Using the null stream to detect strongly lensed gravitational waves

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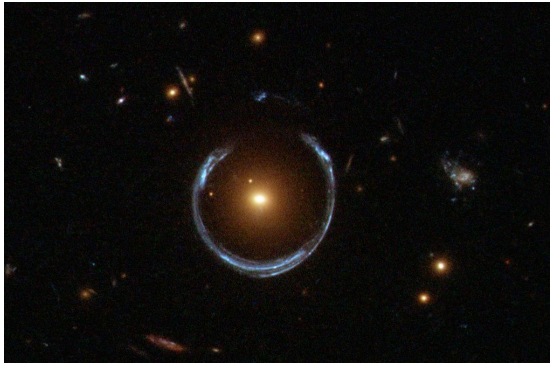
UCLouvain

fnrs FREEDOM TO RESEARCH

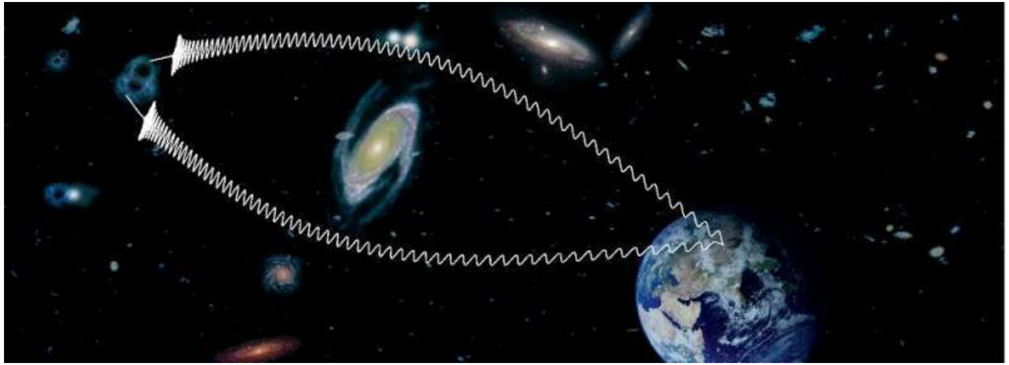
Gravitational wave lensing

- We know EM waves can be deflected due to the spacetime curvature caused by massive bodies along their way

- General relativity predicts the same must happen for gravitational waves



Credits: Hubble



Credits: UC Santa Barbara

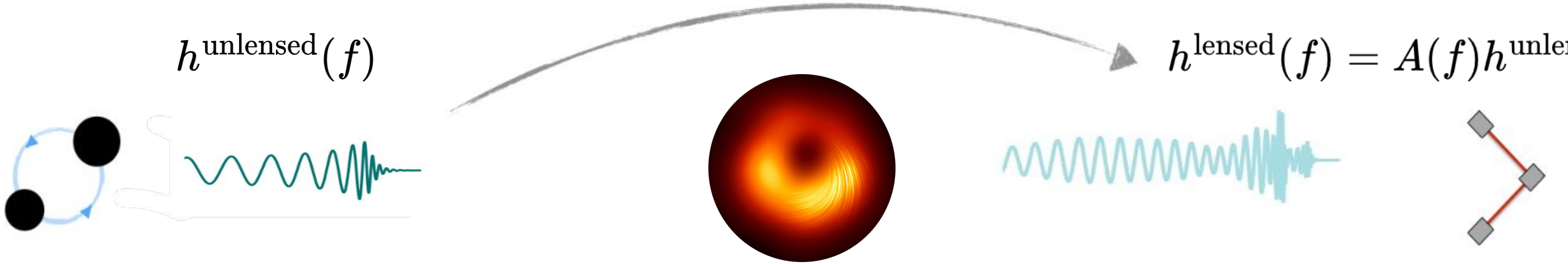
Gravitational wave lensing

Effect of lens on gravitational wave captured by **amplification factor**

$$A(f)$$

$$h^{\text{unlensed}}(f)$$

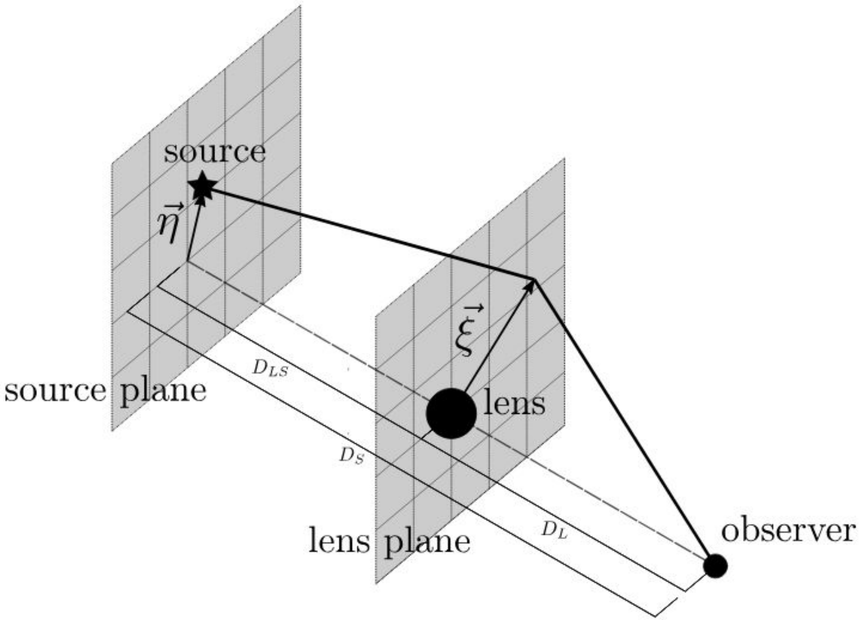
$$h^{\text{lensed}}(f) = A(f)h^{\text{unlensed}}(f)$$



Amplification factor

Amplification factor can be written as a Fresnel diffraction integral:

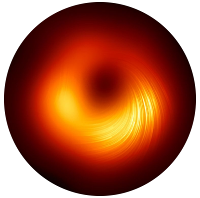
$$A(f) \propto \int d\vec{\xi}\, e^{2\pi i f\, t_d(\vec{\xi},\vec{\eta})}$$



Source: Cheung et al. 2021

Different regimes of lensing

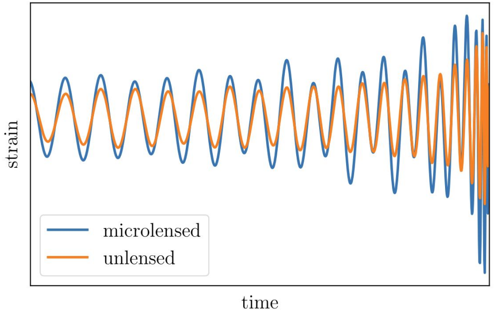
Wave optics: $\lambda_{GW} \sim R_S$



Full integral has to be solved

↓

frequency dependent modulation of signal



Geometric optics: $\lambda_{GW} \ll R_S$

Stationary phase approximation applies

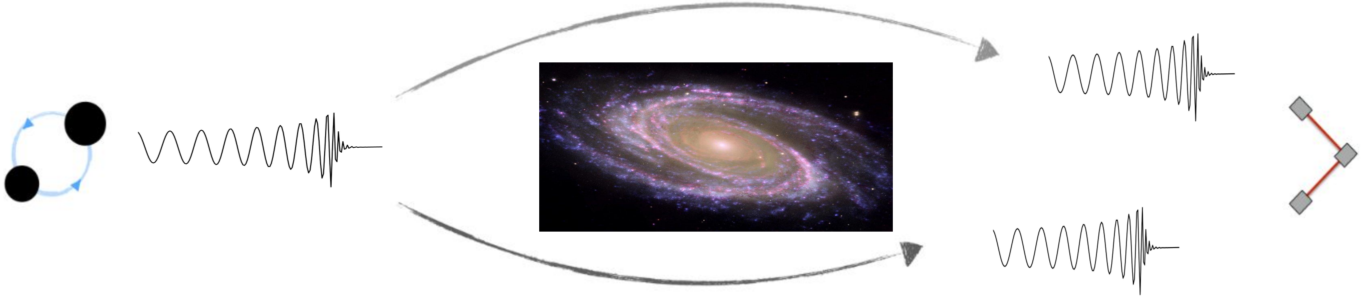
↓

Only contributions of points where time delay is stationary: $$\frac{\partial t_d\left(\vec{x},\vec{y}\right)}{\partial \vec{x}} = 0$$

Amplification factor

In geometric optics, the amplification factor reduces to sum of “images”

$$A(f) = \frac{h^{\text{lensed}}(f)}{h^{\text{unlensed}}(f)} = \sum_j^N \sqrt{|\mu_j|}\, e^{2\pi i f \Delta t_j - i\pi n_j}$$



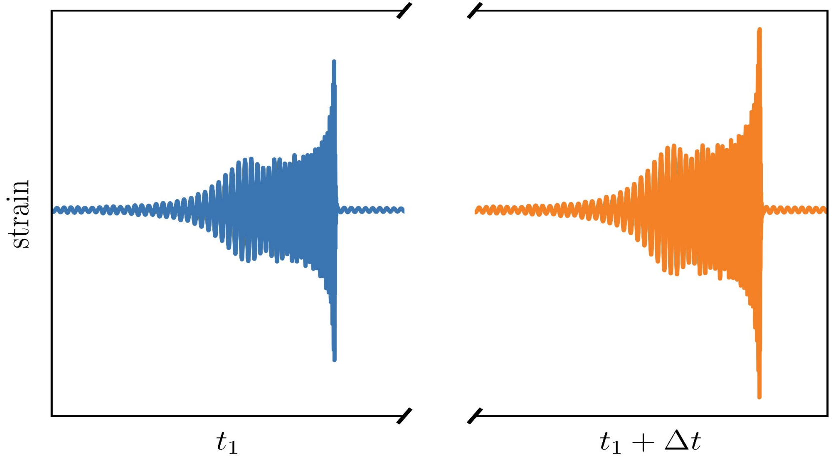
Strong lensing

$$A(f) = \frac{h^{\text{lensed}}(f)}{h^{\text{unlensed}}(f)} = \sum_j^N \sqrt{|\mu_j|} e^{2\pi i f \Delta t_j - i\pi n_j}$$

Morse phase

magnification

time delay



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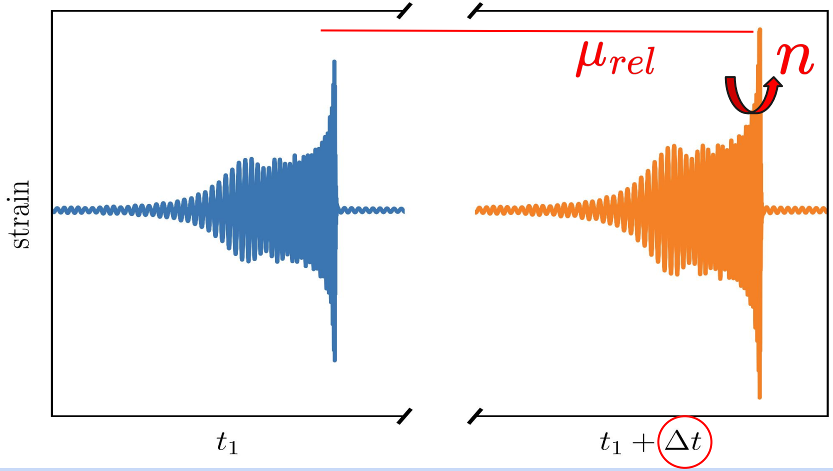
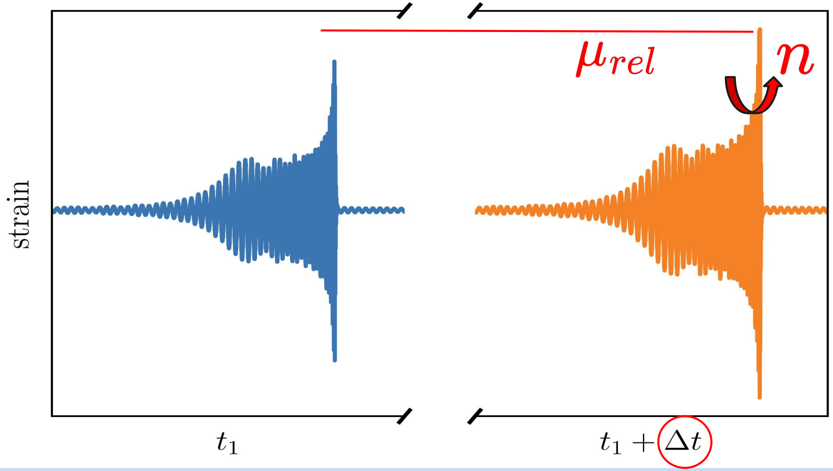


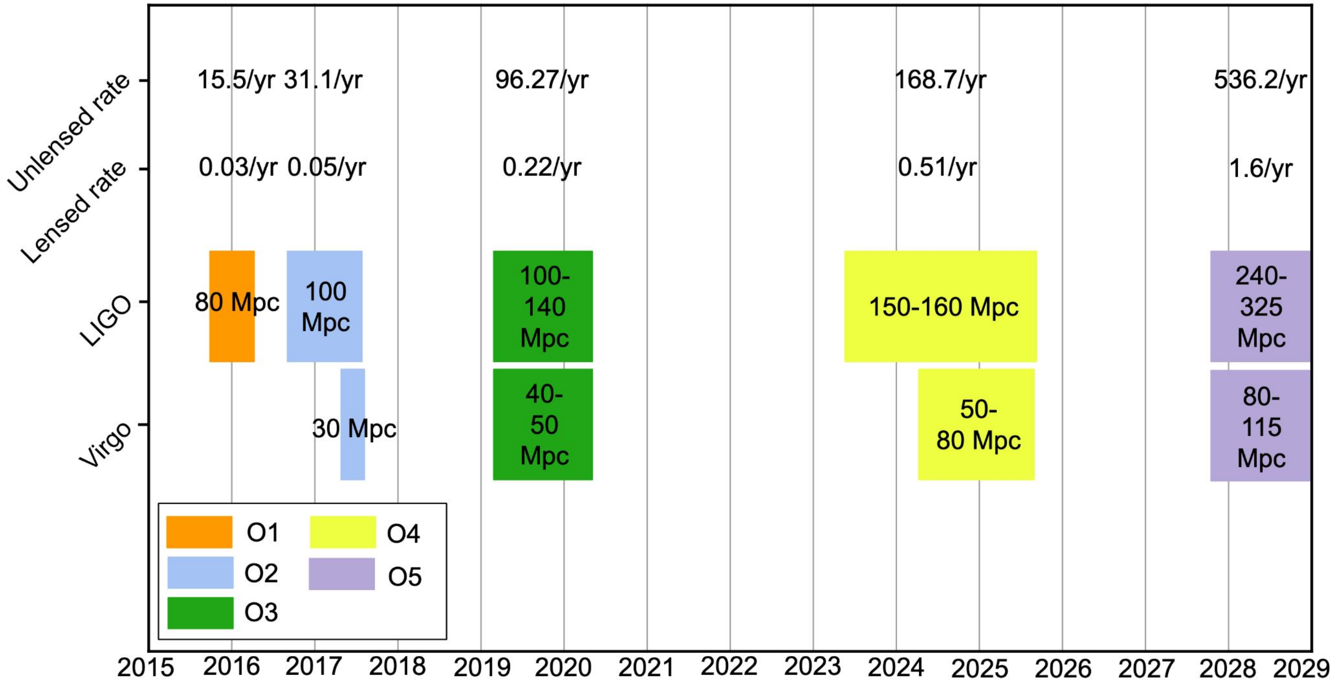
Strong lensing

Repeated magnified and phase shifted copies of the same GW



Strong lensing rate

Based on population of galaxies, estimates of lensing rates can be made (e.g. Ng et al, 2018; Wierda et al, 2021; Xu et al, 2022)

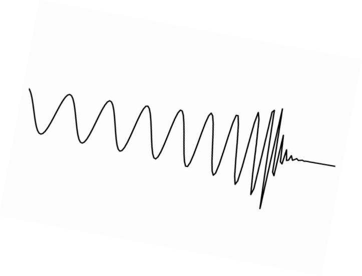


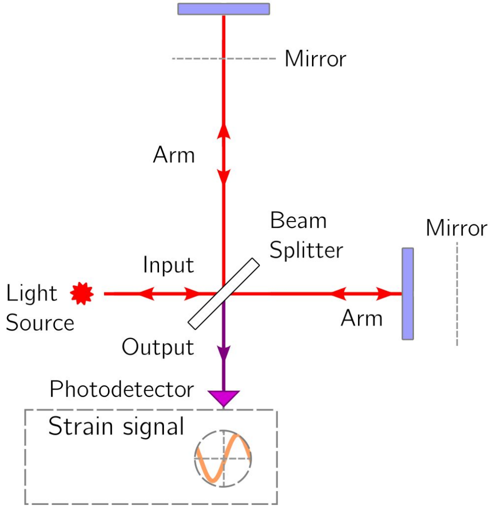
Gravitational wave data

- GW polarizations are coupled to **antenna response functions**

- In detector ρ, we have the data stream

$$d_\rho(t) = F_\rho^+(\alpha, \delta) h_+(t - \Delta t_\rho) + F_\rho^\times(\alpha, \delta) h_\times(t - \Delta t_\rho) + n_\rho(t)$$



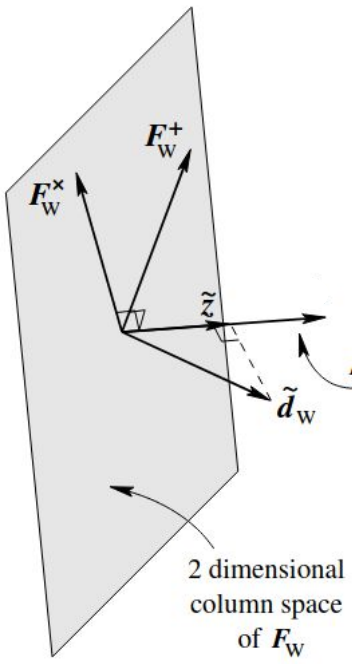


Null stream

- **Null stream:** linear combination of data streams in detector network such that GW signal is cancelled:

Find projection matrix that projects GW signal away →

$$\sum_{\sigma}^{D} P_{\rho\sigma}^{\text{null}} d_{\sigma} = n_{\rho}$$



Credits: Chatterji et al. 2006

Null stream

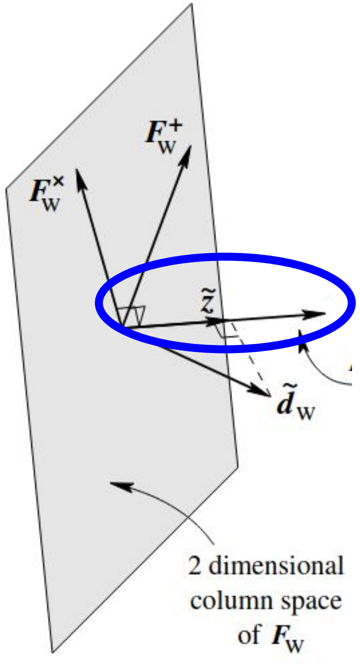
- **Null stream:** linear combination of data streams in detector network such that GW signal is cancelled:

Find projection matrix that projects GW signal away →

$$\sum_{\sigma}^{D} P_{\rho\sigma}^{\text{null}} d_{\sigma} = n_{\rho}$$

- For that, write in matrix form:

$$\tilde{\mathbf{d}} = \mathbf{F}\tilde{\mathbf{h}} + \tilde{\mathbf{n}} \qquad \mathbf{F} = \begin{bmatrix} F_1^{+}(\alpha, \delta, t_c) & F_1^{\times}(\alpha, \delta, t_c) \\ \vdots & \vdots \\ F_D^{+}(\alpha, \delta, t_c) & F_D^{\times}(\alpha, \delta, t_c) \end{bmatrix} \qquad \tilde{\mathbf{h}} = \begin{bmatrix} h_{+} \\ h_{\times} \end{bmatrix}$$

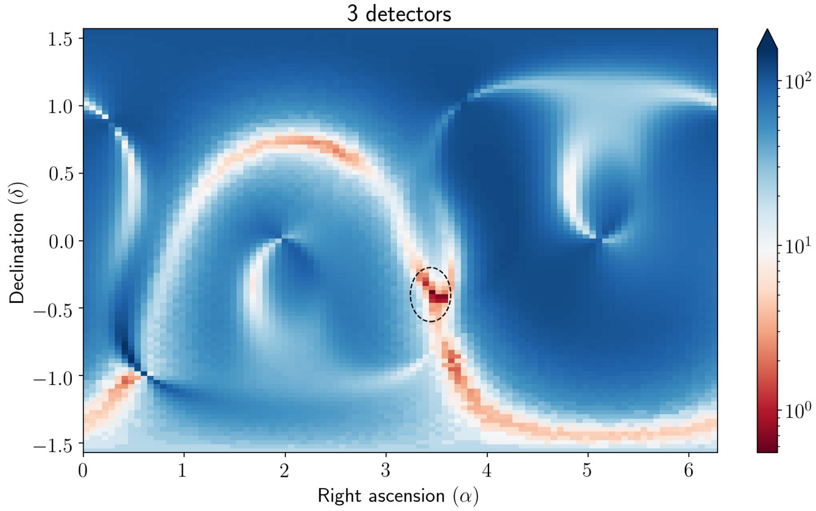


Credits: Chatterji et al. 2006

Null stream

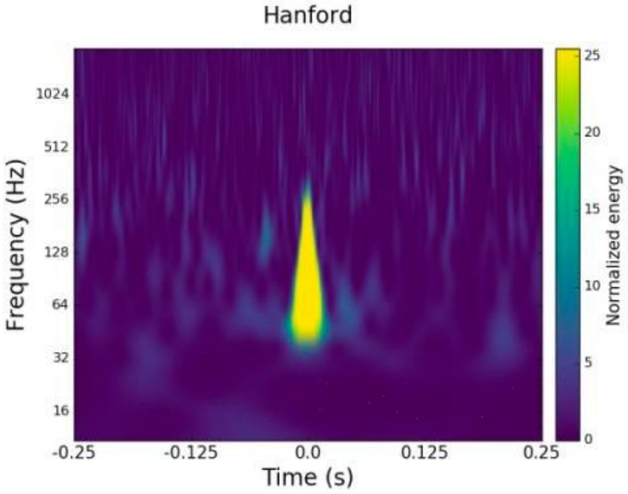
- **Null operator** is then constructed as: $P^{\text{null}} = \mathbf{1} - \mathbf{F}(\mathbf{F}^{\dagger}\mathbf{F})^{-1}\mathbf{F}^{\dagger}$

- **Null energy** is inner product of null stream with itself: $E_{\text{null}} = \tilde{\mathbf{d}}^{\dagger} P^{\text{null}} \tilde{\mathbf{d}}$

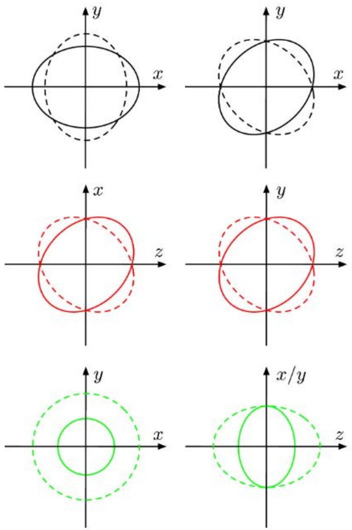


Use cases

Burst search pipelines to veto between glitch and burst by looking at coherent energy [Sutton et al. 2010, Drago et al. 2020]



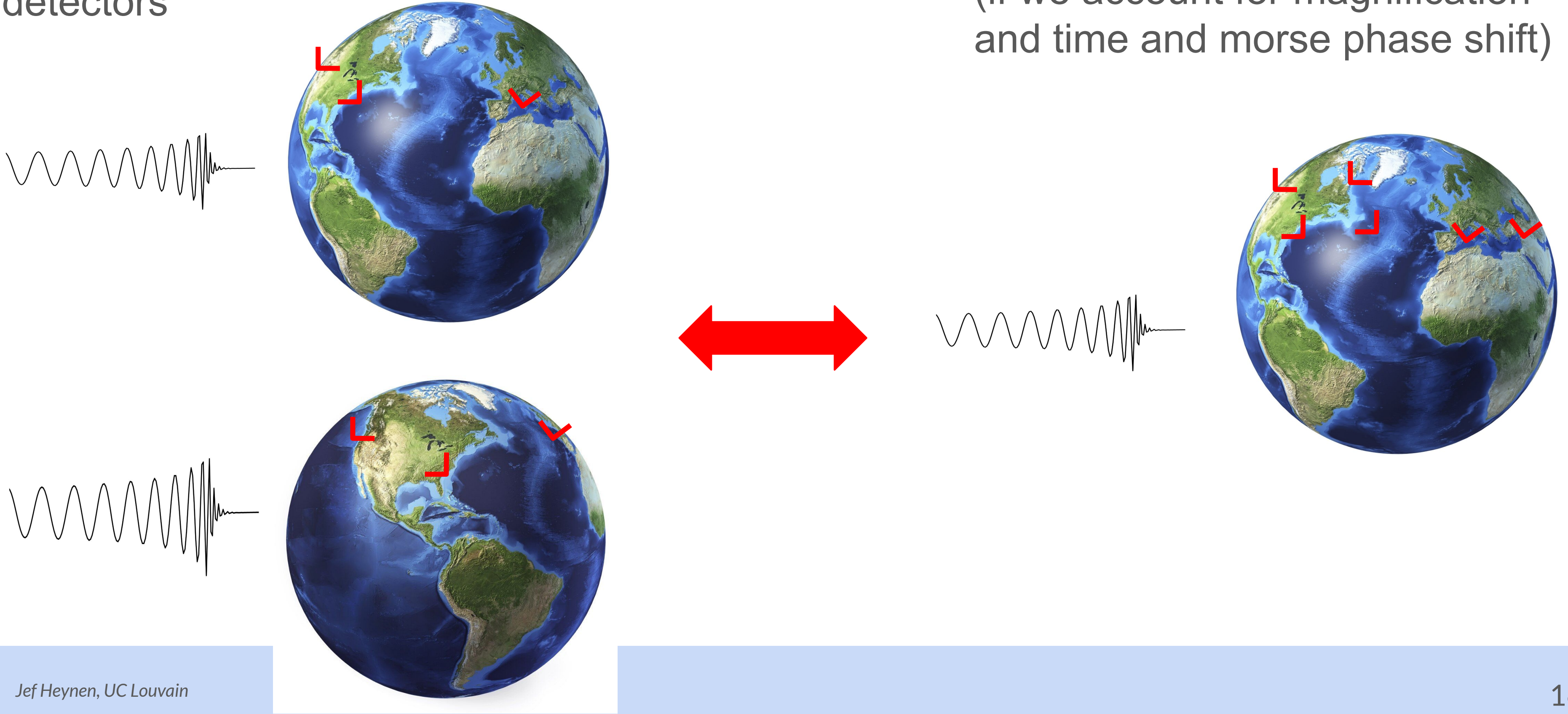
TGR pipelines to search for extra polarizations [Wong et al. 2021]



Strong lensing

Strongly lensed pair detected by D and D' detectors

One detection by D+D'
(if we account for magnification and time and morse phase shift)



Strong lensing null stream

Strongly lensed pair:

$$\tilde{h}_1^L(f) = \sqrt{\mu_1} e^{2\pi i f t_1 - i\pi n_1} \tilde{h}^U(f) \qquad \tilde{h}_2^L(f) = \sqrt{\mu_2} e^{2\pi i f t_2 - i\pi n_2} \tilde{h}^U(f)$$

Construct null projector in the same way as before but with modified **F:**

$$P^{\mathrm{null}} = \mathbf{1} - \mathbf{F}(\mathbf{F}^{\dagger}\mathbf{F})^{-1}\mathbf{F}^{\dagger}$$

$$\mathbf{F} = \begin{bmatrix} F_1^{+}(\alpha,\delta) & F_1^{\times}(\alpha,\delta) \\ \vdots & \vdots \\ F_D^{+}(\alpha,\delta) & F_D^{\times}(\alpha,\delta) \\ \sqrt{\mu_{21}} e^{-i\pi n_{21}} F_{1'}^{+}(\alpha,\delta,t_{21}) & \sqrt{\mu_{21}} e^{-i\pi n_{21}} F_{1'}^{\times}(\alpha,\delta,t_{21}) \\ \vdots & \vdots \\ \sqrt{\mu_{21}} e^{-i\pi n_{21}} F_{D'}^{+}(\alpha,\delta,t_{21}) & \sqrt{\mu_{21}} e^{-i\pi n_{21}} F_{D'}^{\times}(\alpha,\delta,t_{21}) \end{bmatrix}$$

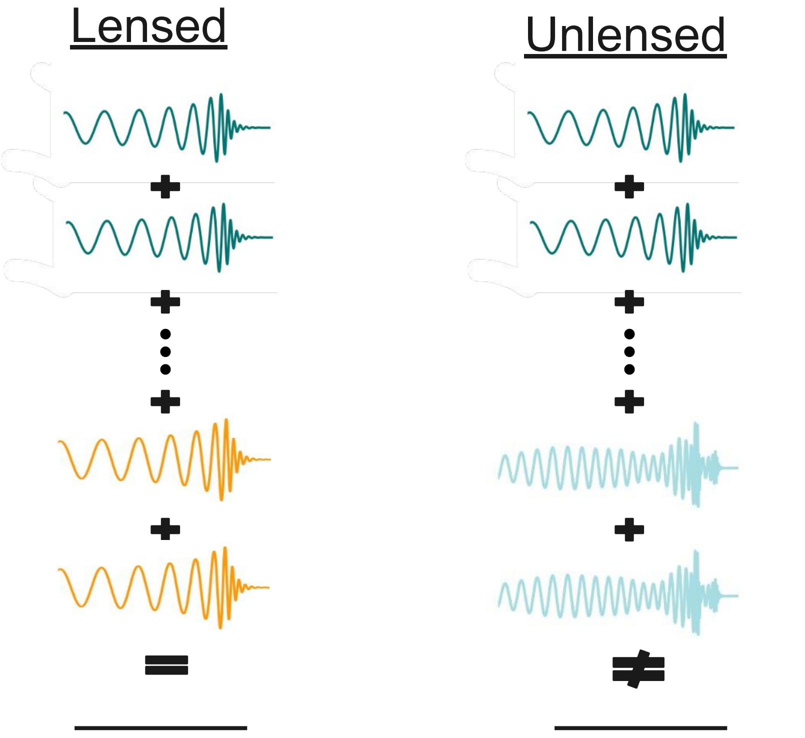
$\rightarrow$ P^{null} will project away strongly lensed GW signal

Strong lensing null stream

- Only valid when the pair is **genuinely lensed** and at **true parameters**

- When pair is unrelated, signal will not be projected away, leaving residual null energy

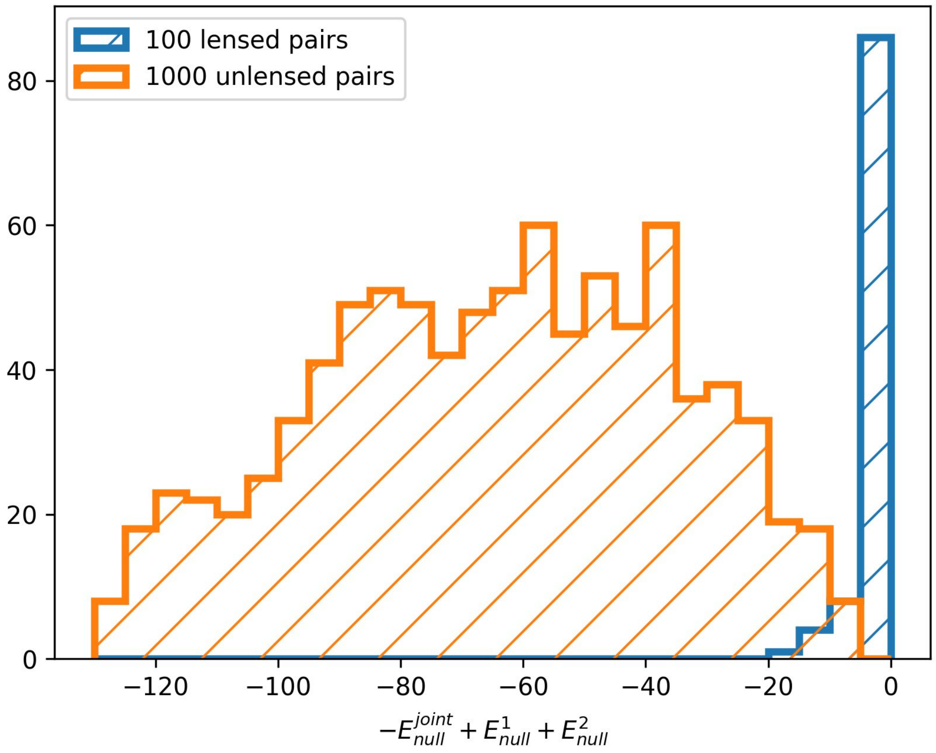
- In case of no noise: $\rightarrow$



$E_{\text{null}} \approx 0$ $E_{\text{null}} > 0$

Validity

- Take a reconstruction of the waveform (here: maximum likelihood waveform obtained from PE)
- Sample over μ_{21}, n_{21}, t_{21}, α, □
- Compute null energy of each sample
- Take mean of obtained distribution

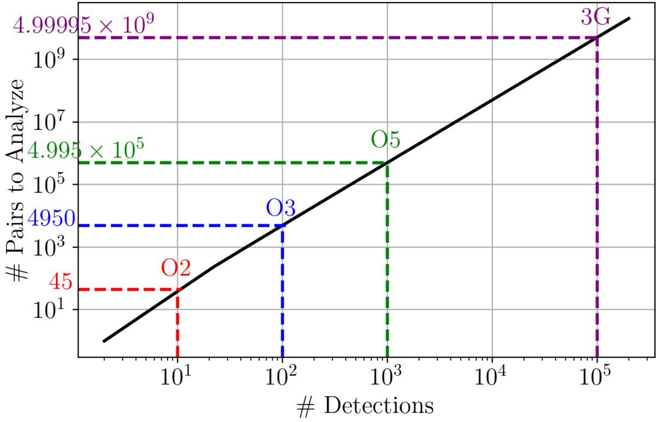


Final objective

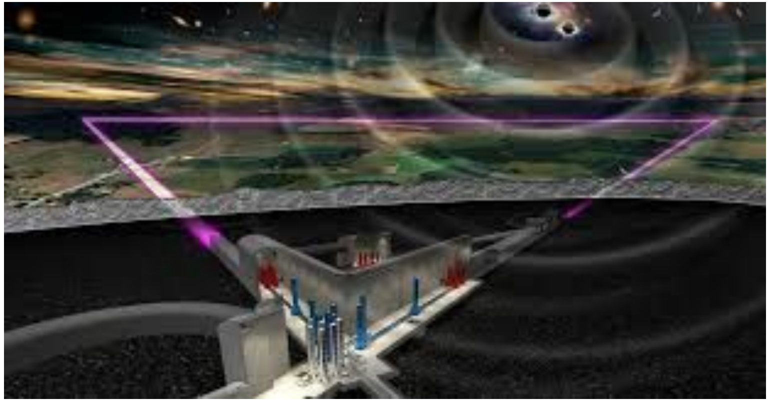
- Make this low-latency pipeline to test lensing hypothesis without relying on PE results

- A quick way to filter out unrelated pairs

- Waveform independent → important for long signals



Credits: Justin Janquart



Credits: Einstein Telescope

PE independence

- Use statistical properties:
    - Under gaussian and stationary noise: $E_{\text{null}} = \tilde{n}_\alpha^\dagger P_{\alpha\beta}^{\text{null}} \tilde{n}_\beta \sim \chi^2_{\text{DoF}}$
- Use network correlation coefficient [Klimenko et al. 2008] as a veto test: $\frac{E_{\text{coh}}}{E_{\text{null}} + |E_{\text{coh}}|}$
- Accurate time-frequency analysis required

Conclusions

- Number of strong lensing candidates will drastically increase the coming years
- Quick way needed to filter out unrelated events
- Strong lensing null stream can be used to look for related pairs
- Making this idea PE independent still work in progress

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Thank you for listening!

Low latency

- Go full Bayesian:
  - Under gaussian and stationary noise: $E_{\mathrm{null}} = \tilde{n}_\alpha^\dagger P^{\mathrm{null}}_{\alpha\beta} \tilde{n}_\beta \sim \chi^2_{\mathrm{DoF}}$
  - Sample with chi-squared likelihood and get Bayes factor
  - Need a good time and frequency analysis to make this efficient
- Go cWB
  - Use network correlation coefficient [Klimenko et al. 2008] as a veto test:

$$\frac{E_{\mathrm{coh}}}{E_{\mathrm{null}} + |E_{\mathrm{coh}}|}$$

Thank you for listening!