# The Doppler boosted LISA response to GWs

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Based on **arXiv:2509.10038**
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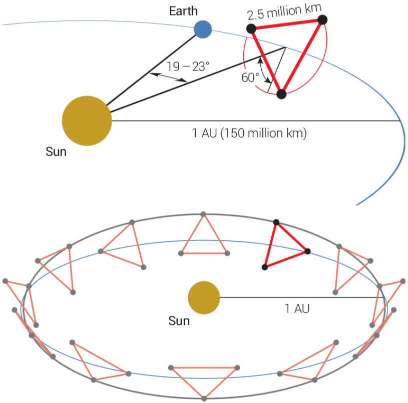
# The LISA instrument

- Next-gen detector in the mHz band
- Constellation of three orbiting spacecraft
- Trailing/leading earth $\Rightarrow$ 1 year orbit
- S/C velocities $\beta = \mathrm{v}/c \sim 10^{-4}$

For the response, the S/C are treated as static during the light travel times.




[LISA: 1702.00786]

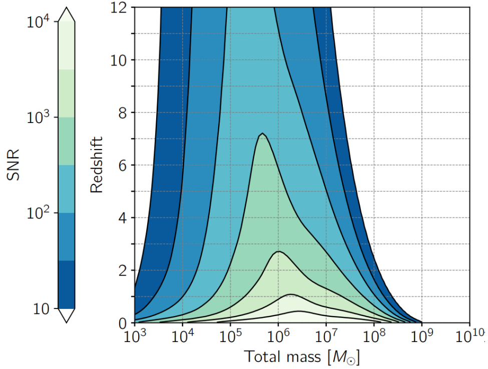
# Massive Black Hole Binaries

- Loudest sources in the LISA band
- Total SNRs up to thousands

Requires leap in modeling:

- Accurate waveform models
- **Accurate instrument simulation**

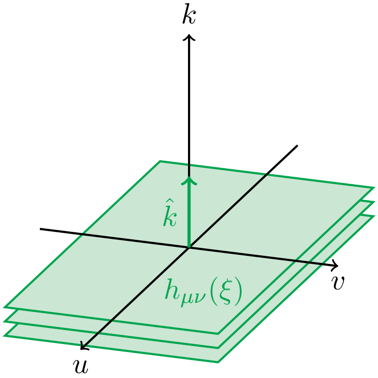
Do we need to include velocity-corrections $\beta \sim 10^{-4}$ in the LISA response to GWs?




**[LISA Redbook: 2402.07571]**

# GW response (stationary spacecraft)

- GW on Minkowski background: $\eta_{\mu\nu} + h_{\mu\nu}(\xi)$
- Plane waves in $\hat{k}$-direction: $\xi = ct - \hat{k} \cdot \vec{x}$




[Cornish+2003, Finn2009, ...]

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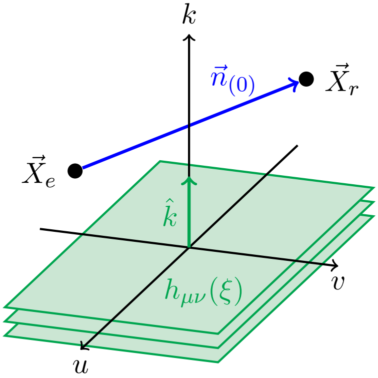


**[Cornish+2003, Finn2009, ...]**

# GW response (stationary spacecraft)

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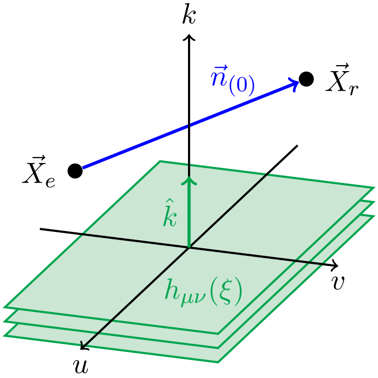
- Plane waves in $\hat{k}$-direction: $\xi = ct - \hat{k} \cdot \vec{x}$
- Send laser from test mass $\vec{X}_e$ to $\vec{X}_r$
- GW imparts time delay (or contraction) $\delta t$
- Frequency shifts $y = (\nu_e - \nu_r)/\nu_e$

$$y_{\mathrm{GW}} = \frac{1}{2} \frac{\hat{n}^l_{(0)} \hat{n}^m_{(0)}}{1 - \hat{k} \cdot \hat{n}_{(0)}} \left( h_{lm}(\xi_r) - h_{lm}(\xi_e) \right)$$

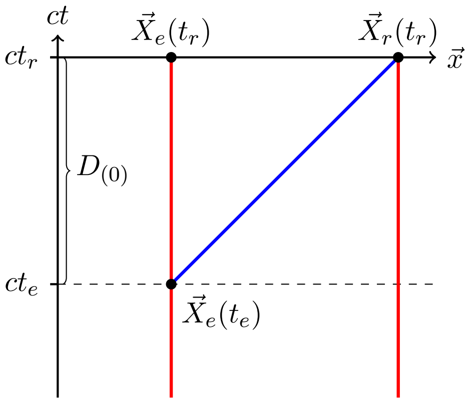



**[Cornish+2003, Finn2009, ...]**

# Adding velocity $\beta$ in the mix

$\mathcal{O}(1)$: **Background, Stationary S/C**

- Instantaneous separation $D_{(0)}$



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$\mathcal{O}(1)$: Background, Stationary S/C

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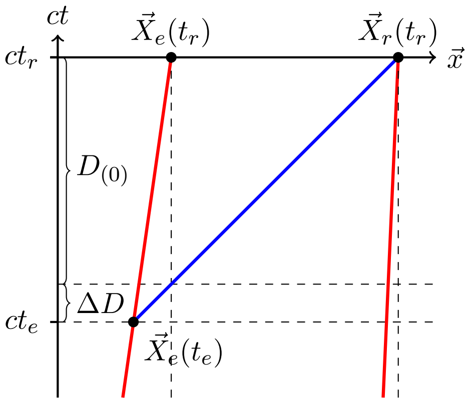
$\mathcal{O}(\beta)$: **Background, Moving S/C**

- Displacement $\Delta D \propto \beta_e$
- Adjusted line-of-sight
- Doppler shift



# Adding velocity $\beta$ in the mix

$\mathcal{O}(1)$: Background, Stationary S/C

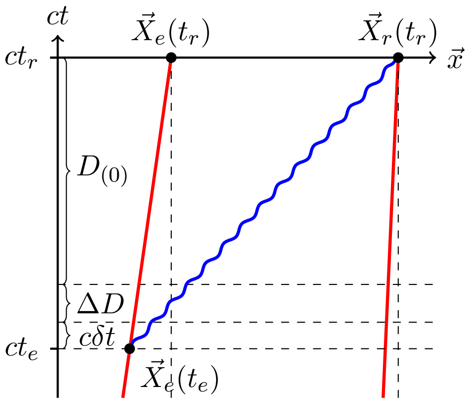
- Instantaneous separation $D_{(0)}$

$\mathcal{O}(\beta)$: Background, Moving S/C

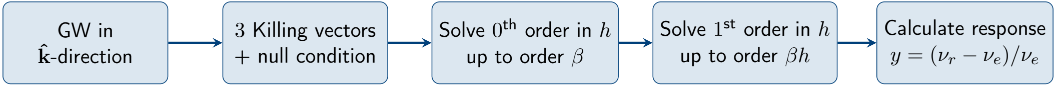
- Displacement $\Delta D \propto \beta_e$
- Adjusted line-of-sight
- Doppler shift

$\mathcal{O}(h\beta)$: **Background+GW, Moving S/C**

- Small shift $c\delta t \propto h\beta$
- New line-of-sight again
- Doppler shift + GW response



# Derivation modified response




- Determine laser's null geodesic $\sigma^\mu(\lambda)$
- Boundaries set by $\vec{X}_r(t_r)$ *(fixed)* and $\vec{X}_e(t_e)$ *(perturbative)*
- Determine observed frequencies $\nu_r$ and $\nu_e$

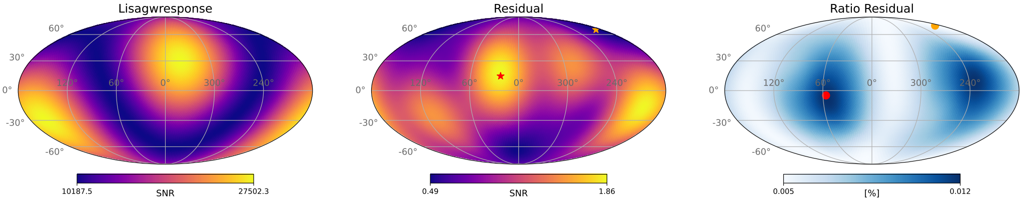
$$\dot{\sigma}(\lambda) = \underbrace{\hat{n}_{(0)}}_{\text{straight line} \quad \mathcal{O}(\beta)} + \underbrace{\hat{n}_{(1)}}_{\text{accumulated GW} \quad \mathcal{O}(h\beta)} + \underbrace{\dots h_{lm}(\xi(\lambda))}_{\text{local GW} \quad \mathcal{O}(h\beta)}$$

**[2509.10038]**

# Doppler corrections to the response

$$
\begin{aligned}
y_{\mathrm{GW}} = &-\underbrace{\frac{1}{2}\frac{\hat{n}^l_{(0)}\hat{n}^m_{(0)}}{1-\hat{k}\cdot\hat{n}_{(0)}}[h_{lm}(\xi_r)-h_{lm}(\xi_e)]}_{\text{Standard response}} + \underbrace{\frac{1}{2}\frac{\hat{n}^l_{(0)}\hat{n}^m_{(0)}}{1-\hat{k}\cdot\hat{n}_{(0)}}[\vec{\beta}_r\cdot\hat{k}\,h_{lm}(\xi_r)-\vec{\beta}_e\cdot\hat{k}\,h_{lm}(\xi_e)]}_{\text{localized redshift}} \\
&-\underbrace{\frac{1}{2}\frac{\hat{n}^l_{(0)}\hat{n}^m_{(0)}}{1-\hat{k}\cdot\hat{n}_{(0)}}\hat{n}_{(0)}\cdot\vec{\beta}_e[h_{lm}(\xi_r)-h_{lm}(\xi_e)]}_{\text{point-ahead correction}} - \underbrace{\frac{1}{2}\frac{\hat{n}^l_{(0)}\hat{n}^m_{(0)}}{1-\hat{k}\cdot\hat{n}_{(0)}}\hat{n}_{(0)}\cdot(\vec{\beta}_r-\vec{\beta}_e)h_{lm}(\xi_e)}_{\text{point-ahead correction}} \\
&-\underbrace{\eta_{ij}(\beta^i_r-\beta^i_e)\hat{n}^j_{(1)}}_{\text{modified Doppler shift}} + \mathcal{O}(\beta^2 h).
\end{aligned}
$$

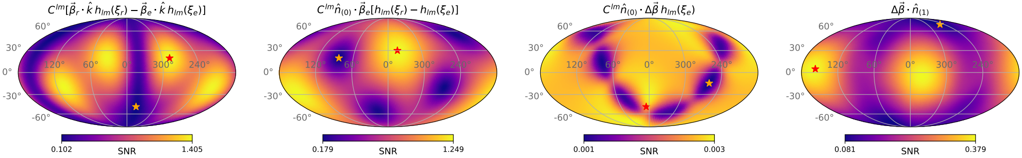
# Impact on total SNR (sky dependence)




Skymap for $M_{\rm tot} = 5 \times 10^6\,\mathrm{M}_\odot$ at $z = 1$, with $q = 1$, $\chi_1 = \chi_2 = 0.7$

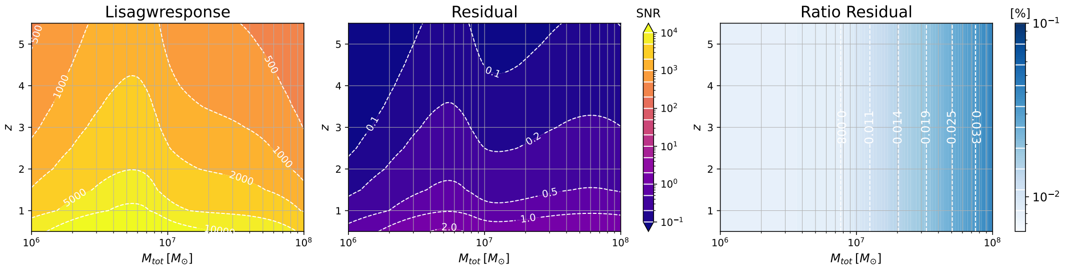
- Maximum residual SNR $\sim 2$
- Differences up to $0.01\%$
- Note this source is *very* loud

# Sky-dependence individual terms




Skymap for $M_{\rm tot} = 5 \times 10^6\,{\rm M}_\odot$ at $z = 1$, with $q = 1$, $\chi_1 = \chi_2 = 0.7$

# Impact on total SNR (mass dependence)




- Differences up to $0.04\%$
- Increase in difference for high mass systems
- High mass sources spend short time in the LISA band

# Conclusions

- Residuals up to SNR $\sim 2$ and differences of $\sim 0.04\%$
- Same accuracy as the state-of-the-art NR simulations
- Not likely to impact intrinsic parameter estimation
- Relevant for high mass sky-localization? (break degeneracy)

Prospects

- Full Bayesian analysis is difficult
- Simulate with modified response, analyze with existing tools
- Fischer Analysis (JAX version of `lisagwresponse`)