Quantum Symmetric Pairs, Hecke Algebras, and Representations: Exploring Spherical Functions (Q-SPHERE 2026)

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Radboud University

Book of Abstracts

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Stokman: Quasi-polynomial analogs of Askey-Wilson polynomials

Author: Jasper Stokman[1]

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I will first introduce quasi-polynomial analogs of the nonsymmetric and symmetric Askey-Wilson polynomials using an explicit representation of a rank one double affine Hecke algebra. In the second part of the talk, I will explain how they are related to Mizhan-Rahman's associated Askey-Wilson polynomials. The second part of the talk is joint work with Mikhail Isachenkov.

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Gaudillot-Estrada: A Mackey Analogy for real semisimple quantum groups

Author: Yvann Gaudillot-Estrada[1]

[1] *Université de Lorraine, FR*

For a given real semisimple group G, the Mackey analogy consists of a collection of explicit relationships between the groups algebra of G and that of its Cartan motion group G_0. The weakest of these relationships is the Connes-Kasparov isomorphism $K_*(C^*(G_0)) \cong K_*(C^*_r(G))$. In this talk, on the basis of small dimensional examples, I will explain why this analogy may also hold for real semisimple quantum groups, which have been introduced by De Commer.

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Watanabe: Quantizations of coordinate algebras of symmetric pair subalgebras

Author: Hideya Watanabe[1]

[1] *Rikkyo University, JP*

It is known that the quantum coordinate algebra of a Kac–Moody algebra and its crystal basis admit Peter–Weyl type decompositions.
Also, Kashiwara proved that, for finite type, the crystal basis is isomorphic to the crystal basis of the modified quantum group as bicrystals.
The main topic of this talk is quantum symmetric pair analogues of these results.
In particular, I will show you some examples of "bi-icrystals" of type A.
This talk is partly based on a joint work with Mao Hoshino.

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Wang: Weight modules for gl_2 \times \gl_2

Authors: Catharina Stroppel[1]; Liao Wang[2]

[1] *MPIM Bonn, DE*

[2] *University of Bonn, DE*

We develop a theory of weights for the quantum symmetric pair $(\mathfrak{gl}_4, \mathfrak{gl}_2 \times \mathfrak{gl}_2)$ of type AIII. We define "magical" operators that are compatible with weight spaces (wrt. Letzter's Cartan subalgebra) and use them to study Verma modules and irreducible quotients. We then prove the existence of weight bases in tensor products by explicitly constructing some highest weight vectors. These constructions allow us to mimic the important aspects of the classical finite dimensional representation theory. Applications include a definition of rational representations, the BGG resolution, a Clebsch–Gordan formula, the Harish-Chandra isomorphism and central characters, as well as a classification and description of all irreducible polynomial representations.

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Langen: Uniform bounds on the Dunkl kernel

Author: Lukas Langen[1]

[1] *Universität Paderborn, DE*

In Dunkl theory the Dunkl kernel replaces the classical exponential function in its predominant role in harmonic analysis. For regular spectral parameters, we present upper bounds for the Dunkl kernel and its derivatives which are uniform in the spatial variable. These estimates generalize sharp uniform upper bounds for spherical functions of Cartan motion groups and classical Bessel functions. The proof is based on an asymptotic study of the differential system satisfied by the Dunkl kernel with respect to parabolic subgroups of the Weyl group. As a consequence we obtain results regarding the Lebesgue density of the representing measure of Dunkl's intertwining operator.

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Terwilliger: The q-Onsager algebra and its finite-dimensional irreducible modules

Author: Paul Terwilliger[1]

[1] *University of Wisconsin-Madison, US*

This talk is about the q-Onsager algebra O_q. The algebra O_q is defined by two generators and two relations called the q-Dolan/Grady relations. We will describe the finite-dimensional irreducible O_q-modules V that satisfy a mild assumption. We will show that the O_q-generators act on V as a tridiagonal pair. We will describe the tridiagonal pairs and the related tridiagonal systems, using the concept of a tetrahedron diagram. We will classify up to isomorphism the tridiagonal systems, and explain which ones come from an O_q-module.

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Vlaar: On quantum affine symmetric pairs (part 1)

Author: Bart Vlaar[1]

[1] *BIMSA, CN*

The Yang–Baxter equation and the reflection equation, or boundary Yang–Baxter equation, are fundamental identities in quantum integrable systems, governing factorizable particle interactions on a line and on a half-line, respectively. While the Yang–Baxter equation is deeply connected to quantum groups, solutions of the reflection equation, called K-matrices, arise naturally from quantum symmetric pairs. In this two-part talk, we will survey recent joint work and open problems on quantum symmetric pairs, with special emphasis on affine type.

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Appel: On quantum affine symmetric pairs (part 2)

Author: Andrea Appel[1]

[1] *Università degli Studi di Parma, IT*

The Yang–Baxter equation and the reflection equation, or boundary Yang–Baxter equation, are fundamental identities in quantum integrable systems, governing factorizable particle interactions on a line and on a half-line, respectively. While the Yang–Baxter equation is deeply connected to quantum groups, solutions of the reflection equation, called K-matrices, arise naturally from quantum symmetric pairs. In this two-part talk, we will survey recent joint work and open problems on quantum symmetric pairs, with special emphasis on affine type.

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Van Horssen: Shift operators for non-symmetric Heckman-Opdam polynomials and non-symmetric Macdonald-Koornwinder polynomials

Author: Max van Horssen[1]

Co-author: Maarten van Pruijssen[2]

[1] *KU Leuven, BE*

[2] *Radboud Universiteit, NL*

We present an algebraic construction of shift operators for the non-symmetric Heckman-Opdam polynomials and the non-symmetric Macdonald-Koornwinder polynomials. To each linear character of the finite Weyl group, we associate forward and backward shift operators, which are differential-reflection and difference-reflection operators that satisfy certain transmutation relations with the Dunkl-Cherednik operators. In the Heckman-Opdam case, the construction recovers the non-symmetric shift operators of Opdam and Toledano Laredo for the sign character.

This talk is based on joint work with Maarten van Pruijssen (arXiv:2602.06784).

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Rösler: Limits of Bessel functions for root systems as the rank tends to infinity

Author: Margit Rösler[1]

[1] *Universität Paderborn, DE*

In this talk, we consider the asymptotic behaviour of Dunkl-type Bessel functions associated with root systems of type A and type B with positive multiplicities as the rank tends to infinity.
To obtain limits, one has to take sequences of spectral parameters which are of Vershik-Kerov type, i.e. tend to infinity in a suitable way. The situation is similar to the case of Heckman-Opdam polynomials whose limits were studied by Okounkov and Olshanski more than 20 years ago. Nowadays, there is renewed interest in such topics within the area of integrable probability.

We characterize both the possible limit functions as well as the spectral sequences for which limits of Bessel functions can be obtained, both in the cases of type A and type B.
For multiplicities related to group cases, these results have an interpretation in the context of asymptotic harmonic analysis in the sense of Olshanski.

The talk is based on joint work with Dominik Brennecken

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Meereboer: Kostant's branching law for quantum symmetric pairs

Author: Stein Meereboer[1]

[1] *Radboud Universiteit, NL*

Kostant's branching law for a symmetric pair $(\mathfrak{g}, \mathfrak{k})$ determines the multiplicity of a irreducible $\mathfrak{k}$-module inside irreducible $\mathfrak{g}$-modules. In this talk I will explain how to derive this branching law using the language of Watanabe's integrable modules for quantum symmetric pairs. This is joint work in progress with Stefan Kolb.

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Song: Representations of quantum symmetric pairs at roots of 1

Author: Jinfeng Song[None]

[1] *Hong Kong University of Science and Technology*

The celebrated work of De Concini–Kac–Procesi established that the representation theory of quantum groups at roots of unity is closely linked to the conjugacy classes of the underlying group. In this talk, we extend this approach to quantum symmetric pairs.

For an i-quantum group $U^\imath$ associated with an involution θ, we consider a De Concini–Kac type integral form and study its specialization $U^\imath$ at an odd root of unity. We show that the simple modules of $U^\imath$ are parametrized by θ-twisted conjugacy classes of the underlying group and provide an explicit upper bound on their dimensions. Moreover, this bound is attained for simple modules corresponding to generic twisted conjugacy classes. Along the way, we construct a finite-dimensional coideal subalgebra of the Lusztig small quantum group.

This is joint work with Weinan Zhang.

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Van Haastrecht: Principal series induced from Heisenberg parabolic subgroups

Author: Robin van Haastrecht[1]

[1] *Göteborgs universitet/Chalmers University of Technology, SE*

Let G be a semisimple Lie group and K the maximal compact subgroup. Principal series representations of G induced from a parabolic subgroup $P = MAN$ whose unipotent radical N is abelian have been well studied. The analysis of these representations exploits the fact that (K, L) has the structure a symmetric pair, where $L = K \cap M$. The next easiest case is when N has the structure of a Heisenberg group. We study principal series representations induced from Heisenberg parabolic subgroups and we use the Peter-Weyl theorem for circle bundles over a symmetric space of K to analyze these representations. We study reducibility, complementary series, and unitary subrepresentations, with a focus on $SL(n+2,\mathbb{R})$ and $G_{2(2)}$. This talk is based on joint work with Jan Frahm, Clemens Weiske and Genkai Zhang.

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Yuncken: Crystallization of function algebras on semisimple groups.

Author: Robert Yuncken[1]

[1] *Université de Lorraine, FR*

The structure theory of the finite dimensional representations of a quantized enveloping algebra undergoes a massive simplification when the deformation parameter q goes to 0. This is the fundamental observation in the theory of crystal bases of Kashiwara and Lusztig. On might ask if there is a similar simplification of the Pontrjagin dual, meaning a crystallisation of the quantized algebra of functions on a compact semisimple Lie group. The answer is that the crystal limit is a combinatorial algebra associated to symbolic dynamics. I will describe this crystal limit, as well as its analogue for the noncompact group SL(2,C).
(Joint work with Marco Matassa and Aidan Sims.)

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Liu: Representation Theory of very non-standard quantum so(2N)

Author: Xinyang Liu[1]

[1] *Newcastle University, UK*

We study finite-dimensional representations of the quantum group analogue of $\mathfrak{so}_{2N}$ inside $\mathfrak{so}_{2N+1}$ appearing in the theory of quantum symmetric pairs. Using a Verma module approach, we classify finite-dimensional simple modules in terms of highest weights. These highest weights are joint eigenvalues of the Letzter-Cartan subalgebra. Using a modified $\mathfrak{sl}_2 \times \mathfrak{sl}_2$-argument, we explicitly determine the highest weights of finite-dimensional irreducible representations.

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De Groot: An indefinite spectral triple for SU(1,1)

Author: Jort de Groot[1]

[1] *Universiteit van Amsterdam, NL*

Spectral triples provide a noncommutative analogue of spin manifolds and play a central role in noncommutative geometry. While the compact Riemannian case is well understood, far less is known in the noncompact pseudo-Riemannian setting. In this talk, I will present the construction of an indefinite spectral triple for the Lie group SU(1,1), highlighting the role of representation theory in the construction. Moreover, I will explain how this example acts as a stepping stone towards a spectral triple for the quantum group SUq(1,1). This talk is based on arXiv:2601.22171.

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Kobayashi: Stability regions of branching multiplicities

Author: Toshiyuki Kobayashi[1]

[1] *University of Tokyo, JP*

In this talk, we discuss a conceptual perspective on when and how branching multiplicities change in restrictions of representations. We develop a framework for the stability of branching multiplicities arising in the restriction of finite- and infinite-dimensional representations of real reductive Lie groups.

Focusing on pairs whose complexifications are of type (gl(n+1), gl(n)) and (o(n+1),o(n)), we show that branching multiplicities are locally constant on explicitly described convex regions in the joint parameter space of infinitesimal characters, and can change only upon crossing certain hyperplanes ("fences").

To illustrate the structure concretely, we describe these regions explicitly in the case of tensor products for sl(2), and in connection with this, Pevzner's talk will discuss the parameter dependence of the blow-up of branching multiplicities in relation to the parameter dependence of Jacobi polynomials.

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Pevzner: On blow-up phenomena for fusion rules

Author: Michael PEVZNER[1]

[1] *FJ-LMI and Reims University, JP-FR*

Controlling multiplicities in branching problems is a central question
in representation theory. In the setting of infinite-dimensional representations of real reductive Lie groups, analytic methods based on symmetry breaking operators provide one explanation of a surprising blow-up phenomena mentioned in the talk by T. Kobayashi.
We shall present this approach considering the basic case of $\mathfrak{sl}_2$ with a particular emphasis on the interplay with the classical theory of special functions.

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Opdam: Non symmetric shift operators

Authors: Eric Opdam[1]; Valerio Toledano Laredo[2]

[1] *Universiteit van Amsterdam, NL*

[2] *Northeastern University, US*

(Joint work with Valerio Toledano Laredo)

The well known "Dunkl operators" associated to a finite real reflection group constitute a commutative parameter family of deformations of the directional derivatives in Euclidean space. These operators are "differential-reflection" operators. Heckman and Cherednik have defined trigonometric versions of Dunkl's operators. The interest for these
operators lies in their deep ties to Macdonald polynomials and hypergeometric functions, to the Calogero-Moser quantum integrable system, and to the representation theory
of Hecke algebras.

"Hypergeometric shift operators" are tools to study Weyl group symmetric structures and functions in these contexts. In the present talk we present a theorem of existence and uniqueness of ''nonsymmetric shift operators" for the Dunkl operators themselves. These nonsymmetric
shift operators are differential-reflection operators which "shift"
the parameters of the Dunkl operators by integers by means of a "transmutation relation".

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Schlösser: Inner Product Methods for Matrix Spherical Functions

Author: Philip Schlösser[1]

[1] *Radboud Universiteit, NL*

Given a quantum symmetric pair (U, B), it is a well-known fact that (with some caveats) the (elementary) zonal spherical functions (ZSF) restrict to symmetric Macdonald polynomials.
We use the Haar functional on U's dual to construct an inner product for the matrix spherical functions (MSF). Since the MSF form a free module over the ZSF, we can interpret this inner product to be related to an inner product with a matrix weight that is closely related to the Macdonald weight.

This inner product can be used to identify MSF with intermediate Macdonald polynomials in several example cases.

This talk is based on the preprint 2511.23367, which is joint work with Stein Meereboer.

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Labriet: Bivariate Jacobi polynomials via the rank 2 Jacobi algebra

Author: Quentin Labriet[1]

[1] *Université de Montréal, CA*

Bivariate Jacobi polynomials is a family of orthogonal polynomials on the triangle that are solutions to a second order differential equation. This talk will present an algebraic interpretation for this family of polynomials based on the representation theory of the so-called rank 2 Jacobi algebra.

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Shimeno: Matrix-valued spherical functions on semisimple Lie groups

Author: Nobukazu Shimeno[1]

[1] *Kwansei Gakuin University, JP*

Harish-Chandra's *c*-function on a real semisimple Lie group gives the leading coefficient of the zonal spherical function and determines the Plancherel measure for the spherical transform. Gindikin and Karpelevič gave an explicit formula for the *c*-function. Moreover, Heckman and Opdam developed a theory of hypergeometric functions associated with root systems, which are generalizations of zonal spherical functions.

In the case of spherical functions for non-trivial *K*-types, explicit formulae for c-functions and spherical inversions have been known for a few cases, including the case of one-dimensional *K*-types. In this talk, I will explain that for certain class of *K*-types, associated elementary spherical functions can be written by Opdam's non-symmetric hypergeometric functions. As corolaries, we have explicit formulae of *c*-functions and inversion formulae for the spherical transforms.

This talk is based on joint work with Hiroshi Oda.

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Dal Martello: A crystallographic take on (rank n) GDAHA

Author: Davide Dal Martello[1]

[1] *University of Padua, IT*

Aiming for a revival of the dormant theory of **crystallographic complex reflection groups**, we give Coxeter-like reflection presentations for the top family of such groups. These new presentations behave à la Coxeter—encoding many of the group's properties at a glance—and further achieve the **braid theorem**, allowing to deform into the **generic Hecke algebra**. As part of a final showcase of the theory's promising future, the whole **GDAHA** family is rethought of as an elementary example in this new lingo.

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Koornwinder: Local extension and automorphisms of rank 1 DAHA and related special functions

Author: Tom H. Koornwinder[1]

[1] *Universiteit van Amsterdam, NL*

Consider the rank 1 DAHA of type $\check{C}_1C_1$ with generators T_1, T_0, Z, Z^{-1} and depending on the Askey–Wilson parameters a, b, c, d, q. Its basic representation is a faithful representation on the space of Laurent polynomials in z. We extend the generators Z, Z^{-1} to all rational functions in Z. Then the basic representation extends to a faithful representation on the space of rational functions in z. Then the mapping sending a rational function $f(z)$ to $f(z^{-1})$ can be lifted to the extended DAHA, and similarly for the mapping sending $f(z)$ to $f(qz)$.

A DAHA automorphism is a parameter transformation $(a, b, c, d) \to (a'.b', c', d')$ together with an algebra isomorphism, depending on a, b, c, d, from the (extended) DAHA $H_{a,b,c,d}$ to $H_{a',b',c',d'}$. For each parameter transformation we define explicit isomorphisms $H_{a,b,c,d} \to H_{a',b',c',d'}$ which we call canonical. This can be done in particular for the $a \leftrightarrow c$ flip discussed in Mazzocco's lecture.

More general isomorphisms $H_{a,b,c,d} \to H_{a',b',c',d'}$ factorize as a canonical isomorphism followed by a DAHA automorphism $H_{a',b',c',d'} \to H_{a',b',c',d'}$, which turns out to be a conjugation in many examples.

Many special DAHA automorphisms can be easily read off from the relations for the DAHA generators, but they are usually not canonical. We discuss some examples, and also their factorization. Some examples imply transformation formulas of non-symmetric Askey–Wilson polynomials (or functions), which functions occur as eigenfunctions of $Y = T_1 T_0$ in the basic representation.

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De Martino: A Quantized Metaplectic Howe Duality in Rank One

Author: Marcelo De Martino[1]

[1] *Forward College, PT*

Dirac-type operators and Howe dualities provide a uniform approach to decomposition problems and symmetry algebras in the orthogonal and symplectic settings. Quantizing these structures is subtle and in the orthogonal Dirac setting competing frameworks do not currently agree.

In this talk I focus on the symplectic (metaplectic) Howe duality in the first nontrivial case, rank one, and present an explicit and computable quantization, based on joint work with M. Brito. The main outcome is a clean quantum duality in which both sides are Drinfeld–Jimbo sl(2)-type quantum groups (with different deformation parameters), realized via Hayashi's deformed Weyl algebra. I will outline the resulting quantum analogues of the classical realization, Fischer-type structure, monogenics, and first-order symmetries, and conclude with the main obstacles to extending the construction beyond rank one.

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Isachenkov: Spin graph functions for the Lorentz group

Author: Mikhail Isachenkov[1]

[1] *Universiteit van Amsterdam, NL*

Spin graph functions of Reshetikhin-Stokman are generalizations of vector-valued spherical functions that arise in many contexts in harmonic analysis and physics. In this talk I will consider a special class of spin graph functions for the (identity connected component of the) Lorentz group $\mathrm{SO}(n,1)_e$, which is relevant for the analysis of Euclidean four-point local correlation functions in $(n-1)$-dimensional conformal field theory. I will present a construction, mapping such smooth spin graph functions to smooth vector-valued functions on a certain Cartan subgroup of the Lorentz group, and discuss the pushforward under that map of the algebra of invariant differential operators acting on the spin graph functions. This in particular provides an explicit representation of the corresponding algebra of quantum Hamiltonians by endomorphism-valued differential operators. The talk is based on the joint work with Edward Berengoltz and Jasper Stokman.

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Lamers: The spin-Ruijsenaars-Macdonald system

Author: Jules Lamers[1]

[1] *University of Glasgow, UK*

In this talk I will revisit the spin-Ruijsenaars-;Macdonald system, given by a matrix-valued generalisation of Macdonald operators that arises in the context of induced modules of double affine Hecke algebra and fits in a quantum-affine version of Schur-;Weyl duality. After reviewing the construction of this system, I will outline recent developments, including elliptic generalisations and applications to quantum spin chains with long-range interactions.

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Kolb: Short star products for quantum symmetric pairs

Author: Stefan Kolb[1]

[1] *Newcastle University, UK*

Short star products are filtered deformations of graded algebras satisfying a truncation condition first considered by Beem-Peelaers-Rastelli and further developed by Etingof and Stryker. In this talk I will explain that quantum symmetric pair coideal subalgebras are realized as short star products on quantum horospherical subalgebras. The shortness property allows for immediate conceptual interpretations of antiautomorphisms and bar-involutions which had previously been constructed via the quasi K-matrix. Moreover, this perspective allows us to express the quasi K-matrix in terms of the quasi R-matrix of Drinfeld and Lusztig. The talk is based on joint work with Milen Yakimov.

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Mazzocco: Quantum middle convolution on rank 1 DAHA

Author: Marta Mazzocco[1]

[1] *Universitat Politècnica de Catalunya, ES*

The Riemann–Liouville fractional integral interpolates iterated integration to non-integer orders and turns a hypergeometric solution of rank p into one of rank $p+1$. Katz recast this into the so-called \emph{middle convolution}, an operation on local systems on the punctured Riemann sphere that preserves the rigidity index. Dettweiler and Reiter subsequently made the operation explicit at the matrix level, as an action either on the residues of a Fuchsian system (additive middle convolution) or on the monodromy representation (multiplicative middle convolution). In this talk I will first recall the Riemann–Liouville transform and both the additive and the multiplicative middle convolutions of Dettweiler–Reiter, and then describe a noncommutative analogue of the multiplicative case — a \emph{quantum middle convolution} — built by enlarging the standard Hecke triple of the rank-one DAHA of type $\check{C}_1C_1$ to a cyclic Noumi–Stokman-type quadruple and performing an explicit two-step quotient by the resulting invariant subspaces. Applied to the basic representation, this construction yields an explicit intertwiner between the basic representations with parameters (a, b, c, d) and (c, b, a, d). On the spherical subalgebra this operation reduces to the identity, recovering the $W(D_4)$ action on the Askey–Wilson / Zhedanov algebra; on the full DAHA basic representation it provides a new and explicit incarnation of the parameter symmetry $a \leftrightarrow c$.

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De Commer: A Kazhdan-Lusztig theorem in type B

Author: Kenny De Commer[1]

[1] *Vrije Universiteit Brussel, BE*

Braided monoidal categories are governed by braid groups of type A and the associated Yang-Baxter equation. Given a semisimple compact Lie group G, the Kazhdan-Lusztig theorem gives a non-trivial equivalence between two particular braided monoidal unitary categories: one constructed from a non-trivial associator on the category of unitary G-representations, using solutions to the so-called KZ-equation, and one constructed as the representation category of the quantization of G. In this talk, we will explain how the Kazhdan-Lusztig theorem has an analogue 'in type B', with quantum groups replaced by coideals and braided monoidal categories replaced by braided module categories, which are governed by braid groups of type B and the associated reflection equation. This is based on joint work with S. Neshveyev, L. Tuset and M. Yamashita.