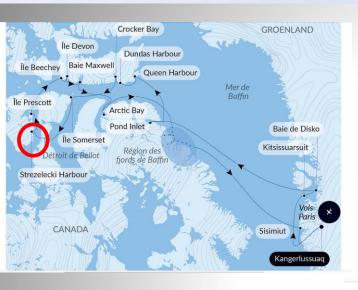


Preamble Crocker Bay GROENLAND Dundas Harbour Île Beechey Baie Maxwell Queen Harbour Mer de **Arctic Circle Expedition** "North-West Passage" Pond Inlet Baie de Disko Kitsissuarsuit Détroit de Bellot 24/08-14/09 Strezelecki Harbour CANADA Kangerlussuaq

Preamble



In the news: 7 September 2025

Dutch ship grounds in Canadian Arctic

Rescue Mission Underway for Freighter Stuck Arctic Sea Route

Published Sep 08, 2025 at 4:24 PM EDT

Dutch Freighter With 16 People Onboard Runs Aground In Canada's Northwest Passage

Freighter incident highlights Arctic challenges

3 Lectures

Lecture 1:

- Introduction to neutrinos
- History of neutrino physics and open questions.
- Neutrino oscillation physics (Introduction)

Lecture 2:

- Neutrino oscillation physics (Experimental)
- Neutrino properties
- Searches for the 4th generation

Lecture 3:

- Astrophysical neutrinos
- Neutrinos at the LHC
- Next generation of neutrino experiments

Quarks





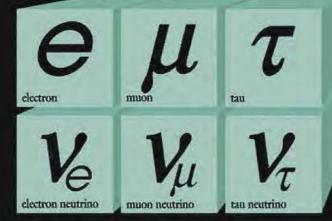
Forces











Leptons

Neutrinos

Neutrinos are still mysterious particles

- Have only (left handed) weak interactions
- Are mass-less in the (minimal) SM .. untill 1998
- Are the only neutral fermions in the SM
- Could be Majorana or Dirac fermions
- Neutrinos are produced everywhere
 - Solar neutrinos
 - Atmospheric neutrinos
 - Neutrinos from supernova explosions
 - Primordial neutrinos from the Big Bang
 - Nuclear reactor created neutrinos
 - Accelerator created neutrinos
 - Geoneutrinos, Radioactive decays, even from your body...

Neutrinos

Interaction of neutrinos with matter is very weak!





 $(1 \text{ MeV} = 10^6 \text{ eV})$

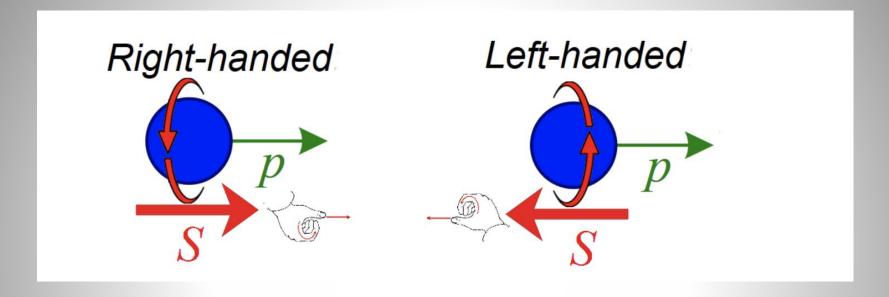
And all of us also emit ~a few M V/day through

$$^{40}{
m K} \rightarrow ^{40}{
m Ca} + e^+ + \bar{\nu}_e$$

Note:

At any point in time there are ~100,000 solar neutrinos in you (even 200x more when accounting for cosmological v's)

Left-Right Handed Particles



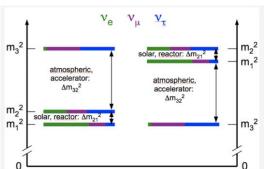
All particles have left- and right-handed versions Neutrinos are always left-handed...

Only left handed particles interact in the weak force

Neutrinos

Neutrino experiments today -> Open Questions!

- Neutrino mass values?
- Neutrino mass hierarchy? Normal or Inverted?
- CP violation in the lepton sector? Are neutrinos key the baryon asymmetry in the Universe?

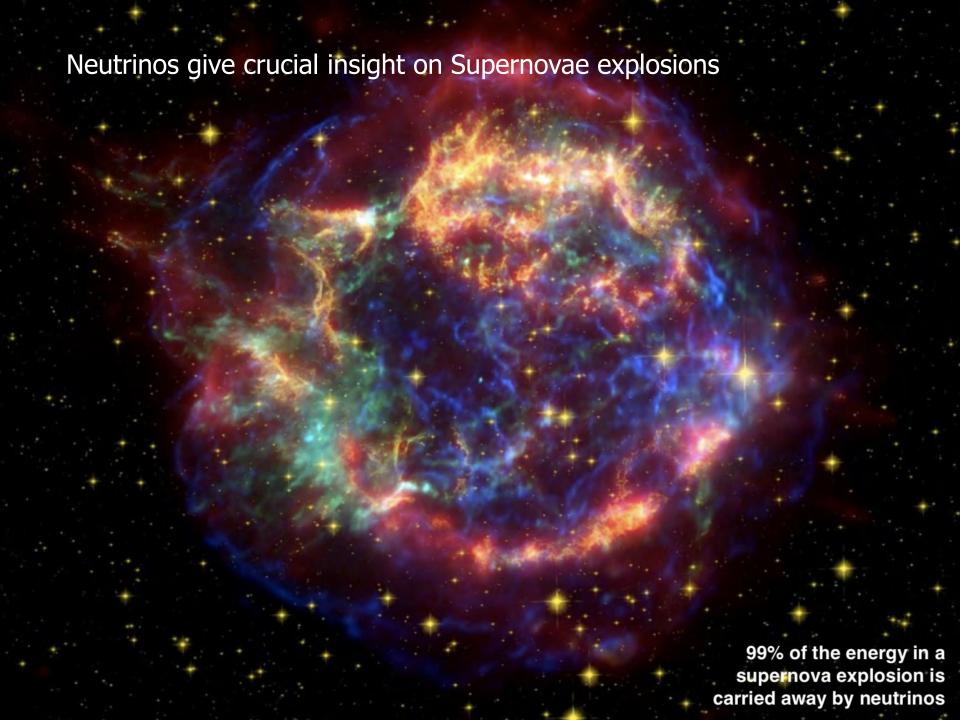


- Are neutrinos their own antiparticles? -> LNV processes
- Do right-handed/sterile/heavy neutrinos exist?
- Are there non-standard neutrino interactions?
- Neutrinos and Dark Matter?
- Testing of CPT...
- Neutrinos are Chameleons: They can change flavour!!



Neutrinos are an essential part of our Universe and our very existence, and can provide answers to some of the key fundamental questions today





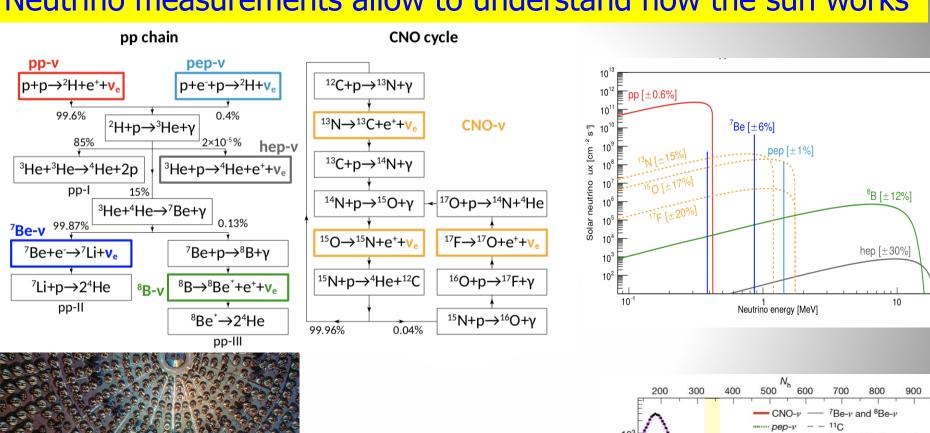
Neutrinos allow us to to look into the heart of the sun



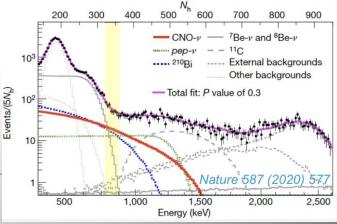
(with a flux of ~1011/cm2/sec at the Earth)

Solar Neutrinos

Neutrino measurements allow to understand how the sun works

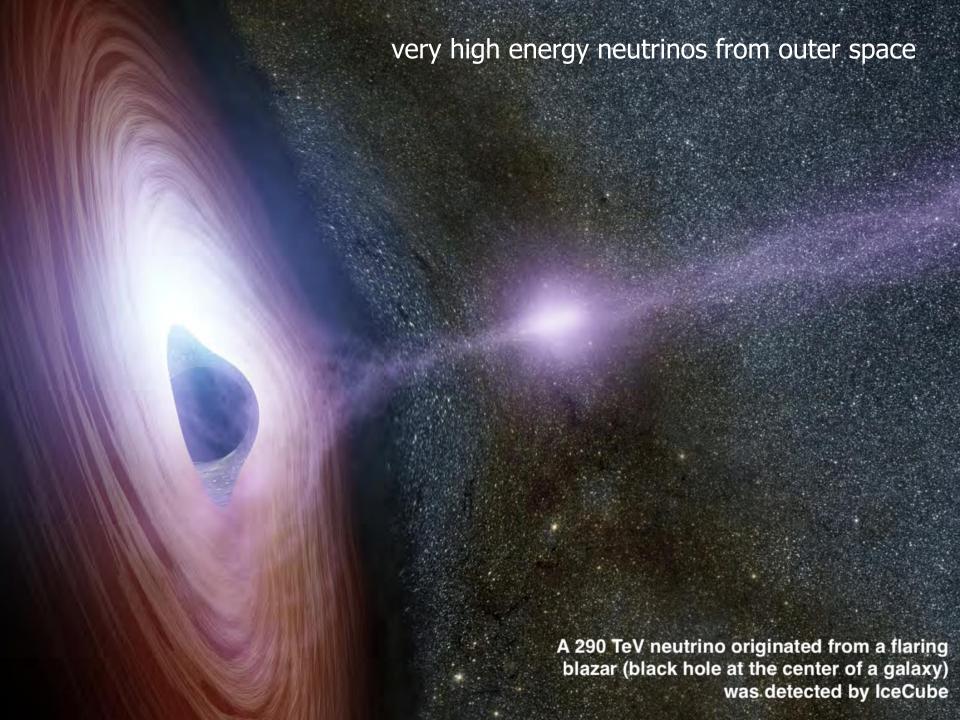


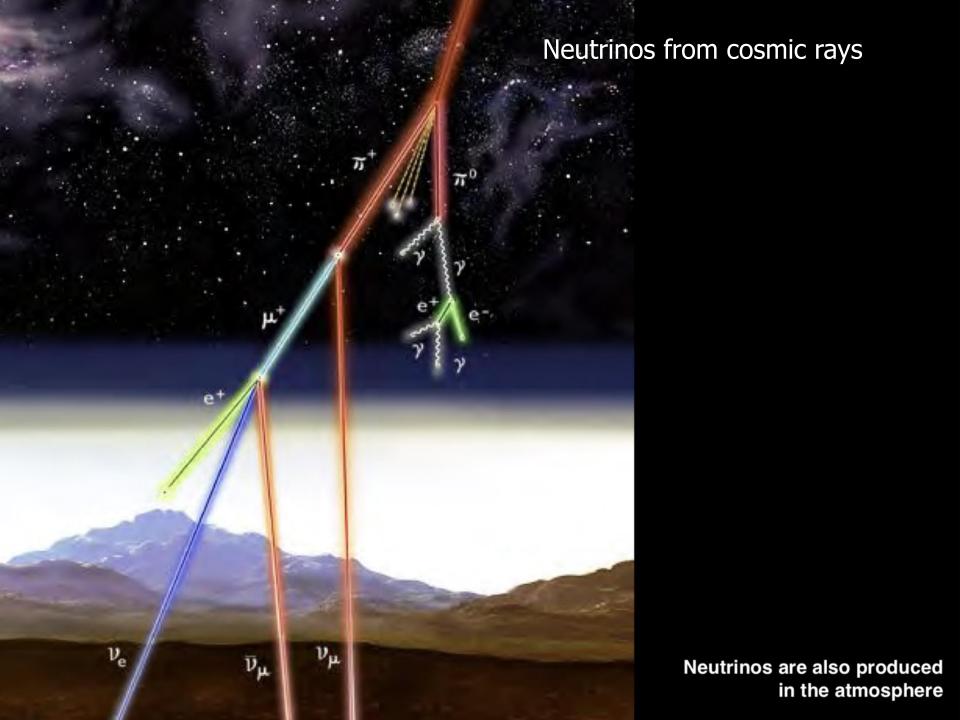
2020: Borexino measured the CNO cycle -> Nature 587 (2020) 577





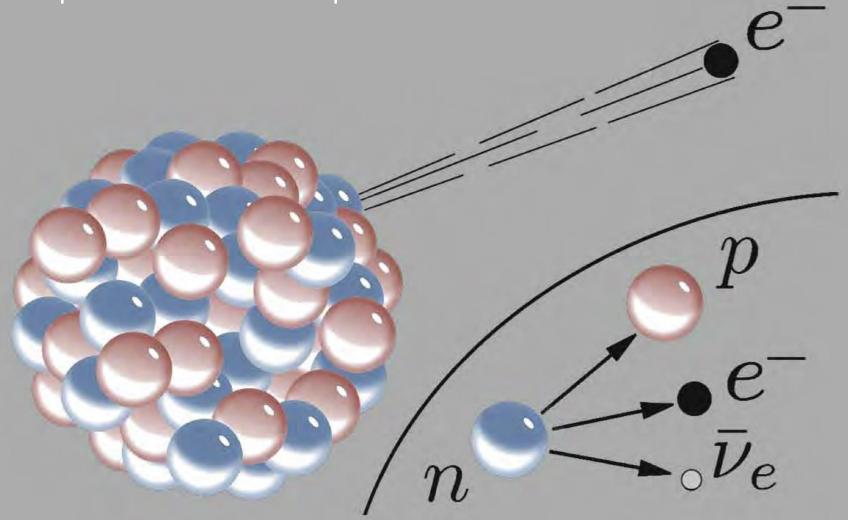
January 2022 issue



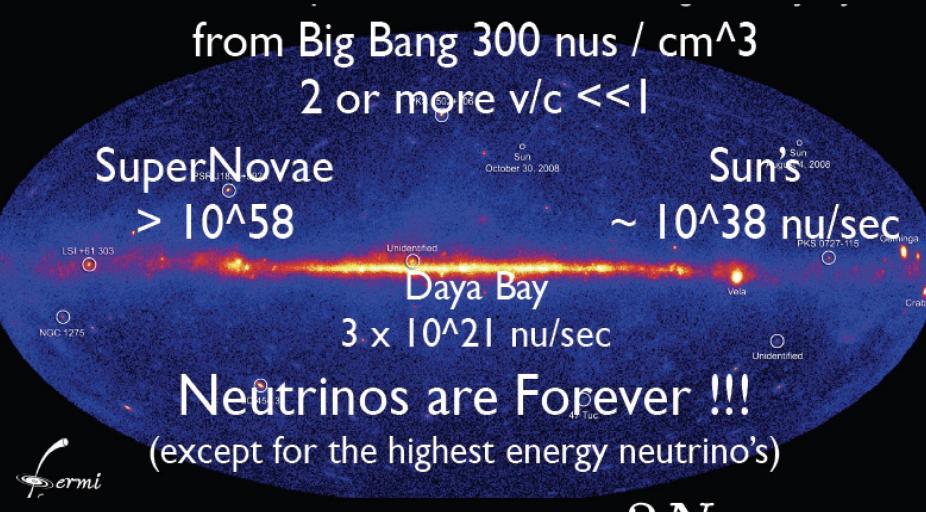




Radioactive beta-decay
The process that led to the postulation of the neutrino



Neutrinos are Everywhere!

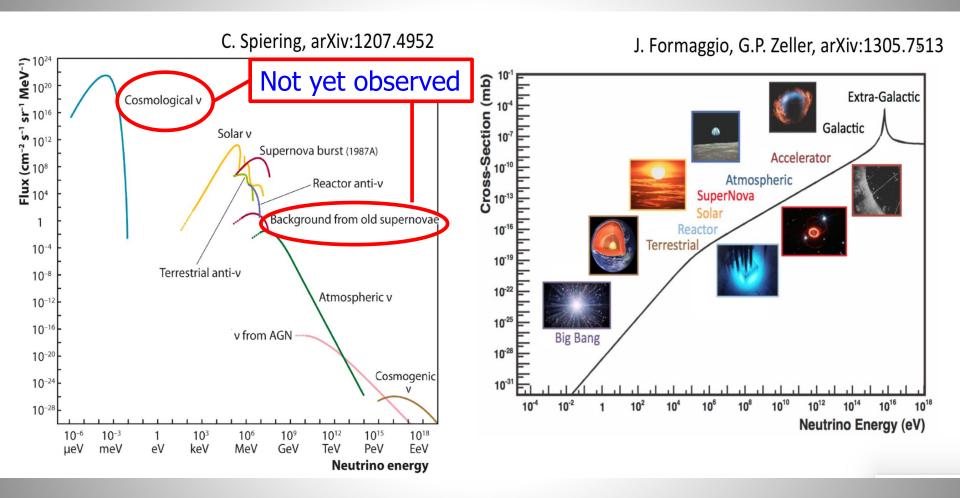


therefore in the Universe:

 $\frac{\partial N_{\nu}}{\partial t} > 0$

Neutrinos are the most abundant matter particles in our Universe

Neutrino Sources, Flux and Cross Sections

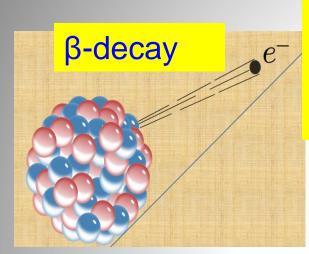


Cosmological and background from old supernovae neutrinos not yet observed!



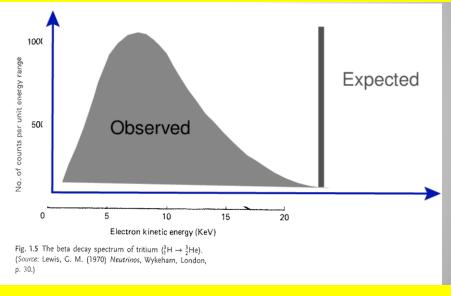


Neutrinos were introduced in 1930!



If the process is A -> B + electron, the energy of the electron should be at a fixed value. This is not the case! Energy-momentum not conserved in Beta-decays?





Pauli proposed instead the process:

$$n \rightarrow p^+ + e^-$$

But he believed we could never detect this particle!!

Neutrinos are known to us since 1930!

Physikalisches Institut der Eidg. Technischen Hochschule Zürich

Zirich, 4. Des. 1930 Cloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst ansuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen versweifelten Ausweg verfallen um den "Wechselsats" (1) der Statistik und den Energiesats zu retten. Mamlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und won Lichtquanten musserdem noch dadurch unterscheiden, dass sie mist mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen fineste von derselben Orossenordnung wie die Elektronenmasse sein und jedenfalls nicht grosser als 0,01 Protonemasse .- Das kontimuierliche Spektrum ware denn verständlich unter der Annahme, dass beim beta-Zerfall mit dem blektron jeweils noch ein Neutron emittiert derart, dass die Summe der Energien von Neutron und Elektron konstant 1st.

Pauli Letter Collection, CERN

Pauli did not believe energy-momentum conservation was violated He proposed a desperate way out: a new 'invisible' particle He called it the neutron.

He also stayed away from the conference because of a ball in Zurich...

Neutrinos are known to us since 1934!

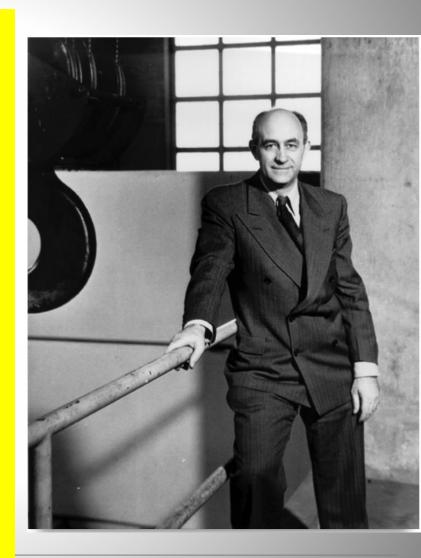
1934

Enrico Fermi, father of the world's first nuclear reactor, coined the term "neutrino" which is Italian for "little neutral"

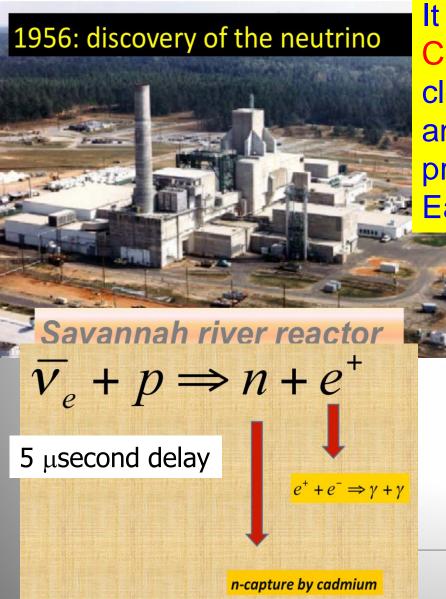
He proposed a theory for β -decay including the neutrino, a first formulation of the weak force...

This is one of the keystone papers for the later development of the Standard Model

Funny enough his paper got refused by Nature magazine (criticism: nothing practical in this paper)



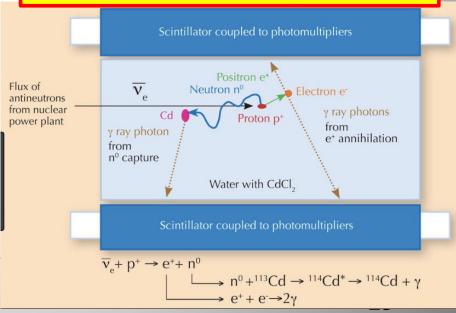
The Discovery of the Neutrino



It took 26 years to detect this particle. Cowan and Reines put a detector close to the reactor in South Carolina and observed the inverse beta decay process (few events/hour)

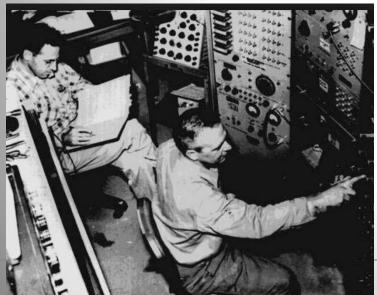
Early reactors gave 10¹⁹ neutrinos/sec

The neutrino really exists!



The Discovery of the Neutrino



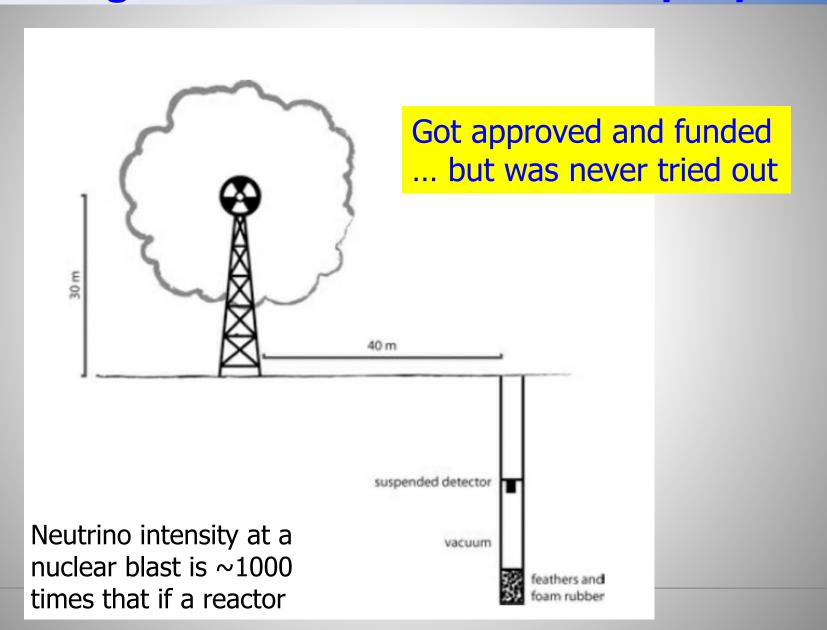


This was however not the first idea of Cowan and Reines.

They had originally proposed (and got approved for) putting an experiment close to an even more intense source of neutrinos nml 100m distance from an atomic blast!

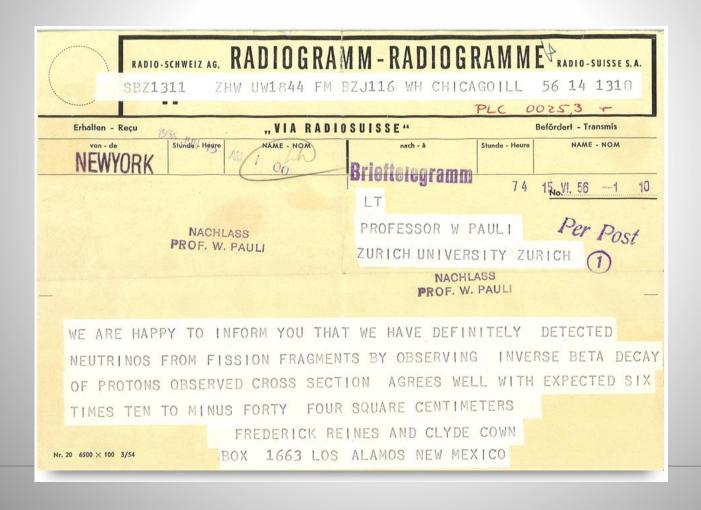
They abandoned that idea when the realized there were certain 'practical problems' for the detector... (to survive)

The "Original" Cowan and Reines proposal



The Discovery of the Neutrino

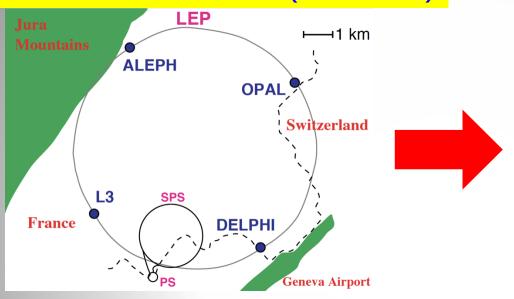
1956: the first experimental evidence from project "Poltergeist", informing Wolfgang Pauli..

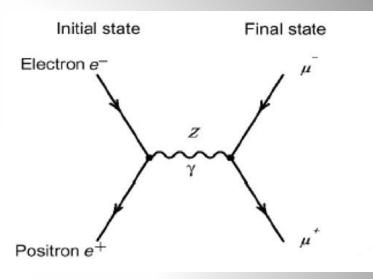


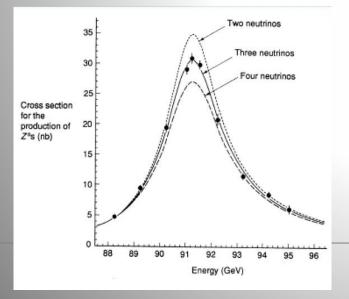
How Many Different Neutrinos?

LEP e+e- collider at CERN (1988-2000)

Detailed study of the Z-boson









The width of the Z-boson gives the number of neutrinos

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_l + \frac{N_v}{v}\Gamma_v$$
$$N_v = 2.99 \pm 0.02$$

LEP: three active neutrinos with mass<45 GeV

More Neutrino Personalities

1937: Ettore Majorana
He postulated that neutrinos could be their
own antiparticles. This special class of particles
came to bear his name: Majorana particles

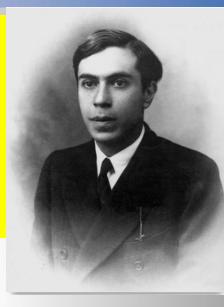
Majorana disappeared in 1938 on a boat trip from Sicily



He hypothesized that neutrinos may oscillate, or change from one type to another and would go on to develop that theory over the years as more flavors were discovered.

He also predicted that supernovae, the giant explosion of a dying star, would release an enormous amount of energy in the form of neutrinos

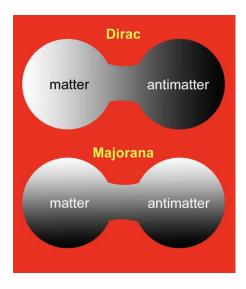
Pontecorvo disappeared ... to the east block in 1950

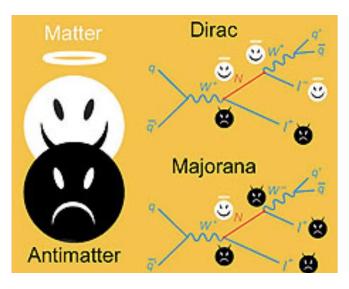




Majorana Neutrinos

- A Majorana fermion is a fermion that is its own antiparticle
 A Dirac fermion particle and antiparticles are not the same
- Fermions with electric charge (ie all fermions except neutrinos) are by definition Dirac fermions
- Neutrinos COULD be Majorana Fermions, but not demonstrated yet -> The goal of neutrinoless double beta decay experiments
- Neutrino mass -> allows for Majorana mass terms





Left Handed Neutrinos

1957: Goldhaber, Grodzins and Synar discovered that neutrinos emitted from nuclei had the peculiar property that their spin vector pointed in the opposite direction to its motion. Using Europium they found only left handed and never right handed neutrinos were emitted in weak decays.

$$h_{\mathcal{V}} := \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} = -1$$



Maurice Goldhaber

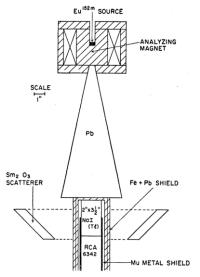


Fig. 1. Experimental arrangement for analyzing circular polarization of resonant scattered γ -rays. Weight of Sm₂O₄ scatterer: 1850 grams.

$$e^- + ^{152} \text{Eu} \longrightarrow ^{152} \text{sm}^* + \frac{\nu}{\gamma}$$

$$\downarrow \qquad \qquad \downarrow^{152} \text{sm} + \frac{\gamma}{\gamma}$$

Helicity of Neutrinos*

M. GOLDHABER, L. GRODZINS, AND A. W. SUNYAR Brookhaven National Laboratory, Upton, New York (Received December 11, 1957)

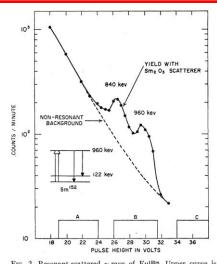
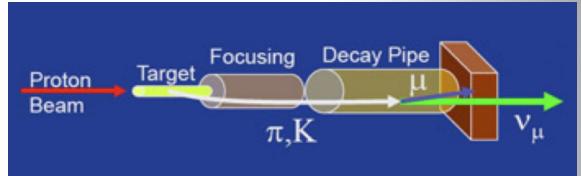


Fig. 2. Resonant-scattered γ rays of Eu^{150m}. Upper curve is taken with arrangement shown in Fig. 1 with unmagnetized iron. Lower curve shows nonresonant background (including natural background).

Neutrinos in the 1960s

1962: Lederman, Schwartz and Steinberger discovered the existence of second type of neutrino at the AGS in Brookhaven: the muon neutrino



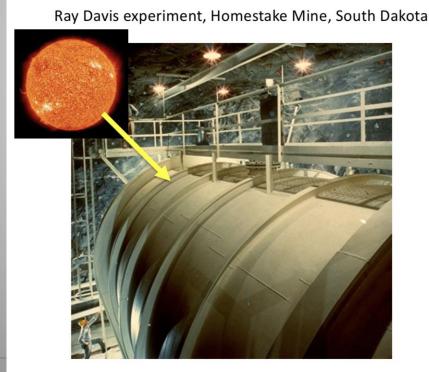
1968: Davis and Bahcall and the solar neutrino problem. Only 1/3 of the expected (electron) neutrino rate was observed. What was wrong?



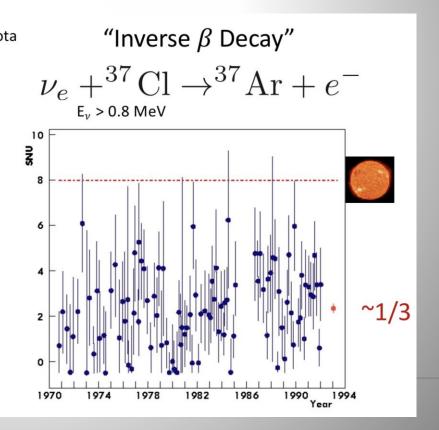
Davis and Bahcall



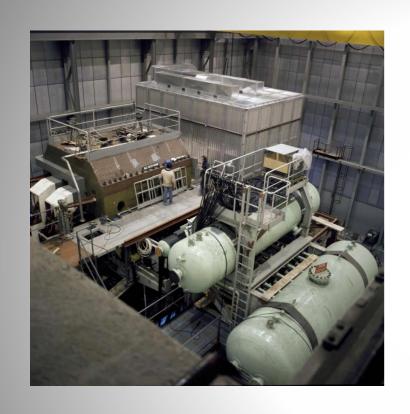
- Filter out argon and search for ³⁷Ar decay
- Detecting ~5 atoms of ³⁷Ar per day in 390,000 litres of C₂Cl₄

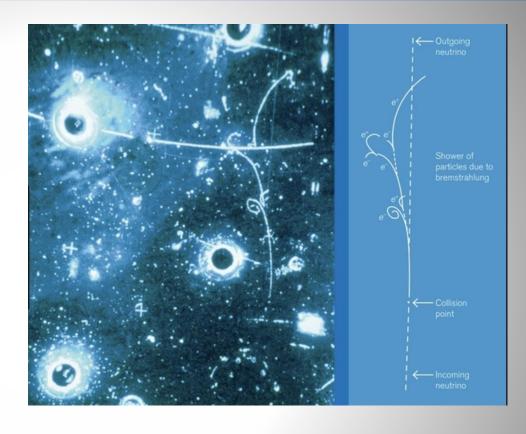


Filled with 390,000 litres of cleaning fluid (C₂Cl₄)



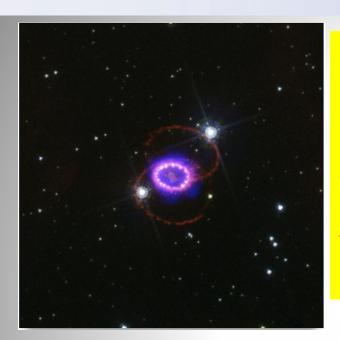
Neutrinos in the 1970s





1973: Discovery of the "neutral currents" as predicted from the Electroweak Theory: neutrino + electron -> neutrino + electron A triumph for the emerging Standard Model!

Neutrinos in the 1980s



1987: A supernova, a dying star, exploded in the Large Magellanic Cloud. Most of the energy is released as neutrinos. The Kamiokande and IMB experiments —both large experiments conceived to detect proton decays— saw a dozen of neutrino events during the burst of O(10) seconds. The neutrinos arrive at the earth before the light does (and could trigger an SN observation)

1987: Kamiokande (Japan) and IMB (US) detect atmospheric neutrinos. Echoing the solar neutrino problem: the experiment found a smaller ratio of muon neutrinos to electron neutrinos than expected. This became the atmospheric neutrino anomaly



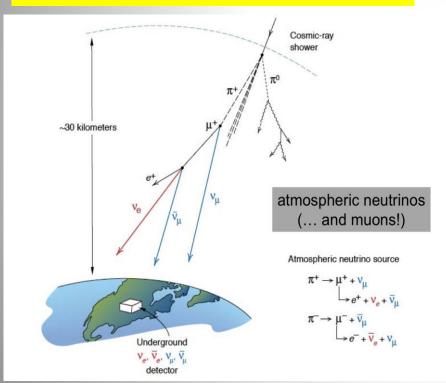
Neutrino Anomalies at the Time

- Solar neutrinos
 - Only about 1/3 of expected neutrino flux observed (electron neutrinos)
 - Depends on uncertainties of modelling of the Sun, detector effects?
- Atmospheric neutrinos
 - Muon neutrino disappearance increases with distance traveled
 - Direct evidence for neutrino disappearance
- What happens to the neutrinos?
 - Perhaps the neutrinos are decaying?
 - Need a mechanism for flavour change and a complete set of measurement for all flavours

Atmospheric Neutrinos

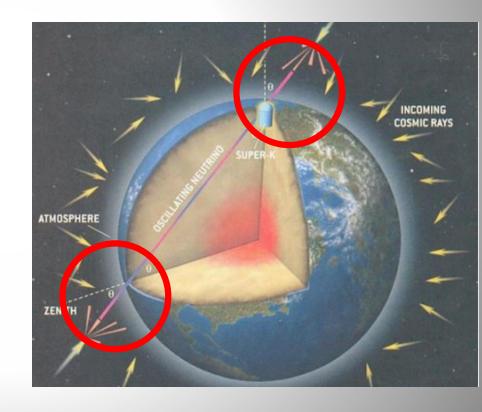
Cosmic rays hit the atmosphere at 30 km height.

These produce particles that decay and give neutrinos



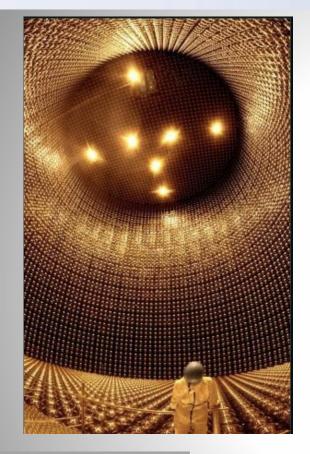
Some neutrinos are produced close to the detector.

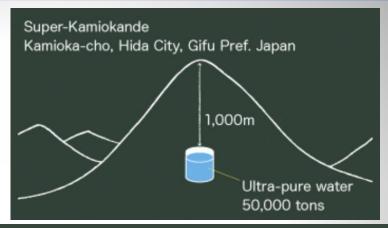
Others thousands of km away from it

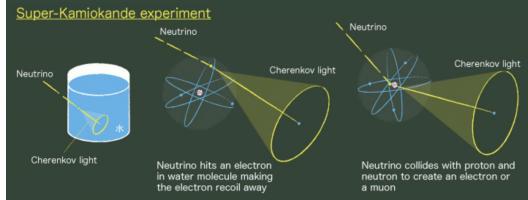


Neutrinos hitting the detector 'from below' travelled much longer than others

SuperKamiokande

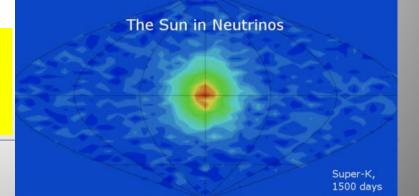






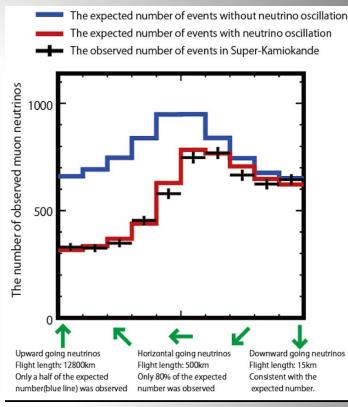


50,000 tons of ultra-pure water, watched by 13,000 photomultipliers



Neutrinos Oscillate! (1998)





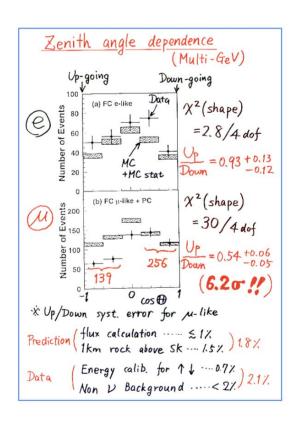
1998: The Super-Kamiokande experiment in Japan used a massive underground detector filled with ultrapure water.

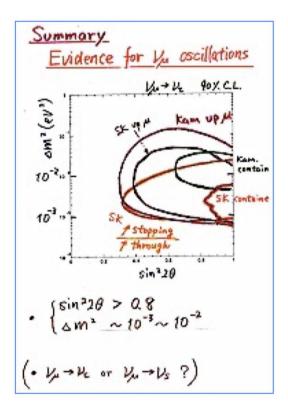
They announced first evidence of neutrino oscillations. The experiment showed that muon neutrinos disappear as they travel through the earth to the detector It also offered an explanation for the observed solar neutrino discrepancy.

Neutrinos Oscillate! (1998)

1998: Nobel-worth discovery of oscillation effects

[Takaaki Kajita for Super-Kamiokande, slides at Neutrino '98 conference]



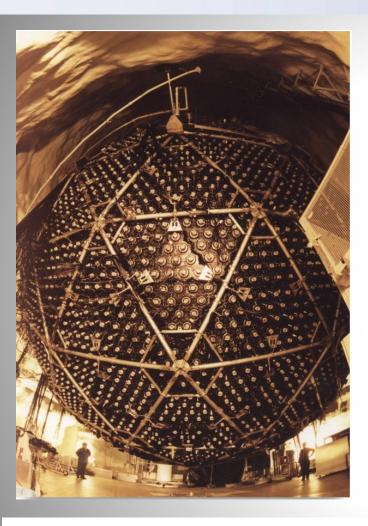


E. Lisi Re-interpretation Workshop 17/2/2021

Initial interpretation in terms of simple 2v ($v_{\mu} \rightarrow v_{\tau}$) oscillations

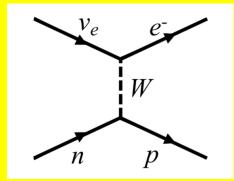
Neutrino Oscillations first firmly established with atmospheric neutrinos

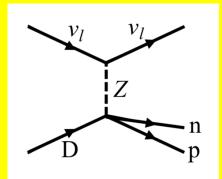
Neutrinos in 2000+



1000 tons of heavy water in a 6m radius vessel, viewed by 9600 photomultiplier tubes

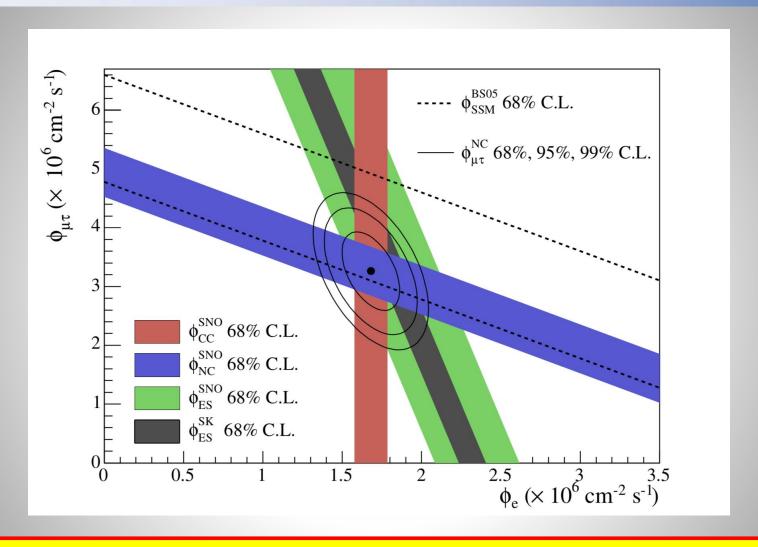
2002: The Sudbury Neutrino Observatory (SNO) used heavy water in a detector deep underground in Canada, announced conclusive evidence on solar neutrino oscillations, by measuring the sum of all neutrino interactions as well.





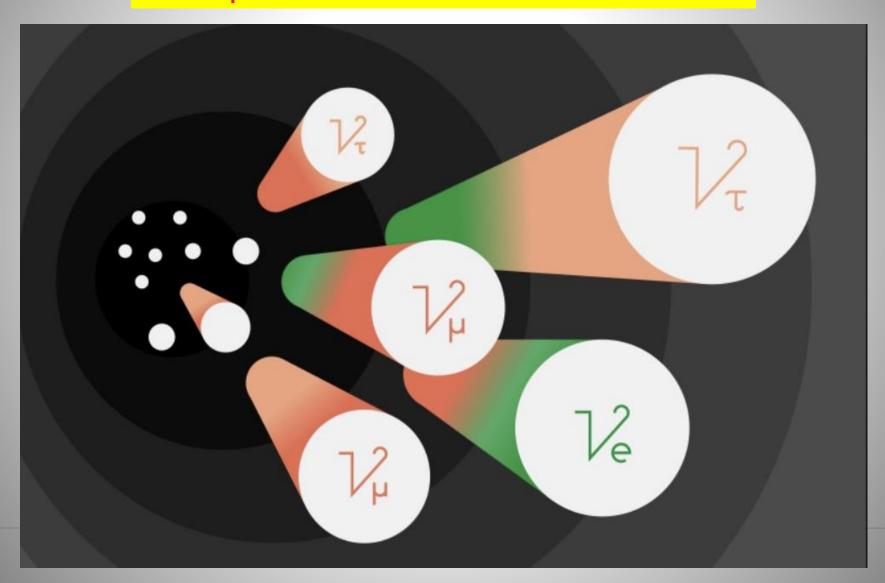
This was the final answer to Ray Davis' solar neutrino problem: Neutrinos from the sun transformed from the electron variety onto other flavors as they travelled to earth

SNO Demonstrates Flavor Change

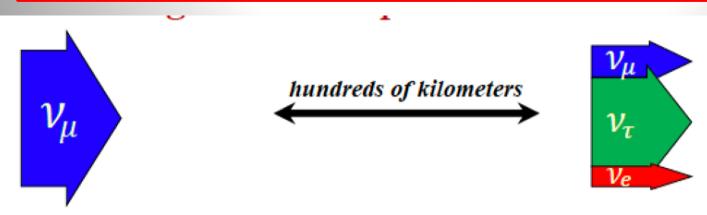


SNO could show conclusively the 2/3 missing electron neutrinos appear with a different flavor in the detector -> Neutrino oscillations!!

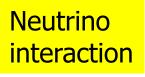
Atmospheric and solar neutrinos oscillate!!



- Important discovery in 1998: neutrino oscillations
- Neutrino oscillation is a quantum mechanical phenomenon whereby a neutrino created with a specific lepton flavor (electron, muon, or tau) can later be measured to have a different flavor. The probability of measuring a particular flavor for a neutrino varies between 3 known states as it propagates through space
- Neutrino oscillations only possible if neutrinos have a nonzero mass! Neutrino oscillations -> Neutrinos have mass!!



Each flavour state is a linear combination of mass states:



$$|\nu_{\alpha}\rangle = \sum_{i} U_{\alpha i}^{*} |\nu_{i}\rangle$$

Neutrino travel through space

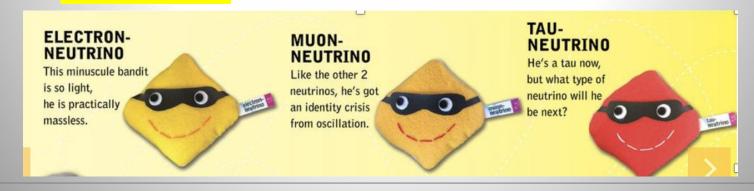
Flavour state $\alpha = e, \mu, \tau$

PMNS lepton mixing matrix

Mass state i = 1, 2, 3

Flavor states

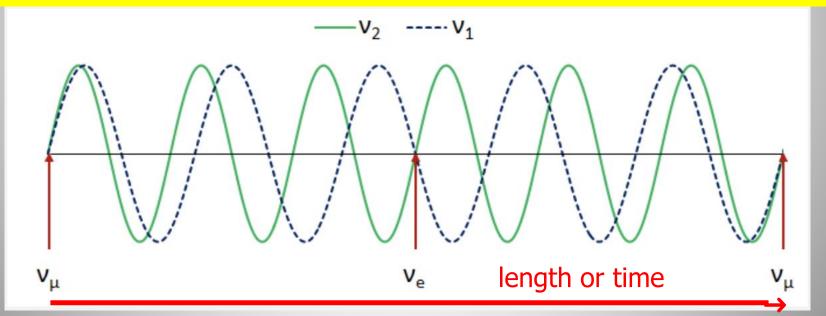
(*) Pontecorvo-Maki-Nakagawa-Sakata Matrix



The bizarre world of Quantum Mechanics: particles and waves

Take that the neutrino particle is a hybrid of two mass states v1 and v2 as it travels through space the associated waves of these mass states advance at a different rate

Hence the picture looks as follows: (propagation as a superposition of two masses)



The neutrinos change identity (flavor) along the way...!!

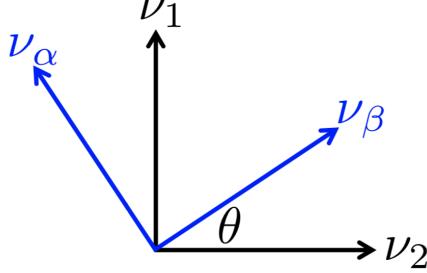
Two Flavour Oscillations

Flavour states

"Rotation Matrix"

Mass states

$$\begin{pmatrix} \nu_{\alpha} \\ \nu_{\beta} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_{1} \\ \nu_{2} \end{pmatrix}$$



$$|\nu(t=0)\rangle = |\nu_{\alpha}\rangle = \cos\theta |\nu_{1}\rangle + \sin\theta |\nu_{2}\rangle$$

Two Flavour Oscillations

$$|\nu(t)\rangle = e^{i(E_1t - pL)}\cos(\theta)|\nu_1\rangle + e^{i(E_2t - pL)}\sin(\theta)|\nu_2\rangle$$

plane wave

$$\langle \nu_{\beta} | \nu(t) \rangle = \sin(\theta) \cos(\theta) (e^{i(E_2 t - pL)} - e^{i(E_1 t - pL)})$$

$$E pprox p + rac{m_i^2}{2E}$$
 and $t = rac{L}{c}$ ultra-relativistic

$$\langle \nu_{\beta} | \nu(t) \rangle = \sin(\theta) \cos(\theta) \left(e^{i\frac{m_2^2 L}{2E}} - e^{i\frac{m_1^2 L}{2E}} \right) = \sin(\theta) \cos(\theta) e^{i\frac{\Delta m_i^2 L}{2E}}$$

$$P(\nu_{\alpha} \to \nu_{\beta}) = \langle \nu_{\beta} | \nu(t) \rangle^{2} = \sin^{2}(2\theta) \sin^{2}\left(\frac{\Delta m_{i}^{2} L}{2E}\right)$$

L: distance travelled

E: energy of the neutrino

Neutrino oscillations is a pure Quantum Mechanical effect
The effect depends on the mass difference between flavor states

$$V_{\mu}$$

$$V_{1}, V_{2}$$

$$V_{1}, V_{2}$$

$$V_{\mu} \text{ or } V_{\tau}$$

$$V_{\tau} = \left(\frac{\cos \theta + \sin \theta}{\cos \theta}\right) \left(\frac{v_{1}}{v_{2}}\right) \quad P(v_{\mu} \rightarrow v_{\tau}) = \sin^{2}(2\theta) \sin^{2}\left(\frac{1.27\Delta m^{2}L}{E_{\nu}}\right)$$
• Measure prob.
$$- \text{ Survival} \quad 0.8 \quad 0.6$$

$$- \text{ Appearance} \quad 0.8 \quad 0.6$$
• Result
$$- \text{ Mixing angle} \quad 0.8 \quad 0.2 \quad 0.2$$

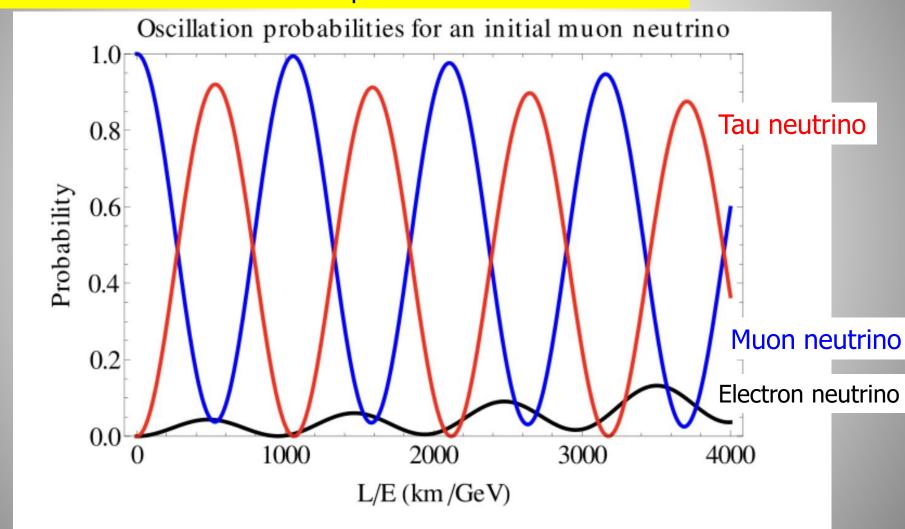
$$- \text{ Mass differences} \quad 0.2 \quad 0.2$$

- $\Delta m_{21}^2 = m_2^2 m_1^2 \approx 8 * 10^{-5} \text{ eV}^2 => \text{ wavelength of } \sim 100 \text{km}$
- $|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \approx$ 2 * 10⁻³ eV² => wavelength of ~1km

Absolute mass values? Mass hierarchy?

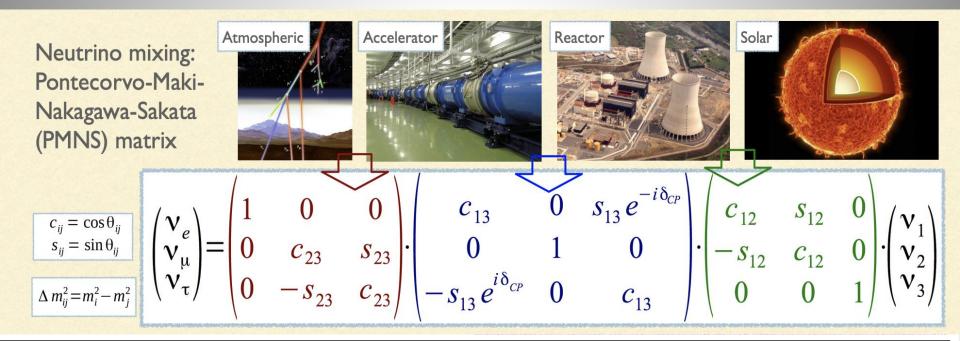
Oscillation with 3 Flavors

Pattern for the case of atmospheric neutrino oscillations



- Since 20 years an active field of study and data from many experiments collected:
 - Long baseline accelerator experiments (LBL)
 - Short baseline reactor experiments
 - Atmospheric neutrinos
 - Solar Neutrinos
 - Neutrinoless double beta decay experiments

LBL experiments in the US and Japan



+ interactions in matter → effective terms ~ G_F · E · density

Neutrino masses

- Neutrino oscillations only work when neutrinos have masses.
- The oscillations are sensitive to differences in (squared) mass values only.

 Why do Neutrino masses cause a problem in the Standard Model?

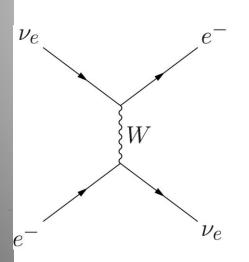
End of Lecture 1

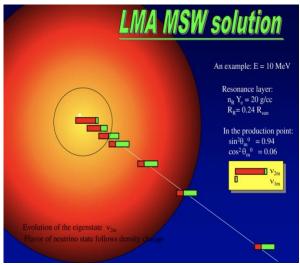
- Neutrinos are found to oscillate and hence have a tiny mass, as found in atmospheric neutrinos and neutrinos from the sun
- How small are the masses?
- What generates the neutrino mass?
- How do the neutrinos mix in the mass states?
- Is the neutrino a Majorana particle?
- ⇒Lecture 2: Oscillation parameter measurements and accelerator CPV tests, experiments to measuring neutrino properties; are there more than 3 neutrinos?

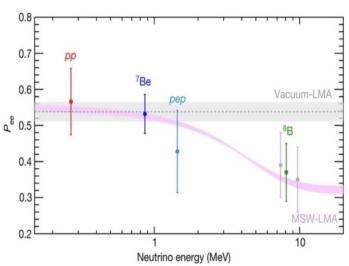
Backup

Note: MSW or Matter Effect

- When neutrinos travel over long distances through dense matter (Sun, Earth), their propagation is modified through coherent forward scattering off electrons (...like light in matter)
- This effect modifies the flavour oscillation probability (Mikhaev, Smirnov, Wolfenstein). Once the neutrino leaves the sun it is in a pure mass eigenstate consisting predominantly of the muon and tau flavors; no more further oscillation until it reaches earth.
- The MSW effect predicts a flavor conversion of solar neutrinos, that is independent of the distance between the sun and earth, of a factor 3 for the electron neutrinos (without any fine tuning)







Matter Effects

• The probability for ν_e appearance:

$$P(\nu_{\mu} \to \nu_{e}) \simeq \sin^{2}\theta_{23}\sin^{2}2\theta_{13}\frac{\sin^{2}(\Delta_{31} - aL)}{(\Delta_{31} - aL)^{2}}\Delta_{31}^{2}$$

$$+\sin 2\theta_{23}\sin 2\theta_{13}\sin 2\theta_{12}\frac{\sin(\Delta_{31} - aL)}{(\Delta_{31} - aL)}\Delta_{31}\frac{\sin(aL)}{(aL)}\Delta_{21}\cos(\Delta_{31} + \delta_{CP})$$

$$+\cos^{2}\theta_{23}\sin^{2}2\theta_{12}\frac{\sin^{2}(aL)}{(aL)^{2}}\Delta_{21}^{2},$$

$$\Delta_{ij} = \Delta m_{ij}^{2}L/4E_{\nu}, \ a = G_{F}N_{e}/\sqrt{2},$$

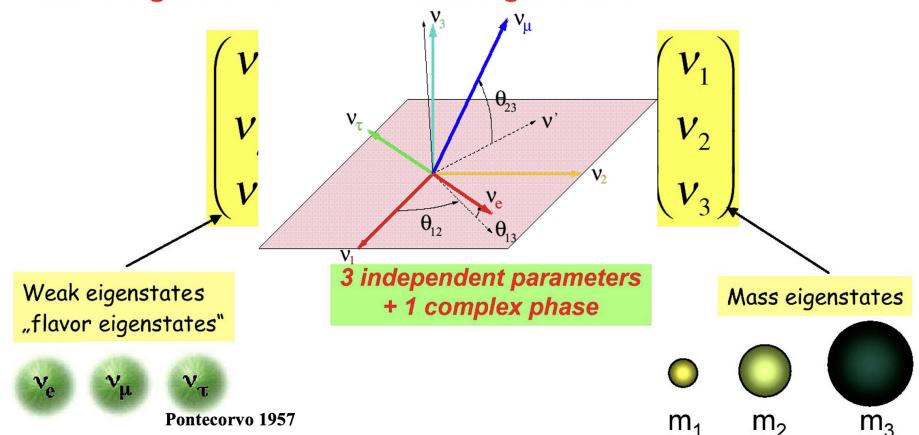
- both δ_{CP} and a (matter effect) switch signs in going from the $\nu_{\mu} \to \nu_{e}$ to the anti-neutrino process
- The origin of the matter effect asymmetry is simply the presence of electrons and absence of positrons in the Earth.

neutrino definitions

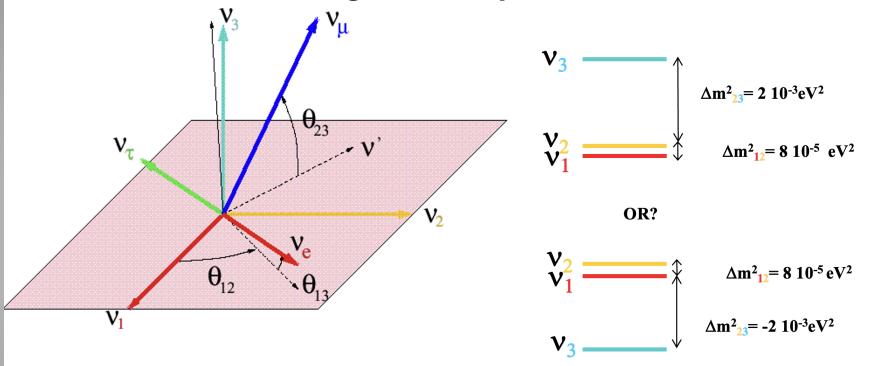
the electron neutrino is present in association with an electron (e.g. beta decay) the muon neutrino is present in association with a muon (pion decay) the tau neutrino is present in association with a tau ($W\rightarrow \tau \nu$ decay) these flavor-neutrinos are not (as we know now) quantum states of well defined **Mass** (neutrino mixing)

the mass-neutrino with the highest electron neutrino content is called ν_1 the mass-neutrino with the next-to-highest electron neutrino content is ν_2 the mass-neutrino with the smallest electron neutrino content is called ν_3

If neutrinos are massive particles, then it is possible that the mass eigenstates and the weak eigenstates are not the same:



The neutrino mixing matrix: 3 angles and a phase δ



$$\mathbf{U_{MNS}}: \left(\begin{array}{ccc} \sim \frac{\sqrt{2}}{2} & \sim -\frac{\sqrt{2}}{2} & \sin \theta_{\mathbf{13}} \ e^{i\boldsymbol{\delta}} \\ \sim \frac{1}{2} & \sim \frac{1}{2} & \sim -\frac{\sqrt{2}}{2} \\ \sim \frac{1}{2} & \sim \frac{1}{2} & \sim \frac{\sqrt{2}}{2} \end{array} \right)$$

Unknown or poorly known phase δ , sign of Δm_{23}^2



Neutrino masses occur via processes which are intimately related to the Higgs boson what are the couplings of the H(125) to neutrinos?

Let us follow the steps of the Standard Model to construct a minimal neutrino mass model

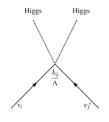
Adding neutrino masses to the Standard model 'simply' by adding a Dirac mass

→ right-handed neutrino

$$m_{D}\overline{v_{L}}v_{R}$$
 N_{D} N_{D}

m_D is the Higgs **Yukawa coupling** (like everybody else). Then the right handed neutrinos are sterile, (**except** that they couple to both the Higgs boson and gravitation). Things become more interesting: **a Majorana mass term** arises (So-called **Weinberg Operator**) using the Higgs boson and the neutrino Yukawa coupling:

Origin of neutrino mass:



 $\begin{array}{c|c} H & & & & \\ & Y_N^{\dagger} & & & \\ & & M_N & & \\ L & & & & \\ \end{array}$

Majorana mass term is extremely interesting as this is the **particle-to-antiparticle transition** that we want in order to explain

the Baryon asymmetry of the Universe (+ CP violation in e.g. neutrinos)

 $\mathbf{M_R} \ \overline{\nu_R^c} \ \nu_R$

+ restores SU(2) symmetry!

Pilar Hernandez, Silvia Pascoli Granada 2019-05

Having two mass terms per family, neutrinos undergo level splitting → Mass eigenstates

See-saw type I:

$$\mathcal{L} = \frac{1}{2} (\bar{\nu}_L, \, \bar{N}_R^c) \left(\begin{array}{cc} 0 & m_D \\ m_D^T & M_R \end{array} \right) \left(\begin{array}{c} \nu_L^c \\ N_R \end{array} \right)$$

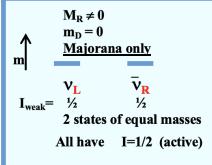
 $M_R \neq 0$ $m_D \neq 0$ Dirac + Majorana
mass terms

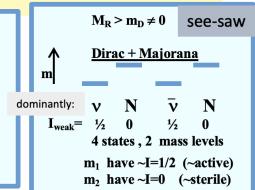
$$\tan 2\theta = \frac{2\,m_D}{M_R - 0} \qquad \ll 1$$

$$m_\nu = \frac{1}{2} \left[(0 + M_R) - \sqrt{(0 - M_R)^2 + 4\,m_D^2} \right] \qquad \simeq -m_D^2/M_R$$

$$M = \frac{1}{2} \left[(0 + M_R) + \sqrt{(0 - M_R)^2 + 4\,m_D^2} \right] \qquad \simeq M_R$$
 general formula if $m_D \ll M_R$

 $M_{R} = 0$ $m_{D} \neq 0$ Dirac only, (like e- vs e+): $I_{weak} = \begin{array}{ccc} V_{L} & V_{R} & \overline{V_{L}} & \overline{V_{R}} \\ I_{2} & 0 & 1/2 & 0 \\ 4 & states of equal masses \\ Some have I=1/2 (active) \\ Some have I=0 (sterile) \\ \end{array}$





Electroweak Charge

Electroweak charges of Standard Model particles

Spin J	Particle(s)	Weak charge $Q_{\sf w}$	Electric charge Q or Q_ϵ	Weak isospin T_3		Weak hypercharge $Y_{ m w}$		Z boson coupling	
								$= 2 Q_{L} + 2 Q_{R}$	LEFT
		1/2		e ⁻ , μ ⁻ , τ ⁻ electron, muon, tau ^[i]	$-$ 1 + 4 $\sin^2 heta_{ m w}$ $pprox 0$	-1	$-\frac{1}{2}$	0	-1
1/2	u, c, t up, charm, top ^[i]	$+1 - \frac{8}{3}\sin^2\theta_{\rm W}$ $\approx +\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{1}{2}$	0	$+\frac{1}{3}$	+ 4/3	$1 - \frac{4}{3}\sin^2\theta_{\rm w}$ $\approx + \frac{2}{3}$	$-\frac{4}{3}\sin^2\theta_{\rm W}$ $\approx -\frac{1}{3}$
1/2	d, s, b down, strange, bottom ^[i]	$-1 + \frac{4}{3}\sin^2\theta_{\rm W}$ $\approx -\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$+\frac{1}{3}$	$-\frac{2}{3}$	$-1 + \frac{2}{3}\sin^2\theta_{\rm W}$ $\approx -\frac{5}{6}$	$+\frac{2}{3}\sin^2\theta_{\rm W}$ $\approx +\frac{1}{6}$
1/2	$ u_e, \nu_\mu, \nu_\tau $ neutrinos ^[i]	+1	0	$+\frac{1}{2}$	0 [ii]	-1	0 [ii]	+1	O [ii]
1	g, γ, Z ⁰ , gluon ^[iii] , photon, and Z boson, ^[iv]	O [iv]							
1	W ⁺ W boson ^[v]	$+2 - 4 \sin^2 \theta_{\rm w}$ $\approx +1$	+1	+1		0		+2 – 4 $\sin^2 \theta_{\rm w}$ $pprox$ +1	
0	Higgs boson	-1	0	$-\frac{1}{2}$		+1		-1	