The role of renormalisation in the fine-tuning discussion

January 10th 2022

Research performed in collaboration with

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1. Naturalness

Setting: - Standard Model (SM) could be an effective low-energy theory originating from an underlying theory that emerges at a high-energy scale $\Lambda \gg \Lambda_{ew}$

- SM sector couples (directly/indirectly) to particles at the high scale OR Λ acts as a natural cutoff

Question: are the SM mass parameters stable against high-scale effects?

⤷ If some mass parameters are not stable, why are these masses not of $\mathcal{O}(\Lambda)$?

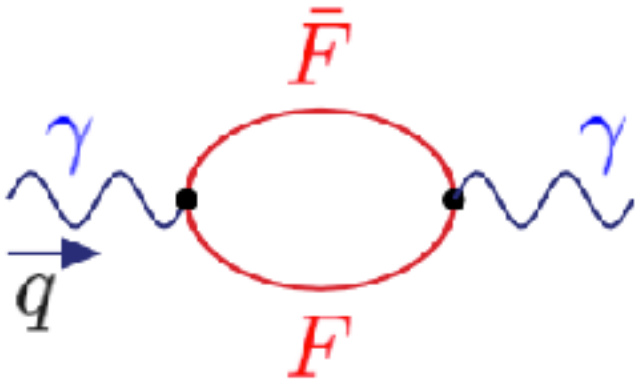
Naturalness

The mass of a particle is called naturally small if setting the associated Lagrangian parameter to zero enhances the symmetry of the system

such a symmetry-breaking mass is protected against large quantum corrections, resulting in $\delta m \propto m$

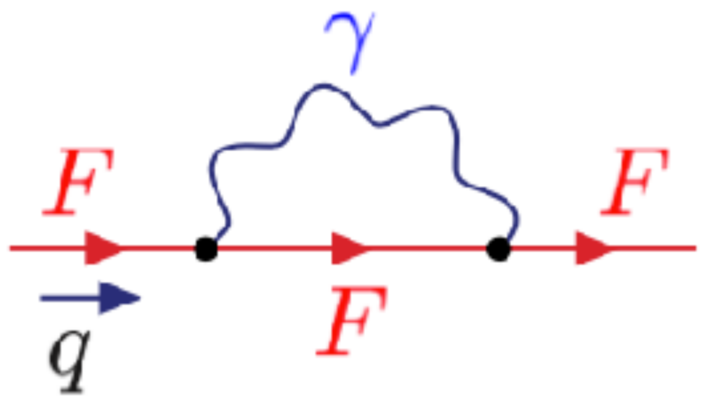
Examples: $\mathscr{L}_{QED} = \bar{\psi}_F (i\not{D} - m_F)\psi_F - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + i|e|Q_F A_\mu$



$\rightarrow$ $\delta m_\gamma = 0$ due to $U(1)_{em}$ gauge invariance

$\Rightarrow$ photon remains massless!



$$\to \quad \delta m_F \propto m_F \underbrace{\int \frac{d^4\ell}{\ell^2\,[(\ell+q)^2 - m_F^2]}}_{\text{LD (log. divergent)}} \Bigg|_{q^2=m_F^2} \propto m_F \log(\Lambda/m_F)$$

using Λ as a cutoff of the loop integral

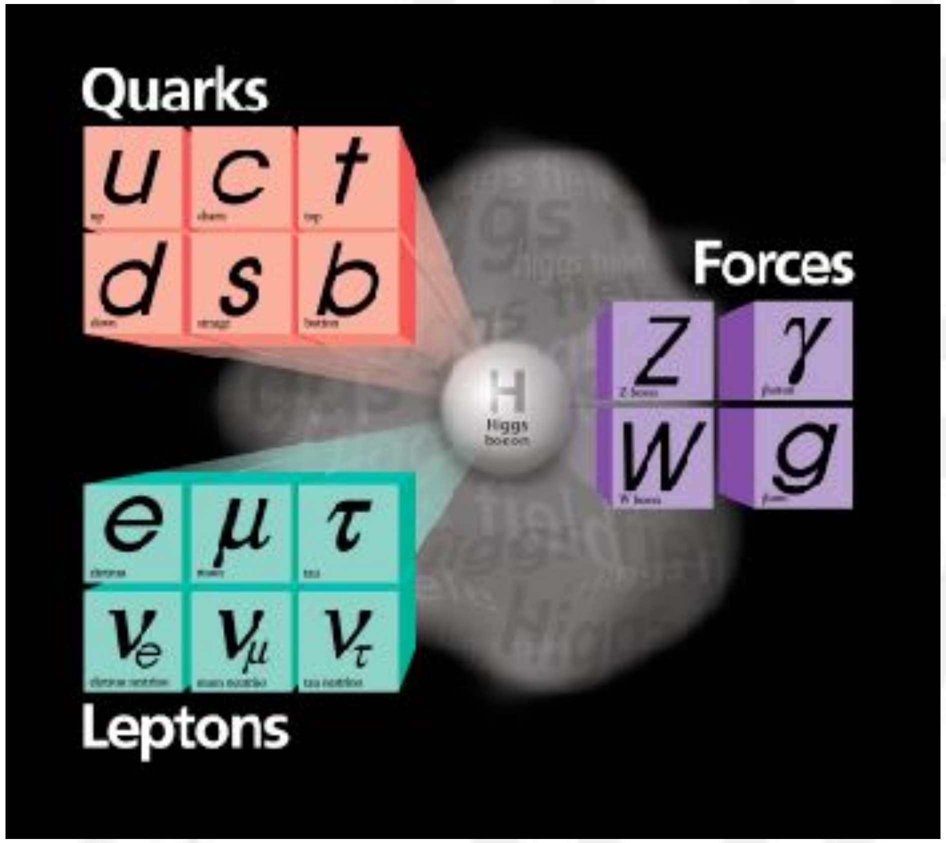
$m_F = 0:$ theory invariant under chiral transformations

$$\psi(x) \to \exp(i\theta\gamma^5)\,\psi(x) \quad , \quad \bar{\psi}(x) \to \bar{\psi}(x)\,\exp(i\theta\gamma^5)$$

$m_F \neq 0:$ chiral invariance broken by the mass term $\propto m_F$, but the original symmetry protects m_F (i.e. $\delta m_F \propto m_F$ rather than $\delta m_F \propto \Lambda$) as the symmetry is restored if the original Lagrangian parameter $m_F \to 0$!

Naturalness and the Standard Model

The masses of the fermions and gauge bosons are protected by symmetry, with the Higgs vacuum expectation value v and the Yukawa couplings g_f being the symmetry-breaking parameters

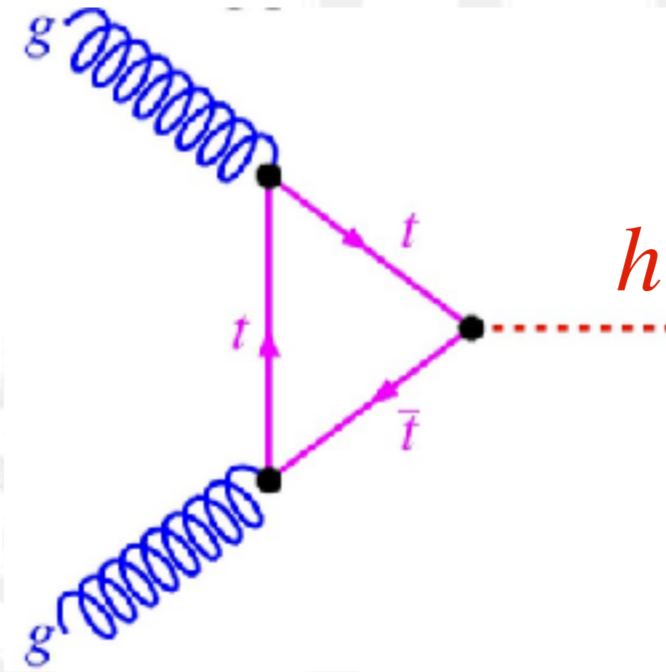


However, the Higgs-boson mass m_h is not protected by an enhanced symmetry if $m_h \to 0$

⇒ δm_h tends to be governed by the highest mass scale in the theory that couples (at tree/loop level) to the Higgs boson!

The upside of non-decoupling high-scale physics

Higgs production from gluon fusion @ LHC:



- no direct gluon-gluon-Higgs coupling
- heavy coloured particles in loop favoured

$$\sigma(gg \to h) \approx \frac{\alpha_s^2}{576\,\pi}\left(\frac{m_t/v}{m_t}\right)^2$$ in the SM

$$\sigma(gg \to h) \approx \frac{\alpha_s^2}{576\,\pi}\left(\frac{m_t/v}{m_t} + \frac{m_{t'}/v}{m_{t'}} + \frac{m_{b'}/v}{m_{b'}}\right)^2$$ for an extra quark generation

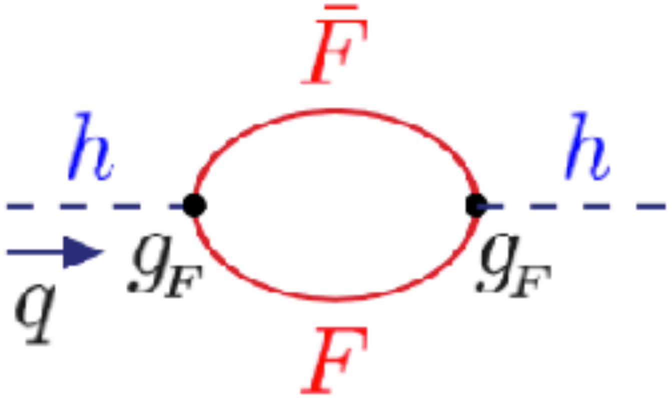
Experimental results for Higgs production @ LHC:

an extra SM-like generation of heavy quarks is excluded!

2. Fine tuning of the Higgs-boson mass

Consider the following Yukawa theory

$$\mathscr{L}_{Yu} = \bar{\psi}_F\,(i\partial\!\!\!/ - m_F)\,\psi_F + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 - g_F h\,\bar{\psi}_F\psi_F \qquad (\,g_F = m_F/v\,)$$



$$= -\,g_F^2 \int \frac{d^4\ell}{(2\pi)^4}\,\frac{Tr\,(\,[\,\ell\!\!\!/ + q\!\!\!/ + m_F]\,[\,\ell\!\!\!/ + m_F]\,)}{[\ell^2 - m_F^2]\,[(\ell + q)^2 - m_F^2]}$$

$$= -\,\frac{2g_F^2}{(2\pi)^4}\left\{\int \frac{d^4\ell}{\ell^2 - m_F^2} + \int \frac{d^4\ell}{(\ell + q)^2 - m_F^2} + \int \frac{d^4\ell\,(4m_F^2 - q^2)}{[\ell^2 - m_F^2]\,[(\ell + q)^2 - m_F^2]}\right\}$$

QD (quadratically divergent) LD (log. divergent)

Consequence: δm_h^2 contains terms of $\mathcal{O}(g_F^2\Lambda^2)$ and $\mathcal{O}(g_F^2 m_F^2)$!

Taken at face value: a 125 GeV Higgs-boson mass seems not to be stable against high-scale effects

This is referred to as the fine-tuning or hierarchy problem: to have $m^2_{h,phys} = m^2_{h,bare} + \delta m^2_h = O(\Lambda^2_{ew} \ll \Lambda^2)$ seems to require an extreme amount of fine tuning!

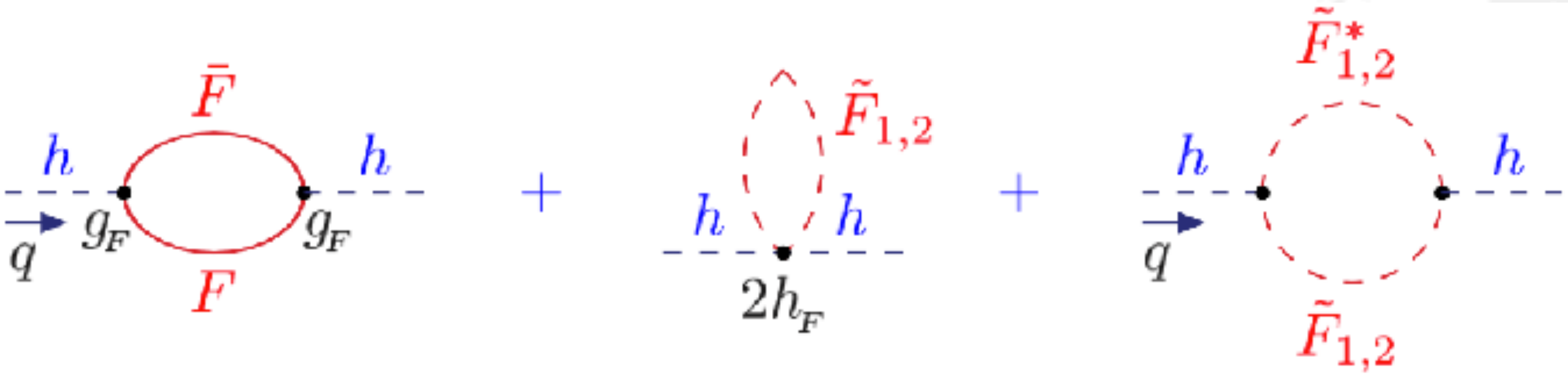
Guiding principle for model-building: a symmetry or a theoretical concept is needed to make this plausible, e.g.

- Reduce the scale hierarchy by lowering the Planck scale to $\mathcal{O}$(TeV), as done in extra-dimensional gravity
- The Higgs boson is composite rather than elementary
- Invoke supersymmetry (SUSY)

The SUSY approach

Each fermionic/bosonic degree of freedom in the theory has a bosonic/fermionic counterpart: e.g. fermion $F \leftrightarrow$ scalars $\tilde{F}_{1,2}$

Idea: fermionic and bosonic loop corrections come with opposite signs and could cancel each other



- Quadratic divergences cancel if $h_F = g_F^2$, as required by SUSY
- In that case $\delta m_h^2 = \mathcal{O}([m_{\tilde{F}_{1,2}}^2 - m_F^2]\, g_F^2 \log[\Lambda/m_h]) + \mathcal{O}(m_h^2 \log[\Lambda/m_h])$

  The first term vanishes for exact SUSY ($m_{\tilde{F}_1}^2 = m_{\tilde{F}_2}^2 = m_F^2$) and constitutes a source of fine-tuning in broken SUSY!

3. The role of renormalisation

This was all before performing the renormalisation

- Regularisation: - loop-momentum cutoff $\int d^4\ell_E \rightarrow \int_{\ell_E \leq \Lambda} d^4\ell_E$

  - dimensional regularisation $\int d^4\ell \rightarrow \mu^{4-n}\int d^n\ell$

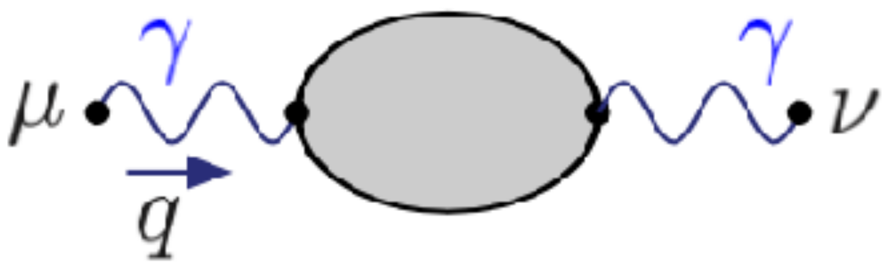
  We will use the latter to preserve the Ward identities!

  Special UV-divergent quantity: $\Delta = \frac{-2}{n-4} - \gamma_E + \log(4\pi)$

- Renormalisation schemes:
  - ★ $\overline{\text{MS}}$ scheme: remove divergences $\propto \Delta$ and absorb them in the Lagrangian parameters
  - ★ On-shell (OS) scheme: apply renormalisation conditions to the poles and residues of full propagators

Photon propagator

Consider again the QED example: $\mathscr{L}_{QED} = \bar{\psi}_F\,(i\not{D} - m_F)\,\psi_F - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$



$= \; -\,i\,\dfrac{g_{\mu\nu} - q_\mu q_\nu / q^2}{q^2 - \Sigma_\gamma(q^2)}$ + terms with $q_\mu\,, q_\nu$

effectively $\longrightarrow$ $\dfrac{-ig_{\mu\nu}}{q^2 - \Sigma_\gamma(q^2)}$ with $\Sigma_\gamma(q^2) = q^2\,\Pi(q^2)$ due to Ward identity

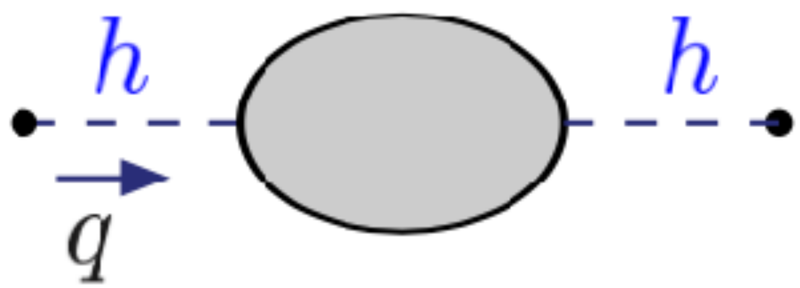
- Pole remains at $q^2 = 0$ $\Rightarrow$ photon indeed remains massless!
- Residue at $q^2 = 0$: $Z_\gamma \equiv \left[\,1 - \Pi(0)\,\right]^{-1}$
- Renormalisation: $\Pi_{ren}^{\overline{MS}}(0) \;=\; \Pi_{ren}^{OS}(0) \;=\; 0,\;\; Z_\gamma$ absorbed in photon field

  Heavy fermions decouple: $\Pi_{ren}(\,|\,q^2\,| \ll m_F^2) = \mathcal{O}(q^2/m_F^2)$

Fine-tuning revisited

Consider a Yukawa theory involving a very heavy fermion

$$\mathcal{L}_{Yu} = \bar{\psi}_F\,(i\partial\!\!\!/ - m_F)\,\psi_F + \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) - \tfrac{1}{2}m_h^2 h^2 - g_F h\,\bar{\psi}_F\psi_F \qquad (\,g_F = m_F/v \gg 1\,)$$



$$= \frac{i}{q^2 - m^2_{h,bare} - \Sigma_h(q^2)}$$

$\Sigma_h(q^2)$ contains divergent and finite terms of $\mathcal{O}(m_F^2 g_F^2)$ and $\mathcal{O}(g_F^2)$

$\overline{\text{MS}}$ renormalisation scheme:

- The bare mass absorbs the leading divergent terms:

  $m^2_{h,bare} + \delta m^2_{h,\overline{MS}} = m^2_{h,\overline{MS}}$, but finite terms of $\mathcal{O}(m_F^2 g_F^2)$ remain

  that lead to a fine-tuned $m^2_{h,\overline{MS}} - m^2_{h,\,physical\ pole} = \mathcal{O}(m_F^2 g_F^2)\,!$

OS renormalisation: two renormalisation conditions are imposed

1) The renormalised mass coincides with the physical pole mass:

$$m^2_{h,\,OS} \equiv m^2_{h,\,phys} \quad \text{with} \quad q^2 - m^2_{h,bare} - \Sigma_h(q^2)\Big|_{q^2=m^2_{h,\,phys}} = 0$$

2) The residue of this pole is defined to be unity

Consequence: the first condition removes all $\mathcal{O}(m_F^2 g_F^2)$ terms
and the second condition removes all $\mathcal{O}(g_F^2)$ terms!

Near the pole: $$\frac{i}{q^2 - m^2_{h,bare} - \Sigma_h(q^2)} \to \frac{i}{q^2 - m^2_{h,\,phys}} + \text{regular terms,}$$

with only $\mathcal{O}(g_F^2/m_F^2)$ regular terms remaining!

No fine-tuning in the OS case!

4. Conclusions and outlook

- Most masses in the SM are protected against high-scale effects
- This does not hold for the Higgs-boson mass
- If the Higgs-boson couples to heavy particles
  - ★ m_h is fine-tuned in the $\overline{MS}$ renormalisation scheme
  - ★ m_h is not fine-tuned in the on-shell renormalisation scheme

In progress/remaining questions:

- Is the $\overline{MS}$ scheme not able to properly separate energy scales?
- How does this relate to the Appelquist-Carazzone theorem?

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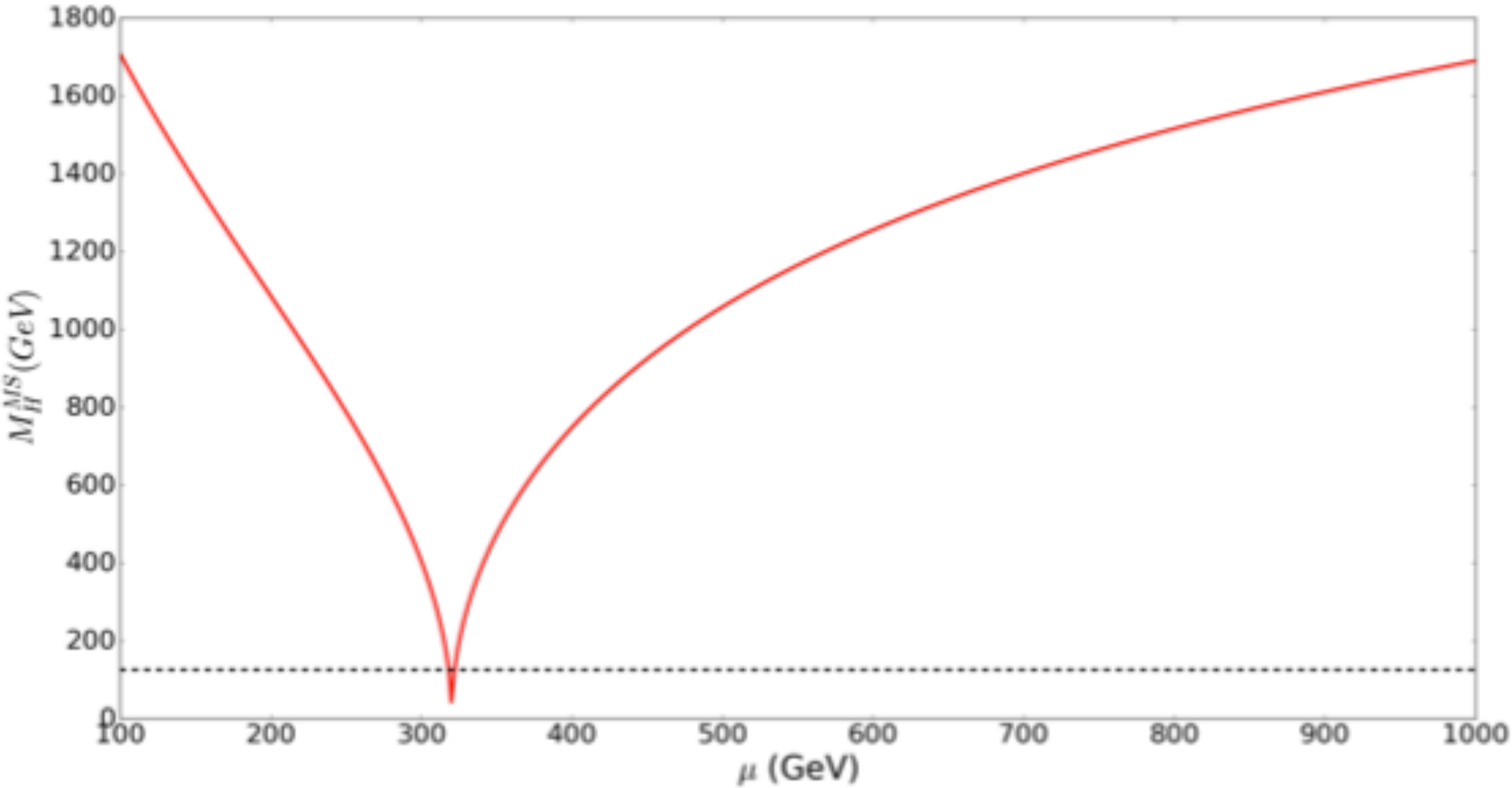
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$M_H^{MS}(GeV)$
μ (GeV)