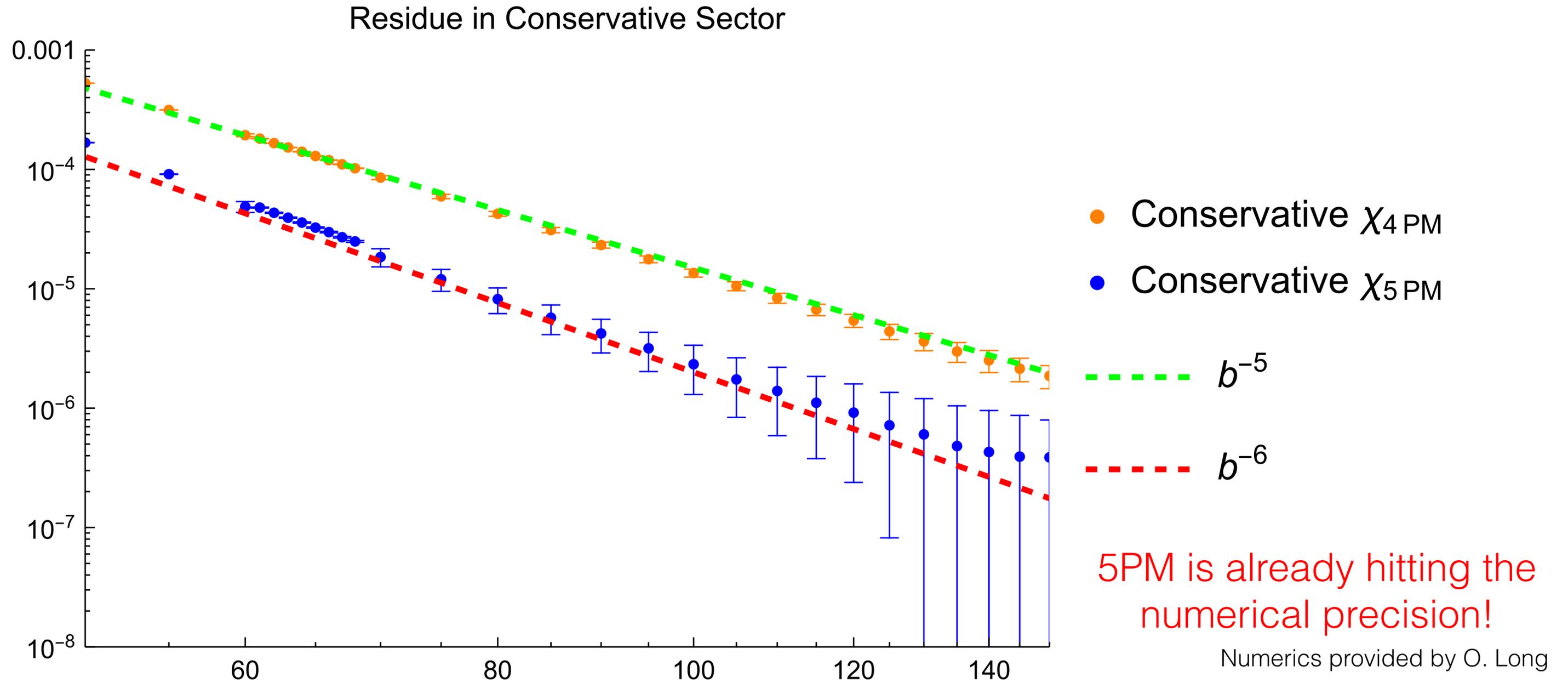
Numerics provided by O. Long

Comparisons with literature

Dissipative sector compared against Barack et al. [15] up to 4PM + Jones and Ruf [18] for the horizon flux at 3PM.

The conservative sector is trickier because it contains two undetermined parameters, i.e. Wilson coefficients, at 4PM, from [15].

$$\delta\chi_1^{\text{cons}} = 0,$$

$$\delta\chi_2^{\text{cons}} = -\frac{\pi}{4}G\frac{m_2^2}{b^2},$$

$$\delta\chi_3^{\text{cons}} = -\frac{4}{3}G^2\frac{\sigma(1+2\sigma^2)}{(\sigma^2-1)}\frac{m_2^3}{b^3},$$

$$\delta\chi_4^{\text{cons}} = \pi G^3\frac{3m_2^4}{8(\sigma^2-1)b^4}\left\{ -\left[4\mathcal{M}_4^{\text{t}}\log\left(\frac{\sqrt{\sigma^2-1}}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\text{rem}}\right]\right.$$

$$\left. + c_1(\sigma^2 - 5) + \left(c_2(\bar{\mu}) - \frac{31}{3} + 2\log\left[2b^2 e^{2\gamma_{\text{E}}}\bar{\mu}^2\right]\right)(\sigma^2-1)\right\}.$$

Results

$$c_1 = \frac{1}{6} \qquad\qquad c_2(\overline{\mu}) = -\frac{11}{6} - 4\gamma - 2\log(2M^2\overline{\mu}^2)$$

c_1 agrees with numerical fits but $c_2(\overline{\mu})$ is rather different, but the fit is quite unstable.

However…

Comparisons with numerics

Our values of c_1 and $c_2(\bar{\mu})$ were recently used in

Black hole scattering near the transition to plunge: Self-force and resummation of post-Minkowskian theory

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To construct a Post-Minkowskian resummation of the conservative scattering angle to compare against the numerics, showing the validity of our results.