$$\boxed{\text{Topology Induced 1-order phase transitions in 4D CDT}}$$

arXiv 2202.07392
(to appear in JHEP)

J. Gizbert-Studnicki, A. Görlich, D. Nemeth
and me

First some trivial remarks about phase transitions of Lattice systems. (Lattice spin systems)

$\boxed{\text{Second order transitions}}$

order parameter, e.g. $\langle s_i \rangle = \langle \phi \rangle$

$\langle \phi \rangle$ is usually continuous at the critical T_c

But $\Delta \phi$ will diverge:

$$\chi \sim \frac{1}{N} \sum_{i,j} \langle (s_i - \langle s_i \rangle)(s_j - \langle s_j \rangle) \rangle \sim \frac{1}{|T-T_c|^{\gamma}}$$

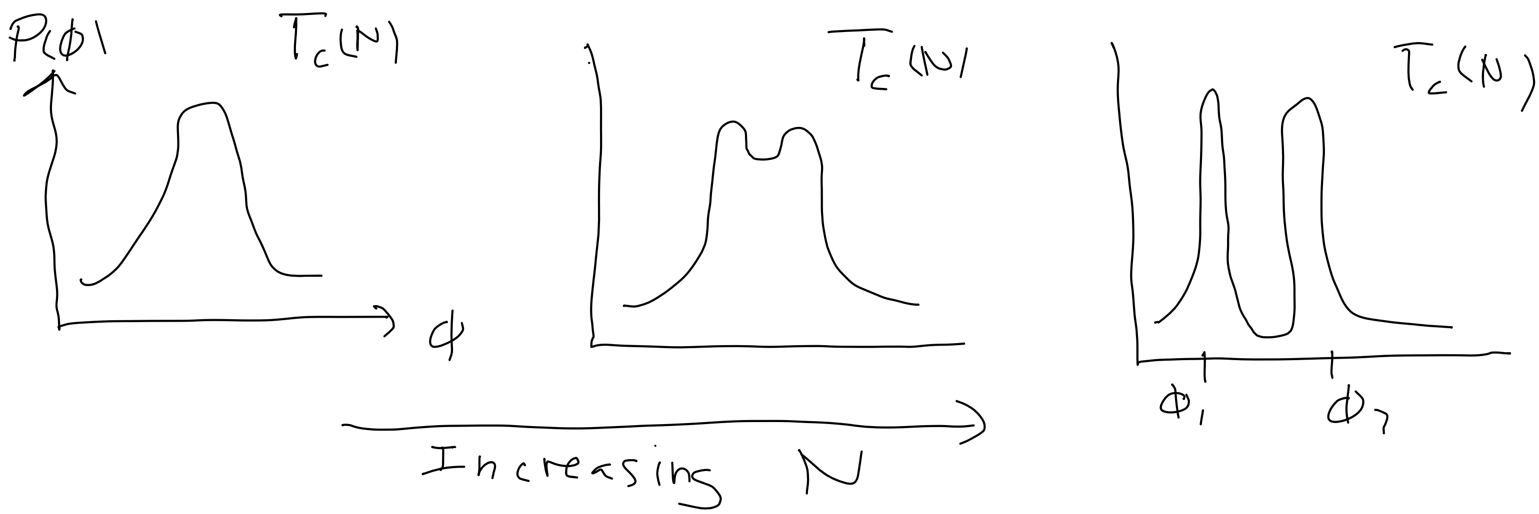
This is due to a divergent spin-spin correlation length $\xi(T) \sim \frac{1}{|T-T_c|^{\nu}}$ at T_c

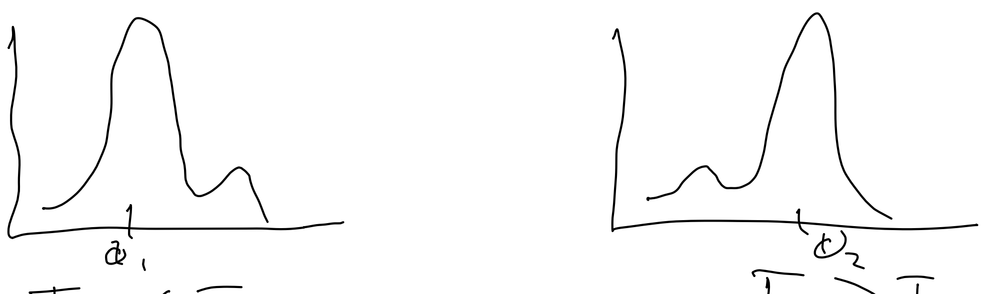
and this is the reason we can talk about an underlying continuum field theory.

On a finite lattice of size N there is strictly speaking no phase transitions, only pseudo-critical $T_c(N)$

$$T_c(N) = T_c(\infty) - \frac{c}{N^{\Delta}}, \quad \Delta = \frac{1}{\nu D}$$

First order transitions





$T < T_c$ $\qquad$ $T > T_c$

There will be a jump of the order parameter ϕ at T_c and the phenomenon of hysteresis

And no divergent correlation length

Let us move to QG, which we treat as a lattice system in CDT. However, the main difference to the above is that the lattice itself becomes the dynamical system

It makes it unclear to what extent one can use the spin analogy (what is a correlation length when the lattice fluctuates?)

Nevertheless it seems to work in 2d toy models and we will go ahead in 4d

The action:

$$S = -\frac{1}{G} \int d^4\xi \sqrt{g(\xi)} \left(R(\xi) - 2\Lambda \right)$$

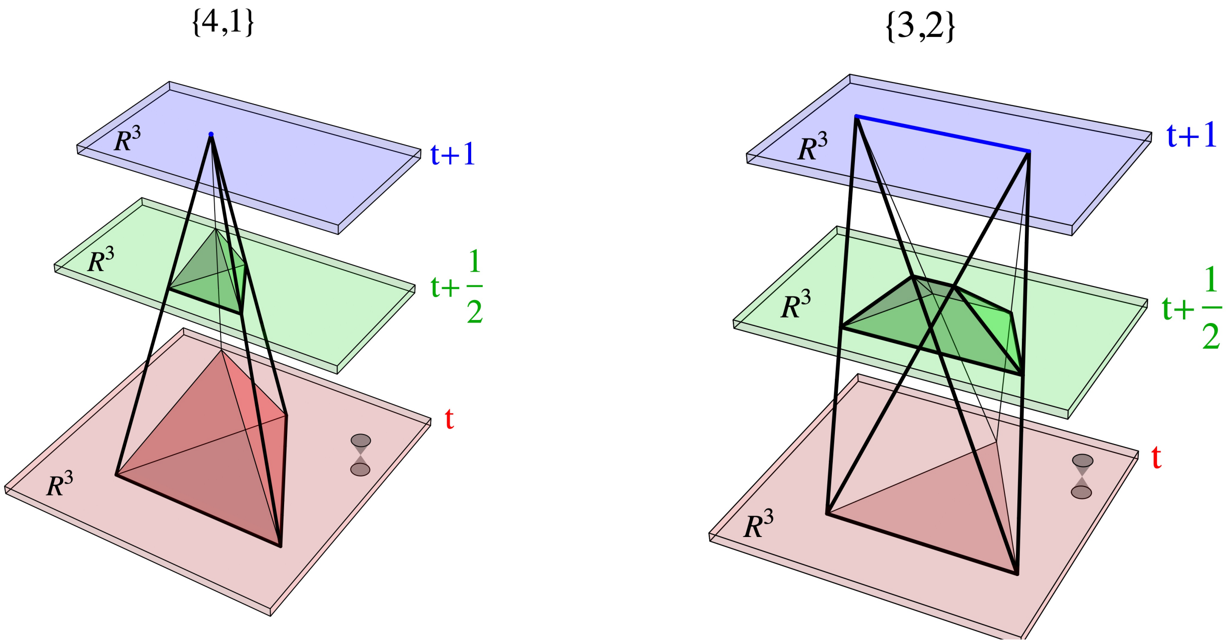
Discretize to 4d lattice made from gluing together equilateral 4d simplices

$$S = -k_0 N_0 + k_4 N_4 \qquad \text{(EDT)}$$

The action only depends on global N_0, N_4

$$K_0 \sim \frac{1}{G} \quad , \quad K_4 \sim \frac{\Lambda}{G}$$

Actually in CDT we put in more structure: a time foliation



$$S = -(K_0 + 6\Delta)N_0 + K_4(N_{23} + N_{14}) + \Delta N_{14}$$

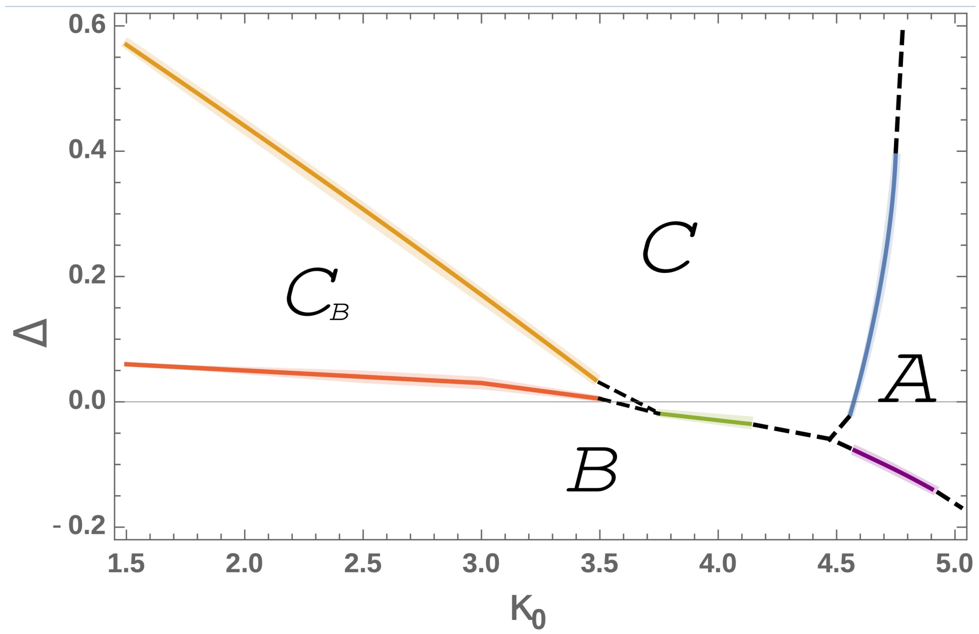
Δ reflects a possible asymmetry at lattice level between space and time. We will treat it as a coupling constant.

$$Z = \sum e^{-S[K_0, K_4, \Delta]}$$

We perform MC-simulations for fixed $N_4 = N_{4,1} + N_{3,2}$

We thus have two coupling constants K_0, Δ

Phase diagram:

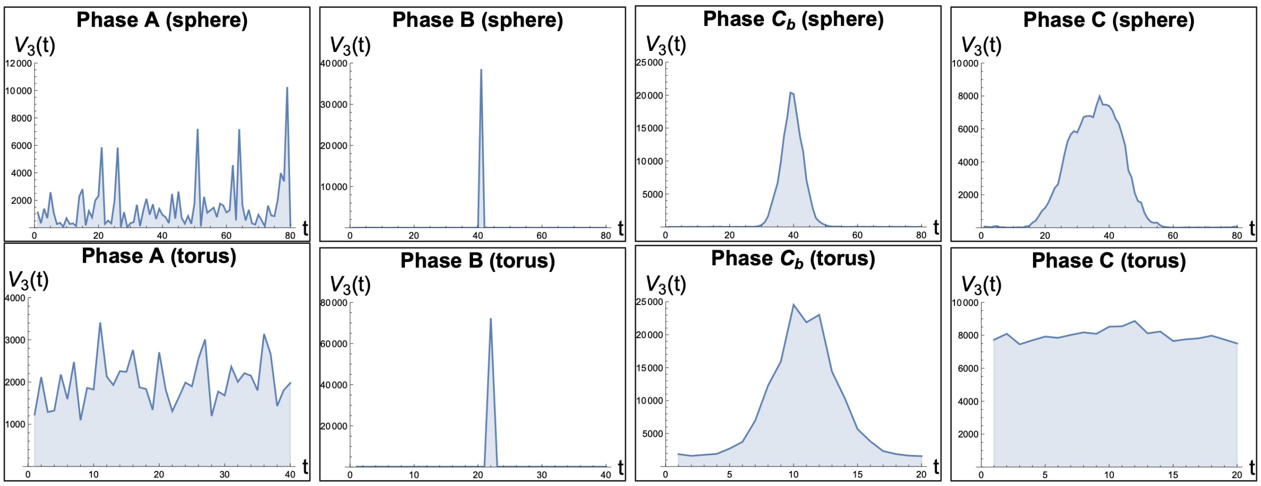


We put in the topology of spacetime by hand (preserved in MC simulations)

The phase diagram seems independent of this topology, but not the order of the phase transitions !

I will not discuss what order

parameters we use or how in detail we determine the order of the transitions. Instead I will show the geometries in the different phases as a generalized order parameter.



Topologies put in by hand:

$$S^3 \times S^1 \quad \text{and} \quad (S^1)^3 \times S^1$$

space time space time

The spatial volume profiles:

$V(t) = N_{14}(t)$ do not respect the

time S topology

We should rather speak about an "effective" topology

$$\boxed{\text{Whenever the effective topology is not changing continuously across a phase transition line, the transition seems to be first-order}}$$

This depends on the underlying topology of space.

For continuum physics the interesting transitions are:

$$C \longrightarrow C_b \quad (1), \qquad C \to B \quad (2)$$

(1) is second order for $S^3 \times S^1$

but first order for $T^3 \times S^1$

(2) is first order for $T^3 \times S^1$

and unknown for $S^3 \times S^1$

Conclusion: There seems to be no continuum limit for underlying topology $T^3 \times S^1$

If this is the correct model of QG:

Space cannot have the T^3 topology !?