Equation of State at finite temperature in the era of new nuclear physics and multi-messenger constraints

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Physics across scales

Nuclei and Neutron Star and Equation of state

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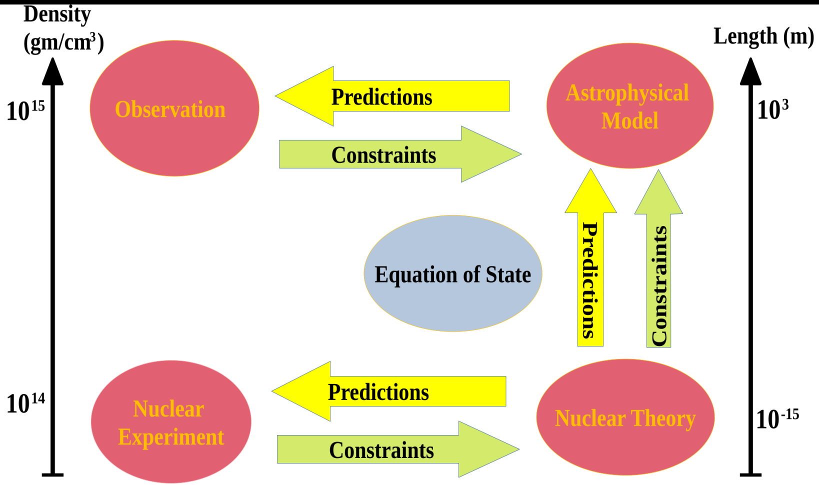

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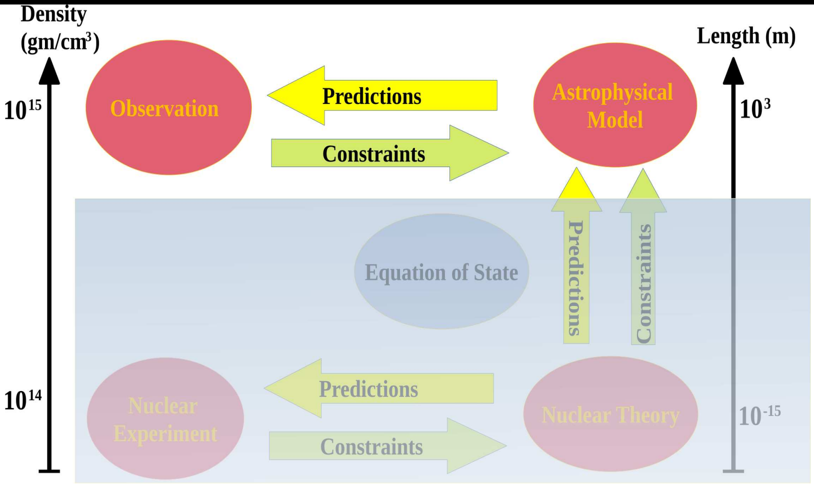
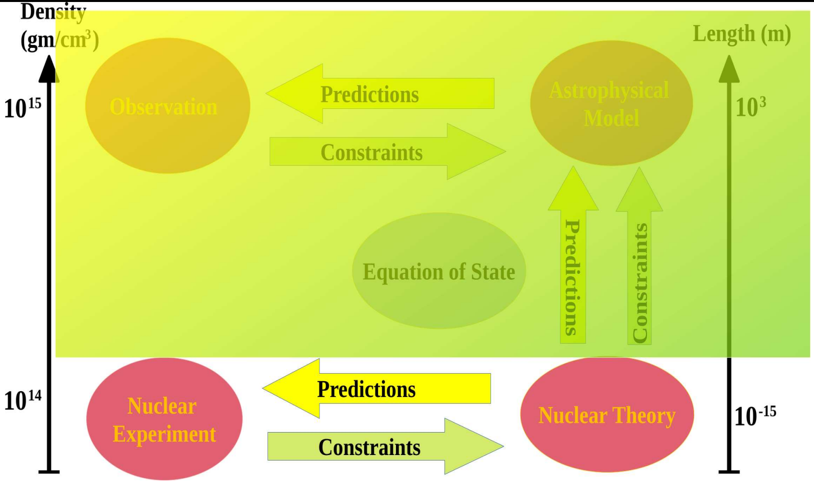
Nuclei and Neutron Star and Equation of state



$$\mathcal{H}\psi = \mathrm{E}\psi \rightarrow < \psi|\mathcal{O}|\psi > \implies \mathrm{P}(\epsilon).$$

Physics across scales

Nuclei and Neutron Star and Equation of state

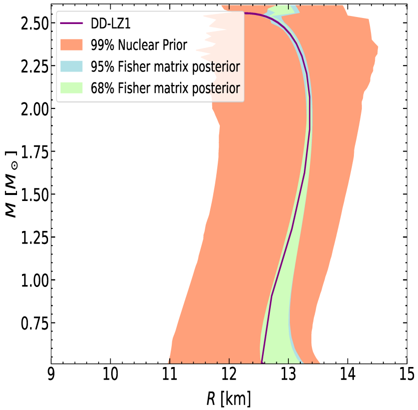


$$P(\epsilon) \to G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}.$$

NS Radius from GW detection

Einstein Telescope case

Solve Tolman-Oppenheimer-Volkoff (TOV) equations, we can construct the unique M-R or Λ-M(R) relations.

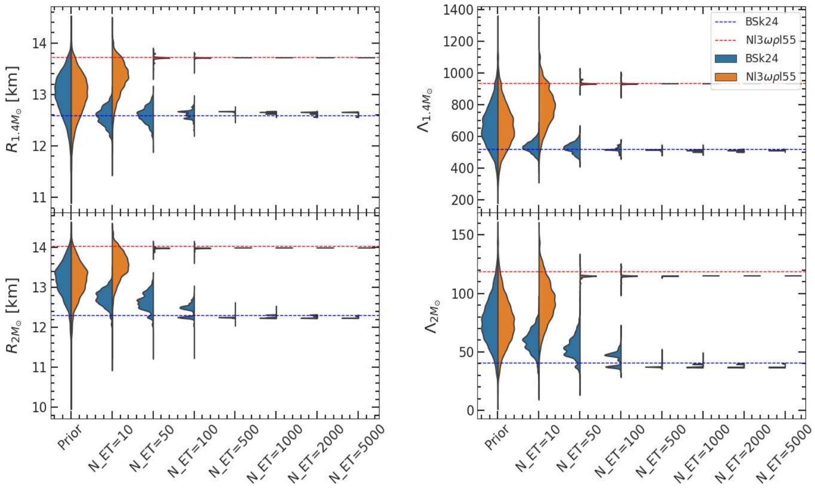


ET CoBA study, Branchesi, CM et.al., JCAP 07, 068 (2023)

Information from the inspiral:

- The joint tidal $\tilde{\Lambda}$ not Λ_1, Λ_2
- The chirp mass $\mathcal{M}_c$ not m_1, m_2
- The GW170817 case.
- Prospect of many NS-NS in ET with 3G detectors

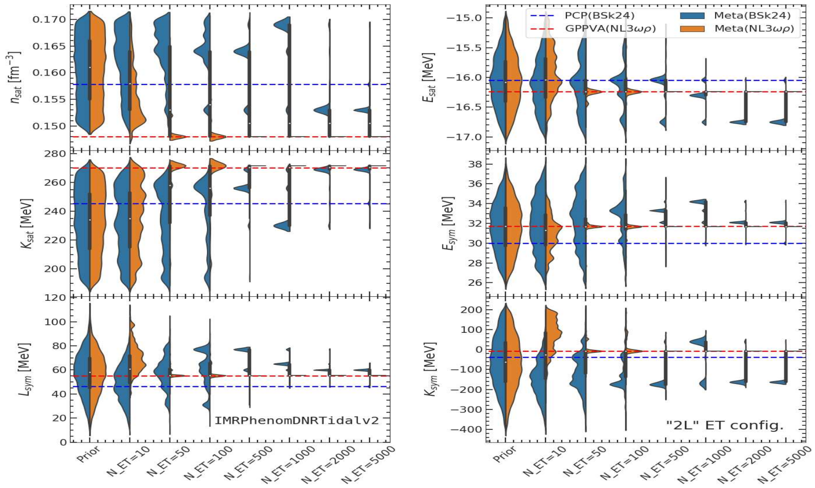
N_{det}	$R_{1.4M_\odot}{}^{+\Delta R_+}_{-\Delta R_-}$ [km]
Prior	$12.983^{+0.420}_{-0.420}$
54	$13.163^{+0.221}_{-0.227}$
592	$13.146^{+0.122}_{-0.136}$
5970	$13.107^{+0.148}_{-0.037}$



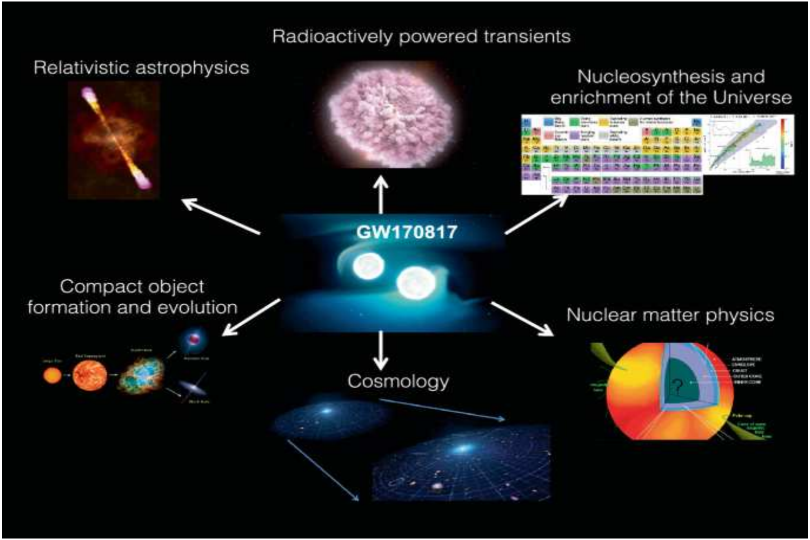
CM et.al., PRD 108, 122006 (2023)

Stacking GW data to learn Nuclear Physics

Einstein Telescope Case



CM et.al., PRD 108, 122006 (2023)



Branchesi 2023. Conf. article springer.

Effects of temperature

Merger simulations

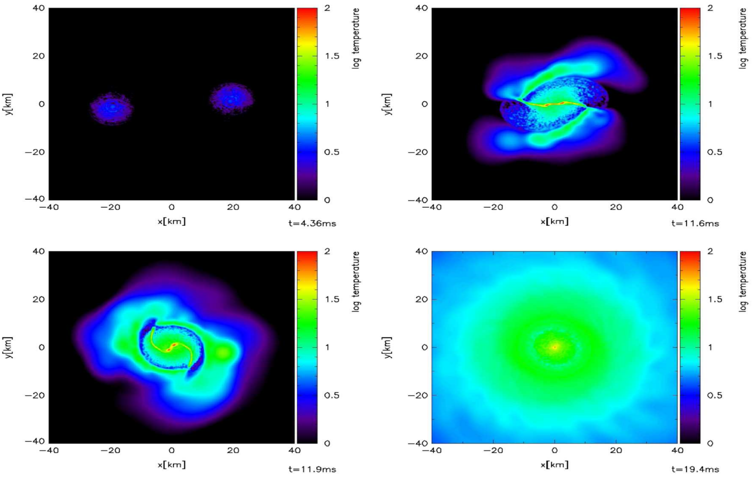
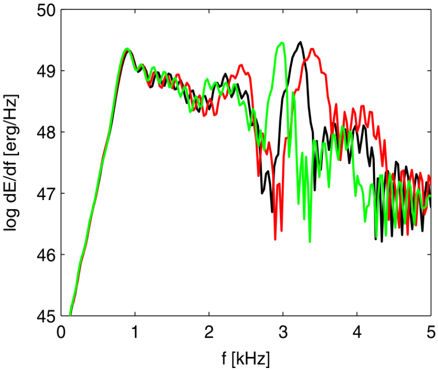


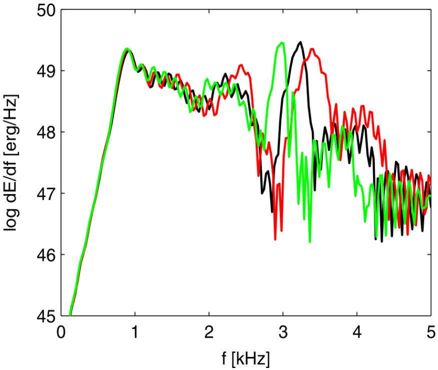
Fig Courtesy:
Bauswein *et. al.* PRD 82, 084043 (2010)



- Luminosity spectra for $M_1 = M_2 = 1.35 M_\odot$ binary system

Effects of temperature

Merger simulations



- Luminosity spectra for $M_1 = M_2 = 1.35 M_{\odot}$ binary system
- Ideal gas index: $\Gamma_{th}(n, T, x_p) = \frac{P_{th}}{n\mathcal{E}_{th}} + 1$.
- Red:$\Gamma_{th} = 1.5$; Green:$\Gamma_{th} = 2$; **Black**: Full temp dep. The frequencies can vary from 50-250 Hz.

- End-to-end NS merger simulations =>
  Hydrodynamics, Nucleosynthesis, radiative transfer
  [See Just *et. al.* ApJL 951, 12 (2023), MNRAS 510, 2804 (2022), MNRAS 510, 2820 (2022)]
- Reaction rates can be calculated with BSkG3 inputs including the masses.
- For the hydrodynamics part the EoS of hot dense matter is needed.

Main Features:

- Finite Nuclei: HFB equations on the grid, fitted on whole mass table.
  $\sigma_{\mathrm{M}}^{rms} = 0.631$ MeV
  $\sigma_{r_{\mathrm{ch}}} = 0.0237$ fm
  $\sigma_{\mathrm{V}_{\mathrm{fiss}}} = 0.33$ MeV
- Nuclear matter:
  Constraints on effective mass, neutron matter, pairing gaps.
- Neutron stars:
  Fulfill the observational constraint on massive pulsars.

Energy Density Functional

Brussels Skyrme model

- At baryon density n, asymmetry $\delta\left(=\frac{\mathrm{n_n}-\mathrm{n_p}}{\mathrm{n}}\right)$, temperature T,

$$
\begin{aligned}
&\mathcal{F} \equiv \mathcal{E} - \mathrm{T}\mathcal{S}, \\
&\text{where } \mathcal{E} = \sum_{\mathrm{q}} \frac{\hbar^2}{2\mathrm{M}_\mathrm{q}^*}\tau_\mathrm{q} + \frac{1}{8}\mathrm{t}_0\left\{3 - (2\mathrm{x}_0+1)\delta^2\right\}\mathrm{n}^2 \\
&+\frac{1}{48}\mathrm{t}_3\left\{3 - (2\mathrm{x}_3+1)\delta^2\right\}\mathrm{n}^{\alpha+2}. \\
&\text{with } \frac{\hbar^2}{2\mathrm{M}_\mathrm{q}^*} = \frac{\hbar^2}{2\mathrm{M}_\mathrm{q}} + f\left(\mathrm{n},\delta\right)
\end{aligned}
$$

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$$+\frac{1}{48}\mathrm{t_3}\left\{3 - (2\mathrm{x_3}+1)\delta^2\right\}\mathrm{n}^{\alpha+2}.$$

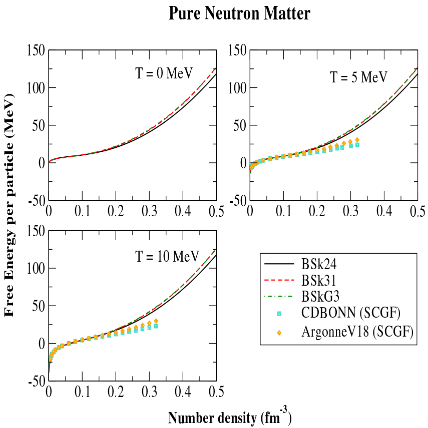
$$\text{with } \frac{\hbar^2}{2\mathrm{M}^*_{\mathrm{q}}} = \frac{\hbar^2}{2\mathrm{M_q}} + f(\mathrm{n}, \delta)$$

- The density and kinetic density involves Fermi integrals (q=n,p):

$$\mathrm{n_q} = \frac{1}{2\pi^2}\left(\frac{2\mathrm{M}^*_{\mathrm{q}}}{\hbar^2}\right)^{\frac{3}{2}}\mathrm{T}^{\frac{3}{2}}\mathrm{I}_{\frac{1}{2}}(\nu_{\mathrm{q}}); \quad \tau_{\mathrm{q}} = \frac{1}{2\pi^2}\left(\frac{2\mathrm{M}^*_{\mathrm{q}}}{\hbar^2}\right)^{\frac{5}{2}}\mathrm{T}^{\frac{5}{2}}\mathrm{I}_{\frac{3}{2}}(\nu_{\mathrm{q}})$$

where, $\mathrm{I}_\sigma(\nu_{\mathrm{q}}) = \int_0^\infty \frac{x^\sigma}{1+\exp(x-\nu_{\mathrm{q}})}dx$

Free energy

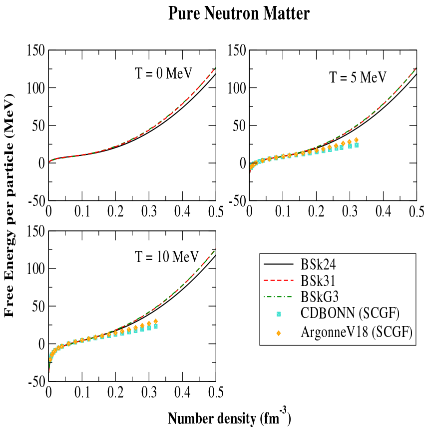


CM and N. Chamel

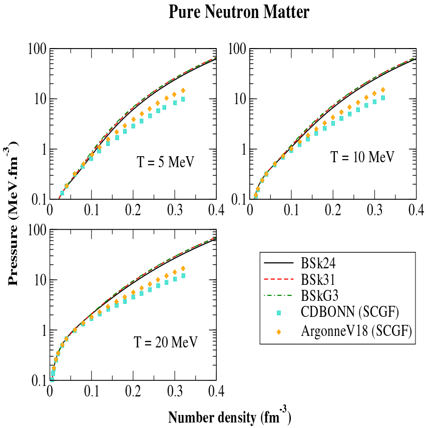
Comparison with Ab-initio models

Pure Neutron Matter

Free energy



Pressure



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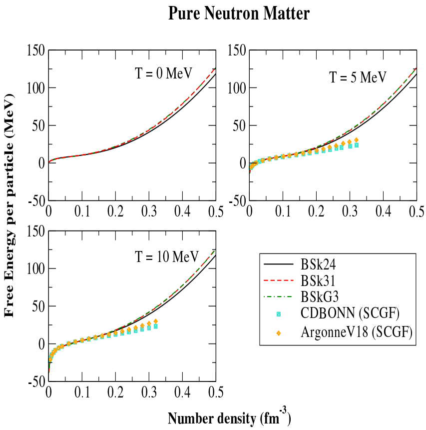
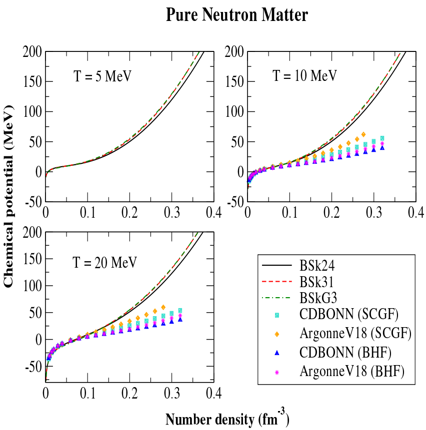


Chemical Potential



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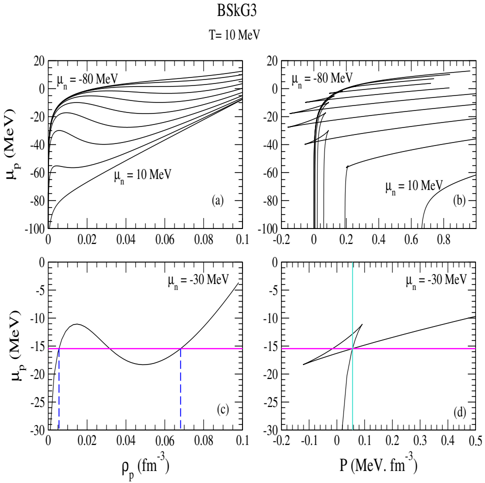
Gibb's construction



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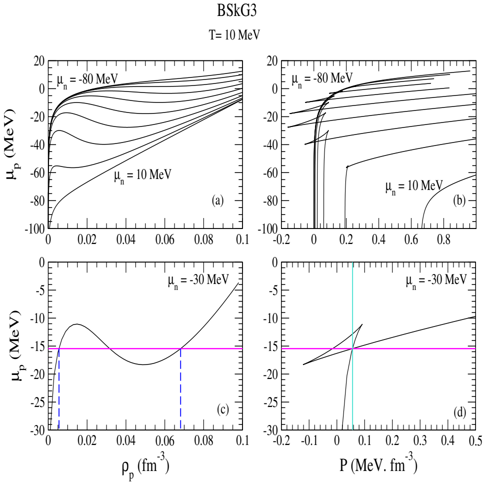

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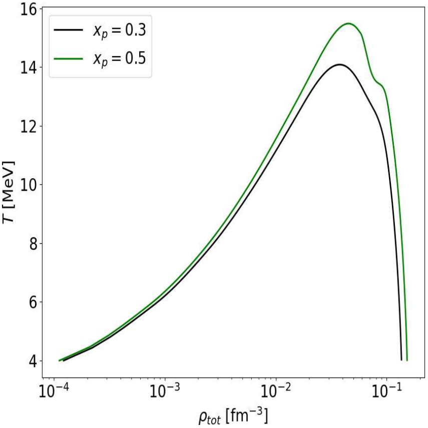
Coexistence



WS cell at finite temperature

Thomas Fermi (fast approximation of HFB)

Form of the density:

$$\mathrm{n_q(r) = n_{B,q} + \frac{n_{0,q}}{1 + \exp\left\{\left(\frac{C_q - R_{WS}}{r - R_{WS}}\right)^2 - 1\right\}\exp\left(\frac{r - C_q}{a_q}\right)}}$$

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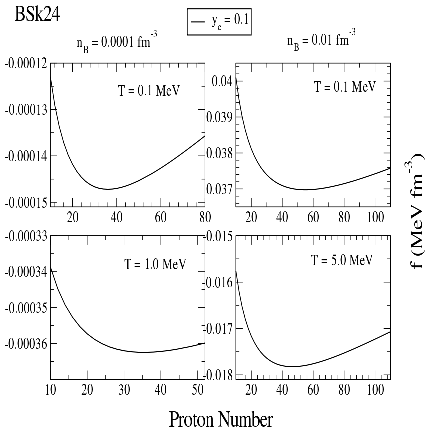
Chemical potential for damped profile [Chamel, **Shchechilin**, and Chugunov, PRC 111, 015805 (2025).]:

$$\tilde{\mu}_\mathrm{q} = \frac{3}{\mathrm{R_{WS}^3}}\int_0^{\mathrm{R_{WS}}} \mathrm{dr}\cdot \mathrm{r}^2 \frac{\partial\mathcal{F}(\mathrm{r})}{\partial \mathrm{n_q(r)}},$$

$$\mathrm{n_q} = \frac{1}{2\pi^2}\left(\frac{2\mathrm{M_q^*}}{\hbar^2}\right)^{\frac{3}{2}}\mathrm{T}^{\frac{3}{2}}\mathrm{I}_{\frac{1}{2}}(\nu_\mathrm{q});\quad \tau_\mathrm{q} = \frac{1}{2\pi^2}\left(\frac{2\mathrm{M_q^*}}{\hbar^2}\right)^{\frac{5}{2}}\mathrm{T}^{\frac{5}{2}}\mathrm{I}_{\frac{3}{2}}(\nu_\mathrm{q});$$

$$\mathrm{S_q} = \frac{5}{3\mathrm{T}}\frac{\hbar^2}{2\mathrm{M_q^*}}\tau_\mathrm{q} - \nu_\mathrm{q}\mathrm{n_q}.$$

Given ρ, T, y_e



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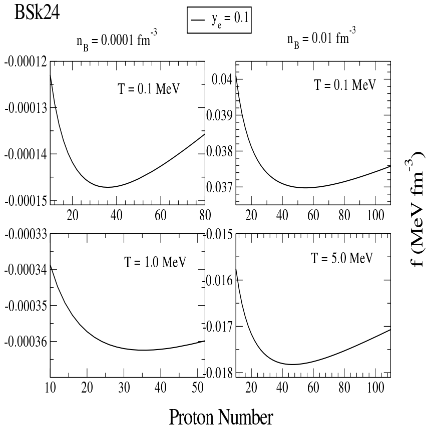
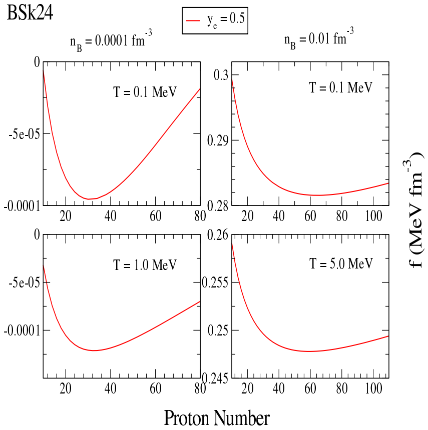
Clusters at finite temperature

Full grid

Given ρ, T, y_e





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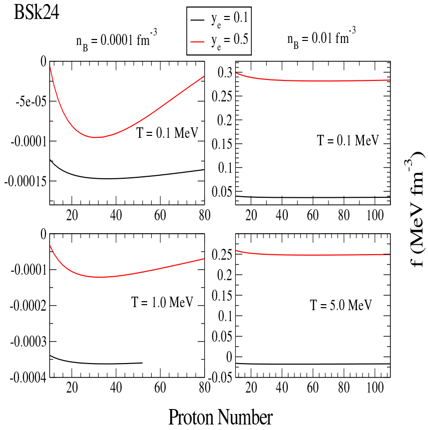
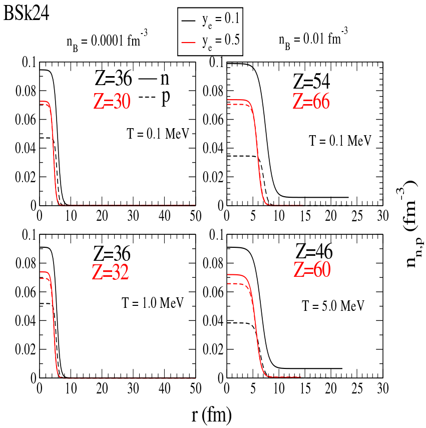


Profiles



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Prospects

- Unified equation of state at finite temperature.

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- Merger simulations using BSkG models are being explored.