Extra Dimensions

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The Hierarchy Problem (again!)

- Ratio of gravitational (Newton) and weak (Fermi) interaction constants:

$$G_N/G_F \sim 10^{-34}$$

- Number sensitive to the cut-off energy Λ

Naturalness

- A theory is 'natural' if its underlying parameters are all of the same order of magnitude in appropriate units.
  - ➡ In the SM, the GN/GF ratio is unnatural: 34 orders of magnitude
  - ➡ Other naturalness problems:
    - ➡ Vacuum energy density: 120 orders of magnitude
    - ➡ Fermion masses hierarchy: 11-12 orders of magnitude
- Is the world ‘natural’? Is fine-tuning OK?

Gravity - Quick Recap

- Described by General Relativity
  - ➡ Gravity as a geometric property of spacetime
  - ➡ Einstein field equation (EFE) determine the spacetime metric for a given distribution of matter within it:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor representing the EFE solution, Λ is the cosmological constant, G is Newton's gravitational constant, c is the speed of light in vacuum and $T_{\mu\nu}$ is the stress-energy tensor.

  - ➡ 4x4 symmetric tensors, each with 10 independent components.
  - ➡ EFE reduce to Newton's law of gravitation where the gravitational field is weak and velocities are much less than the speed of light.

Linearised Gravity

- Effective theory for when gravitational field is weak (used e.g. in gravitational waves and weak-field gravitational lensing calculations)
- Solution is Minkowski metric + small deviations:

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu}$$

- The deviation can be written in terms of a linear perturbation of a canonically normalised massless spin-2 field, the graviton,

$$\delta g_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\mathrm{P}}}$$

- In semi-classical treatment, $h_{\mu\nu}$ describes a massless spin-2 particle with two propagating degrees of freedom.

Extra Dimensions - Historical Overview

- Theodor Kaluza, 1919-1921:
  - ➡ Classical extension of General Relativity to 5D
  - ➡ 15 component metric tensor: 5D Einstein equation yields:
    - 4D Einstein field equations
    - Maxwell equations for electromagnetic field
    - One equation for scalar field (‘radion' or 'dilaton')
    - Cylinder boundary conditions: no component of 5D metric depends on 5th dimension

- Oskar Klein, 1926:
  - ➡ Quantum interpretation
  - ➡ Hypothesis: 5th dimension small (10^{-30} cm), closed and periodic, explaining cylinder boundary conditions
  - ➡ Derive quantisation of electric charge in terms of integer multiples of 5th dimension momentum
  - ➡ Also contributed to classic theory by providing a normalised 5D metric
- 1940's :
  - ➡ Independent groups in Germany, France, and Switzerland finished solving the Kaluza equations under the cylinder conditions

https://www.nature.com/articles/118516a0.pdf

Modifying Gravity

- Gravity follows:

$$f(r) = -G_N \frac{m_1 m_2}{r^2} \equiv -\frac{1}{M_{Pl}^2} \frac{m_1 m_2}{r^2} \qquad D = 4 \text{ dimensional}$$

$$f(r) = -G_D \frac{m_1 m_2}{r^{2+\delta}} \equiv -\frac{1}{M_D^{(2+\delta)}} \frac{m_1 m_2}{r^{(2+\delta)}} \qquad D = 4 + \delta \text{ dimensional}$$

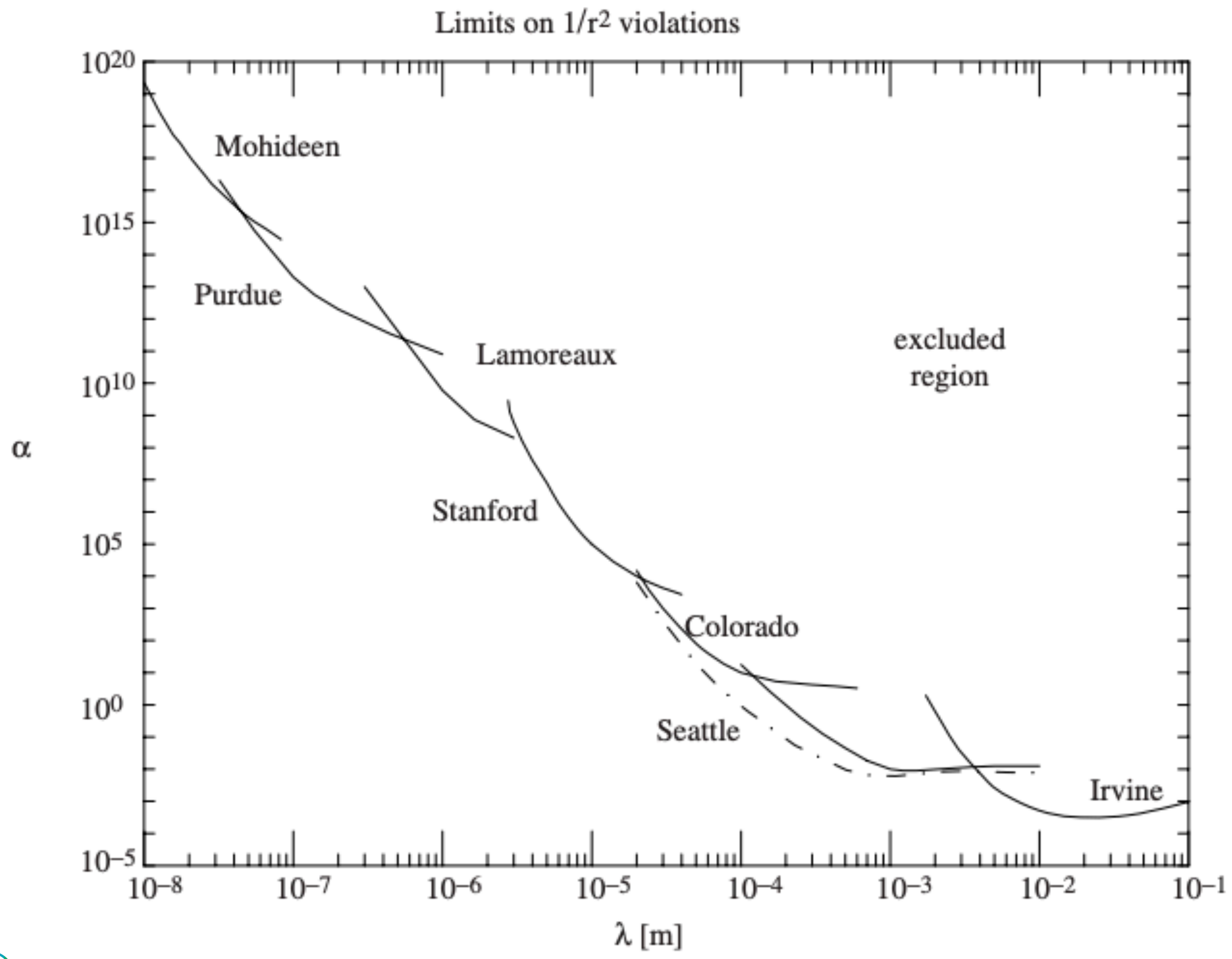
➡ Modification from inverse square law can be parameterised with massive gravitons contribution following Yukawa description with range parameter λ and strength α:

$$V(r) = V_N(r)(1 + \alpha e^{-r/\lambda})$$

with $V_N(r)$ as the Newtonian potential ~ 1/r

How precise do you think we have directly measured gravity?

- Experimental limits on inverse square law:



- Delicate torsion balance experiments, summarised here, sensitive to sub mm effects (0.2 mm - 80 μm).

Hair diameter: average 100 μm

New Nature article March 2021, torsion balance for 1mm radius spheres, masses <100mg

- Working out the numbers from the D-dimensional forces for δ extra flat dimensions to bring down the gravity cutoff M_D to the TeV scale:

$$= \begin{cases} \sim 0.4 \text{ light years} & \delta = 1 \\ \sim 1 \text{ mm} & \delta = 2 \\ \sim 5 \times 10^{-7} \text{cm} & \delta = 3 \\ \sim 10^{-9} \text{cm} & \delta = 4 \end{cases}$$

with R the distance where gravity would deviate from $\sim 1/r^2$

- From our 0.1 mm measured limit, $\delta = 1$ and $\delta = 2$ are excluded, but $\delta \geq 3$ is still allowed.

- More complicated metrics for the space time with extra dimensions will have more freedom in the number and size of extra dimensions

Particles in Extra Dimensions

- Gravitons - full extra-dimensional spacetime (*bulk)*
- Other particles
  - ➡ Confined in 4D
    - ➡ Large extra dimensions (ADD) models
    - ➡ Randall-Sundrum (RS1 and RS2) models
  - ➡ In bulk
    - ➡ Universal Extra Dimensions (UEH)
    - ➡ Bulk Randall-Sundrum

Kaluza-Klein Mechanism

- For simplicity, write it in 5 (4+1) dimensions
  - ➡ x_μ four space-time coordinates; y is 5-th dimension;
  - ➡ 5-dimensional spacetime index: capital latin letters A, B, …
  - ➡ Extra-dimension flat and non-compact
  - ➡ Relativistic dispersion relation:

$$p_A p^A = p_\mu p^\mu - p_y^2 = m^2$$

  - ➡ For a massless particle:

$$p_A p^A = p_\mu p^\mu - p_y^2 = 0$$

$$p_A p^A = p_\mu p^\mu - p_y^2 = \cancel{m^2}\ {}_{0}$$

- Two observers: one in 4D ($y = 0$) and one in 5D

5-D observer

Alice

Single massless particle that can have different momenta

4-D observer
y = 0

Bob

Continuum of particles with different masses!

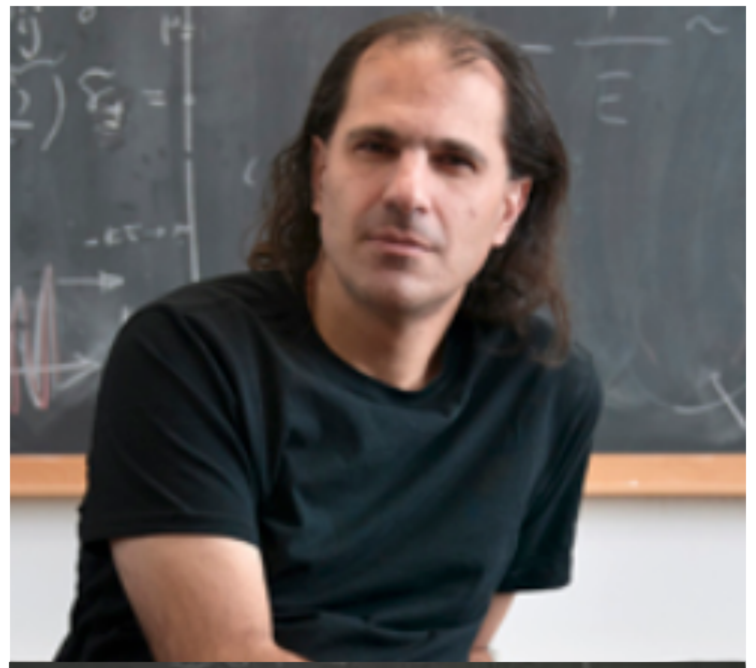
$$p_\mu p^\mu = p_y^2$$

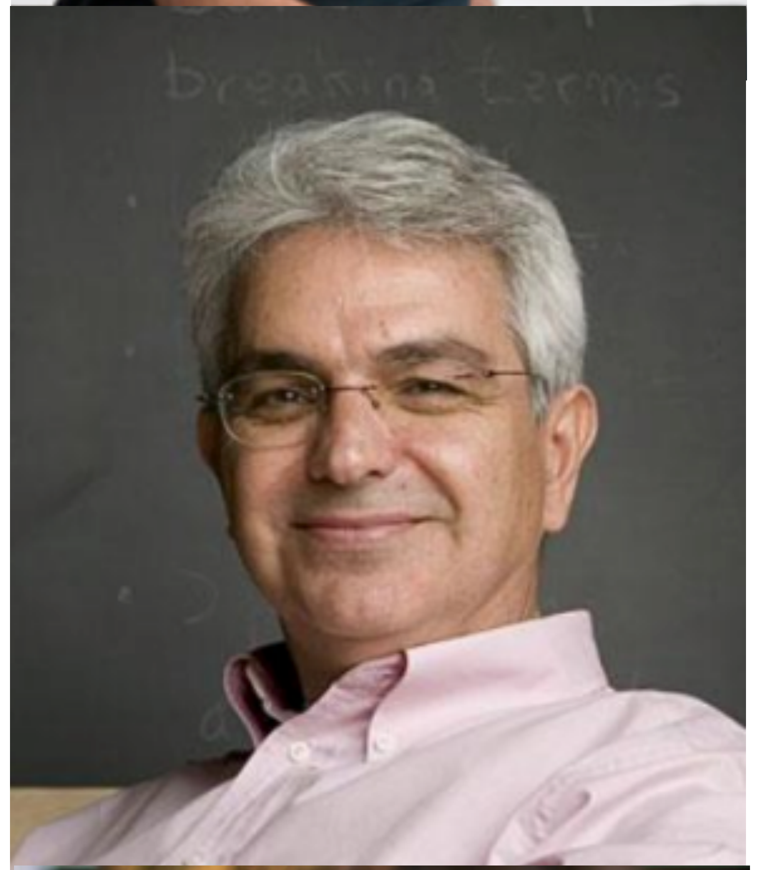
$$m^2 = p_y^2$$

- Boundary condition in the 5-th component (i.e. extra dimension is compact), p_y is quantised
  - ➡ Discrete mass states in 4D (Kaluza-Klein (KK) towers)

Large Extra Dimensions

- Proposed by Arkani-Ahmed, Dimopoulos and Dvali in 1998 (ADD model)
- Changed paradigm on hierarchy problem
  - ➡ From: Two fundamental scales: weak interaction (~100 GeV), realm of quantum field theories; and the Planck scale (10^{19} GeV), realm of quantum gravity / string theory
  - ➡ To: Gravity and gauge forces united at the weak scale, the only scale in the theory
    - Solves hierarchy problem!
    - Does not lead to any stark disagreement with experimental observations





- Main model assumptions:
  - ➡ A new number δ, $\delta \geq 2$, of compact extra spatial dimensions which are large compared to the weak scale m_{EW} (flat metric)
  - ➡ The Planck scale is now related to M_{Pl} (4D) by:

$$M_{Pl}^2 \sim M_D^{(2+\delta)} R^\delta$$

R^δ is the volume of the δ-dimensional manifold, which needs to be large enough so enough gravity 'leaks' into the extra dimensions. To comply with $M_D \sim m_{EW}$ and $M_{Pl} \sim 10^{19}$ GeV,

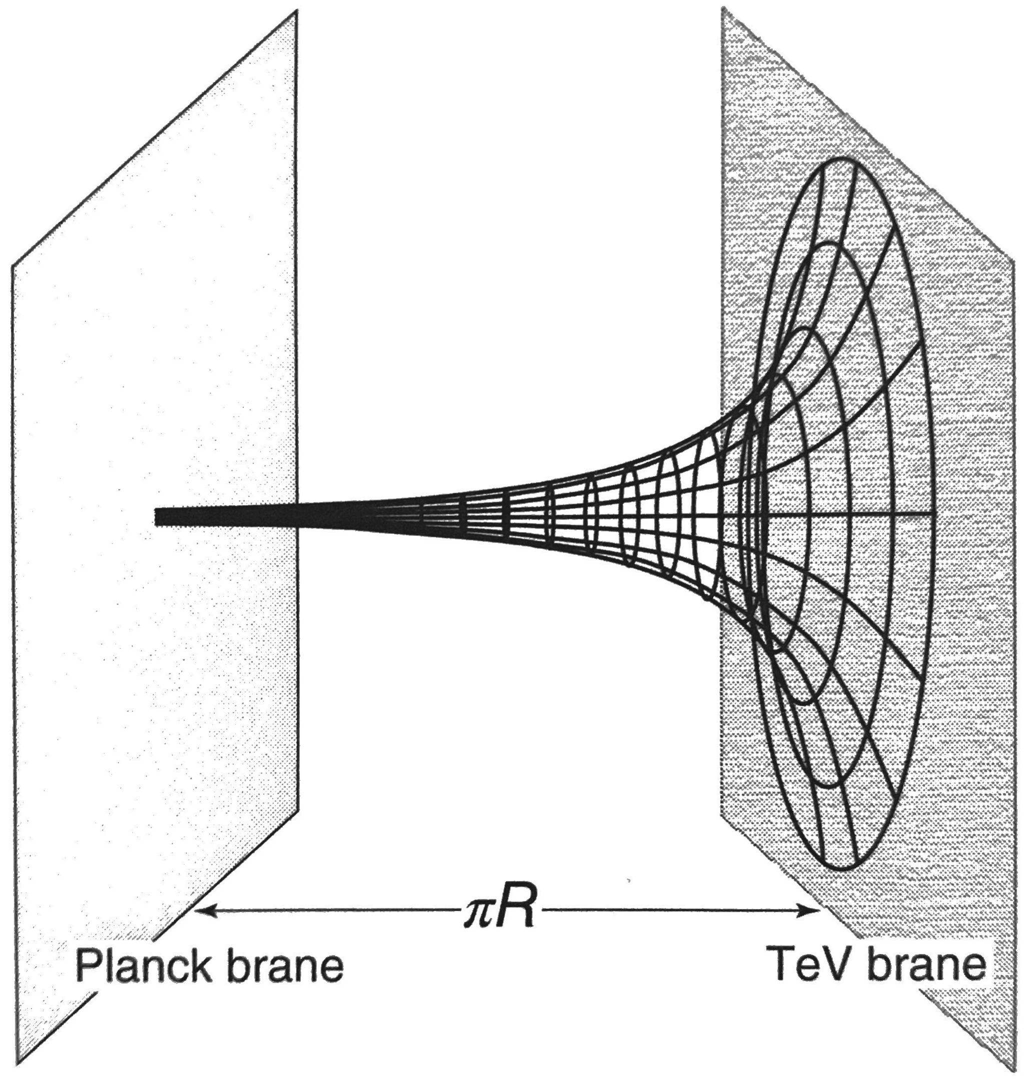
$$R \sim 10^{\frac{30}{n}-17}\mathrm{cm} \times \left(\frac{1\,\mathrm{TeV}}{m_{EW}}\right)^{1+\frac{2}{\delta}}$$

This is the same calculation used to compute R size as a function of δ in the previous slides.

  - ➡ Gravitons propagate in all dimensions, but SM fields are localised in a 4-D manifold of weak scale 'thickness' in the extra dimensions

Warped Extra Dimensions

- Proposed by Lisa Randall and Raman Sundrum in 1999 (RS model)
- 5-D theory
  - ➡ Extra dimension (y) compactified in an orbifold S^1/Z_2, $y \in [0, \pi R]$ (infinite in RSII)
  - ➡ Bulk is an anti-de Sitter space (negative curvature, negative cosmological constant Λ)
  - ➡ Gravity in 5-D, but mostly localised at $y = 0$ (Planck or gravitational brane)
  - ➡ SM particles confined on a (3+1)-D brane at $y = \pi R$ (TeV or weak brane)
- Hierarchy between Planck and TeV scales generated by curvature in extra dimension
  - ➡ Solves hierarchy problem by having all fundamental scales at Planck scale

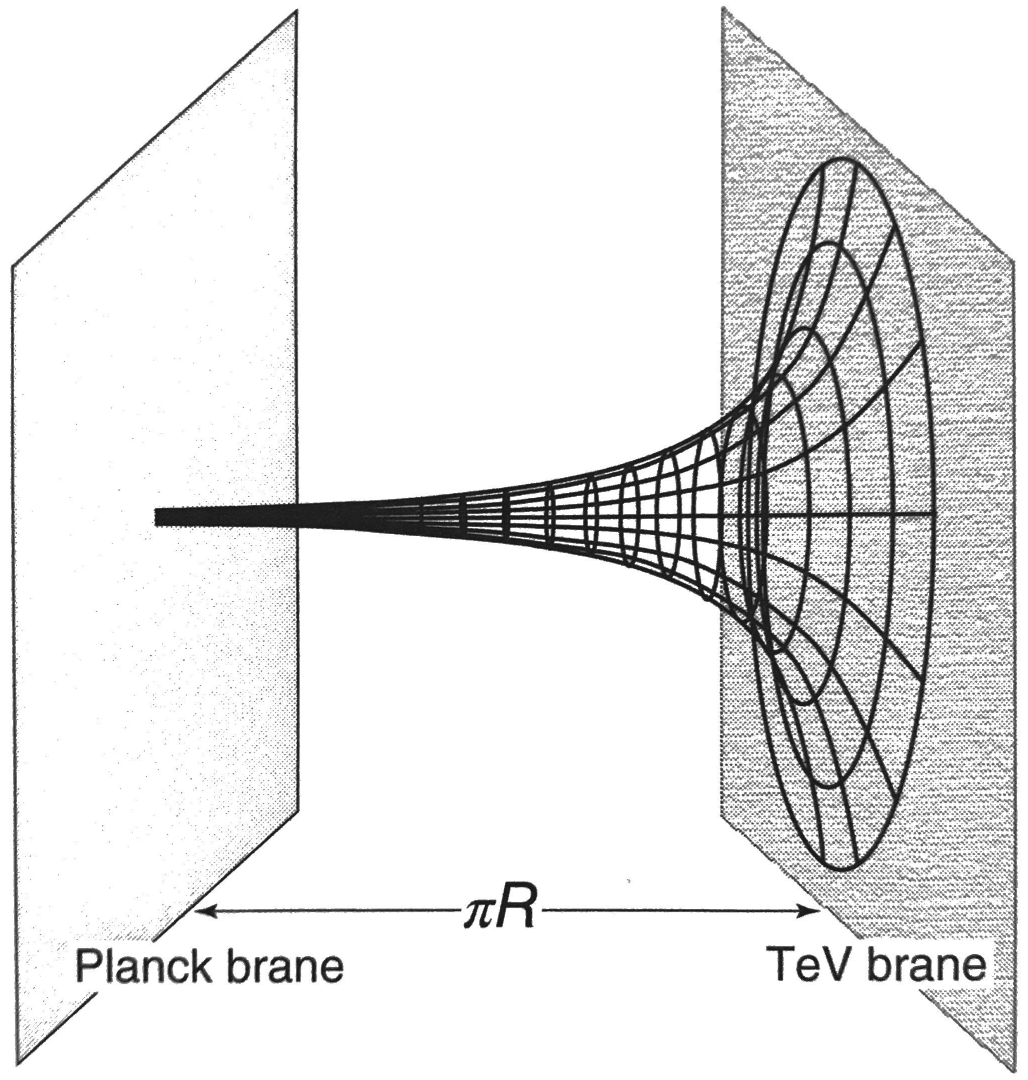


- Metric solution to Eistein's equation in 5D:

$$ds^2 = e^{-ky} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$$

➡ e^{-ky} is the 'warp factor': determines how 4D scales changes as a function of position in the extra dimension

- $y = 0$: $e^{-k.0} = 1$ (ultraviolet boundary) - unchanged scale
- $y = \pi R$: $e^{-k.\pi R}$ (infrared boundary) - exponentially suppressed scale



$$m = m_0 e^{-k\pi R}$$

➡ The action in 5-dimensions gives rise to the effective 4D Lagrangian:

$$\mathscr{L} = -\underbrace{\frac{1}{\bar{M}_P} T^{\mu\nu} h^{(0)}_{\mu\nu}}_{\text{Usual gravity massless graviton}} - \underbrace{\frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}}_{\text{KK massive gravitons}}$$

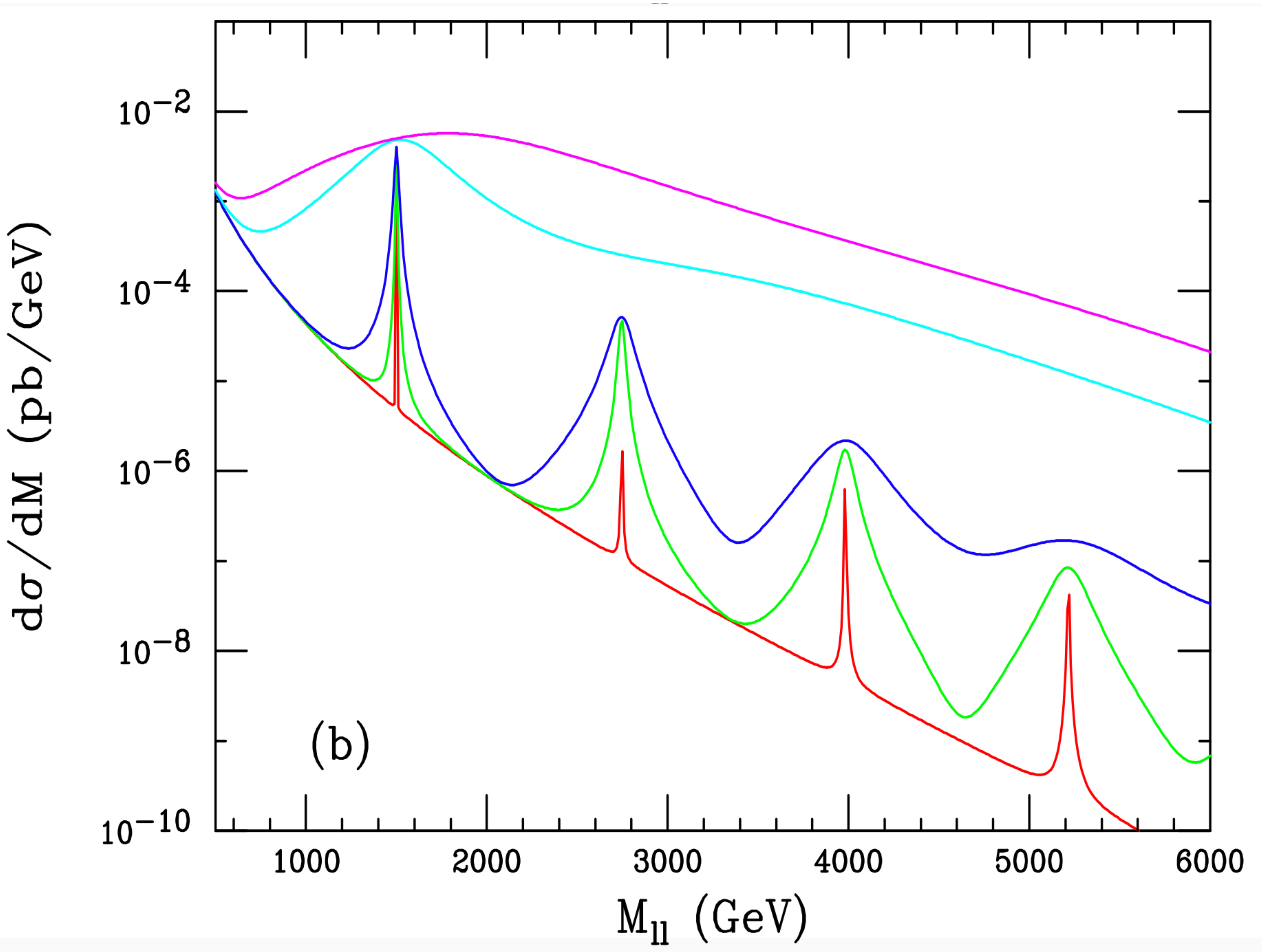
RS Gravitons in Colliders

- Gravitons can be produced in colliders via fermion-antifermion (electron-positron, quark-antiquark) or gluon-gluon fusion
- Allowed decays to gluons, vector bosons, photons, lepton pairs, quark pairs, Higgs pairs
- Mass and width of first massive graviton enough to fully determine model predictions:

$$m_1 = \frac{k x_1 \Lambda_\pi}{\bar{M}_P}$$

$$\Gamma_1 = \rho m_1 x_1^2 \left(\frac{k}{\bar{M}_P} \right)^2$$

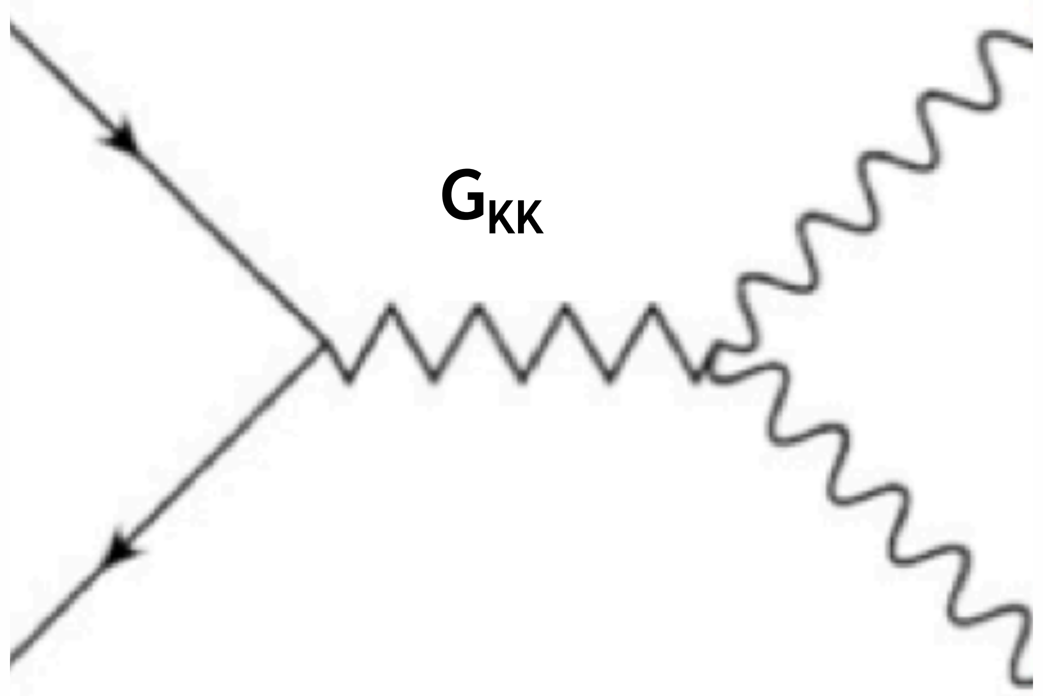
ρ is a constant which depends on the open channels

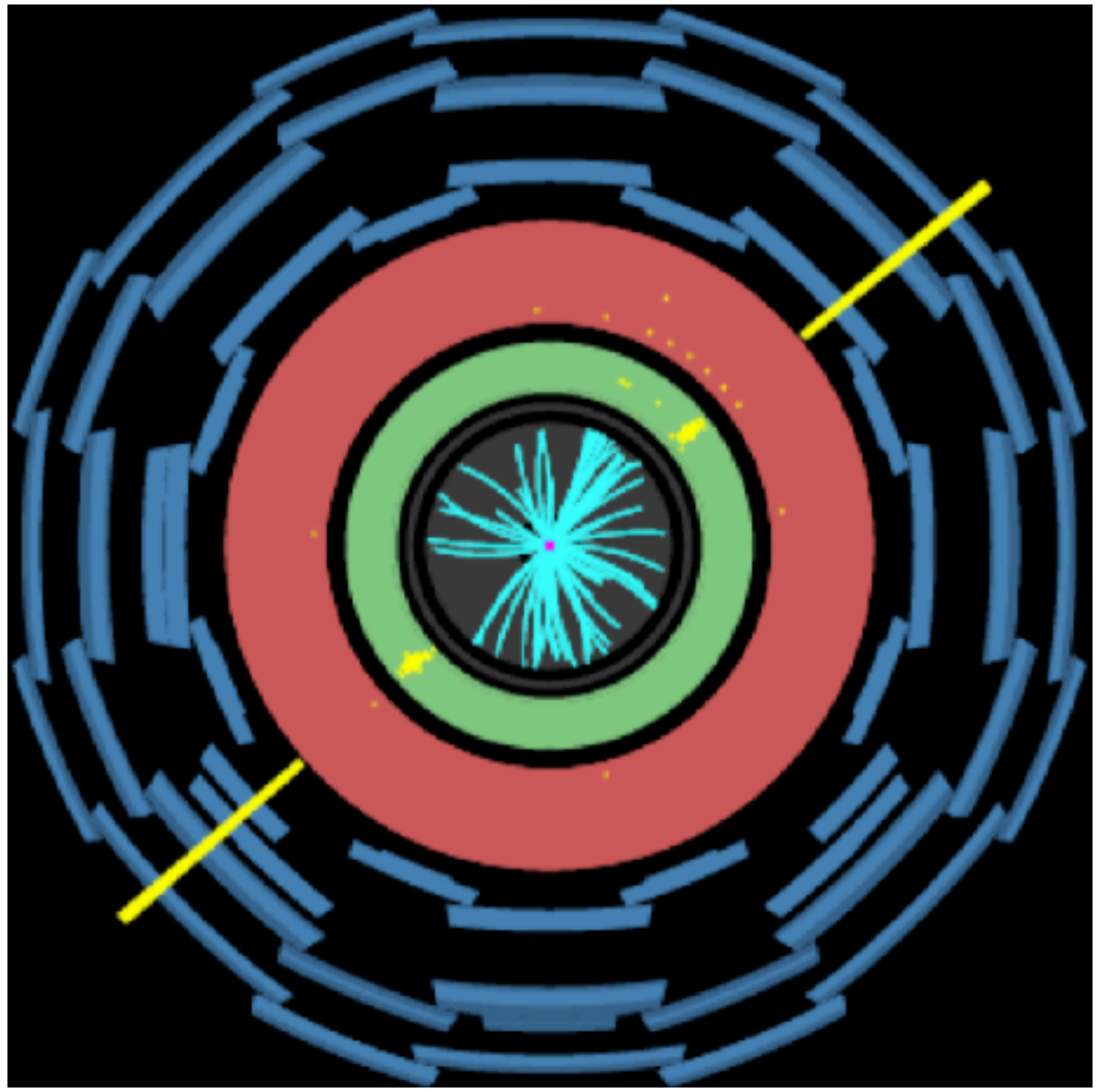


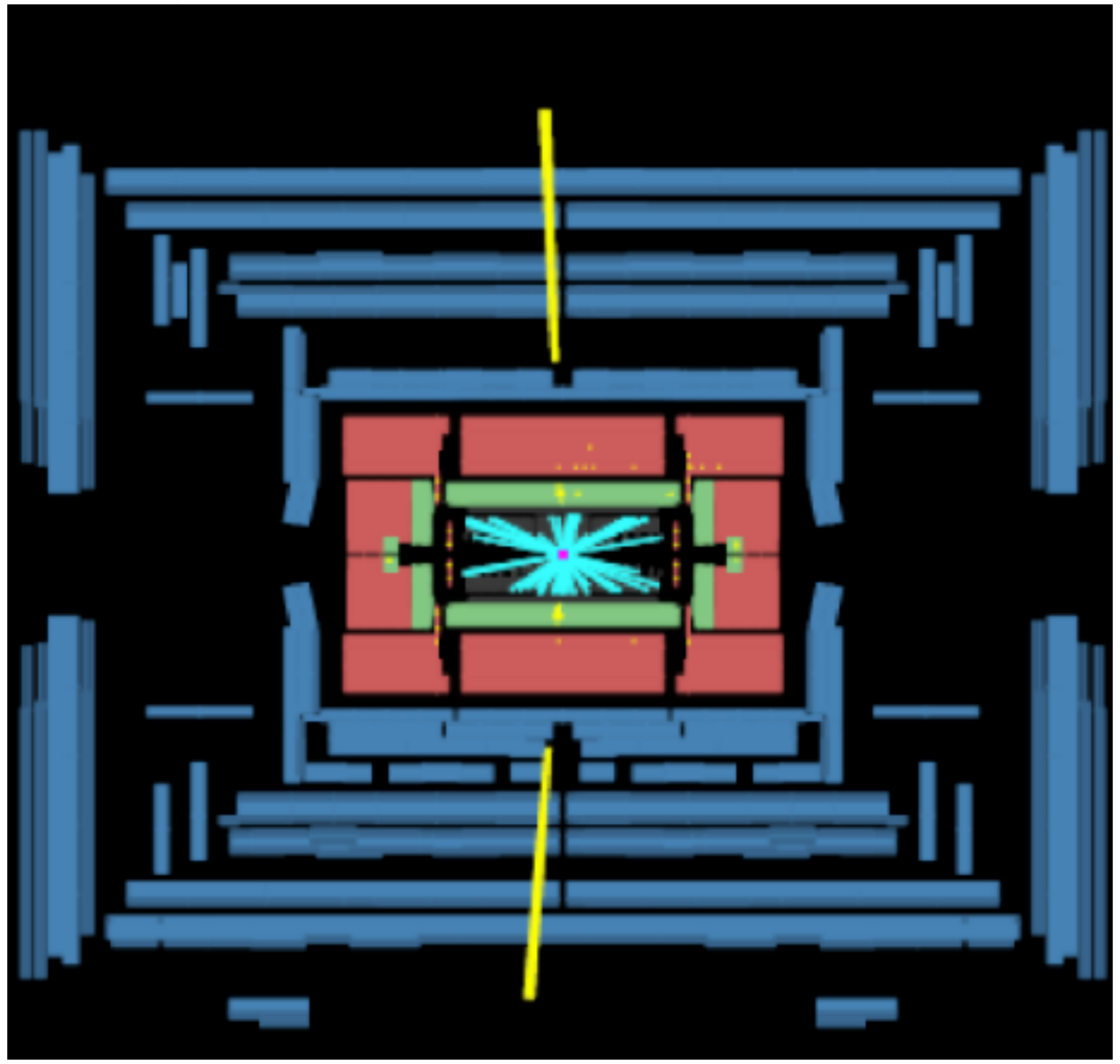
Drell-Yan production of a 1.5 TeV G_{KK} and its subsequent tower states at a 14 TeV LHC.
From top to bottom, the curves are for k/M_P =1, 0.5, 0.1, 0.05, and 0.01, respectively

LHC Searches for RS Gravitons

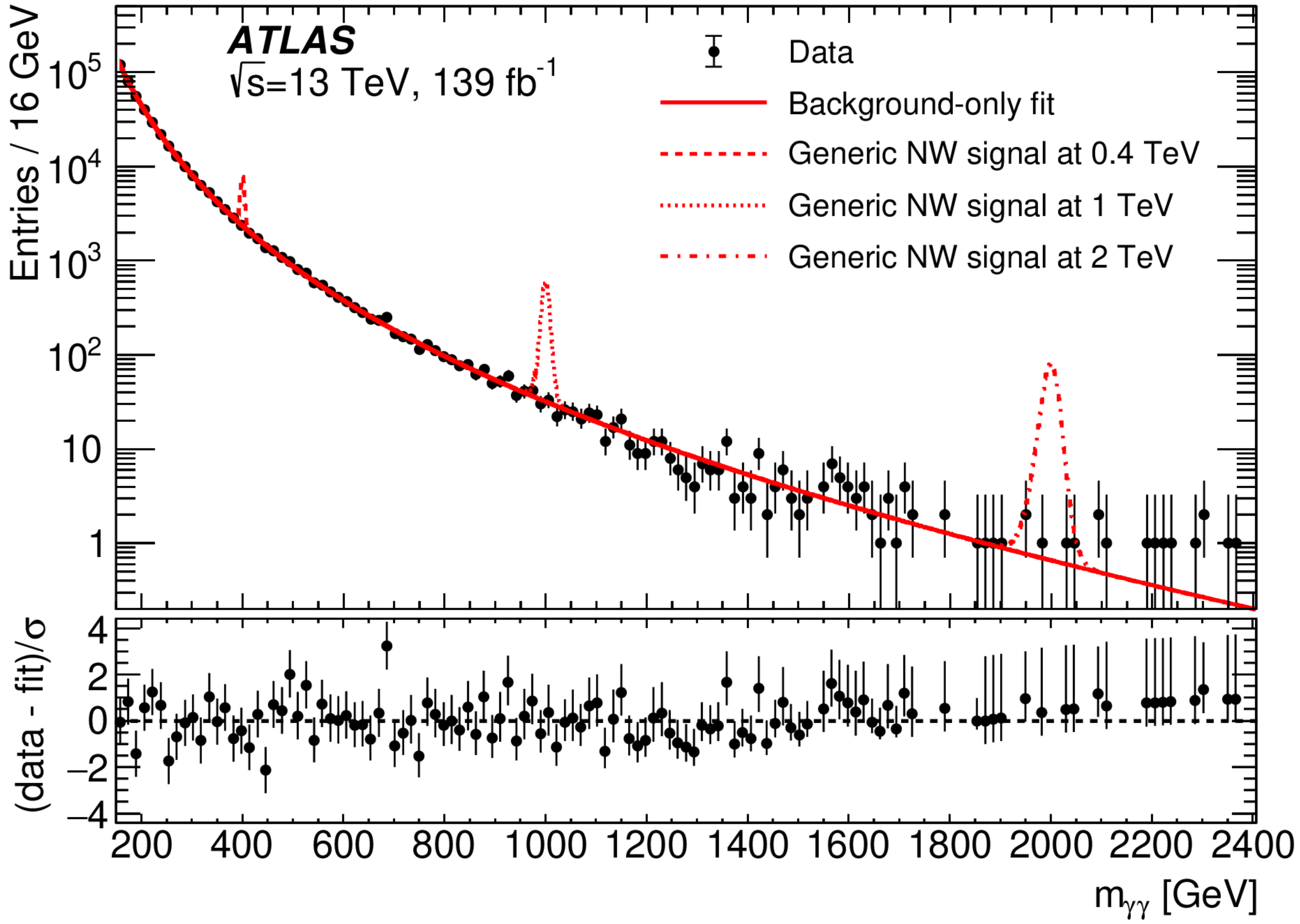
- $G_{KK} \to \gamma\gamma$

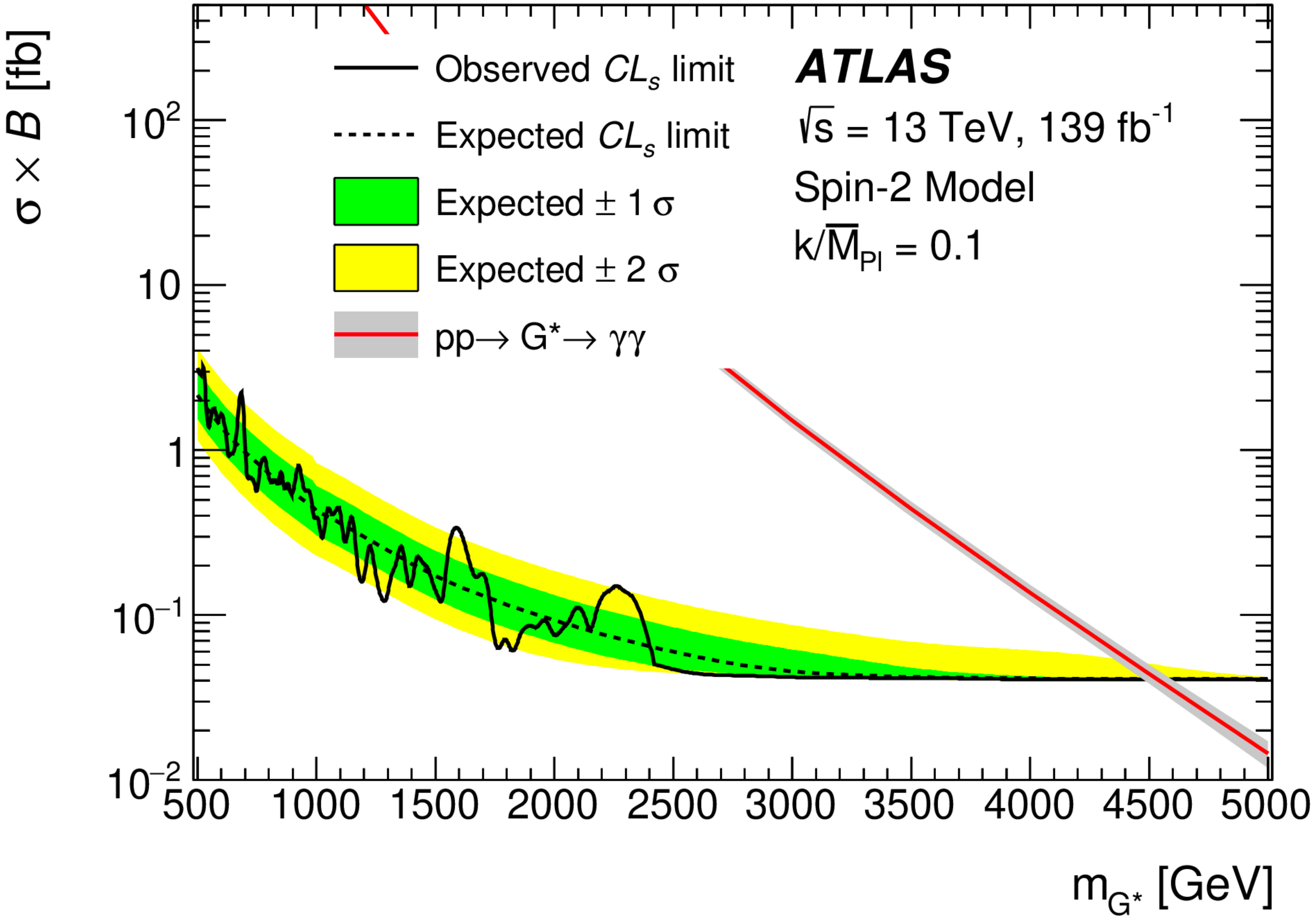






- Exclusion limits:
  - ➡ ATLAS: $G_{KK} > 2.2$, 3.9 and 4.5 TeV for $k/M_P = 0.01, 0.05, 0.1$





Summary

- The Hierarchy Problem could be an interesting guide to search for new physics
- Theories predicting extra dimensions can solve the Hierarchy Problem by:
  - ➡ Bringing the Planck scale down to TeV (Large Extra Dimensions)
  - ➡ Having a “natural" mechanism that explains the difference in strength between gravity and the other forces by warping the space-time
- Gravitons are looked for at the LHC
  - ➡ Other way to search for extra dimensions is to look for microscopic Black Holes! In next lecture 🧐