Symmetries and observables in (quantum) gravity

HEP Dept Seminar,
24 Jan 2022

Renate Loll @RU

Preview

From the broad perspective of "symmetry", I will examine how gravity (= *General Relativity*, Einstein's theory of dynamical spacetime) differs from the other fundamental interactions, why this poses difficulties in constructing a (nonperturbative) ***quantum theory of gravity***, and how we have been learning to deal with them.

<u>My talk today will be about</u>

- standard classical formulation of gravity: analogies and differences with local gauge field theories
- diffeomorphism invariance, background-independence and observables
- (Causal) Dynamical Triangulations as "exact", nonperturbative lattice implementation of these concepts

Modelling spacetime: back to the roots …

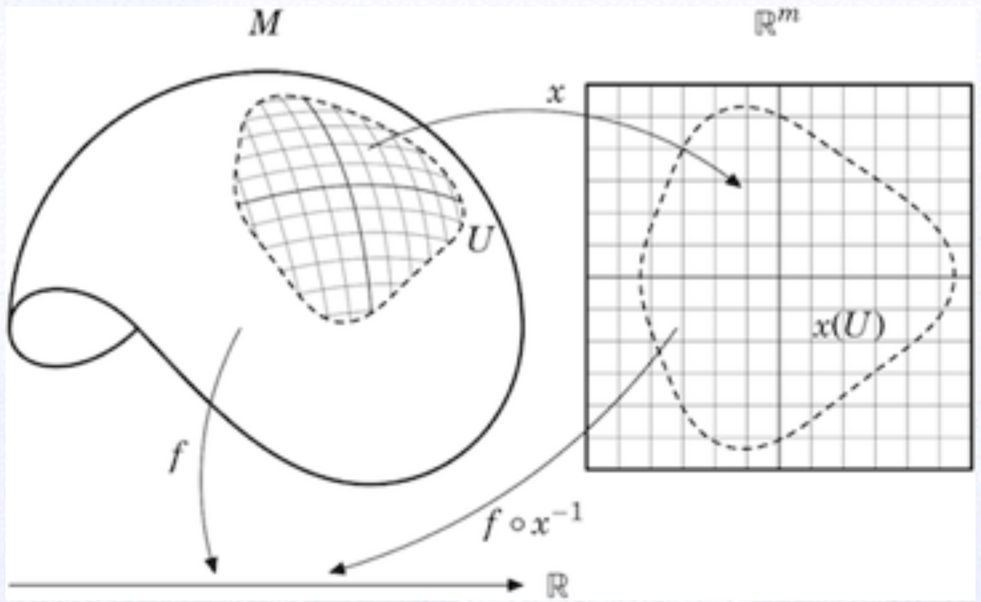
"Über die Hypothesen, welche der Geometrie zu Grunde liegen" (B. Riemann, Göttingen, 1854):

- laying the foundations of Riemannian geometry: *n*-dimensional manifolds with a metric structure, characterized by the infinitesimal line element

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

- guided by "experience", physical (Newtonian) considerations
- "my considerations may not apply in the immeasurably small" — may have to revisit them in the light of new physical observations
- contemplates "discrete manifolds" (as opposed to continuous ones) — counting is more intuitive than measuring
- foreshadows the spectacular success of (pseudo-)Riemannian differentiable manifolds in GR (and our love affair with coordinates)

Setting the stage for General Relativity (GR)



differentiable manifold *M* and a coordinate chart

- In classical GR, smooth manifolds $(M, g_{\mu\nu})$ with *Lorentzian* metrics $g_{\mu\nu}(x)$ of signature $(-+++)$ provide convenient, powerful models of spacetime.
  Using local coordinate charts U, we can compute just like on R^4.
- Geometric properties are encoded in the Riemann curvature tensor $R^{\kappa}{}_{\lambda\mu\nu}(x)$.

- tensors transform according to the rules of tensor calculus; e.g. the metric transforms nontrivially under $x \mapsto y(x)$:

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu = g_{\mu\nu}(x(y)) \frac{\partial x^\mu(y)}{\partial y^\rho} \frac{\partial x^\nu(y)}{\partial y^\sigma} dy^\rho dy^\sigma =: \tilde{g}_{\rho\sigma}(y) dy^\rho dy^\sigma$$

- coordinates are arbitrary; for a given metric (e.g. with isometries), can use this "freedom of choice" to obtain a simple functional form

Physics vs. gauge

- classically, we do not care much about the redundancy $g_{\mu\nu} \sim \tilde{g}_{\mu\nu}$
- however, to interpret the dynamics correctly, we must distinguish between **physics (= potentially observable) and gauge (= mere "coordinate effects")**
- infamous "pitfalls" from the early days of GR:
  - long-standing confusion about the physical status of *r = 2GM* (aka the event horizon) in the Schwarzschild solution
  - are gravitational waves real? (Einstein and Rosen, 1936)

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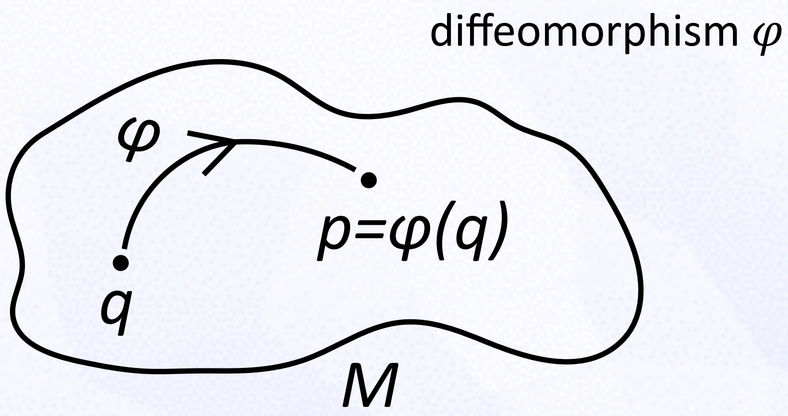
Einstein to J. Tate (editor Physical Review), 27 Jul 1936:

"Dear Sir, we (Mr. Rosen and I) had sent you our manuscript for publication and had not authorized you to show it to specialists before it is printed. I see no reason to address the – in any case erroneous – comments of your anonymous expert. On the basis of this incident I prefer to publish the paper elsewhere. Respectfully, A. Einstein"

Diffeomorphism invariance

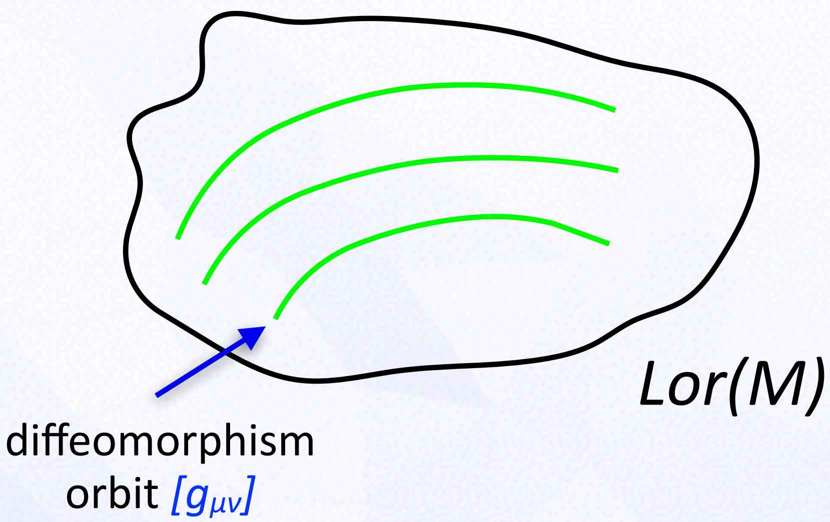


- (global) diffeomorphism is a C^∞-map $\varphi: M \to M$ with inverse φ^{-1}
- "structure-preserving" maps of differentiable manifolds
- form ∞-dimensional group $Diff(M)$ under composition
- in local coordinate charts, tensor components transform as usual
- denote equations of motion $\mathcal{F}[\, \Phi, \Sigma \,] = 0$, where Φ are dynamical fields, Σ is nondynamical (background) structure
- a diffeomorphism-invariant theory satisfies

$$\mathcal{F}[\, \Phi, \Sigma \,] = 0 \quad \Leftrightarrow \quad \mathcal{F}[\, \varphi \cdot \Phi, \Sigma \,] = 0, \text{ for all } \varphi$$

- true for Einstein equations: φ maps solutions to solutions, no Σ
- gravity is *background-independent*: since metric "gets moved along" with φ, "the point x" is not a physically meaningful concept

Gravity vs. gauge field theory



- the dynamics of GR takes place on the space of geometries $G(M)$, where

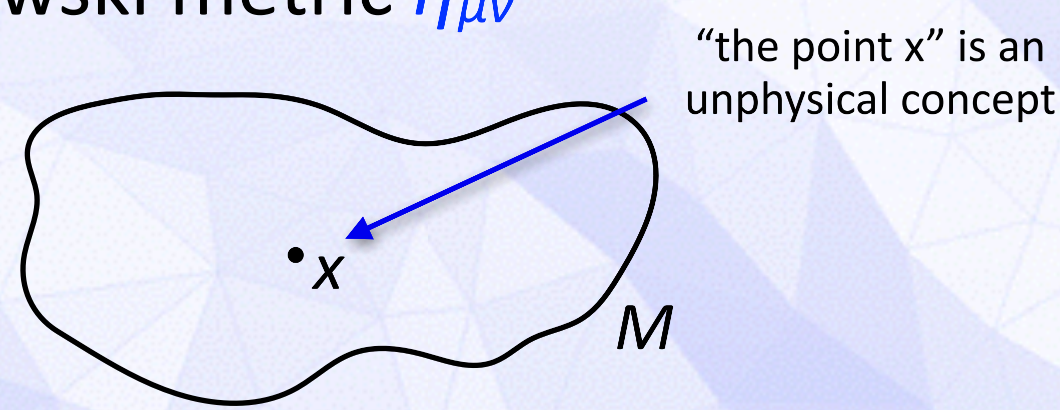
$$[g_{\mu\nu}(x)] \in G(M) := Lor(M)/Diff(M)$$

- analogue of nonabelian gauge field theory

$$[A_\mu{}^a(x)] \in \mathcal{A}^{su(N)}(M)/\mathcal{G}(SU(N))$$

.

- in both cases, physics is invariant along gauge orbits
- however, $\mathcal{G}(SU(N))$ acts pointwise at x in internal space, and there *is* a fixed background structure, the Minkowski metric $\eta_{\mu\nu}$



- observables in Yang-Mills theory are local scalars, like $F^{\mu\nu}F_{\mu\nu}$, but because of background independence, observables in gravity are <u>nonlocal</u> integrals of scalars, like $\int_M d^4x\sqrt{g}\,R(x)$

Challenges of quantum gravity

- because gravity is not perturbatively renormalizable, we must construct a quantum theory *nonperturbatively*
- diffeomorphisms must be represented in the quantum theory, and the associated redundancy must be removed somehow, otherwise the gravitational path integral has infinities
- the physical configuration space *G(M)* is not linear or "nice"; how can we parametrize it (e.g. by gauge-fixing) in the full theory?
- how can we regularize and renormalize without breaking diffeomorphism-invariance?
- once we have addressed these issues, we must still construct nonlocal ***quantum observables*** and understand the dynamics and properties of ***quantum spacetime*** in terms of them
- somewhat miraculously, CDT quantum gravity provides a blueprint for handling diffeomorphism invariance *and* quantum observables

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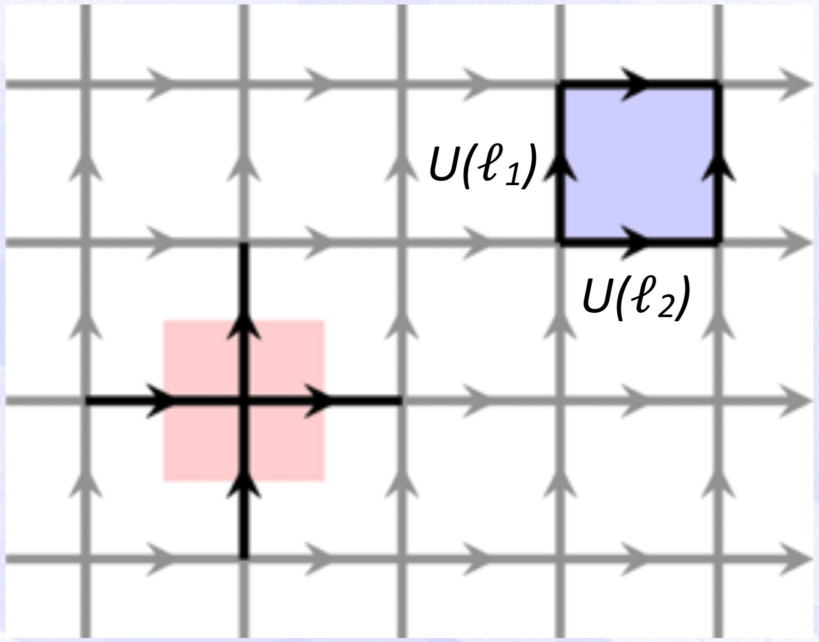
Taking a cue from lattice gauge field theory

- lattice gauge theory has been immensely successful in helping us understand and quantify the nonperturbative regime of QCD
- general philosophy: lattice acts as regulator, with UV cutoff *a*; search for a continuum limit by approaching a second-order phase transition in the limit *a —> 0* while renormalizing bare couplings appropriately; attain "universality" (independence of regularization details)

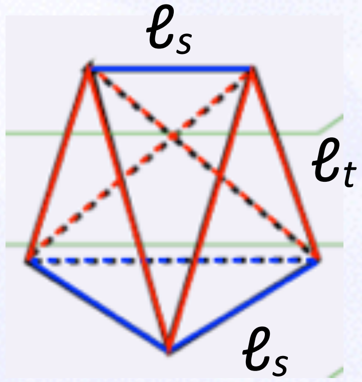


a cubic lattice representing flat spacetime, with gauge fields living on edges and vertices

- the fundamental lattice variables are edge holonomies $U(\ell)=P\exp\int_\ell A$
- they still transform under *SU(N)* at their end points, lying at the vertices
- key: the gauge transformations are "exact", still form a group, despite the regularization

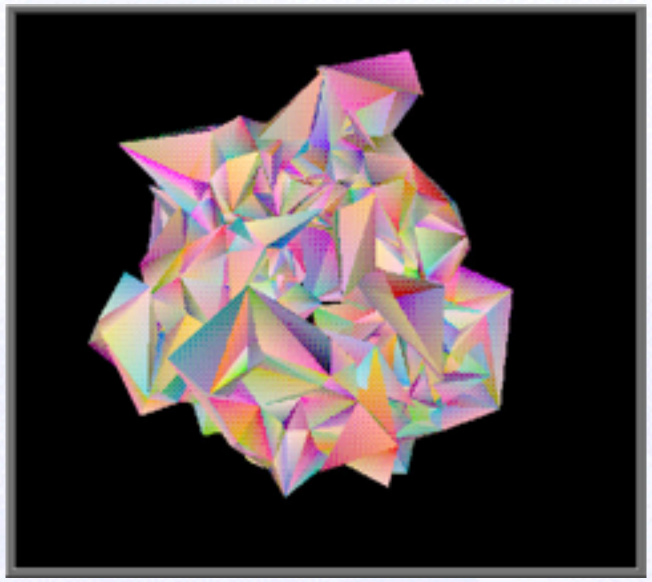
Putting quantum gravity on a lattice, correctly

Strategy: approximate curved spacetimes by simplicial manifolds, following the profound, but underappreciated idea of "General Relativity without Coordinates" (Regge, 1961).



- 'piecewise flat' gluings of 4D triangular building blocks (four-simplices) describe intrinsically curved spacetimes
- Geometry is specified *uniquely* by the edge lengths ℓ of the simplices and how they are 'glued' together. **No coordinates are needed.** One is working directly on (a regularized version of) *G(M)*.
- The full power of this idea is unleashed in the *quantum* theory, using a (C)DT path integral over dynamical, equilateral "lattices" ($\ell = a$ up to global time vs. space scaling, for a UV cut-off a).
- The nonperturbative gravitational path integral has no coordinate redundancies. The MC simulations are **relabeling invariant**.

Next, let the fun begin (but not today …)



triangulated model of quantum space

The philosophy of *defining* the nonperturbative theory as the continuum limit of a (dynamical) lattice theory can be implemented in gravity!

Complete, analytically solved toy models of quantum gravity in 2D (where there are continuum theories to compare!).

Real progress in constructing nonperturbative quantum gravity in 4D (with no continuum theory to compare). Nonlocal quantum observables (various "dimensions", spatial volume, volume correlators, quantum Ricci curvature) have been constructed and measured.

CDT reviews: J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, Physics Reports 519 (2012) 127, arXiv: 1203.3591; RL, Classical and Quantum Gravity 37 (2020) 013002, arXiv:1905.08669

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Thank you!