UNIVERSITÉ LIBRE DE BRUXELLES

UNIVERZITA KARLOVA
Matematicko-fyzikální fakulta

Gravitational waves from the quasi-spherical inspiral of a spinning body in Kerr spacetime

Viktor Skuopý, **Gabriel Andres Piovano**, Vojtěch Witzany

arXiv:2506.20726

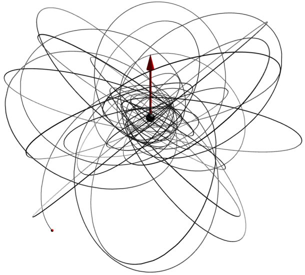
Belgian-Dutch Gravitational Wave Meeting, 27th-28th October 2025
Radboud University

Extreme-mass ratio inspirals (EMRIs)

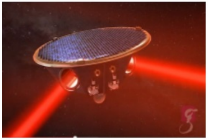
Primary mass M: $10^6 - 10^9 M_\odot$ Secondary mass μ: $1 - 100 M_\odot$

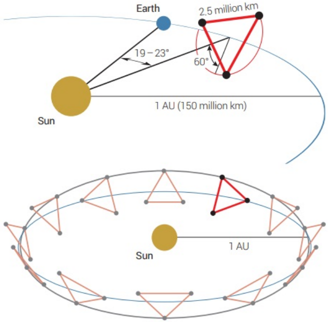
$M/\mu = 1/q \sim 10^5$ orbits in 1 year!

binary parameters measured with extreme precision
$\implies$ important for astrophysics and fundamental physics



Credit: Maarten van de Meent

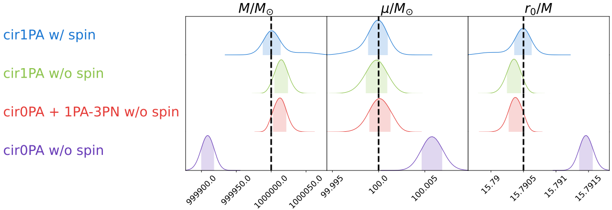


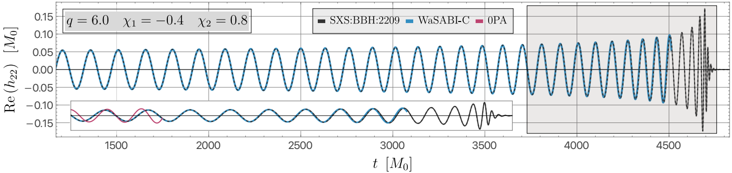


Why do we want to include the small body spin?

$$\Phi_{\rm GW} = \underbrace{q^{-1}\mathcal{C}^{(0)}}_{\text{adiabatic}} + \underbrace{q^{-1/2}\mathcal{C}^{(1/2)}}_{\text{resonances}} + \underbrace{q^{0}\mathcal{C}^{(1)}}_{\text{post-1-adiabatic}} + \mathcal{O}(q)$$

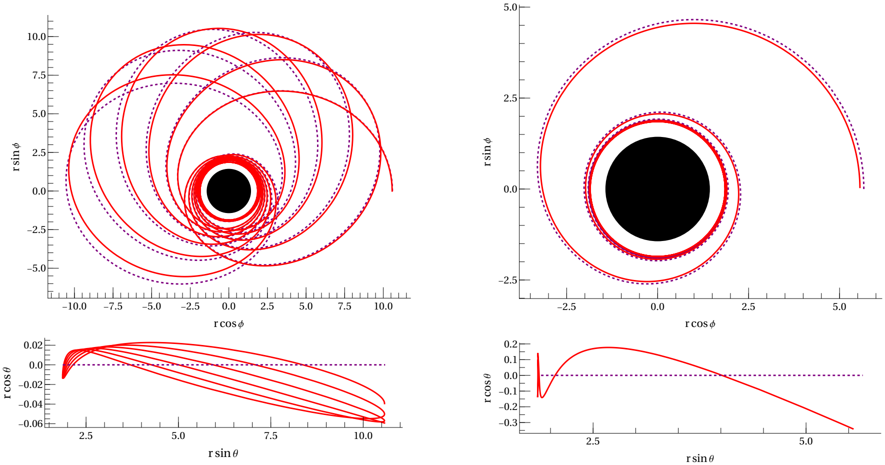
$\mathcal{C}^{(1)}$ conservative 1SF, dissipative 2SF, secondary spin χ





Top: Burke, Piovano+ (2023). Bottom: hybrid waveform (Honet et al, 2025), `WaSABI-C`

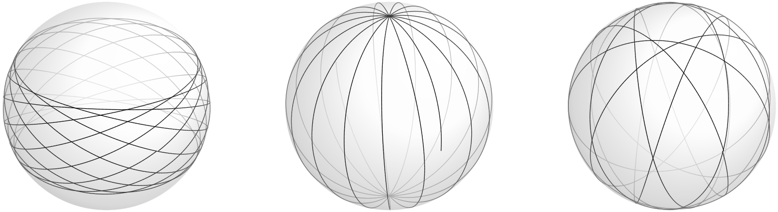
Why do we want to include the small body spin?



Near-equatorial orbits for a spinning (fill) and non-spinning particle (dashed)
Left: zoom-whirl orbit. Right: homoclinic orbit ("A plunge too far")
Piovano, 2025

Quasi-spherical inspirals in Kerr spacetime

- Boyer Linquist radius $r =$ const and inclination $x \neq \pm 1$.
  Precession orbital plane (Teo, 2021)
- "spherical evolves into spherical" (Kennefick&Ori, 1996)
- Adiabatic inspiral (Hughes, 2001), adiabatic+1SF (Lynch+, 2024)



Examples of spherical geodesic orbits (Credit: Teo, 2021)

We study quasi-spherical inspiral for a spinning test-body

Outline of the presentation

1. Equations of motion for a spinning body
2. Gravitional fluxes and radiation reaction equations
3. Waveforms!
4. Conclusions and future perspective

Equations of motion for a spinning body

(linearized) MPD equations of motion

Mathisson-Papapetrou-Dixon (MPD) equations of motion at order $\mathcal{O}(q\chi)$

$$\frac{D^2 x_{\rm g}^\mu}{\mathrm{d}\tau^2} = 0\,, \qquad \frac{D^2 \delta x^\mu}{\mathrm{d}\tau^2} = -\frac{1}{2} R^\mu{}_{\nu\kappa\lambda}\frac{\mathrm{d}x_{\rm g}^\nu}{\mathrm{d}\tau} S^{\kappa\lambda}\,, \qquad \frac{DS^\mu}{\mathrm{d}\tau} = 0\,.$$

with $x^\mu = x_{\rm g}^\mu + q\chi_\parallel \delta x^\mu + q\chi_\perp \delta x^\mu$ and $z = \cos\theta$

$$\sigma = S/(\mu M) = q\chi \ll 1$$

Numerically solved by Drummond&Hughes (2022), Skuopý+ (2023)

Easier to solve a 1st order system (Witzany, 2019)

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = V_{\rm g}^t(r_{\rm g}, z_{\rm g}) + q\Big(\chi_\parallel \delta V_\parallel^t(r_{\rm g}, z_{\rm g}) + \chi_\perp \delta V_\perp^t(r_{\rm g}, z_{\rm g}, \psi)\Big)$$

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 = R_g(r_{\rm g}) + q\Big(\chi_\parallel \delta R_\parallel(r_{\rm g}, z_{\rm g}) + q\chi_\perp \delta R_\perp(r_{\rm g}, z_{\rm g}, \psi)\Big)\,,$$

$$\left(\frac{\mathrm{d}z}{\mathrm{d}\lambda}\right)^2 = Z_g(z_{\rm g}) + q\Big(\chi_\parallel \delta Z_\parallel(r_{\rm g}, z_{\rm g}) + q\chi_\perp \delta Z_\perp(r_{\rm g}, z_{\rm g}, \psi)\Big)$$

$$\frac{\mathrm{d}\phi_s}{\mathrm{d}\lambda} = V_{\rm g}^\phi(r_{\rm g}, z_{\rm g}) + q\Big(\chi_\parallel \delta V_\parallel^\phi(r_{\rm g}, z_{\rm g}) + q\chi_\perp \delta V_\perp^\phi(r_{\rm g}, z_{\rm g}, \psi)\Big)$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\lambda} = \Psi_r(r_{\rm g}) + \Psi_z(z_{\rm g})$$

Solved by

Skuopý&Lukes-Gerakopoulos, (2021,2022) ; Witzany&Piovano, 2023 ; Piovano+, (2024); Skuopý&Witzany (2024)

Semi-analytic solutions for the orbits

Analytic parts

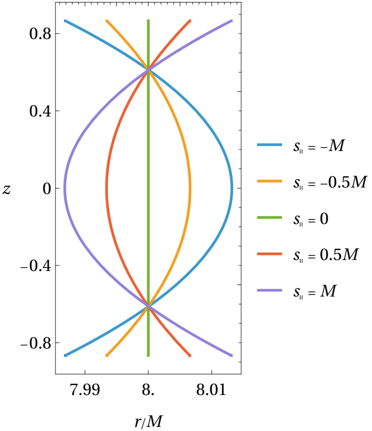
Given in terms of Legendre elliptic integrals

- $\psi(\lambda)$ (Marck, 1983 ; van de Meent, 2020)
- Shifts radial motion $\delta r(\lambda)$
- Correction frequencies $\delta\Upsilon_t, \delta\Upsilon_r, \delta\Upsilon_z, \delta\Upsilon_\phi$
- Shifts constants of motion $\delta E, \delta L_z, \delta K$

Semi - Analytic parts

Given as Fourier series

- Shift polar trajectory $\delta z(\lambda)$
- Shifts $\delta t(\lambda), \delta\phi(\lambda)$



$a = 0.95,\ p_{\rm g} = 8,\ x_{\rm g} = 0.5$
$r = r_{\rm g} + q\chi_\parallel \delta r$

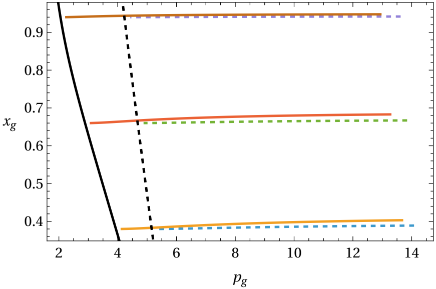
Used Drummond - Hughes (DH) spin gauge (2022): $\langle \delta r \rangle = \langle \delta z \rangle = 0$

Gravitional fluxes and radiation reaction equations

Evolution orbital elements

GW fluxes fully linearized: $\mathcal{F}^{E,L_z} = \mathcal{F}_{\mathrm{g}}^{E,L_z} + q\chi_{\parallel}\delta F^{E,L_z}$

$$p(t) = p_{\mathrm{g}}(t) + q\chi_{\parallel}\delta p(t) \qquad x(t) = x_{\mathrm{g}}(t) + q\chi_{\parallel}\delta x(t)$$



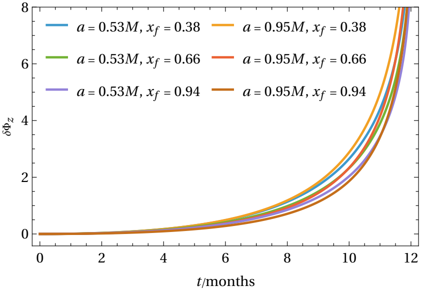
Evolution $p_{\mathrm{g}}, x_{\mathrm{g}}$ for $a = 0.95$ (solid), 0.525 (dashed) and $x_{\mathrm{f}} = 0.38, 0.66, 0.94$.

Inclination evolve slowly (Hughes, 2001)

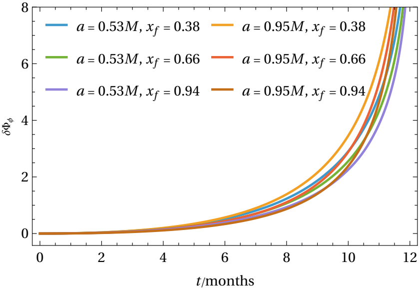
Evolution orbital phases

$$\Phi_z(t) = \Phi_{z\mathrm{g}}(t) + q\chi_{\parallel}\delta\Phi_z(t)$$

$$\Phi_\phi(t) = \Phi_{\phi\mathrm{g}}(t) + q\chi_{\parallel}\delta\Phi_\phi(t)$$



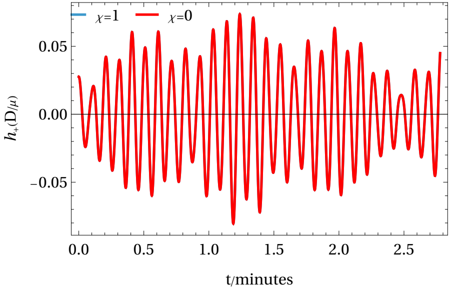
Shift polar phase $q\delta\Phi_z$. $\chi_{\parallel} = 1$

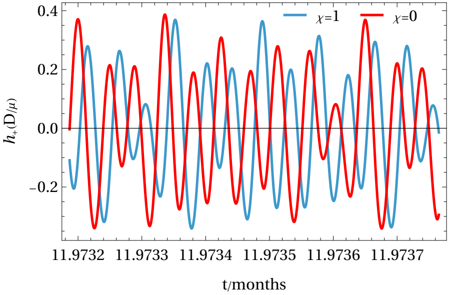


Shift azimuthal phase $q\delta\Phi_\phi$. $\chi_{\parallel} = 1$

Waveforms!

Time domain waveform



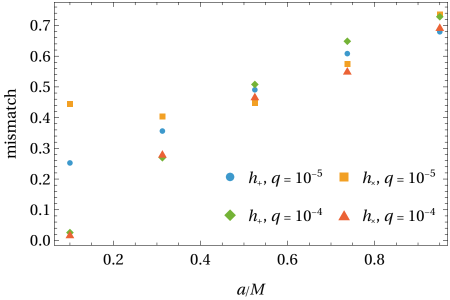


Time-domain waveforms for spinning and non-spinning particle

Frequency-domain waveform computed with Stationary Phase Approximation

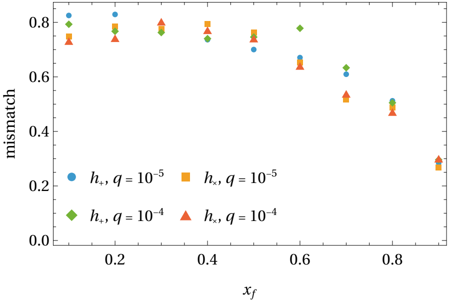
Is the small body spin detectable?

Mismatches waveforms for a spinning and spinless secondary...



...vs primary spin

$x_f = 0.66$, $\chi_\parallel = 1$



...vs inclination

$a = 0.95$, $\chi_\parallel = 1$

Lindblom criterion: two waveforms distinguishable if

$$\mathcal{M} > \frac{\#\ \text{parameters}}{2\text{SNR}^2} \simeq 1.5 \times 10^{-2} \qquad \text{for SNR} = 20$$

Conclusions and future perspective

Conclusions

What we have...

a waveform model for adiabatic inspiral of spinning secondary on quasi-spherical orbits

...and what we observed

Secondary spin effect seems detectable for inclined orbits (in agreement with Fisher forecast with kludge of Cui+,2025)

Future work

- "flux" correction Carter constant (Grant, 2024; Witzany+, 2024; Mathews&Pound, 2025)
- complete inspiral spinning body (the Final frontier)

My main quest:

implement our model into FEW. Perform Bayesian Analysis on χ. Get shiny plots...

Final notes and acknowledgments

- Data available in https://zenodo.org/records/15783358

  (Part) of the code available in
  https://github.com/gabriel-andres-piovano/spherical_inspiral_spinning _particle

- More generic orbits? Check out here:
  https://github.com/gabriel-andres-piovano/Spinning-Body-Hamilton-Jacobi
- Huge thanks to Viktor and Vojtěch for this collaboration
- this work is proudly powered by the almighty BHPToolkit

  https://bhptoolkit.org/

  Don't' be a pirate!

  Use only genuine parts of the BHPToolkit certified by the quality logo[1]

  Be an angel!

  Cite and contribute to the BHPToolkit ~~and the companion paper~~

- Feel free to contact me at gabriel.andres.piovano@ulb.be

[1] unless you REALLY know what you are doing...

Final notes and acknowledgments

- Data available in https://zenodo.org/records/15783358

  (Part) of the code available in
  https://github.com/gabriel-andres-piovano/spherical_inspiral_spinning _particle
- More generic orbits? Check out here:
  https://github.com/gabriel-andres-piovano/Spinning-Body-Hamilton-Jacobi
- Huge thanks to Viktor and Vojtěch for this collaboration
- this work is proudly powered by the almighty BHPToolkit

  https://bhptoolkit.org/

  Don't' be a pirate!

  Use only genuine parts of the BHPToolkit certified by the quality logo[1]

  Be an angel!

  Cite and contribute to the BHPToolkit ~~and the companion paper~~
- Feel free to contact me at gabriel.andres.piovano@ulb.be

Thank you for you attention!

[1] unless you REALLY know what you are doing...

Backup slides

Calculation GW fluxes

- GW fluxes fully linearized: $\mathcal{F}^{E,L_z} = \mathcal{F}_{\mathrm{g}}^{E,L_z} + q\chi_{\parallel}\delta F^{E,L_z}$
- Calculation derivatives wrt $p_{\mathrm{g}}, x_{\mathrm{g}}$ of geodesic trajectories, adiabatic fluxes
- Relative error summation is 10^{-7} (10^{-4}) for fluxes (derivatives fluxes)
- Interpolation equidistant grid on (a, v, x), with

$$p = \frac{6 + \Delta p}{v^2\left(1 + v\left(\sqrt{\frac{6+\Delta p}{r_{\mathrm{ISSO,g}}(a,x)+\Delta p}} - 1\right)\right)^2}$$

- We used *Hermite interpolation* with derivatives
- Interpolation error fluxes on (p, x) for a on grid: 10^{-6}...
- Interpolation error fluxes in between a is $10^{-8} \sim 10^{-2}$ (too few points in a)

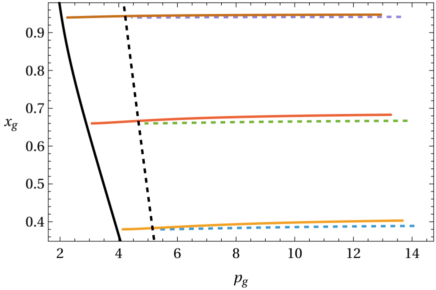
Evolution orbital elements

$$p(t) = p_{\mathrm{g}}(t) + q\chi_{\parallel}\delta p(t) \qquad x(t) = x_{\mathrm{g}}(t) + q\chi_{\parallel}\delta x(t)$$

Evolution equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} p_{\mathrm{g}} \\ x_{\mathrm{g}} \end{pmatrix} = -q\mathbb{J}_{\mathrm{g}}^{-1}\begin{pmatrix} \mathcal{F}_{\mathrm{g}}^{E} \\ \mathcal{F}_{\mathrm{g}}^{J} \end{pmatrix} \qquad \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \delta p \\ \delta x \end{pmatrix} = \begin{pmatrix} \partial_p \delta E_{\mathrm{g}} & \partial_x \delta E_{\mathrm{g}} \\ \partial_p \delta L_{z\mathrm{g}} & \partial_x \delta L_{z\mathrm{g}} \end{pmatrix}\begin{pmatrix} \delta p \\ \delta x \end{pmatrix} + \begin{pmatrix} \delta \dot{p} \\ \delta \dot{x} \end{pmatrix}$$

$$\begin{pmatrix} \delta \dot{p} \\ \delta \dot{x} \end{pmatrix} = -q\mathbb{J}_{\mathrm{g}}^{-1}\left(\delta\mathbb{J}\begin{pmatrix} \dot{p} \\ \dot{x} \end{pmatrix} + q\begin{pmatrix} \delta\mathcal{F}^{E} \\ \delta\mathcal{F}^{E} \end{pmatrix}\right)$$



Evolution $p_{\mathrm{g}}, x_{\mathrm{g}}$ for $a = 0.95$ (solid), 0.525 (dashed) and $x_{\mathrm{f}} = 0.38, 0.66, 0.94$.

Inclination evolve slowly (Hughes, 2001)

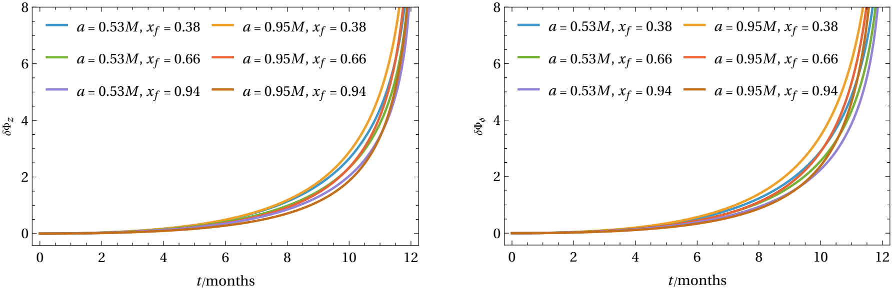
Evolution orbital phases

$$\Phi_z(t) = \Phi_{zg}(t) + q\chi_\parallel \delta\Phi_z(t) \qquad \Phi_\phi(t) = \Phi_{\phi g}(t) + q\chi_\parallel \delta\Phi_\phi(t)$$

Evolution equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\Phi_{zg}\\ \Phi_{\phi g}\end{pmatrix} = \begin{pmatrix}\Omega_{zg}(t)\\ \Omega_{\phi g}(t)\end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\delta\Phi_z\\ \delta\Phi_\phi\end{pmatrix} = \begin{pmatrix}\partial_p\Omega_z & \partial_x\Omega_z\\ \partial_p\Omega_\phi & \partial_x\Omega_\phi\end{pmatrix}\begin{pmatrix}\delta p\\ \delta x\end{pmatrix} + \begin{pmatrix}\delta\Omega_z\\ \delta\Omega_\phi\end{pmatrix}$$



Shift polar phase $q\delta\Phi_z$. $\chi_\parallel = 1$ Shift azimuthal phase $q\delta\Phi_\phi$. $\chi_\parallel = 1$

Does "spherical still evolve into spherical"?

Gold Observation (Quasi-spherical adiabatic inspiral spinning particle)

$$\dot{e} \sim e \quad \text{for} \;\; e \ll 1$$

1st argument. Evolution radial action J_r
In the DH gauge, spin correction radial action $\delta J_r \sim \mathcal{O}(e^2)$. Then $J_r = J_{r\mathrm{g}} + q\chi\delta J_r$

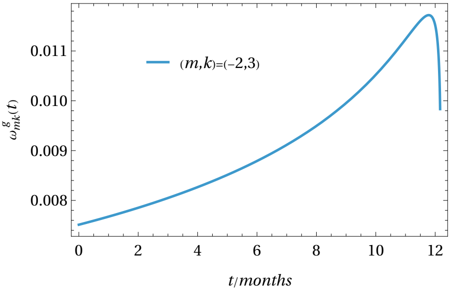
$$\frac{\mathrm{d}J_r}{\mathrm{d}t} \sim e \implies \dot{e} \sim e$$

2nd argument. Virtual Geodesic formalism

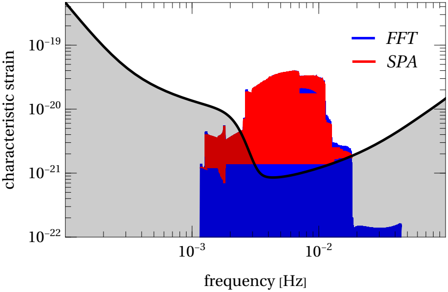
$$x^\mu = \tilde{x}^\mu(\tilde{C}) + q\chi\delta x^\mu(x^\mu_{\mathrm{g}}) \quad \text{(see Viktor's talk)}$$

Virtual Geodesic $\tilde{x}^\mu(\tilde{C})$ stays spherical $\implies$ $e \sim q\chi$ during inspiral

Frequency domain waveform



Example non-monotonic mode.
$a = 0.9$, $x_f = 0.75$

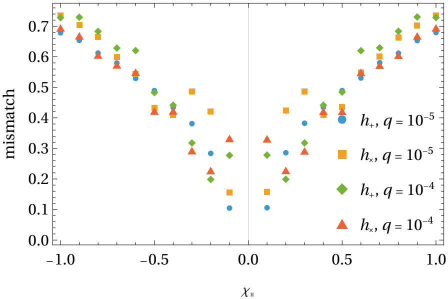


SPA vs FFT
$a = 0.95$, $x_f = 0.66$

We implemented extended Stationary Phase Approximation of Hughes+, 2021

Is the small body spin detectable?

Mismatches waveforms for a spinning and spinless secondary...



...vs secondary spin

$a = 0.95$, $x_f = 0.66$