

# QNMs of black holes encircled by a gravitating thin disk

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- 2 QNMs of a perturbed black hole
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Gravitational field of **static** and **axially symmetric** vacuum spacetimes is described by

$$ds^2 = -e^{2\nu} dt^2 + \rho^2 e^{-2\nu} d\phi^2 + e^{2\lambda-2\nu} (d\rho^2 + dz^2), \quad (1)$$

where  $t, \rho, \phi, z$  are the Weyl cylindrical coordinates and  $\nu(\rho, z), \lambda(\rho, z)$ .

Vacuum Einstein equations then

$$\Delta\nu = 0 \quad (2)$$

$$\lambda_{,\rho} = \rho(\nu_{,\rho}^2 - \nu_{,\phi}^2) \quad (3)$$

$$\lambda_{,z} = 2\rho\nu_{,\rho}\nu_{,z}, \quad (4)$$

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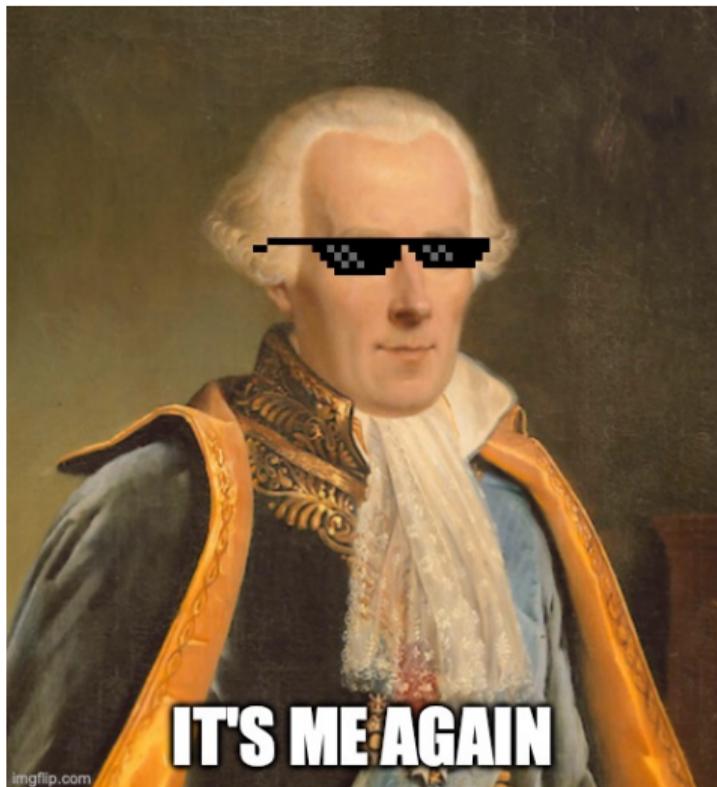
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$\nu$  is the counterpart of the Newtonian  
**gravitational potential**

But,  $\lambda$  is also present in GR

→ nonlinear (quadratic in  $\nu$ )

Consider **two** distinct **solutions**, described by  $\nu_1$ ,  $\nu_2$ , and  $\lambda_1$ ,  $\lambda_2$  respectively

Their common gravitational field is given by

$$\nu = \nu_1 + \nu_2, \quad (5)$$

$$\lambda = \lambda_1 + \lambda_2 + \lambda_{int}, \quad (6)$$

where  $\lambda_{int}$  satisfies

$$\lambda_{int,\rho} = 2\rho(\nu_{Schw,\rho}\nu_{disk,\rho} - \nu_{Schw,z}\nu_{disk,z}), \quad (7)$$

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We wish to study a **Schwarzschild black hole** described by

$$\nu_{\text{Schw}} = \frac{1}{2} \ln \left( \frac{R_+ + R_- - 2M}{R_+ + R_- + 2M} \right), \quad (9)$$

$$\lambda_{\text{Schw}} = \frac{1}{2} \ln \left[ \frac{(R_+ + R_-)^2 - 4M^2}{4R_+R_-} \right], \quad (10)$$

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$$R_{\pm} = \sqrt{\rho^2 + (|z| \mp M)^2}, \quad (11)$$

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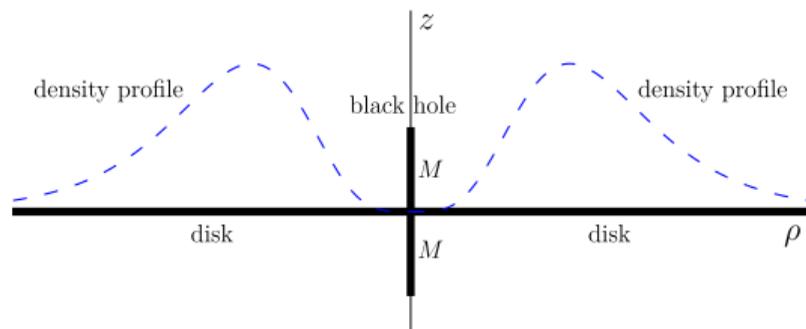
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## The SBH+disk model Kotlařík and Kofroň (2022):



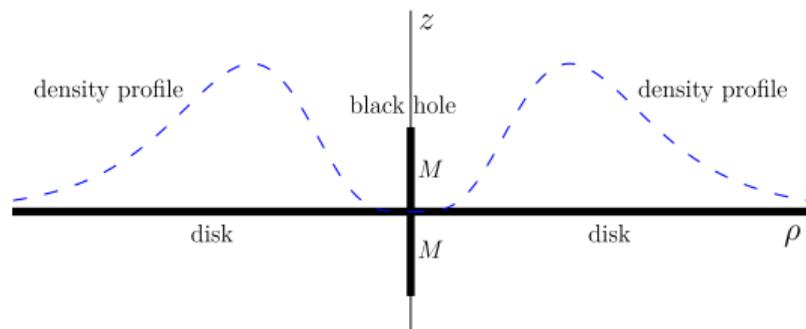
- a **family** of thin disks; potential first considered by Vogt and Letelier (2009)
- both metric functions  $\nu$ ,  $\lambda$  found analytically in **closed forms** (polynomials and square roots)

$$\text{Newtonian disk density} \propto \frac{\mathcal{M} b^{2m+1} \rho^{2n}}{(\rho^2 + b^2)^{m+n+3/2}}, \quad m, n \in \mathbb{N}_0, \quad (12)$$

where for every  $m, n$ , there are 2 free parameters:

- $\mathcal{M}$  is the total mass of the disk,
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Two simple physical interpretations

- 1 ideal fluid with surface density  $\sigma$  and azimuthal pressure  $P$  (set of solid hoops)
- 2 two identical counter-orbiting dust streams with surface densities  $(\sigma_+ = \sigma_- \equiv \sigma/2)$  following circular geodesics

Both characteristics  $\sigma$  and  $P$  are encoded in the jump of the normal derivative of the gravitational potential

$$\sigma + P = \frac{\nu_{,z}(z = 0^+)}{2\pi} e^{\nu-\lambda}, \quad P = \frac{\nu_{,z}(z = 0^+)}{2\pi} \rho \nu_{,\rho} e^{\nu-\lambda}.$$

For a wide range of parameters, the disk in SBH+disk modes satisfies all energy conditions.

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ring a black hole

→ field dynamics  $\Rightarrow$  the whole system is  
dissipative

→ exponential decay of waves

→ **quasinormal modes**



Consider a simpler task: perturbations of **massless scalar field**, which are governed by the Klein-Gordon equation

$$\square\psi = 0. \quad (13)$$

The wave equation is separable on the Schwarzschild background in the Schwarzschild coordinates  $(r, \theta)$ :

$$\rho = \sqrt{r(r-2M)} \sin\theta, \quad z = (r-M) \cos\theta. \quad (14)$$

In particular, the ansatz

$$\psi = \frac{\psi_{\omega\ell m_z}(r)}{r} Y_{\ell m_z}(\theta, \phi) e^{-i\omega t}, \quad (15)$$

where  $Y_{\ell m_z}$  are the spherical harmonics, leads to the famous Regge-Wheeler-type equation for a scalar field.

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However, the presence of the (axially symmetric) disc breaks this convenient property.

Cano, Fransen, and Hertog (2020): “almost separable” systems, projection method.

Chen, Chiang, and Tsao (2022): Schwarzschild black hole perturbed by a small axially symmetric deformation.

Suppose  $\epsilon \equiv \mathcal{M}/M \ll 1$ , the wave equation again leads to the Regge-Wheeler form

$$\frac{d^2 \Psi_{\omega l m_z}}{dr_*^2} + \left[ \omega^2 - V_{\text{eff}}(r) \right] \Psi_{\omega l m_z} = 0, \quad (16)$$

with the appropriately modified tortoise coordinate  $r_*$ , and

$$V_{\text{eff}}(r) = V_{\text{eff}}^{\text{Sch}}(r) + \epsilon V_{\text{eff}}^{\text{corr}}(r). \quad (17)$$

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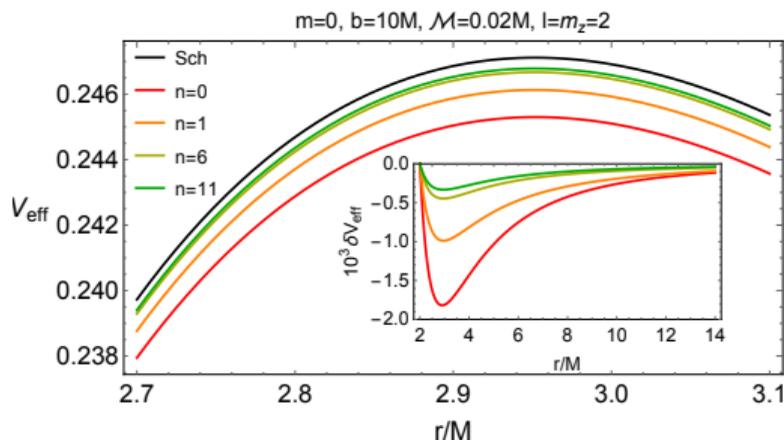
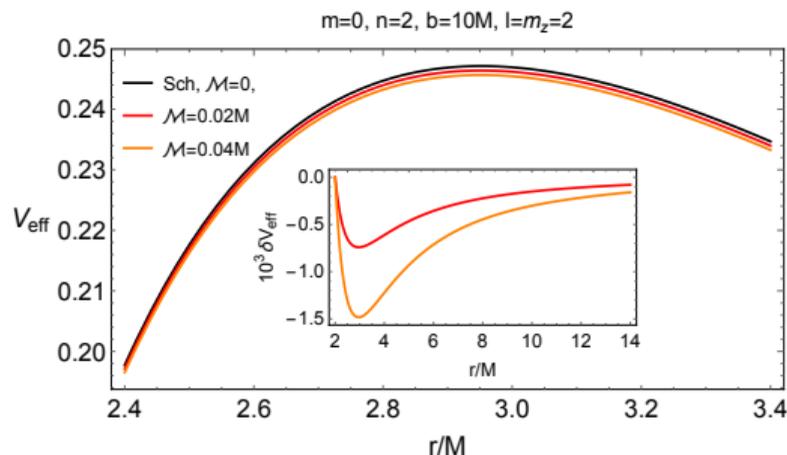
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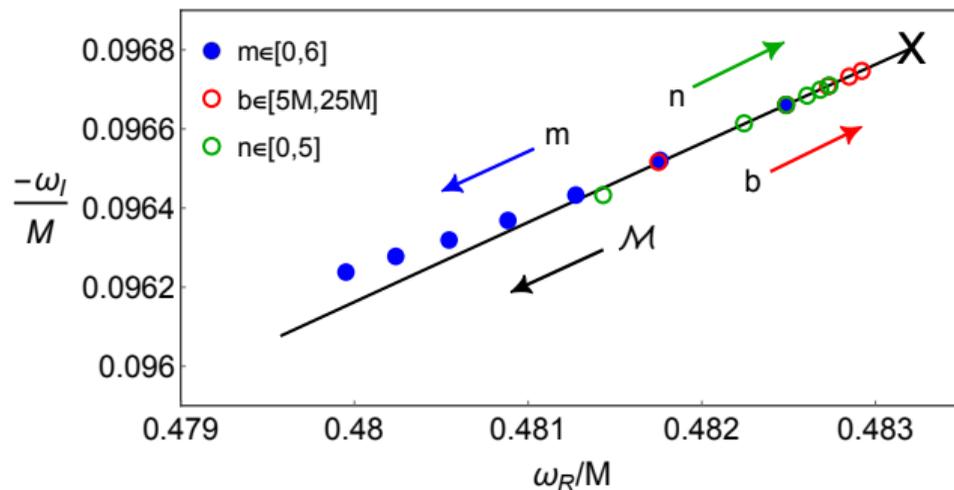
The disk is described by 4 parameters: 2 positive real  $\mathcal{M}, b$ , and a pair  $(m, n)$ .

$$\text{Newtonian disk density} \propto \frac{\mathcal{M} b^{2m+1} \rho^{2n}}{(\rho^2 + b^2)^{m+n+3/2}} \quad (18)$$



# Universal behaviour?

QNM frequencies computed by 3-order WKB method (AIM for consistency check):



Newtonian disk density

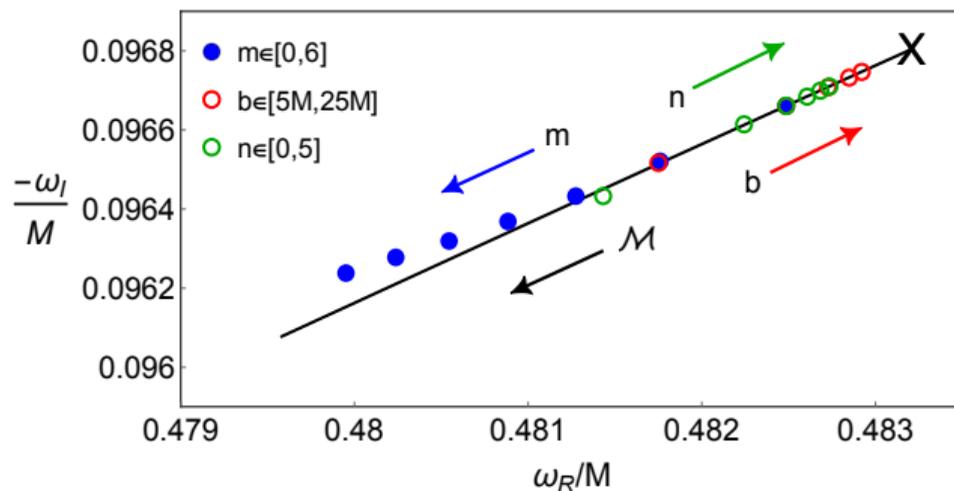
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Remarks:

- BH immersed in spherically symmetric matter exhibits similar behavior (Cardoso et al., 2022; Konoplya, 2021)
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## Brief summary

- QNMs of a scalar field propagating in the SBH+disk background
- disk flattens the effective potential
- QNMs seems to follow a **universal relation**
- can we **disentangle** environmental effects from those induced by alternative theories of gravity?

-  Kotlařík, P. and D. Kofroň (2022). “Black Hole Encircled by a Thin Disk: Fully Relativistic Solution”. *ApJ* 941(1), p. 25.
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Konoplya, R. A. (2021). “Black Holes in Galactic Centers: Quasinormal Ringing, Grey-Body Factors and Unruh Temperature”. *Physics Letters B* 823, p. 136734.

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